The Timing and Returns of Mergers and Acquisitions in Oligopolistic Industries*

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Abstract

This paper embeds a dynamic industry equilibrium model in a real options framework to examine the interaction between product market competition and takeover activity. Industry equilibrium generates endogenous synergy gains. Heterogeneous firms have an incentive to merge because restructuring decisions are motivated by operating and strategic benefits. The unified framework predicts that (i) merger activities are more likely in more concentrated industries or in industries that are more exposed to industrywide shocks; (ii) returns to merger and rival firms arising from restructuring are higher in more concentrated industries; (iii) increased industry competition delays the timing of mergers; (iv) in sufficiently concentrated industries, bidder competition induces a bid premium that declines with product market competition; and (v) mergers are more likely and yield larger returns in industries with higher dispersion in firm size.

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1 Introduction

Despite extensive research on mergers and acquisitions, some important issues in the takeover process remain unclear. Most models tend to focus on firm characteristics to explain why firms should merge or restructure and do not endogenize the timing and terms of a takeover deal or the synergy gains from merging. In addition, most real options models of investment under uncertainty consider symmetric firms in a competitive industry when they study product market competition (see, e.g., Dixit and Pindyck (1994)), while real options models of mergers and acquisitions typically abstract from industry characteristics.

In this paper, we embed takeovers in a dynamic industry equilibrium model with asymmetric firms that allows us to study the effect of a non-competitive product market on: (i) the endogenous synergy gains from merging; (ii) the joint determination of the timing and terms of mergers; and (iii) the returns to both merging and rival firms.[1] Notably, the model also provides novel insights into how the interaction between industry competition and bidder competition for a scarce target influences bid premiums in takeover deals. To our knowledge, this paper presents the first unified framework to formally examine the link between firm heterogeneity, industry structure, and the timing, terms, and returns of takeovers, and to relate the outcome of takeover contests with multiple bidders to the nature of industry competition.

There is ample empirical evidence on various aspects of industry characteristics affecting mergers and acquisitions in practice. Mitchell and Mulherin (1996), Andrade, Mitchell, and Stafford (2001), Andrade and Stafford (2004), and Harford (2005) find that takeover activity is driven by industrywide shocks, and is strongly clustered by industry. Borenstein (1990) and Kim and Singal (1993) document that airline fares on routes affected by a merger increase significantly over those on routes not affected by a merger. Kim and Singal (1993) and Singal (1996) also identify a positive relation between airfares and industry concentration. Eckbo (1985) documents that acquirers and targets earn abnormal returns, while Singal (1996) reports in addition that abnormal returns to rival firms are positively related to changes in industry concentration. This non-exhaustive list of empirical research strongly supports the notion that mergers and acquisitions not only affect product market outcomes and vice versa but also have substantial anticompetitive effects.[2]

[1] Weston et al. (1990) and Weston et al. (1998) provide detailed reviews of the literature on mergers and acquisitions. Research in industrial organization, such as Farrell and Shapiro (1990), studies the welfare implications of mergers, but not the determinants of merger timing, merger terms, and merger returns, or bid premiums in control transactions, which are central to the analysis of this paper.

[2] Eckbo (1983), Eckbo (1985), and Eckbo and Wier (1985) find little evidence that challenged horizontal mergers are anticompetitive. McAfee and Williams (1988) question whether event studies can detect anticompetitive mergers.

Against the backdrop of the industry-level evidence, we study horizontal mergers of two firms in an industry with a single bidder or with multiple bidders. To analyze the role of industry characteristics for the takeover process, we embed an asymmetric oligopolistic industry structure similar to that in Perry and Porter's (1987) static model in a dynamic model of mergers and acquisitions. We do not assume *exogenous* synergy gains as in many other models, such as economies of scale or efficiency-enhancing capital reallocation. Instead, we specify a tangible asset that helps firms to produce output at a given average cost. The merged entity operates the tangible assets of the two merging partners, so it is larger and its average and marginal costs are lower. In addition to a different cost structure for the merged firm, all firms face a different industry structure after a merger of two firms. Recognizing these economic effects, we derive closed-form solutions to industry equilibria within a dynamic model of takeovers in which the merger benefits, the merger timing, and the merger terms are determined endogenously by both cost reduction and changes in product market competition.

There are several recent cases of mergers that match our modeling approach closely. These cases were investigated by the Department of Justice and approved if they were not likely to reduce competition substantially despite of the substantial change of the cost structure and the industry structure. For example, Whirlpool acquired Maytag in 2006 and gained market power in the appliance industry. Whirlpool also substantiated large cost savings. Another example is the $2.6 billion deal of 2008 between two major U.S. air carriers, Delta and Northwest. This merger reduced the combined carrier's operating costs without being viewed to damage competition substantially.

Our model delivers a number of empirical predictions, which are in line with the available evidence. First, we introduce shocks to industry demand and demonstrate that a merger occurs the first time the demand shock hits a trigger value from below. In other words, control transactions emerge as an endogenous product market outcome from rising demand shocks in the model and hence cyclical product markets generate procyclical takeover activity. Second, our real options model formalizes the folklore that mergers are more likely in industries that are more exposed to or more sensitive to industrywide demand shocks. All of these empirical implications of the model are consistent with the evidence reported by, e.g., Mitchell and Mulherin (1996) and Maksimovic and Phillips (2001).

Third, cumulative merger returns are higher for the smaller merging firm (e.g. target) than the larger merging firm (e.g. acquirer) when the firms have identical merger costs. The intuition for this interesting finding is that the smaller firm benefits relatively more from a merger in our industry

Andrade and Stafford (2004), Holmstrom and Kaplan (2001), Shleifer and Vishny (2003) differentiate the takeover activity in the 1960s and 1970s and the more recent merger waves of the 1980s and 1990s.

equilibrium and hence enjoys a larger relative synergy gain (i.e. return). Because mergers typically involve acquisitions of a small firm by a large firm (see, e.g., Andrade, Mitchell, and Stafford (2001) or Moeller, Schlingemann, and Stulz (2004)), this implication supports the available evidence without reliance on additional assumptions, such as asymmetric information (see, e.g., Rhodes-Kropf and Visvanathan (2004)) or misvaluation (see, e.g., Shleifer and Vishny(2003)).

Moreover, our model has several novel testable implications regarding the timing and terms of mergers, the returns to merging firms, and the bid premium in contested deals, which demonstrate the importance of relaxing the assumption of exogenous syngery gains. First, the model's industry equilibrium reveals that cumulative merger returns are determined by an anticompetitive effect, two size effects, and a hysteresis effect. While the latter two effects are shared by other dynamic models of mergers, the anticompetitive effect is unique to our analysis.[3] Due to this effect, we find that returns to merging and rival firms arising from restructuring are higher in more concentrated industries. Intuitively, returns are under certain conditions positively related to anticompetitive profit gains, which are positively related to industry concentration.

Second, and perhaps surprisingly, increased product market competition delays the timing of mergers in our real options model. Notably, this result is *contrary* to some conclusions in recent research on irreversible investment under uncertainty. Grenadier (2002), for instance, emphasizes that more industry competition increases the opportunity cost of waiting to invest and thus accelerates option exercise. In his symmetric industry model, anticompetitive profits result from exogenously reducing the number of identical firms that compete in the industry. Firms are allowed to be asymmetric in our model, however, and anticompetitive profits result from endogenous synergy gains of combining two firms. Crucially, these synergy gains are lower in more competitive industries, *ceteris paribus*, because a merger of two firms has a smaller effect on output price increases in these industries. These lower synergy gains from exercising the merger option reduce the opportunity cost of waiting to merge. Thus, by relaxing some key features of previous work, our real options model implies that firms in more competitive industries may optimally exercise their option to merge later.

Third, our analysis provides new insights into the role of dispersion in firm size for timing and returns of mergers. We find that, all else equal, takeovers are more likely in industries in which firm size is more dispersed. In our industry framework, the endogenous synergy gains from merging increase when the size difference of the two merging partners' tangible assets rises. Because of this economic effect, the model also predicts that returns to merging and rival firms arising from

[3]The anticompetitive effect also affects the merger terms. In Section 3.1, we show that a large acquirer (small target) demands a lower (higher) ownership share in the merged firm if the industry is less competitive.

restructuring are higher in industries with more dispersion in firm size.

Fourth, our model sheds light on if and how multiple sources of competition (i.e. bidder competition and industry competition) in our real options framework influence *bid premiums* in control transactions. One might be tempted to conclude that bidder competition generally raises the price of a contested deal irrespective of industry competition. However, we demonstrate that in our model the degree of industry competition plays a central role for if and how a bid premium emerges in equilibrium. Suppose a large firm bidder and a small firm bidder compete for a small firm target. When their merger costs are similar, the large firm bidder wins the takeover contest, which is consistent with the stylized fact that targets are on average smaller than acquirers. Notably, only in a sufficiently concentrated industry, bidder competition speeds up the takeover process and leads the large firm acquirer to pay a bid premium to discourage the competing small firm competitor. This bid premium decreases with industry competition. By contrast, in a more competitive industry, the small firm bidder does not matter, and the equilibrium outcome corresponds to the one without bidder competition.

Our work contributes to a growing body of research using the real options approach to analyze the dynamics of mergers and acquisitions. Lambrecht (2004), Morellec and Zhdanov (2005), Hackbarth and Morellec (2008), and Margsiri, Mello, and Ruckes (2008) analyze the timing and terms of takeovers under exogenous synergy gains in that takeovers provide either economies of scale or result in a more efficient allocation of resources. To isolate the effect of industry competition on mergers as much as possible, our model abstracts from these exogenous synergy gains. Only Lambrecht briefly analyzes a duopoly model and shows that market power enhances symmetric firms' exogenous synergy gains. In contrast, we consider an oligopolistic Cournot–Nash equilibrium with an arbitrary number of asymmetric firms competing with each other in the product market which allows for endogenous synergy gains. More recently, Leland (2007) considers purely financial synergies in motivating acquisitions when timing is exogenous, while Morellec and Zhdanov (2008) explore interactions between financial leverage and takeover activity with endogenous timing. None of these papers considers the relation of dynamic industry equilibrium and takeover activity.

Other related models study the link between incumbents' incentives to merge and outsiders' incentives to enter an industry; see, e.g., Werden and Froeb (1998), Pesendorfer (2005), Bernile, Lyandres, and Zhdanov (2008), and Toxvaerd (2008). Unlike our study, these papers typically focus on merger waves, which are under certain conditions deterred by the threat of entry. Instead, we focus on the endogenous incentives of two firms in an industry to merge subject to industry competition without entry. Our paper is also related to Gorton, Kahl, and Rosen (2008), who propose a theory of mergers that combines managerial merger motives with an industry level regime shift that may lead to some value-increasing merger opportunities. As in our paper, they emphasize the importance of the distribution of firm sizes within an industry. Unlike our paper, they do not consider industry competition and its interaction with bidder competition in a dynamic environment. All the papers cited in this paragraph are silent on the role of industry structure for bid premiums, timing, and returns, which are the focus of our analysis.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 analyzes the merger timing, terms and returns with a single bidder. Section 4 studies mergers with multiple bidders. Section 5 concludes. Proofs are relegated to an appendix.

2 Model

We incorporate an asymmetric oligopolistic industry structure into a real options framework. After outlining the framework's assumptions, we characterize industry equilibrium when asymmetric firms play Cournot–Nash strategies. We then examine the incentive to merge when restructuring decisions are motivated by operating and strategic benefits.

2.1 Assumptions

Time is continuous, and uncertainty is modeled by a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$. To construct a dynamic equilibrium model of firms' restructuring decisions, we consider an industry populated by infinitely lived firms whose assets generate a continuous stream of cash flows. The industry consists of N heterogeneous firms that produce a single homogeneous product, where $N \geq 2$ is an integer. Each firm i initially owns an amount $k_i > 0$ of physical capital.

To focus on the dynamics of mergers and acquisitions and to keep the industry analysis tractable, we follow authors such as Perry and Porter (1985) and Shleifer and Vishny (2003) and assume that firms can grow through takeovers, but not through internal investments.[4] In addition, we assume that capital does not depreciate over time. The industry's total capital stock is in fixed supply and equal to K. Thus, the industry's capital stock at each time t satisfies:

$$\sum_{i=1}^{N} k_i = K. \tag{1}$$

[4]See e.g. Bernile et al. (2008) or Margsiri et al. (2008) on takeovers and entry or takeovers and internal growth.

The cost structure is important in the model. We denote by $C(q,\kappa)$ the cost function of a firm that owns an amount κ of the capital stock and produces output q. The output q is produced with a combination of the fixed capital input, κ, and a vector of variable inputs, z, according to a smooth concave production function, $q = F(z,\kappa)$. Then the cost function $C(q,\kappa)$ is obtained from the cost minimization problem. To isolate the role of product market competition on mergers, assume that the production function F has constant returns to scale. This implies that $C(q,\kappa)$ is linearly homogeneous in (q,κ). For analytical tractability, we adopt the quadratic specification of the cost function, $C(q,\kappa) = q^2/(2\kappa)$. This cost function may result from the Cobb-Douglas production function $q = \sqrt{\kappa z}$, where z may represent labor input. Note that both the average and marginal costs decline with the capital asset κ. Salant, Switzer, and Reynolds (1983) show that if average cost is constant and independent of firm size, a merger may be unprofitable in a Cournot oligopoly with linear demand. It is profitable if and only if duopolists merge into monopoly. As Perry and Porter (1985) point out, the constant average cost assumption does not provide a sensible description of mergers.

The capital asset plays an important role in the model. It allows us to address the industry asymmetries caused by mergers of a subset of firms. A merged firm combines the capital assets of the two entities to produce output. It faces a different optimization problem immediately after restructuring because of its altered cost function and because of new strategic considerations that arise from the change in industry structure.

We suppose the industry's inverse demand at time t is given by the linear function:

$$P(t) = a\,Y(t) - b\,Q(t), \tag{2}$$

where $Y(t)$ denotes the industry's demand shock at time t observed by all firms, $Q(t)$ is the industry's output at time t, and a and b are positive constants; a represents exposure of demand to industrywide shocks, and b represents the price sensitivity of demand. We assume that the demand shock is governed by the geometric Brownian motion process:

$$dY(t) = \mu\,Y(t)dt + \sigma\,Y(t)\,dW(t),\ Y(0) = y_0, \tag{3}$$

where μ and σ are constants and $(W_t)_{t\geq 0}$ is a standard Brownian motion defined on $(\Omega, \mathcal{F}, \mathcal{P})$.

We assume that all firms in the industry are Cournot-Nash players and that management acts in the best interests of shareholders. We also assume that shareholders are risk-neutral and discount future cash flows by $r > 0$. Therefore, all corporate decisions are rational and value-maximizing choices. To ensure that the present value of profits is finite, we make the following assumption:

Assumption 1 *The parameters μ, σ, and r satisfy the condition $2\left(\mu+\sigma^2/2\right)<r$.*

Following Shleifer and Vishny (2003) among others, we consider a subset of two firms i $(i=1,2)$ with capital stocks k_i, which can negotiate a takeover deal at time $t>0$ if it is in their shareholders' best interest.[5] To this end, we assume that each firm i's strategy space is restricted to the optimal exercise strategy of its merger option. For each firm i, this strategy is given by a threshold y_i^*, such that the merger is executed the first time when $Y(t)$ exceeds y_i^*. The second-stage negotiation problem then reduces to identifying the merger terms, ξ_i, which will induce both firms to exercise their merger option at the globally efficient merger threshold, y^* (see Section 3.3). Finally, control transactions are costly in reality. We therefore assume that each merging firm i incurs a fixed lump-sum cost $X_i>0$ for $i=1,2$. This cost captures fees to investment banks and lawyers as well as the cost of restructuring.

2.2 Industry equilibrium

Let $q_i\left(t\right)$ denote the quantity selected by firm i at time t. Then firm i's instantaneous profit is given by

$$\pi_i\left(t\right)=\left[a\,Y\left(t\right)-b\,Q\left(t\right)\right]q_i\left(t\right)-q_i\left(t\right)^2/\left(2k_i\right), \tag{4}$$

where

$$Q\left(t\right)=\sum_{i=1}^{N}q_i(t) \tag{5}$$

is the industry output at time t. Given the instantaneous profits, we can compute firm value, or the present value of profits:

$$V_i\left(y\right)=\mathbb{E}^y\left[\int_0^\infty e^{-rt}\pi_i\left(t\right)dt\right], \tag{6}$$

where $\mathbb{E}^y[\cdot]$ denotes the conditional expectation operator, given that the current industry shock takes the value $Y(0)=y$.

We define strategies and industry equilibrium as follows. The strategy $\{(q_1^*\left(t\right),...,q_N^*\left(t\right)):t\geq 0\}$ constitutes an industry (Markov perfect Nash) equilibrium if, given information available at date t, $q_j^*\left(t\right)$ is optimal for firm $j=1,...,N$, when it takes other firms' strategies $q_i^*\left(t\right)$ for all $i\neq j$ as given. Because firms play a dynamic game, there could be multiple Markov perfect Nash equilibria, as is well known in game theory. Instead of finding all equilibria, we will focus on the equilibrium in which firms adopt static Cournot strategies. As is well known, the static Cournot strategies

[5] We discuss in section 5 the additional implications that may arise from extending our analysis to multiple mergers and an endogenous sequence of takeover activity, which is a formidable problem in continuous time and clearly beyond the scope of the current paper. Qiu and Zhou (2007) provide some examples within a static model along these lines.

that firms play at each date constitute a Markov perfect Nash equilibrium. The first proposition characterizes this industry equilibrium. (Proofs for all propositions are given in the Appendix).

Proposition 1 *The strategy:*

$$q_i^*(t) = \frac{\theta_i}{1+B}\frac{aY(t)}{b} \tag{7}$$

constitutes a Cournot–Nash industry equilibrium at time t for firms $i = 1, ..., N$, where

$$\theta_i = \frac{b}{b + k_i^{-1}}, \text{ and } B = \sum_{i=1}^{N} \theta_i. \tag{8}$$

In this equilibrium, the industry output at time t is given by

$$Q^*(t) = \frac{B}{1+B}\frac{aY(t)}{b}, \tag{9}$$

and the industry price at time t is given by

$$P^*(t) = \frac{aY(t)}{1+B}. \tag{10}$$

Note that this proposition also characterizes the equilibrium after a merger, once we change the number of firms and the capital stock of the merged firm. A merger brings the capital of two firms under a single authority and thus reduces production cost.

Without loss of generality, we consider that firms 1 and 2 merge. We use the subscript M to denote the merged entity. The merged firm owns capital $k_M = k_1 + k_2$. By Proposition 1, the values of θ_i for the non-merging firms $i \geq 3$ are unaffected by the merger. We can verify that the value of θ for the merged firm satisfies:

$$\max\{\theta_1, \theta_2\} < \theta_M = \frac{\theta_1 + \theta_2 - 2\theta_1\theta_2}{1 - \theta_1\theta_2} < \theta_1 + \theta_2. \tag{11}$$

Thus, the value of B in Proposition 1, which determines total industry output, changes after the merger to:

$$B_M = B + \theta_M - \theta_1 - \theta_2 < B. \tag{12}$$

By equations (9), (10), and (12), we conclude that the merger causes total output to fall and industry price to rise. In addition, we can use equations (12) and (7) to show that:

$$\max\{q_1^*(t), q_2^*(t)\} < q_M(t) < q_1^*(t) + q_2^*(t), \tag{13}$$

where $q_M(t)$ is the output produced by the merged firm at time t. This result implies that the merged firm produces more than either of the two merging firms, but less than the total output level of the two. The analysis highlights the tension of a merger: After a merger, the industry price rises, but the merged firm restricts production. Thus, a merger may not always generate a profit gain.

2.3 Incentive to merge

In order to analyze mergers tractably, we follow Perry and Porter (1985) and consider an oligopoly structure with small and large firms. Specifically, we assume that the industry initially consists of n identical large firms and m identical small firms. Each large firm owns an amount k of capital and incurs merger costs X_l if it engages in a merger. Each small firm owns an amount $k/2$ of capital and incurs merger costs X_s if it engages in a merger. In this case, equation (1) becomes $nk + mk/2 = K$, and hence we can use $m = 2Kk^{-1} - 2n$ to replace m in the analysis below.

We assume that either two small firms can merge (symmetric merger) or a small firm and a large firm can merge (asymmetric merger). In the former case, a merger preserves the two-type industry structure, but the number of small firms and large firms changes. In the latter case, firm heterogeneity increases. That is, all non-merging small or large firms remain identical, but the merged entity owns more physical capital than a large firm, destroying the two-type industry structure.

We do not consider mergers between two large firms or further mergers over time. This assumption makes our analysis tractable and permits us to focus on the key questions of how product market competition interacts with bidder competition and how product market competition influences the timing and terms of mergers as well as merger returns. One justification of our assumption may be related to antitrust law. White (1987, p. 16) writes that: "[the Horizontal Merger] Guidelines use the Herfindahl-Hirschman Index (HHI) as their primary market concentration guide, with concentration levels of 1,000 and 1,800 as their two key levels. Any merger in a market with a post-merger HHI below 1,000 is unlikely to be challenged; a merger in a market with a post-merger HHI above 1,800 is likely to be challenged (if the merger partners have market shares that cause the HHI to increase by more than 100), unless other mitigating circumstances exist, like easy entry. Mergers in markets with post-concentration HHI levels between 1,000 and 1,800 require further analysis before a decision is made whether to challenge." In our model, a merger of two large firms would raise industry concentration to a higher level than a merger of two small firms or a merger between a small firm and a large firm. The industry concentration level following a merger of two large firms is more likely to cross the regulatory threshold, and thus such a merger is more likely to be challenged by antitrust authorities.[6]

[6] For an industry with two large firms and a price sensitivity of $b = 0.5$, our model's pre-merger HHI equals $1,378$ when $k = 0.2$, $K = 1$, and $a = 100$. This concentration measure rises to 1,578 (1,734) following a merger of two small firms (a merger of a small and a large firm) compared to a post-merger HHI is 2,022 after a merger of two large firms.

As an example, the two largest office superstore chains in the United States, Office Depot and Staples, announced their agreement to merge on September 4, 1996. Seven months later, the Federal Trade Commission voted 4 to 1 to oppose the merger on the grounds that it was likely to harm competition and lead to higher prices in "the market for the sale of consumable office supplies sold through office superstores." (see Dalkir and Warren-Boulton (2004).)

Symmetric merger. We first consider a symmetric merger of two small firms. Define:

$$\Delta(n) \equiv \left(b+k^{-1}\right)\left(b+2(Kb+1)k^{-1}\right)-b^2 n>0. \tag{14}$$

Note that the argument n in this equation indicates that there are n large firms in the industry. This notation is useful for our merger analysis below because the number of large firms changes after a merger option is exercised. It follows from equation (1) that $Kk^{-1}>n$, which implies that both $\Delta(n)$ and $\Delta(n+1)$ are positive. We use Proposition 1 and equation (6) to compute equilibrium firm value.

Proposition 2 *Consider a symmetric merger between two small firms in the small-large oligopoly industry. Suppose Assumption 1 holds. The equilibrium value of the type $f=s,l$ firm is given by:*

$$V_f(y;n)=\frac{\Pi_f(n)\,y^2}{r-2\left(\mu+\sigma^2/2\right)}, \tag{15}$$

where

$$\Pi_s(n) = \frac{a^2\left(b+k^{-1}\right)^3}{\Delta(n)^2}, \tag{16}$$

$$\Pi_l(n) = \frac{a^2\left(b+2k^{-1}\right)^2\left(b+k^{-1}/2\right)}{\Delta(n)^2}. \tag{17}$$

After a merger between two small firms, there are $n+1$ identical large firms and $m-2$ identical small firms in the industry. Thus, the value of the large firm after a merger is given by $V_l(y;n+1)$. It follows from Proposition 2 that the benefit from merging is given by:

$$V_l(y;n+1)-2\,V_s(y;n)=\frac{\left[\Pi_l(n+1)-2\,\Pi_s(n)\right]y^2}{r-2\left(\mu+\sigma^2/2\right)}. \tag{18}$$

The term $\Pi_l(n+1)-2\,\Pi_s(n)$ represents the profitability of an anticompetitive merger. For there to be takeover incentives, this term must be positive. After a merger, the number of small firms in the industry is reduced, and hence the industry's market structure is changed. As the output of two small firms prior to a merger exceeds the output of the merged firm, an incentive to merge requires that the increase in industry price be enough to offset the reduction in output of the merged entity. The conditions for takeover incentives may be summarized as follows:

Proposition 3 *Consider a symmetric merger between two small firms. Let $\Delta(n)$ be given in (14) and define the critical value:*

$$\Delta^* \equiv \frac{b^2}{1-A}, \tag{19}$$

where A is given by:

$$A \equiv \left(b + 2k^{-1}\right)\sqrt{\frac{b + k^{-1}/2}{2\left(b + k^{-1}\right)^3}}. \tag{20}$$

(i) If

$$\max_n \Delta(n) = \Delta(0) < \Delta^*, \tag{21}$$

then there will always be an incentive to merge. (ii) If

$$\min_n \Delta(n) = \Delta(K/k - 1) > \Delta^*, \tag{22}$$

then there will never be an incentive to merge. (iii) If

$$\Delta(0) > \Delta^* > \Delta(K/k - 1), \tag{23}$$

then when n is high enough so that $\Delta(n) < \Delta^*$, there will be an incentive to merge.

In Proposition 3, (21) and (23) provide two conditions for takeover incentives in our Cournot–Nash framework.[7] That is, when the increase in price outweighs the reduction in output so that the net effect leads to an increase in instantaneous profits, the two small firms have an incentive to form a large organization. These two conditions depend on the industry demand function through the price sensitivity b, and the size of a large firm k, and on the industry structure through the number n of large firms prior to the restructuring. Moreover, the proposition shows that there could be no incentives to merge if the condition in (22) holds.

It is straightforward to show that there is always an incentive to merge when the industry consists of a small-firm duopoly.[8] A similar result is obtained by Perry and Porter (1985) and Salant et al. (1983) in a static industry model.

Asymmetric merger. After a merger between a large firm and a small firm, the industry consists of $n-1$ identical large firms, $m-1$ identical small firms, and a huge merged firm. The merged entity owns capital $k_M = k + k/2 = 3k/2$. We use Proposition 1 to derive equilibrium firm value after an asymmetric merger:

[7] Similar conditions have been derived by Perry and Porter (1985) in their static model.

[8] One can immediately verify that condition (21) is satisfied for $k = K$, $n = 0$, and $m = 2$, which is a limiting case of our model.

Proposition 4 *Consider an asymmetric merger between a large firm and a small firm in the small-large oligopoly industry. Suppose Assumption 1 holds. After this merger, the equilibrium firm value is given by:*

$$V_f^a(y;n-1)=\frac{\Pi_f^a(n-1)\,y^2}{r-2(\mu+\sigma^2/2)},\ \text{for } f=s,l,M, \tag{24}$$

where

$$\Pi_s^a(n-1) = \frac{a^2\left(b+k^{-1}\right)^3}{\left[\Delta(n)-b^2\left(1+\frac{2}{2+3bk}\right)\right]^2}, \tag{25}$$

$$\Pi_l^a(n-1) = \frac{a^2\left(b+2k^{-1}\right)^2\left(b+k^{-1}/2\right)}{\left[\Delta(n)-b^2\left(1+\frac{2}{2+3bk}\right)\right]^2}, \tag{26}$$

$$\Pi_M^a(n-1) = \frac{3a^2\left(b+2k^{-1}\right)^2\left(b+k^{-1}\right)^2\left(3b+k^{-1}\right)/\left(3b+2k^{-1}\right)^2}{\left[\Delta(n)-b^2\left(1+\frac{2}{2+3bk}\right)\right]^2}. \tag{27}$$

The benefit from an asymmetric merger is given by:

$$V_M^a(y;n-1)-V_s(y;n)-V_l(y;n)=\frac{\left[\Pi_M^a(n-1)-\Pi_s(n)-\Pi_l(n)\right]y^2}{r-2(\mu+\sigma^2/2)}. \tag{28}$$

For there to be an incentive to merge, the expression in equation (28) must be positive. As in Proposition 3, we have the result:

Proposition 5 *Consider an asymmetric merger between a small firm and a large firm in the small-large oligopoly industry. Let $\Delta(n)$ be given in (14) and let the critical value Δ^* take the value:*

$$\Delta^a\equiv\frac{b^2}{1-D}\left(1+\frac{2}{2+3bk}\right), \tag{29}$$

where D is given by:

$$D\equiv\frac{\left(b+k^{-1}\right)\left(b+2k^{-1}\right)}{3b+2k^{-1}}\sqrt{\frac{3\left(3b+k^{-1}\right)}{\left(b+k^{-1}\right)^3+\left(b+2k^{-1}\right)^2\left(b+k^{-1}/2\right)}}. \tag{30}$$

Then parts (i)-(iii) in Proposition 3 apply here.

To facilitate the analysis in Section 3, we make the following assumption.

Assumption 2 *Suppose $\Pi_M^a(n-1)-\Pi_s(n)-\Pi_l(n)>0$ and $\Pi_l(n+1)-2\,\Pi_s(n)<0$ so that there is an incentive to merge between a large firm and a small firm, but no incentive to merge between two small firms.*

A sufficient condition for this assumption is that case (i) or (iii) holds in Proposition 5, but both cases are violated in Proposition 3. We invoke an alternative assumption for the analysis in Section 4.

Assumption 3 *Suppose* $\Pi_l(n+1) - 2\,\Pi_s(n) > 0$ *and* $\Pi_M^a(n-1) - \Pi_s(n) - \Pi_l(n) > 0$ *so that there is an incentive to merge between a large and a small firm and between two small firms.*

A sufficient condition for this assumption is that case (i) or (iii) holds in both Propositions 3 and 5. The following lemma is useful for our later merger analysis:

Lemma 1 *Under Assumptions 2 or 3, the profit differentials* $\Pi_M^a(n-1) - \Pi_s(n) - \Pi_l(n)$ *increase with the parameters* a *and* n*, and the profit ratios* $[\Pi_M^a(n-1) - \Pi_s(n) - \Pi_l(n)]/\Pi_s(n)$ *and* $[\Pi_M^a(n-1) - \Pi_s(n) - \Pi_l(n)]/\Pi_l(n)$ *increase with the parameter* n*.*

To interpret this lemma, recall that the parameter n represents the number of large firms in the industry prior to a merger. This parameter proxies for industry concentration. A higher value of n represents a higher level of industry concentration. The parameter a represents the exposure of industry demand to the industrywide shock. A higher value of a implies that the increase in industry price is higher in response to an increase in the exogenous demand shock. Lemma 1 then demonstrates that under certain conditions the anticompetitive gains from a merger measured in terms of either profit differential or profit ratio are larger in more concentrated industries. In terms of profit differential, these gains are also greater in industries that are more exposed to industrywide shocks.

Example 1. The conditions in Assumption 2 or 3 are not easy to check analytically. We thus use a numerical example to illustrate them and Propositions 3 and 5. The baseline parameter values are: $b = 0.5$, $k = 0.2$, and $K = 1$. By equation (1), these values imply that the numbers of large and small firms must satisfy $2n + m = 10$.

[Insert Figure 1 Here]

Figure 1 shows the profit differentials of symmetric and asymmetric mergers for a wide range of parameter values of b and n. Both profit differentials increase monotonically with the number of large firms n, but non-monotonically with the price sensitivity parameter b. Intuitively, this non-monotonicity results from two opposing effects of a decline in b; that is, it raises price, but reduces output. Thus, change in the price sensitivity parameter has an ambiguous effect on the profit differentials, because it depends on whether the price effect or the quantity effect dominates.

From the lower left end of the surfaces in Figure 1, we find that asymmetric mergers can be profitable, while symmetric mergers are not, so Assumption 2 is satisfied. This happens, for example, when $b = 0.5$, and n takes values from 1 to 5. When $b = 0.4$, and n takes values from 1 to 5, however, both symmetric and asymmetric mergers are profitable, so Assumption 3 is satisfied.

3 Mergers with a single bidder

In this section, we analyze the timing and terms of a merger between a small firm target and a large firm bidder. We suppose Assumption 2 holds, so that two identical small firms do not have an incentive to merge, but a large firm and a small firm do have an incentive to merge. This case is interesting because mergers typically involve acquisitions of a small firm by a large firm. In Andrade, Mitchell, and Stafford's (2001) sample of 4,256 deals over the 1973–1998 period, the median target size is 11.7% of the size of the acquirer. Moeller, Schlingemann, and Stulz (2004) measure relative size as transaction value divided by acquirer's equity value, and report averages of 19.2% (50.2%) for 5,503 small (6,520 large) acquirers between 1980 and 2001. Rhodes-Kropf and Robinson (2008) also document positive and significant differences in market equity valuations of bidders and targets.

We assume the acquirer submits a bid in the form of an ownership share of the merged firm's equity.[9] Given this bid, both acquirer and target shareholders select their value-maximizing merger timing. In equilibrium, the merger timing chosen by the acquirer and the target are the same. The merger offers participants in the deal an option to exchange one asset for another. That is, they can exchange their shares in the initial firm for a fraction of the shares of the merged firm. Thus, the merger opportunity is analogous to an exchange option (Margrabe, 1978). The equilibrium timing and terms of the merger are the outcome of an option exercise game in which each participant determines an exercise strategy for its exchange option. We first solve for the equilibrium, and then show that the equilibrium timing is globally optimal.

[9]This merger mechanism is similar to a friendly merger. In an earlier working paper, we also analyze other control mechanisms, such as a hostile takeover. We do not consider these altneratives here since they do not provide significantly novel new insights.

3.1 Equilibrium

To solve for the equilibrium, we first consider the exercise strategy of the large firm bidder. Let ξ_l denote the ownership share of the large firm in the merged entity. Then $1 - \xi_l$ is the ownership share of the small firm target. The merger surplus accruing to the large firm is given by the positive part of the (net) payoff from the merger: $[\xi_l V_M^a (y; n-1) - V_l (y; n) - X_l]^+$, where $V_M^a (y; n-1)$ and $V_l (y; n)$ are given in Propositions 2 and 4. When it considers a merger, the large firm trades off the stochastic benefit from merging against the fixed cost X_l of merging. Since firms have the option but not the obligation to merge, the surplus from merging has a call option feature.

Let y_l^* denote the merger threshold selected by the large firm. The value of this firm's option to merge, denoted $OM_l\big(y, y_l^*, \xi_l; n\big)$ for $y \leq y_l^*$, is given by:

$$OM_l (y, y_l^*, \xi_l; n) = \mathbb{E}^y \left\{ e^{-r\tau_{y_l^*}} \left[\xi_l V_M^a \left(Y(\tau_{y_l^*}); n-1 \right) - V_l \left(Y(\tau_{y_l^*}); n \right) - X_l \right] \right\}, \tag{31}$$

where $\tau_{y_l^*}$ denotes the first passage time of the process (Y_t) starting from the value y to the merger threshold y_l^* selected by the large firm. By a standard argument (e.g., Karatzas and Shreve, 1999), we can show that

$$OM_l (y, y_l^*, \xi_l; n) = [\xi_l V_M^a (y_l^*; n-1) - V_l (y_l^*; n) - X_l] \left(\frac{y}{y_l^*} \right)^{\beta}, \tag{32}$$

where β denotes the positive root of the characteristic equation

$$0.5\, \sigma^2 \beta\ (\beta - 1) + \mu\, \beta - r = 0. \tag{33}$$

Note that it is straightforward to prove that $\beta > 2$ under Assumption 1. Equation (32) admits an intuitive interpretation. The value of the option to merge is equal to its share of the merger benefits net of the merger costs, $[\xi_l V_M^a (y_l^*; n-1) - V_l (y_l^*; n) - X_l]$, generated at the time of the merger multiplied by a discount factor $(y/y_l^*)^{\beta}$. This discount factor can be interpreted as the Arrow-Debreu price of a primary claim that delivers \$1 at the time and in the state the merger occurs. It can also be regarded as the probability of the demand shock $Y(t)$ reaching the merger threshold y_l^* for the first time from below, given that the current level of the demand shock is y.

The optimal threshold y_l^* selected by the large firm maximizes the value of the merger option (32). Thus, it satisfies the first-order condition:

$$\frac{\partial OM_l(y, y_l^*, \xi_l; n)}{\partial y_l^*} = 0. \tag{34}$$

Solving this equation yields:

$$y_l^* = \sqrt{\frac{\beta X_l}{\beta - 2} \frac{r - 2(\mu + \sigma^2/2)}{\xi_l \Pi_M^a (n-1) - \Pi_l (n)}}. \tag{35}$$

It follows that, as a function of ξ_l, y_l^* declines with ξ_l. This function gives the merger threshold for a given value of the ownership share ξ_l. Figure 2 illustrates this function.

[Insert Figure 2 Here.]

We next turn to the exercise strategy of the small firm target, which can be solved in a similar fashion. The value of the small firm target's option to merge is given by:

$$OM_s(y, y_s^*, \xi_l; n) = [(1 - \xi_l) V_M^a (y_s^*; n-1) - V_s (y_s^*; n) - X_s] \left(\frac{y}{y_s^*}\right)^\beta. \tag{36}$$

The optimal exercise strategy y_s^* selected by the small firm target satisfies the first-order condition:

$$\frac{\partial OM_s(y, y_s^*, \xi_l; n)}{\partial y_s^*} = 0. \tag{37}$$

Solving this equation yields:

$$y_s^* = \sqrt{\frac{\beta X_s}{\beta - 2} \frac{r - 2(\mu + \sigma^2/2)}{(1 - \xi_l) \Pi_M^a (n-1) - \Pi_s (n)}}. \tag{38}$$

This equation implies that, as a function of ξ_l, the merger threshold y_s^* selected by the small firm target rises with ξ_l. Figure 2 charts y_s^* as a function of ξ_l.

[Insert Figure 2 Here]

In equilibrium, the negotiated ownership share must be such that the bidder and the target agree on merger timing, or $y_l^* = y_s^*$. Using this condition, we can solve for the equilibrium timing and terms of the merger. The crossing point of the reaction functions in Figure 2 characterizes the option exercise equilibrium.

Proposition 6 *Consider an asymmetric merger between a large firm and a small firm. Suppose that Assumptions 1 and 2 hold. (i) The value-maximizing restructuring policy is to merge when the industry shock* (Y_t) *reaches the threshold value:*

$$y^* = \sqrt{\frac{\beta (X_s + X_l)}{\beta - 2} \frac{r - 2(\mu + \sigma^2/2)}{\Pi_M^a (n-1) - \Pi_l (n) - \Pi_s (n)}}. \tag{39}$$

(ii) The merger threshold y^* declines with a and n. (iii) The share of the merged firm accruing to the large firm is given by:

$$\xi_l^* = \frac{X_l}{X_s + X_l} + \frac{\Pi_l(n) X_s - \Pi_s(n) X_l}{(X_s + X_l) \Pi_M^a (n-1)}. \tag{40}$$

(iv) The ownership share ξ_l^* declines with n.

Parts (i) and (ii) of Proposition 6 characterize the merger timing. As the values of the option to merge for both firms increase with the realization y of the industry shock, a merger occurs in a rising product market. Thus, consistent with empirical evidence documented by Maksimovic and Phillips (2001), cyclical product markets generate procyclical mergers. This result is also consistent with Mitchell and Mulherin's (1996) empirical finding that industry shocks contribute to the merger and restructuring activities during the 1980s.

As in most real options model, the merger threshold y^* given in (39) determines the merger timing and merger likelihood. A higher value of the merger threshold implies a larger value of the expected time of the merger and a lower probability of merger within a given time horizon.[10] To interpret equation (39), we use Propositions 2 and 4 to rewrite it as:

$$V_M^a(y^*, n-1) - V_l(y^*; n) - V_s(y^*; n) = \frac{\beta (X_s + X_l)}{\beta - 2} > X_s + X_l. \tag{41}$$

Equation (41) implies that, at the time of the merger, the benefit from the merger exceeds the sum of the merger costs $X_s + X_l$. This reflects the option value of waiting. Because mergers and acquisitions are analogous to an irreversible investment under uncertainty, the standard comparative statics results from the real options literature (e.g., Dixit and Pindyck, 1994) apply to merger timing. For example, an increase in the industry's demand uncertainty delays the timing of mergers, and an increase in the drift of the industry's demand shock speeds up the timing of mergers.

Our novel comparative statics results are related to industry characteristics. First, part (ii) of Proposition 6 implies that the optimal merger threshold declines with the parameter a. Since the parameter a represents the exposure of the industry demand to the exogenous shock, one should expect to observe more mergers and acquisitions in industries where demand is more exposed to or more sensitive to exogenous shocks. The intuition behind this result is that an increase in the

[10] The probability that a merger will take place in a time interval $[0, T]$ is given by:

$$\Pr\left(\sup_{0 \le \tau \le T} Y(\tau) \ge y^*\right) = \mathcal{N}\left[\frac{\ln(y_0/y^*) + (\mu - \sigma^2/2) T}{\sigma\sqrt{T}}\right] + \left(\frac{y_0}{y^*}\right)^{-(2\mu - \sigma^2)/\sigma^2} \mathcal{N}\left[\frac{\ln(y_0/y^*) - (\mu - \sigma^2/2) T}{\sigma\sqrt{T}}\right],$$

where $\mathcal{N}$ is the standard normal cumulative distribution function. This probability declines with the merger threshold y^*.

parameter a raises industry demand for a given positive shock. Thus, it increases anticompetitive gains $\Pi_M^a(n-1) - \Pi_l(n) - \Pi_s(n)$ as shown in Lemma 1, thereby raising the benefits from merging. This result is consistent with empirical evidence in Mitchell and Mulherin (1996), who report that the industries experiencing the most merger and restructuring activity in the 1980s were the industries exposed most to industry shocks.

We next turn to the effect of industry concentration on merger timing. Part (ii) of Proposition 6 implies that one should expect to see more mergers and acquisitions in more concentrated (i.e., less competitive) industries. The economic intuition behind this result is as follows. A higher level of pre-merger industry concentration is associated with higher anticompetitive profits by Lemma 1, which raises the incentive to merge, *ceteris paribus*, and hence the potential payoff from exercising the merger option. Importantly, higher anticompetitive profits at the time of a merger lead to a higher opportunity cost of waiting to merge, and so firms in *less* competitive industries will optimally exercise their option to merge earlier.

This implication for the optimal timing of mergers in our Cournot–Nash framework is in sharp contrast to most of the earlier findings in the literature on irreversible investment under uncertainty. Notably, Grenadier (2002) demonstrates that firms in more competitive industries will optimally exercise their investment options earlier in a symmetric industry equilibrium model. Intuitively, *more* industry competition increases the opportunity cost of waiting to invest and thus accelerates investment option exercise. We attribute the difference in our results to differences in the economic modeling of industry competition and structure. In our asymmetric industry equilibrium model, anticompetitive profits result from merging two firms to form a new firm, which alters product market competition endogenously. In Grenadier's (2002) model, anticompetitive profits result from exogenously reducing the number of identical firms that compete for an investment opportunity in the industry. Moreover, Grenadier (2002) studies an incremental investment problem, while we analyze a single discrete option exercise decision. We will consider scarcity of targets that can lead to competition among multiple bidders in Section 4. Bidder competition will hurt the acquirer, as is also shown by Morellec and Zhdanov (2005), and hence may attenuate the delayed option exercise due to the countervailing effect of product market competition that is central to our model.

Parts (iii) and (iv) of Proposition 6 characterize the ownership share. Part (iii) shows that the large firm bidder demands a greater ownership share than the small firm target if the two firms incur identical merger costs $X_l = X_s$. Intuitively, because the large firm has a higher pre-merger firm value, it demands a greater ownership share in our industry equilibrium. When the merger gains are endogenized the pre-merger industry concentration level also influences the ownership share. Part (iv) of Proposition 6 shows that a large merging firm demands a smaller ownership share in more concentrated industries, *ceteris paribus*. The intuition is that the pre-merger profit differential between the large and the small merging partners relative to the value of the merged firm declines with industry concentration. Thus, the large firm does not need to demand a greater share in more concentrated industries.

Example 2. Figure 3 illustrates Proposition 6. The input parameter values are $a = 100$, $b = 0.5$, $K = 1$, $k = 0.2$, $n = 2$, $r = 8\%$, $X_l = 2$, $X_s = 2$, $\mu = 1\%$, and $\sigma = 20\%$. Recall that equation (1) implies that the number of small firms satisfies $m = 2Kk^{-1} - 2n = 6$. Building on the insights from Example 1, the industry's parameter values a, b, K, k, and n, are selected such that a merger between two small firms is not profitable, i.e., $\Pi_l(n+1) - 2\,\Pi_s(n) < 0$, but a merger between a small and a large firm is profitable, i.e., $\Pi_M^a(n-1) - \Pi_s(n) - \Pi_l(n) > 0$. Thus, Assumption 2 holds. To study the role of product market competition for mergers in isolation from frictions shared by other dynamic models of mergers, we use identical restructuring costs for the small firm target and the large firm acquirer.[11] That is, we set $X_f = 2$ for $f = s, l$. The risk-free rate is taken from the yield curve on Treasury bonds. Similarly, the growth rate of industry shocks has been selected to generate a reasonable probability for a merger to arise in this industry. Finally, the value of the diffusion parameter has been chosen to match the time-series volatility of an average S&P 500 firm's asset return (see e.g. Strebulaev (2007)).

As discussed earlier, the non-monotonicity of the profit differential with respect to the price sensitivity results from a tradeoff between a price effect and a quantity effect. Figure 3a displays the impact of this economic phenomenon on the behavior of the merger threshold, i.e., $\partial y^*/\partial b$. Moreover, because the merger gains increase with industry concentration when Assumption 2 is satisfied, the merger threshold decreases with the number of large firms in the industry, n, as shown in Figure 3b. In particular, when n rises from 2 to 5, the merger threshold y^* declines from 2.30 to 1.84. If we set the initial industry shock to $y_0 = 1$, these threshold values imply that the likelihood of a merger over a five-year horizon rises from 5.0% to 14.7%.

[Insert Figure 3 Here]

Another interesting feature of the optimal merger threshold is that it is a decreasing function of k, as shown in Figure 3c. Notice that the variance of firm size, $k_i \in \{k/2, k\}$, in an industry with

[11] Intuitively, larger restructuring costs delay the timing of mergers. We will relax this assumption in Section 4.

N firms is given by:

$$\text{Var}(k_i) = \frac{(K - kn)\, k^3 n}{2(2K - kn)[2K - k(n+1)]}, \tag{42}$$

which also increases with k. Thus, Figure 3c and equation (42) reveal that, all else equal, takeovers are more likely in industries in which firm size is more dispersed. Although we are unable to prove this result theoretically, we have verified that it holds true for a wide range of parameter values. Intuitively, the endogenous synergy gains from merging increase when the size difference of the two merging partners' tangible assets rises. Finally, and as supported by Figure 3, other standard comparative statics results apply within our model, so we do not discuss them.

3.2 Cumulative merger returns

We now turn to cumulative returns resulting from an asymmetric merger. The equity value of a large merging firm before a merger, denoted $E_l(Y(t); n)$, is equal to the value of its assets in place plus the value of the merger option:

$$E_l(Y(t); n) = V_l(Y(t); n) + OM_l(Y(t), y^*, \xi_l^*; n), \tag{43}$$

where $OM_l(Y(t), y^*, \xi_l^*; n)$ is given in equation (32), and y^* and ξ_l^* are given in Proposition 6. Similarly, the equity value of a small merging firm before the merger, denoted $E_s(Y(t); n)$, is given by:

$$E_s(Y(t); n) = V_s(Y(t); n) + OM_s(Y(t), y^*, \xi_l^*; n), \tag{44}$$

where $OM_s(\cdot; n)$ is defined in (36).

We may express the cumulative merger returns as a fraction of the stand-alone equity value $V_f(Y(t); n)$ of the small firm $f = s$ and the large firm $f = l$. That is, the cumulative returns to the small and large merging firms at time $t \leq \tau_{y^*}$ are given by:

$$R_{f,M}(Y(t), n) = \frac{E_f(Y(t); n) - V_f(Y(t); n)}{V_f(Y(t); n)} = \frac{OM_f(Y(t), y^*, \xi_l^*; n)}{V_f(Y(t); n)}, \tag{45}$$

for $f = s, l$. The cumulative return to the merging firm f at the time of the merger announcement is equal to the expression in equation (45) evaluated at $t = \tau_{y^*}$ or $Y(t) = y^*$.

Similarly, we can compute the cumulative merger return to a rival firm at the time of the merger announcement. To do so, we first compute the equity value of a rival firm prior to the announcement of a merger at date $t \leq \tau_{y^*}$. It is equal to the value of assets in place before the merger plus an option value from the merger:

$$V_f(Y(t); n) + \mathbb{E}\left[e^{-r(\tau_{y^*} - t)} \left(V_f^a(Y(\tau_{y^*}); n-1) - V_f(Y(\tau_{y^*}); n)\right) | Y(t)\right], \tag{46}$$

for $f = s, l$. This option value results from the fact that the value of the rival firm becomes $V_f^a(Y(\tau_{y^*}); n-1)$ after the asymmetric merger since there are $n-1$ large firms, $m-1$ small firms, and a huge merged firm in the industry. We then define the cumulative return to a small or large rival firm before the merger as:

$$R_{f,R}(Y(t); n) = \frac{\mathbb{E}\left[e^{-r(\tau_{y^*}-t)}\left(V_f^a(Y(\tau_{y^*}); n-1) - V_f(Y(\tau_{y^*}); n)\right)|Y(t)\right]}{V_f(Y(t); n)}, \tag{47}$$

for $f = s, l$. We focus on the cumulative return at the time of the merger announcement, when $t = \tau_{y^*}$ or $Y(\tau_{y^*}) = y^*$. The next proposition characterizes these returns:

Proposition 7 *Consider an asymmetric merger between a large firm and a small firm. Suppose that Assumptions 1 and 2 hold. (i) The cumulative merger returns to the small and large merging firms at the time of restructuring are given by*

$$R_{f,M}(y^*; n) = \frac{\Pi_M^a(n-1) - \Pi_l(n) - \Pi_s(n)}{\Pi_f(n)} \frac{X_f}{X_s + X_l} \frac{2}{\beta}, \tag{48}$$

for $f = s, l$. (ii) The cumulative merger returns to a small or a large rival firm at the time of restructuring are given by

$$R_{f,R}(y^*; n) = \left[1 - \frac{b^2}{\Delta(n)}\left(1 + \frac{2}{2+3bk}\right)\right]^{-2} - 1, \tag{49}$$

for $f = s, l$. (iii) All the above returns are positive and increase with n.

Proposition 7 highlights several interesting aspects of cumulative merger returns within our Cournot-Nash industry equilibrium framework. First, equation (48) reveals that the cumulative returns to the two merging firms have three types of determinants, including an anticompetitive effect, two size effects, and a hysteresis effect. The hysteresis effect is represented by $2/\beta$, which is a function of the risk-free rate, r, the growth rate of industry shocks, μ, and the volatility of industry shocks, σ. It implies that a higher volatility of industry shocks, σ, leads to higher cumulative returns to both the acquirer and the target. With more uncertainty surrounding the industry, the merger option is exercised when it is deeper in the money, resulting in higher cumulative merger returns. The two size effects are captured by $1/\Pi_f(n)$ and $X_f/(X_s+X_l)$, which respectively reflect the results that the smaller merging firm or the firm with a higher merger cost earns a higher return than the larger merging firm or the firm with a lower merger cost.

Second, and unlike existing real options models of mergers, our model with constant returns to scale does not produce an exogenous synergy effect for merger returns. Instead, we have an anticompetitive and cost reduction effect represented by the term $\Pi_M^a(n-1) - \Pi_l(n) - \Pi_s(n)$, which characterizes the merger benefits as a function of industry equilibria before and after the control transaction. This effect reflects the fact that after a merger, there are fewer small and large firms in the industry, and a huge merged entity emerges. Hence, both the market structure and the competitive landscape of the industry change at the time of the merger. In addition, the huge merged firm combines the tangible assets of the two merging firms, thereby reducing production costs. The fraction $[\Pi_M^a(n-1) - \Pi_l(n) - \Pi_s(n)]/\Pi_f(n)$ in equation (48) commingles the first size effect with the anticompetitive effect, indicating the percentage gain in profits resulting from a control transaction. Notably, this percentage gain differs for a small and a large merging firm as it depends on the firm's size and hence its relative contribution to the merger benefits.

Third, merging firms earn higher returns in more concentrated industries. The intuition is that there is a stronger anticompetitive effect on the equilibrium price after a merger in those industries. As a consequence, merging firms derive higher returns from entering into a takeover.

Fourth, returns to merger and rival firms are invariant to the sensitivity to industrywide demand shocks. Recall that only the anticompetitive effect of returns depends on a. By inspection of equations (25)–(27), notice that all firms' profit multipliers are a quadratic function of a. Thus, even though exposure to industrywide shocks crucially affects the optimal timing of mergers, it does *not* affect the returns from restructuring decisions in our dynamic industry equilibrium framework.

Fifth, equation (49) shows that the cumulative return to a small or a large rival firm at the time of a merger announcement is positive too. Like the cumulative returns to merging firms, the cumulative returns to rival firms increase with industry concentration. The intuition is that the industry's equilibrium price rises after the merger, and rival firms also benefit from this price increase. This benefit increases with industry concentration. Like merger returns, rival returns are therefore higher in more concentrated industries. Notice, however, that the magnitude of rival returns does not depend on firm size, as rival firms do not change their capital stock at the time of a control transaction but only adjust their equilibrium output choices.

Example 3. We illustrate Proposition 7 by a numerical example with baseline parameter values used in Examples 1 and 2. Recall that we let $X_l = X_s$ to focus on the portion of merger returns that is due not simply to different restructuring costs, which is also a feature of other dynamic models of mergers and hence not unique to our framework. Figure 4 graphs the cumulative returns of a small merging firm (dashed line) and a large merging firm (dotted line) given in equation (48) as well as the return to a rival firm (solid line) given in equation (49) as functions of various industry characteristics, such as the price sensitivity of demand, b, and the number of large firms, n. The figure reveals several interesting aspects of the determinants of merger and rival returns, such as firm size, profitability, and the firm's (relative) contribution to the creation of the merger benefit.

First, and somewhat surprisingly, the return of a small merging firm (i.e. target) exceeds the return of a large merging firm (i.e. acquirer). Mathematically, the anticompetitive effect, $\Pi_M^a(n-1) - \Pi_l(n) - \Pi_s(n)$, is scaled by the firm's pre-merger profitability or status quo, $\Pi_f(n)$, in equation (48), so that merger returns reflect a percentage gain in profits that differs for small and large merging firms. Crucially, this percentage gain depends on the pre-merger firm size and hence the firm's relative contribution to the post-merger synergy gains. The intuition for this interesting finding is therefore that the smaller firm benefits relatively more from a merger in our industry equilibrium framework and hence enjoys a larger relative synergy gain (i.e. merger return). As mentioned earlier, mergers typically involve acquisitions of a small firm by a large firm. This implication of the model therefore supports the available evidence without reliance on additional assumptions, such as asymmetric information (see, e.g., Rhodes-Kropf and Visvanathan (2004)) or misvaluation (see, e.g., Shleifer and Vishny(2003)).

Second, merger returns vary non-monotonically with the price sensitivity of demand, b, which is consistent with our findings in the earlier examples. Yet, rival returns increase with the price sensitivity of demand, b. The reason for this interesting difference between merger and rival returns is that industry rivals benefit from the (positive) price effect but are not hurt by the (negative) quantity effect of the Cournot–Nash equilibrium . In fact, because rivals do not change their firm size, k_i, they optimally produce a slightly higher quantity subsequent to a successful takeover deal.

Third, the graph in Figure 4b studies the effect of the number of large firms, n, on merger and rival returns. The figure reveals that a higher level of concentration in the industry (i.e. a larger value of n) leads to higher returns for all firms in the industry.

Fourth, merger and rival returns are increasing in k, as shown in Figure 4c. Recall that the variance of firm size in equation (42) also increases with k. Thus, returns to merging and rival firms arising from restructuring are higher in industries with more dispersion in firm size. Finally, and as confirmed by Figure 4, other comparative statics results are largely due to the hysteresis effect that is shared by other dynamic models of mergers, so we do not discuss them.

[Insert Figure 4 Here]

In our model, total merger returns come from the merger surplus. The merger surplus is generated by two incentives to merge: (i) gaining market power, and (ii) reducing production costs. As Perry and Porter (1985) and Salant et al. (1983) note, the gain in market power alone may not be sufficient to motivate a merger. To illustrate this point, we decompose the total merger surplus (net of merger costs) evaluated at the equilibrium merger threshold y^* into two components:

$$\begin{aligned} & V_M^a\left(y^*;n-1\right)-V_l\left(y^*;n\right)-V_s\left(y^*;n\right)-X_l-X_s \\ = & \underbrace{\left[\tilde{V}_M\left(y^*;n\right)-V_l\left(y^*;n\right)-V_s\left(y^*;n\right)-X_l-X_s\right]}_{S_1}+\underbrace{\left[V_M^a\left(y^*;n\right)-\tilde{V}_M\left(y^*;n\right)\right]}_{S_2}, \end{aligned} \tag{50}$$

where $\tilde{V}_M\left(y^*;n\right)$ represents the value of the merged firm when there is no cost saving. We compute this value using Proposition 1 by assuming that the merged firm uses the large firm's capital stock k to produce output and does not combine the two merging firms' capital assets to reduce production costs. Thus, the expressions S_1 and S_2 on the right-hand side of equation (50) represent the merger surplus attributed to market power and cost savings, respectively.

[Insert Table 1 Here]

Table 1 illustrates these two components numerically for various values of n. One can see the effects of market power on the benefits (and hence returns) from mergers within our framework. First, note that the total merger surplus at the merger threshold is independent of n because it is equal to $2\left(X_s+X_l\right)/\beta$ by equation (41). Second, the large firm and the small firm do not have incentives to merge if the motivation is market power alone. That is, the associated merger surplus (S_1) is negative, even though the value of the merged firm is higher than the value of each merging firm. Third, it is the additional cost savings incentive represented by S_2 that makes the merger profitable. Thus, the merger benefits in our numerical example are attributable largely to cost savings. Finally, the table also reveals that the market power incentive is still present, although it does not motivate the merger by itself. This is because market power effect has a stronger effect as the industry becomes more concentrated (i.e., n is higher). Consequently, the component S_2 of the merger surplus generated by cost savings declines as the industry becomes more concentrated.

3.3 Global optimality

As in other real options models of mergers, the equilibrium merger timing analyzed in Section 3.1 is globally optimal. Formally, consider a central planner who selects the merger timing to maximize the total surplus from the merger. This globally optimal equilibrium solves the following problem:

$$\max_{\tau} \mathbb{E}^{y} \left\{ e^{-r\tau} \left[V_M^a \left(Y_\tau ; n-1 \right) - V_s \left(Y_\tau ; n \right) - V_l \left(Y_\tau ; n \right) - X_s - X_l \right] \right\}. \tag{51}$$

One can verify that the surplus-maximizing policy is characterized by the trigger policy whereby the two firms merge the first time the process reaches the threshold y^* given in Proposition 6. This suggests that, to solve for the equilibrium, one can first solve for the globally optimal merger timing and then solve for the sharing rule so that the globally optimal merger timing is also optimal to each of the two merging partners.

4 Bidder competition and industry competition

So far, we have focused on an asymmetric merger with a single bidder. We now consider two bidders who compete for a small firm target. One bidder is a small firm, and the other is a large firm. We suppose Assumption 3 holds, so that both bidders have incentives to merge with the target. As Fishman (1988) and Morellec and Zhdanov (2005) argue, bidder competition puts the target in an advantageous position and allows target shareholders to extract a higher premium from the bidding firms. Our work is different in that we do not consider asymmetric information, and merger benefits are endogenously derived from an industry equilibrium in our model rather than exogenously specified. Thus, product market competition plays an important role.

Our analysis of Nash equilibria in a takeover contest follows similar steps as in Morellec and Zhdanov (2005). Once the contest is initiated, the two bidding firms submit bids to the target in the form of a fraction of the merged firm's equity to be owned by target shareholders upon the takeover.[12] The bidder who offers the highest monetary value to the target shareholders wins the contest. Given the winner's ownership share, the winning bidder and the target select their merger timing independently. In equilibrium, they must agree on the merger timing.

In our model, a large firm has a production cost advantage over a small firm. Thus, the value of the merged firm is higher when the small firm target merges with a large firm bidder than when it merges with a small firm bidder. Formally, we can use equations (17) and (27) to show that

$$V_M^a \left(y; n-1 \right) > V_l \left(y; n+1 \right). \tag{52}$$

Equation (52) implies that the large firm will win the takeover contest as long as the takeover is profitable to it because it can always slightly bid more than the small firm bidder and deliver more

[12] The target actually cares about the monetary value of a bid. We use ownership share as a device to solve the model, as in Morellec and Zhdanov (2005). Any monetary value of a bid can be transformed into a corresponding ownership share by dividing the monetary value of the bid by the combined firm value.

value to target shareholders.[13] This strategy is costly to the large firm when its merger costs are higher than the small firm bidder's merger costs. In this case, the takeover may not be profitable to the large firm bidder, and it would rather drop out.

From the preceding discussions, we conclude that either the large firm or the small firm may win the takeover contest, depending on the relative effects of production and merger costs. Before analyzing these two possibilities, we define the breakeven share ξ_s^{BE} for the small firm bidder as

$$\xi_s^{BE} V_l\left(y; n+1\right) - V_s\left(y; n\right) - X_s = 0, \tag{53}$$

If the small firm bidder places a bid higher than $1 - \xi_s^{BE}$, then it will realize a negative value by entering the deal. In this case, the small firm bidder would be better off losing the takeover contest. Similarly, we define the breakeven share ξ_l^{BE} for the large firm bidder as

$$\xi_l^{BE} V_M^a\left(y; n-1\right) - V_l\left(y; n\right) - X_l = 0. \tag{54}$$

We now discuss the bidders' strategies by considering the following cases (ignoring cases of ties):

1. The large firm bidder wins the contest. In this case, the losing small firm bidder may still influence equilibrium, depending on whether it is strong or weak.

   (a) The losing small firm bidder is weak in the sense that merging with the large firm bidder and accepting the ownership share $(1 - \xi_l^*)$ is more profitable to the target than merging with the small firm bidder and accepting the ownership share $\left(1 - \xi_s^{BE}\right)$:

   $$\left(1 - \xi_l^*\right) V_M^a\left(y; n-1\right) > \left(1 - \xi_s^{BE}\right) V_l\left(y; n+1\right), \tag{55}$$

   where ξ_l^* is the equilibrium share without bidder competition given in Proposition 6. In this case, for the small firm bidder to win, it must offer an ownership share of at least $\left(1 - \xi_s^{BE}\right)$ to target shareholders. But this implies a negative net payoff to the small firm bidder. Thus, it would rather drop out of the takeover contest. The equilibrium is then the same as in the case of a single bidder.

   (b) The small firm bidder is strong in the sense that merging with the large firm bidder and accepting the ownership share $(1 - \xi_l^*)$ is less profitable to the target than merging with the small firm bidder and accepting the ownership share $\left(1 - \xi_s^{BE}\right)$:

   $$\left(1 - \xi_l^*\right) V_M^a\left(y; n-1\right) < \left(1 - \xi_s^{BE}\right) V_l\left(y; n+1\right). \tag{56}$$

[13] In our model, there is no initial value problem as in Appendix D of Morellec and Zhdanov (2005). This is because the two curves implied by the functions $V_M^a\left(y; n-1\right)$ and $V_l\left(y; n+1\right)$ do not have a crossing point for $y > 0$. In particular, both functions are proportional to y^2.

Under this condition, the small firm bidder has an incentive to bid an amount slightly less than $\left(1-\xi_s^{BE}\right)$ in order to win the contest. Anticipating this bidder's competition, the large firm bidder will place a bid higher than $(1-\xi_l^*)$ until equality holds in (56). We define $\xi_l^{\max}$ as the ownership share satisfying this equality:

$$\left(1-\xi_l^{\max}\right) V_M^a\left(y;n-1\right) = \left(1-\xi_s^{BE}\right) V_l\left(y;n+1\right). \tag{57}$$

The value $\xi_l^{\max}$ is the maximum share that the large firm bidder can extract from the merged firm such that it still wins the takeover contest. If the large firm demands a share higher than $\xi_l^{\max}$, or places a bid lower than $\left(1-\xi_l^{\max}\right)$, the small firm will outbid the large firm and win the contest. Thus, the best response of the large firm is to place a bid $\left(1-\xi_l^{\max}\right)$. Note that the ownership share $\xi_l^{\max}$ of the large firm bidder must be greater than the breakeven share ξ_l^{BE} for the large firm to participate. If this condition is violated, the large firm would rather drop out.

2. The small firm bidder wins the contest. As in Case 1, the losing large firm bidder may influence equilibrium, depending on whether it is strong or weak.

   (a) The losing large firm bidder is weak in the sense that merging with the small firm bidder and accepting the ownership share $(1-\xi_s^*)$ is more profitable to the target than merging with the large firm bidder and accepting the ownership share $\left(1-\xi_l^{BE}\right)$:

   $$\left(1-\xi_s^*\right) V_l\left(y;n+1\right) > \left(1-\xi_l^{BE}\right) V_M^a\left(y;n-1\right), \tag{58}$$

   where ξ_s^* is the equilibrium ownership share when the small firm is the only bidder. Similar to Case 1(a), the losing large bidder is too weak to matter, and the equilibrium is the same as in the case of a single bidder.

   (b) The losing large firm bidder is strong in the sense that merging with the small firm bidder and accepting the ownership share $(1-\xi_s^*)$ is less profitable to the target than merging with the large firm bidder and accepting the ownership share $\left(1-\xi_l^{BE}\right)$:

   $$\left(1-\xi_s^*\right) V_l\left(y;n+1\right) < \left(1-\xi_l^{BE}\right) V_M^a\left(y;n-1\right). \tag{59}$$

   In this case, the winning small firm bidder will raise its bid and offer an ownership share such that the large firm bidder is just willing to drop out. This ownership share $\xi_s^{\max}$ is defined as the value satisfying the equation:

   $$\left(1-\xi_s^{\max}\right) V_l\left(y;n+1\right) = \left(1-\xi_l^{BE}\right) V_M^a\left(y;n-1\right). \tag{60}$$

Given the above bidding strategies, the winning bidder will select a merger time to maximize the option value to merge. The target shareholders also select a merger time to maximize their option value to merge, given the ownership share proposed by the winning bidder. The equilibrium ownership share is determined so that the merger timing selected by the winning bidder and the target shareholders agrees. Figure 5 illustrates equilibria when the large firm bidder wins the takeover contest. The increasing function represents the target's strategy. The solid (dashed) decreasing line represents the large firm bidder's strategy when the small firm bidder is weak (strong). In particular, the dashed line represents y as a function of $\xi_l^{\max}$ derived from equation (57) after we substitute ξ_s^{BE} from equation (53). The crossing points of the reaction functions in Figure 5 represent option exercise equilibria.

[Insert Figure 5 Here.]

Proposition 8 presents the equilibrium solution in the four cases:

Proposition 8 *Consider a small firm bidder and a large firm bidder competing for a small firm target. Suppose Assumptions 1 and 3 hold. Let*

$$\Lambda(n) \equiv \frac{\Pi_M^a(n-1) - \Pi_l(n) - \Pi_s(n)}{\Pi_l(n+1) - 2\Pi_s(n)} \tag{61}$$

denote the relative merger benefits.

(i) If

$$\frac{X_l}{X_s} < \frac{2(\beta-1)\Lambda(n)}{\beta} - 1, \tag{62}$$

then the equilibrium is the same as that with a single large firm bidder described in Proposition 6. The cumulative returns to the winning large firm bidder and the small firm target are given in Proposition 7.

(ii) If

$$\frac{2(\beta-1)\Lambda(n)}{\beta} - 1 < \frac{X_l}{X_s} < \frac{2(\beta-1)\Lambda(n)}{\beta-2} - \frac{\beta}{\beta-2}, \tag{63}$$

then the large firm bidder wins the contest and the small firm bidder is strong. The share of the merged firm accruing to the winning large firm bidder is given by:

$$\bar{\xi}_l^{\max} = 1 - \frac{\beta\Pi_l(n+1) - 2\Pi_s(n)}{2(\beta-1)\Pi_M^a(n-1)}. \tag{64}$$

In addition, the takeover takes place the first time the industry shock (Y_t) reaches the threshold value:

$$y_{cl}^* = \sqrt{\frac{2(\beta-1)}{(\beta-2)}\frac{X_s(r-2(\mu+\sigma^2/2))}{\Pi_l(n+1) - 2\Pi_s(n)}}. \tag{65}$$

The cumulative returns to the winning large firm bidder and the small firm target are given by:

$$R_{l,M}\left(y_{cl}^{*};n\right)=\frac{\Pi_{M}^{a}\left(n-1\right)-\Pi_{l}\left(n\right)-\Pi_{s}\left(n\right)}{\Pi_{l}\left(n\right)}-\frac{\left(\beta-2\right)X_{l}+\beta X_{s}}{2\left(\beta-1\right)X_{s}}\frac{\Pi_{l}\left(n+1\right)-2\,\Pi_{s}\left(n\right)}{\Pi_{l}\left(n\right)},\tag{66}$$

and

$$R_{s,M}\left(y_{cl}^{*};n\right)=\frac{\Pi_{l}\left(n+1\right)-2\,\Pi_{s}\left(n\right)}{\left(\beta-1\right)\Pi_{s}\left(n\right)}.\tag{67}$$

(iii) If

$$\frac{2\left(\beta-1\right)\Lambda\left(n\right)}{\beta-2}-\frac{\beta}{\beta-2}<\frac{X_{l}}{X_{s}}<\frac{2\beta\Lambda\left(n\right)}{\beta-2}-\frac{1}{2},\tag{68}$$

then the small firm bidder wins the contest and the large firm bidder is strong. The share of the merged firm accruing to the winning small firm bidder is given by:

$$\bar{\xi}_{s}^{\max}=1-\frac{\beta X_{s}\left[\Pi_{M}^{a}\left(n-1\right)-\Pi_{l}\left(n\right)\right]+\left(\beta-2\right)X_{l}\Pi_{s}\left(n\right)}{\left[\left(\beta-2\right)X_{l}+\beta X_{s}\right]\Pi_{l}\left(n+1\right)}.\tag{69}$$

In addition, the takeover takes place the first time the industry shock (Y_t) reaches the threshold value:

$$y_{cs}^{*}=\sqrt{\frac{\left(\beta-2\right)X_{l}+\beta X_{s}}{\beta-2}\frac{r-2\left(\mu+\sigma^{2}/2\right)}{\Pi_{M}^{a}\left(n-1\right)-\Pi_{l}\left(n\right)-\Pi_{s}\left(n\right)}}.\tag{70}$$

The cumulative returns to the winning small firm bidder and the small firm target are given by:

$$R_{s,M}\left(y_{cs}^{*};n\right)=\frac{\Pi_{l}\left(n+1\right)-2\Pi_{s}\left(n\right)}{\Pi_{s}\left(n\right)}-\frac{2\left(\beta-1\right)X_{s}}{\left(\beta-2\right)X_{l}+\beta X_{s}}\frac{\Pi_{M}^{a}\left(n-1\right)-\Pi_{l}\left(n\right)-\,\Pi_{s}\left(n\right)}{\Pi_{s}\left(n\right)},\tag{71}$$

and

$$R_{s,M}\left(y_{cs}^{*};n\right)=\frac{2X_{s}}{\left(\beta-2\right)X_{l}+\beta X_{s}}\frac{\Pi_{M}^{a}\left(n-1\right)-\Pi_{l}\left(n\right)-\,\Pi_{s}\left(n\right)}{\Pi_{s}\left(n\right)}.\tag{72}$$

(iv) If

$$\frac{X_{l}}{X_{s}}>\frac{2\beta\Lambda\left(n\right)}{\beta-2}-\frac{1}{2},\tag{73}$$

then the small firm bidder wins the contest and the large firm bidder is weak. The equilibrium is the same as that with a single small firm bidder. The share of the merged firm accruing to the winning small firm bidder is given by $\xi_s^*=1/2$, and the merger threshold is given by:

$$\bar{y}^{*}=\sqrt{\frac{2\beta X_{s}}{\beta-2}\frac{r-2\left(\mu+\sigma^{2}/2\right)}{\Pi_{l}\left(n+1\right)-2\,\Pi_{s}\left(n\right)}}.\tag{74}$$

The cumulative returns to the winning small firm bidder and the small firm target are given by:

$$R_{s,M}\left(\bar{y}^{*};n\right)=\frac{\Pi_{l}\left(n+1\right)-2\Pi_{s}\left(n\right)}{\beta\Pi_{s}\left(n\right)}.\tag{75}$$

Proposition 8 provides a complete characterization of equilibrium for all possible cases. As conditions (68) and (73) indicate, the small firm bidder can win the takeover contest if and only if the relative merger cost X_l/X_s of the large firm bidder is sufficiently high compared to the relative merger benefit $\Lambda(n)$. Otherwise, the large firm bidder will win the takeover contest. We view that this case is more relevant in corporate practice as merger costs include fees to investment banks and lawyers, and should not vary substantially for firms of different sizes (see Gorton et al. (2008)). We have also noted that large firm bidders and small firm targets are consistent with the empirical evidence. As Moeller, Schlingemann, and Stulz (2004) also report that competed deals are more likely for large acquirers than for small acquirers, which implies that large firm bidders tend to win more often in takeover contests, we focus on parts (i) and (ii) of Proposition 8 henceforth.

Part (ii) shows that when the losing small firm bidder is strong, the target can extract additional returns from the winning bidder. That is, if condition (63) holds, the merger return to the winning bidder given in (66) is lower than without the bidder competition given in (48). In addition, the merger return to the target is greater than without bidder competition given in (48).

To interpret parts (i) and (ii) of Proposition 8 further, we define the bid premium resulting from bidder competition as the percentage increase in the target equity value over the case without bidder competition. These equity values are evaluated at the equilibrium time of the merger. Clearly, there is a positive bid premium only if the losing bidder is strong as in part (ii). In this case, we can formally define the bid premium resulting from bidder competition as

$$\frac{[(1-\bar{\xi}_l^{\max})V_M^a(y_{cl}^*;n-1)-(1-\xi_l^*)V_M^a(y_{cl}^*;n-1)]}{(1-\xi_l^*)V_M^a(y_{cl}^*;n-1)}=\frac{\xi_l^*-\bar{\xi}_l^{\max}}{1-\xi_l^*}. \tag{76}$$

We then use condition (63) and the expressions for ξ_l^* and $\bar{\xi}_l^{\max}$ to show that $\xi_l^*>\bar{\xi}_l^{\max}$. Consequently, the bid premium is positive. Note that both condition (63) and the bid premium depend on industry characteristics, such as the industry concentration level n and the price sensitivity of demand parameter b. As a result, there is an interaction between product market competition and bidder competition in our model.

Example 4. A numerical example illustrates the results reported in Proposition 8, focusing on the interesting interactions between bidder competition and industry competition. Baseline parameter values are: $a=100$, $k=0.2$, $K=1$, $b=0.4$, $n=4$, $r=8\%$, $\mu=1\%$, and $\sigma=20\%$. To ensure that the large firm bidder wins the takeover contest and the small firm bidder can be potentially strong as in part (ii) of Proposition 8, we set $X_l=20$ and $X_s=1$ so that condition (63) holds.

[Insert Figure 6 Here]

Figure 6 displays the bid premium in equation (76) as a function of the number of large firms, n, and the price sensitivity of demand, b. The figure highlights several key elements of the different equilibria resulting from jointly analyzing bidder competition and product market competition. First, the left panel shows that the bid premium is first an increasing and then a declining function of the price sensitivity parameter b. When b is low enough, the bid premium is close to zero. When b is high enough, condition (62) holds. As a result, the small firm bidder does not influence the equilibrium bidding process, and there is no bid premium.[14]

Second, the right panel of the figure illustrates our earlier discussion that only when the industry is sufficiently concentrated, bidder competition matters. In the numerical example, bidder competition induces an additional bid premium if there are at least three large firms in the industry. Moreover, in the region where the bid premium is non-trivial it increases with industry concentration. Absent of a formal model, one might have been tempted to conclude that bidder competition generally raises the price of a contested deal irrespective of industry competition. However, this example underlines that the degree of industry competition may play a central role for if and how a bid premium emerges in equilibrium.

5 Conclusion

This paper develops a real options model of mergers and acquisitions that jointly determines the industry's product market equilibrium and the timing and terms of takeovers. The analysis explicitly recognizes the role of product market competition and derives equilibrium restructuring strategies by solving an option exercise game between bidding and target shareholders. Our unified framework's results are generally consistent with the extant empirical evidence. In addition, the model also generates a number of novel testable predictions:

1. Product market competition among heterogeneous firms does not speed up the acquisition process. Instead, we find that increased industry competition delays the timing of takeovers. Thus, mergers are more likely in more concentrated industries.

2. In our model, the relative scarcity of acquirers and targets in the industry is reflected in the

[14] While the bid premium resulting from the interaction of bidder and industry competition in our setting can be higher or lower for different parameter values, it is only one component of the combined run-up and markup observed in corporate practice.

equilibrium equity share of a deal. In particular, a large acquirer (small target) demands a lower (higher) ownership share in the merged firm if the industry is less competitive.

3. Merger returns are higher for the smaller merging firm (e.g. target) than the larger merging firm (e.g. acquirer) when the firms have identical merger costs. Returns to merging and rival firms arising from restructuring are higher in more concentrated industries.

4. Takeovers are more likely in industries in which dispersion in firm size is higher. In addition, returns to merging and rival firms arising from restructuring are higher in industries with more dispersion in firm size.

5. With multiple bidders for a scarce target and similar merger costs, the large firm bidder wins the takeover contest when competing with a small firm bidder. Only in a sufficiently concentrated industry, bidder competition speeds up the takeover process and leads the large firm acquirer to pay a bid premium to discourage the competing small firm competitor. If it exists, this bid premium decreases with industry competition. By contrast, in a sufficiently competitive industry, the small firm bidder does not matter, and the equilibrium outcome corresponds to the one without bidder competition.

The rich set of predictions generated by integrating a real options model of mergers into an industry equilibrium framework suggests that further extensions of our dynamic model will be valuable. Some possible extensions would add complexity to our analysis by refining the link between industry structure and takeover activity. For example, our analysis follows most research in the literature (e.g., Gorton et al. (2008)) by assuming exogenously the initiation of mergers in the sense that the merger structure (i.e., what firm merges with what firm, and which remains independent) is exogenously imposed in the absence of antitrust law. As a result, our model indicates that industry rivals benefit from mergers without having to bear the restructuring costs, leading to a so-called "free-rider problem," i.e., merging firms provide a positive externality to rival firms. This free rider problem may give firms incentives to further delay takeovers in more competitive industries.

More generally, it would be interesting to consider mergers when each firm makes individual merger decisions and responds to mergers by other firms, which is a difficult problem even in a static model (see, e.g., Qiu and Zhou (2007)). In this case, multiple mergers may arise sequentially, and hence the order of mergers may be an important element of the dynamic evolution of the industry's competitive landscape through time. Similarly, modeling new entry into and/or exit from the industry would also have important implications that are beyond the scope of our paper. Prima facie, these extensions could probably generate more or less preemptive behavior, depending on the intensities of entry, exit, etc., and hence generate less or more delays of takeovers in more competitive industries. Research that combines some of these elements with our analysis is likely to be a fruitful avenue for future research in mergers and acquisitions, and more generally, in corporate finance.

Appendix

Proof of Proposition 1: In the Cournot–Nash industry equilibrium, each firm $i = 1, ..., N$ has the objective to

$$\max_{q_i(t)} \pi_i(t) \tag{A.1}$$

while taking other firms' output strategies as given, where $\pi_i(t)$ is defined in equation (4). This maximization problem has the first-order condition for each firm i:

$$a\,Y(t) - b\,Q(t) = \left(b + 2k_i^{-1}\right) q_i(t). \tag{A.2}$$

Using (1) and (5), we can solve the system of first-order conditions (A.2) to obtain the equilibrium expression for $q_i^*(t)$ in equation (7). Aggregating individual firms' output choices yields the industry's optimal output level $Q^*(t)$ in equation (9). The industry's equilibrium price process $P^*(t)$ in equation (10) immediately follows from substituting $Q^*(t)$ into equation (2) and simplifying. ∎

Proof of Proposition 2: Using Proposition 1, we can derive firm i's profits:

$$\pi_i(t) = Pq_i - q_i^2/(2k_i) = \left(\frac{aY(t)}{1+B}\right)^2 \left(1 - \frac{\theta_i}{2bk_i}\right) \frac{\theta_i}{b}. \tag{A.3}$$

We need to substitute the values of θ_i and k_i for a small, a large, or a merged firm to compute their profits and firm values. Substituting $k_s = k/2$ and $k_l = k$, $\theta_s = b/\left(b + 2k^{-1}\right)$, and $\theta_l = b/\left(b + k^{-1}\right)$ into the expression of instantaneous operating profits in equation (A.3) produces the closed-form solutions in equations (16) and (17). Using these equations to evaluate equation (6), we can derive the expression for equilibrium firm value $V_i(y; n)$ for $i = s, l$ that is reported in equation (15). ∎

Proof of Proposition 3: We first find the critical value Δ^* in equation (19) by solving the equation

$$\Pi_l(n+1) - 2\,\Pi_s(n) = 0. \tag{A.4}$$

Economically, equation (A.4) represents a breakeven condition for the incentive to merge. Note that the functional form of $\Delta(n)$ in equation (14) has the useful property:

$$\Delta(n) = \Delta(n+1) + b^2. \tag{A.5}$$

Thus, using equations (16), and (17), we can write

$$\Pi_l\left(n+1\right)-2\,\Pi_s\left(n\right)=\frac{a^2\left(b+2k^{-1}\right)^2\left(b+k^{-1}/2\right)}{\Delta\left(n+1\right)^2}-\frac{2\,a^2\left(b+k^{-1}\right)^3}{\Delta\left(n\right)^2} \tag{A.6}$$

When we insert equations (A.5) and (A.6) into the breakeven condition in equation (A.4), rearrange, and simplify, we obtain:

$$\frac{2\,a^2\left(b+k^{-1}\right)^3}{\Delta\left(n\right)^2}\left[\frac{A^2}{\left(1-b^2/\Delta\left(n\right)\right)^2}-1\right]=0, \tag{A.7}$$

where the positive constant A is given in equation (20). By solving equation (A.7) for $\Delta(n)$, we can determine the critical value Δ^* in equation (19). Since one can easily verify that the term $\Pi_l\left(n+1\right)-2\,\Pi_s\left(n\right)$ increases with the number of large firms, we can therefore derive the three conditions for the incentive to merge in equations (21), (22), and (23). ∎

Proof of Proposition 4: Using Proposition 1, we can show that the post-merger output levels of a small firm and a large firm are given by

$$q_s^a\left(t;n-1\right) = \frac{a\left(b+k^{-1}\right)}{\Delta\left(n\right)-b^2\left(1+\frac{2}{2+3bk}\right)}Y\left(t\right), \tag{A.8}$$

$$q_l^a\left(t;n-1\right) = \frac{a\left(b+2k^{-1}\right)}{\Delta\left(n\right)-b^2\left(1+\frac{2}{2+3bk}\right)}Y\left(t\right), \tag{A.9}$$

and the merged firm produces output at the level

$$q_M^a\left(t;n-1\right)=\frac{\frac{3a}{3b+2k^{-1}}\left(b+k^{-1}\right)\left(b+2k^{-1}\right)}{\Delta\left(n\right)-b^2\left(1+\frac{2}{2+3bk}\right)}Y\left(t\right). \tag{A.10}$$

In addition, the post-merger industry output and price are given by

$$Q^a\left(t;n-1\right) = \frac{a}{b}\left[1-\frac{\left(b+k^{-1}\right)\left(b+2k^{-1}\right)}{\Delta\left(n\right)-b^2\left(1+\frac{2}{2+3bk}\right)}\right]Y\left(t\right), \tag{A.11}$$

$$P^a\left(t;n-1\right) = \frac{a\left(b+k^{-1}\right)\left(b+2k^{-1}\right)}{\Delta\left(n\right)-b^2\left(1+\frac{2}{2+3bk}\right)}Y\left(t\right). \tag{A.12}$$

Substituting the equilibrium output choice $q_s^a\left(t\right)$, $q_l^a\left(t\right)$, and $q_M^a\left(t\right)$ in equations (A.8), (A.9), and (A.10) with $k_M=3k/2$ and $\theta_M=b/\left(b+\frac{2}{3}k^{-1}\right)$ into the expression of instantaneous operating profits in equation (A.3) produces the closed-form solutions in equations (25), (26), and (27). Using these equations to evaluate equation (6), we can derive the expression for equilibrium firm value $V_f^a\left(y;n-1\right)$ for $f=s,l,M$ that is reported in equation (24). ∎

Proof of Proposition 5: The arguments are similar to the proof of Proposition 3. We first find the critical value Δ^a in equation (29) by solving the equation

$$\Pi_M^a(n-1) - \Pi_s(n) - \Pi_l(n) = 0. \tag{A.13}$$

Using equations (16), (17), and (27), we express the breakeven condition (A.13) as follows:

$$\frac{3\left(b+2k^{-1}\right)^2\left(b+k^{-1}\right)^2\left(3b+k^{-1}\right)/\left(3b+2k^{-1}\right)^2}{\left[\Delta(n) - b^2\left(1+\frac{2}{2+3bk}\right)\right]^2} - \frac{\left(b+2k^{-1}\right)^2\left(b+k^{-1}/2\right) + \left(b+k^{-1}\right)^3}{\Delta(n)^2} = 0. \tag{A.14}$$

After rearranging and simplifying, we can rewrite equation (A.14) as

$$\frac{\left(b+k^{-1}/2\right)\left(b+2k^{-1}\right)^2\left(b+k^{-1}\right)^3}{\Delta(n)^2}\left[\frac{D^2}{\left(1-b^2\left(1+\frac{2}{2+3bk}\right)/\Delta(n)\right)^2} - 1\right] = 0, \tag{A.15}$$

where the positive constant D is given in equation (30). By solving equation (A.15) for $\Delta(n)$, we can determine the critical value Δ^a in equation (29). Since one can easily verify that the term $\Pi_M^a(n-1) - \Pi_s(n) - \Pi_l(n)$ increases with the number of large firms, we can therefore derive the three conditions for the incentive to merge that correspond to equations (21), (22), and (23). ∎

Proof of Lemma 1: We use Propositions 2 and 4 to show that

$$\begin{aligned}
& \Pi_M^a(n-1) - \Pi_s(n) - \Pi_l(n) \qquad\qquad \text{(A.16)} \\
= \; & \frac{3a^2\left(b+2k^{-1}\right)^2\left(b+k^{-1}\right)^2\left(3b+k^{-1}\right)/\left(3b+2k^{-1}\right)^2}{\left[\Delta(n) - b^2\left(1+\frac{2}{2+3bk}\right)\right]^2} \\
& - \frac{a^2\left(b+k^{-1}\right)^3}{\Delta(n)^2} - \frac{a^2\left(b+2k^{-1}\right)^2\left(b+k^{-1}/2\right)}{\Delta(n)^2} \\
= \; & \frac{a^2\left(b+k^{-1}\right)^3 + a^2\left(b+2k^{-1}\right)^2\left(b+k^{-1}/2\right)}{\left[\Delta(n) - b^2\left(1+\frac{2}{2+3bk}\right)\right]^2} \\
& \times \left\{D^2 - \left[1 - \frac{b^2\left(1+\frac{2}{2+3bk}\right)}{\Delta(n)}\right]^2\right\},
\end{aligned}$$

where D is defined in equation (30). Under Assumptions 2 or 3, the expression in curly brackets in equation (A.16) is positive. In addition, since $\Delta(n)$ declines with n, we know that $\Pi_M^a(n-1) - \Pi_s(n) - \Pi_l(n)$ increases with n.

We can show the profit ratio $\left[\Pi_M^a(n-1) - \Pi_s(n) - \Pi_l(n)\right]/\Pi_f(n)$ is equal to a positive constant times the expression

$$\left[1 - \frac{b^2\left(1 + \frac{2}{2+3bk}\right)}{\Delta(n)}\right]^{-2} \left\{D^2 - \left[1 - \frac{b^2\left(1 + \frac{2}{2+3bk}\right)}{\Delta(n)}\right]^2\right\}, \tag{A.17}$$

which, under Assumptions 2 or 3, increases with n. ∎

Proof of Proposition 6: Using equation (31), we can derive

$$OM_l\left(y, y_l^*, \xi_l; n\right) = \left[\xi_l V_M^a\left(y_l^*; n-1\right) - V_l\left(y_l^*; n\right) - X_l\right] \mathbb{E}^y\left[e^{-r\tau_{y_l^*}}\right], \tag{A.18}$$

for some undetermined threshold $y_l^* \geq y$. By Karatzas and Shreve (1999), we know that

$$\mathbb{E}^y\left[e^{-r\tau_{y_l^*}}\right] = \left(\frac{y}{y_l^*}\right)^\beta, \tag{A.19}$$

where β is the positive root of the characteristic equation (33). Thus, we obtain equation (32). Solving the first-order condition (34), we obtain equation (35). Similarly, we can derive equation (38). We can then use equations (35) and (38) to determine the equilibrium sharing rule ξ_l^* in (40) by solving the equation $y_l^* = y_s^*$ for ξ_l. Substituting ξ_l^* into either (35) or (38) yields the option value-maximizing merger threshold reported in equation (39). Finally, the comparative statics results in parts (ii) and (iv) of the proposition follow from Lemma 1 and Propositions 2 and 4. ∎

Proof of Proposition 7: Evaluating equations (45) and (47) at the merger threshold $Y(t) = y^*$ and using the results from Propositions 4 and 6, we find the cumulative merger returns in equations (48) and (49). Finally, consider part (iii). By Lemma 1 and Assumptions 2 and 3, the cumulative merger returns to the small and large merging firms are positive and increase with n. Since $\Delta(n) > 0$ and drops with n by equation (14), it follows from equation (49) that the cumulative returns to rival firms are also positive and increase with n. ∎

Proof of Proposition 8: We first consider Case 1(a), where the large firm bidder wins the contest, and the small firm bidder is too weak to matter. In this case, the equilibrium is the same as that with a single large firm bidder. The equilibrium merger threshold y^* and ownership share ξ_l^* of the large firm bidder are given in Proposition 6. Using equation (53), we can derive the breakeven share of the small firm bidder at the merger threshold y^* :

$$\begin{aligned}\xi_s^{BE} &= \frac{X_s + V_s(y^*; n)}{V_l(y^*; n+1)} = \frac{X_s + \Pi_s(n)(y^*)^2 / \left(r - 2\left(\mu + \sigma^2/2\right)\right)}{\Pi_l(n+1)(y^*)^2 / \left(r - 2\left(\mu + \sigma^2/2\right)\right)} \\ &= \frac{(\beta - 2) X_s \left[\Pi_M^a - \Pi_l(n) - \Pi_s(n)\right] + \beta(X_s + X_l)\Pi_s(n)}{\beta(X_s + X_l)\Pi_l(n+1)}.\end{aligned} \tag{A.20}$$

For condition (55) to hold, we must have

$$(1 - \xi_l^*)\Pi_M^a > \left(1 - \xi_s^{BE}\right)\Pi_l(n+1). \tag{A.21}$$

Substituting equation (A.20) for ξ_s^{BE} and equation (40) for ξ_l^* into equation (A.21) yields equation (62). We thus obtain part (i) of the proposition.

We next consider Case 1(b), where the large firm bidder wins the contest, and the small firm bidder matters. Using equations (53) and (57) as well as Propositions 2 and 4, we can derive the merger threshold selected by the winning large firm bidder as a function of $\xi_l^{\max}$:

$$y = y_B\left(\xi_l^{\max}\right) \equiv \sqrt{\frac{X_s\left(r - 2\left(\mu + \sigma^2/2\right)\right)}{\Pi_l(n+1) - \Pi_s(n) - \left(1 - \xi_l^{\max}\right)\Pi_M^a(n-1)}}. \tag{A.22}$$

Using equation (38), we can derive the merger threshold selected by the small firm target's given the winning bidder's demanded ownership share $\xi_l^{\max}$:

$$y = y_T\left(\xi_l^{\max}\right) \equiv \sqrt{\frac{\beta X_s}{\beta - 2}\frac{r - 2\left(\mu + \sigma^2/2\right)}{\left(1 - \xi_l^{\max}\right)\Pi_M^a(n-1) - \Pi_s(n)}}. \tag{A.23}$$

In equilibrium, the winning bidder and the target must agree on the merger timing in that $y_B\left(\xi_l^{\max}\right) = y_T\left(\xi_l^{\max}\right) \equiv y_{cl}^*$. We can then solve the system of equations (A.22) and (A.23) for the winning bidder's ownership share $\bar{\xi}_l^{\max}$ and the merger threshold y_{cl}^* when the losing small firm bidder is strong. They are given in (64) and (65).

For Case 1(b) to happen, condition (62) must be violated. We thus obtain the first inequality in (63). In addition, the large firm bidder must be willing to participate. That is, we require $\bar{\xi}_l^{\max} > \xi_l^{BE}$, and use (54) to compute the breakeven share at the equilibrium trigger y_{cl}^* :

$$\xi_l^{BE} = \frac{X_l + V_l\left(y_{cl}^*; n\right)}{V_M^a\left(y_{cl}^*; n-1\right)} = \frac{(\beta - 2) X_l\left[\Pi_l(n+1) - 2\Pi_s(n)\right] + 2(\beta - 1) X_s \Pi_l(n)}{2(\beta - 1) X_s \Pi_M^a(n-1)}, \tag{A.24}$$

where y_{cl}^* is given in equation (65). Substituting equation (A.24) and equation (64) for $\bar{\xi}_l^{\max}$ into the condition $\bar{\xi}_l^{\max} > \xi_l^{BE}$, we obtain the second inequality in (63). Evaluating equation (45) for $f = s$ and for $f = l$ at the merger threshold $Y(t) = y_{cl}^*$ in equation (65), when the large firm bidder receives the share $\bar{\xi}_l^{\max}$ in equation (64), and using the results from Propositions 2 and 4, we find the cumulative merger returns in equations (66) and (67). We thus obtain part (ii).

In Cases 2(a) and 2(b) the large firm bidder loses the contest. In these cases, we must have $\bar{\xi}_l^{\max} < \xi_l^{BE}$. We thus have the first inequality in condition (68). We first consider Case 2(a), where the losing large firm bidder is weak. In this case, the equilibrium is the same as that with a single small firm bidder. We can easily derive the equilibrium share as $\xi_s^* = 1/2$, and the equilibrium merger threshold $\bar{y}^*$ is given by equation (74).

We now verify condition (58) in equilibrium. We must derive the breakeven share ξ_l^{BE} at the equilibrium merger threshold $\bar{y}^*$. Using (54), we have

$$\xi_l^{BE} = \frac{X_l + V_l\left(\bar{y}^*; n\right)}{V_M^a\left(\bar{y}^*; n-1\right)} = \frac{\left(\beta - 2\right) X_l\left[\Pi_l\left(n+1\right) - 2\Pi_s\left(n\right)\right] + 2\beta X_s \Pi_l\left(n\right)}{2\beta X_s \Pi_M^a\left(n-1\right)}, \tag{A.25}$$

where $\bar{y}^*$ is given by (74). We now substitute $\xi_s^* = 1/2$ and (A.25) for ξ_l^{BE} into the condition (58). After simplifying, we obtain condition (73). Evaluating (45) for $f = s$ at the merger threshold $Y(t) = \bar{y}^*$ in (74) when the small firm bidder receives the share $\xi_s^* = 1/2$, and using the results from Proposition 2, we find the cumulative merger return in (75). We thus obtain part (iv).

Finally, in Case 2(b) the losing large firm bidder is strong, which happens when condition (73) is violated. We thus obtain the second inequality in (68). To derive the equilibrium in this case, we first use (60) to derive the small firm bidder's merger threshold as a function of his bid $\xi_s^{\max}$:

$$\bar{y}_B\left(\xi_s^{\max}\right) = \sqrt{\frac{X_l\left(r - 2\left(\mu + \sigma^2/2\right)\right)}{\Pi_M^a\left(n-1\right) - \Pi_l\left(n\right) - \left(1 - \xi_s^{\max}\right)\Pi_l\left(n+1\right)}}.$$

We next derive the merger threshold selected by the target shareholders as a function of the winning small firm's bid $\xi_s^{\max}$. By a similar argument as in Section 3.1, we can show that

$$\bar{y}_T\left(\xi_s^{\max}\right) = \sqrt{\frac{\beta X_s}{\beta - 2}\frac{r - 2\left(\mu + \sigma^2/2\right)}{\left(1 - \xi_s^{\max}\right)\Pi_l\left(n+1\right) - \Pi_s\left(n\right)}}.$$

In equilibrium, the merger thresholds must be the same in that $\bar{y}_B\left(\xi_s^{\max}\right) = \bar{y}_T\left(\xi_s^{\max}\right) \equiv y_{cs}^*$. Solving this equation yields $\bar{\xi}_s^{\max}$ and y_{cs}^* given in (69) and (70). Evaluating equation (45) twice for $f = s$ at the merger threshold $Y(t) = y_{cs}^*$ in (70) when the small firm bidder receives the share $\bar{\xi}_s^{\max}$ in equation (69), and using the results from Propositions 2 and 4, we find the cumulative merger returns in equations (71) and (72). We thus obtain part (iii) and complete the proof. ∎

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Table 1. Decomposition of total merger surplus. The expressions $V_M^a(y^*; n-1)$, $V_l(y^*; n)$, and $V_s(y^*; n)$ represent values of the merged firm, the pre-merger large firm, and the pre-merger small firm evaluated at the merger threshold. The expression $\tilde{V}_M(y^*; n)$ represents the value of the merged firm evaluated at the merger threshold if it uses the the large firm's capital stock to produce output. We decompose the merger surplus (net of merger costs) $S \equiv V_M^a(y^*; n-1) - V_s(y^*; n) - V_l(y^*; n) - X_s - X_l$ into two components. One component, defined as $S_1 = \tilde{V}_M(y^*; n) - V_s(y^*; n) - V_l(y^*; n) - X_s - X_l$, is attributed to market power only. The other component, defined as $S_2 = V_M^a(y^*; n-1) - \tilde{V}_M(y^*; n)$, is attributed to cost savings. Parameter values are $K = 1$, $k = 0.2$, $b = 0.5$, $r = 0.08$, $\mu = 0.01$, $\sigma = 0.20$, and $X_l = X_s = 2$.

	$V_M^a(y^*; n-1)$	$\tilde{V}_M(y^*; n)$	$V_s(y^*; n)$	$V_l(y^*; n)$	S_1	S_2	S
$n = 2$	183,482	130,376	61,395	122,053	–53,076	53,106	30
$n = 3$	155,969	110,845	52,187	103,747	–45,094	45,124	30
$n = 4$	135,527	96,333	45,345	90,147	–39,163	39,193	30

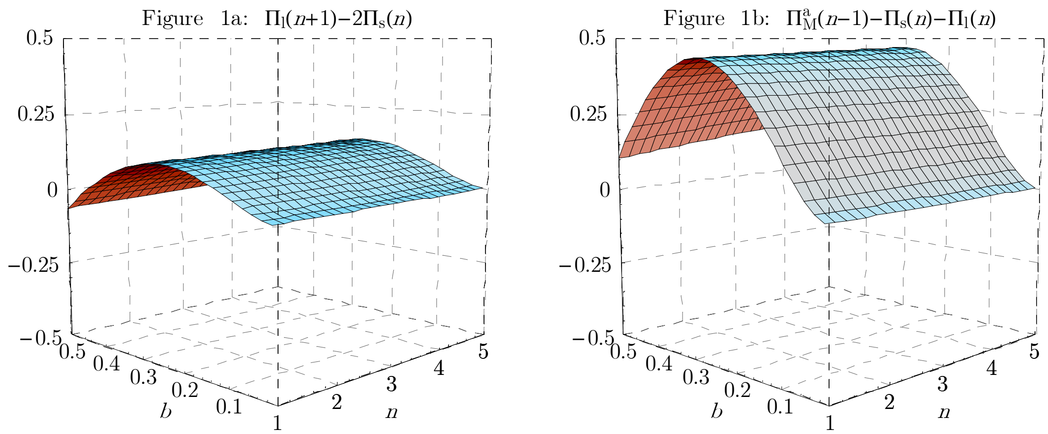

Figure 1. Profit differentials in symmetric and asymmetric mergers.

This figure depicts the profit differentials $\Pi_l\,(n+1) - 2\Pi_s\,(n)$ and $\Pi^a_M\,(n-1) - \Pi_s\,(n) - \Pi_l\,(n)$ as functions of the price sensitivity parameter b and the number of large firms n when $a = 100$, $K = 1$, and $k = 0.2$.

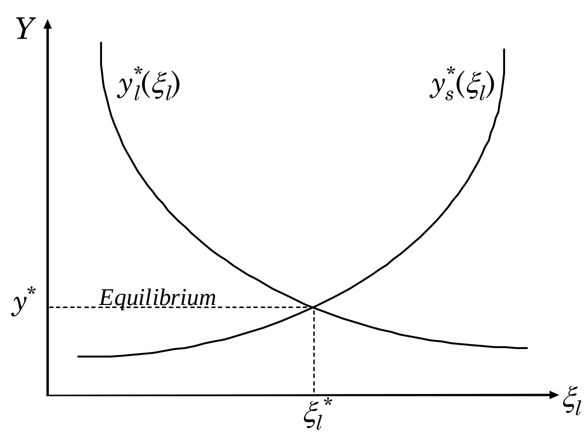

Figure 2. Reaction functions with a single bidder.

The figure plots the reaction functions of the bidder and the target as a function of the bidder's ownership share. The decreasing (increasing) line represents the acquirer's (target's) strategy. The crossing point of the reaction functions represents the option exercise equilibrium of Proposition 6.

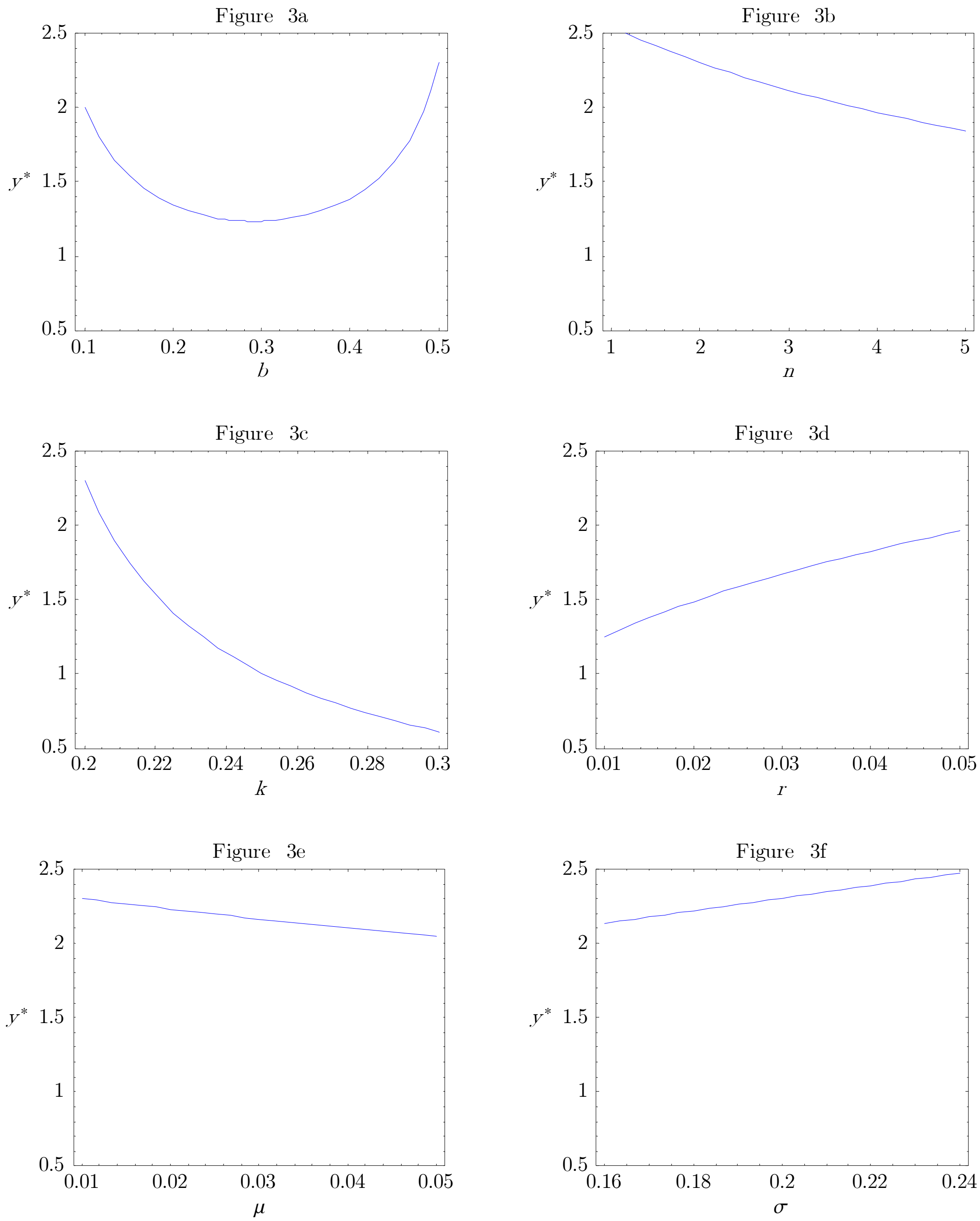

Figure 3. Merger timing.

This figure plots the merger threshold y^* as a function of the price sensitivity of demand, b, the number of large firms, n, the size of the large firm's tangible asset, k, the risk-free rate, r, the growth rate of industry shocks, μ, and the volatility of industry shocks, σ. Parameter values are $a = 100$, $b = 0.5$, $K = 1$, $k = 0.2$, $n = 2$, $r = 8\%$, $X_l = 2$, $X_s = 2$, $\mu = 1\%$, and $\sigma = 20\%$.

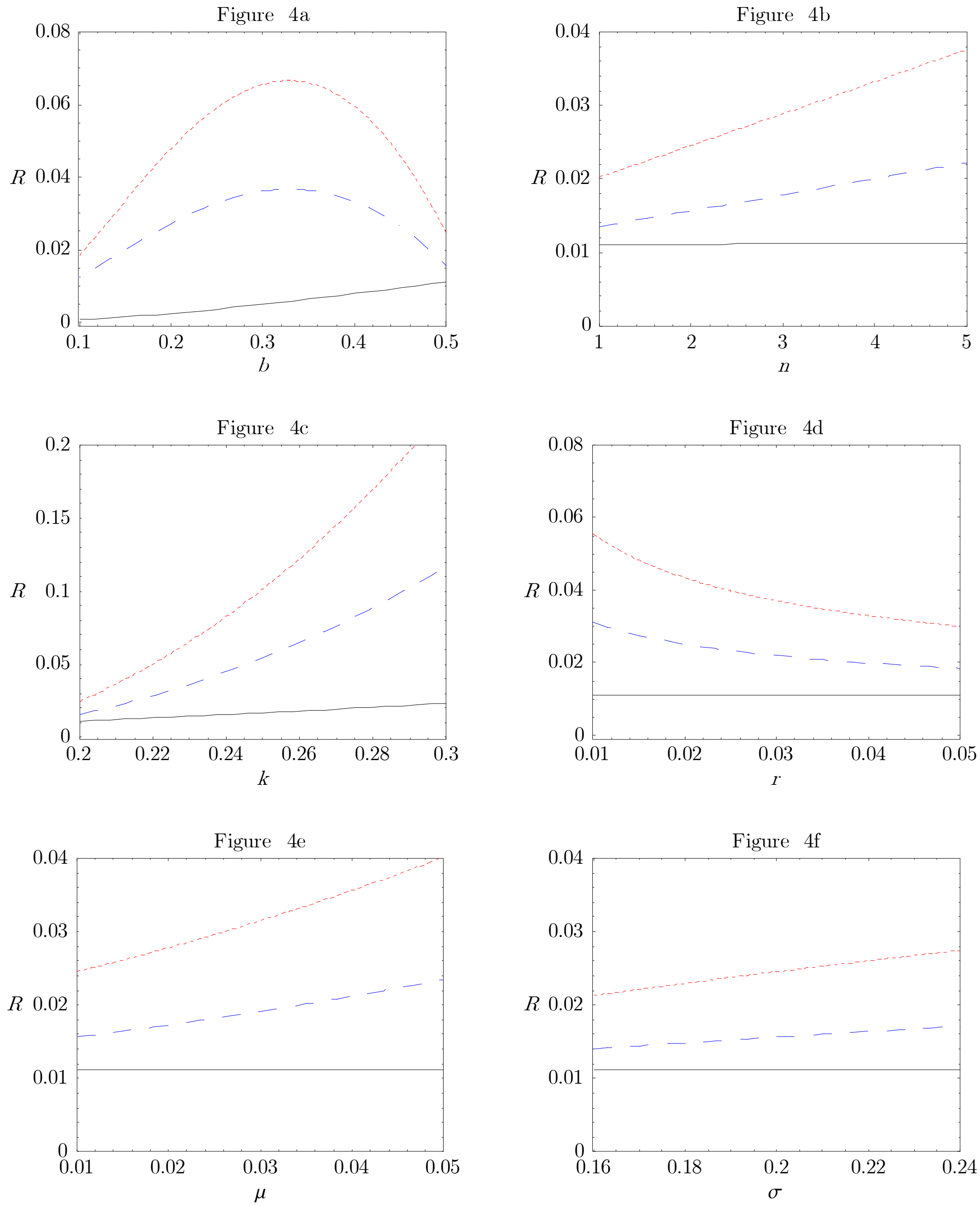

Figure 4. Cumulative merger returns.

This figure depicts the cumulative returns of a small merging firm (dotted line), a large merging firm (dashed line), and a rival firm (solid line) as a function of the price sensitivity of demand, b, the number of large firms, n, the size of the large firm's tangible asset, k, the risk-free rate, r, the growth rate of industry shocks, μ, and the volatility of industry shocks, σ. Parameter values are $a = 100$, $b = 0.5$, $K = 1$, $k = 0.2$, $n = 2$, $r = 8\%$, $X_l = 2$, $X_s = 2$, $\mu = 1\%$, and $\sigma = 20\%$.

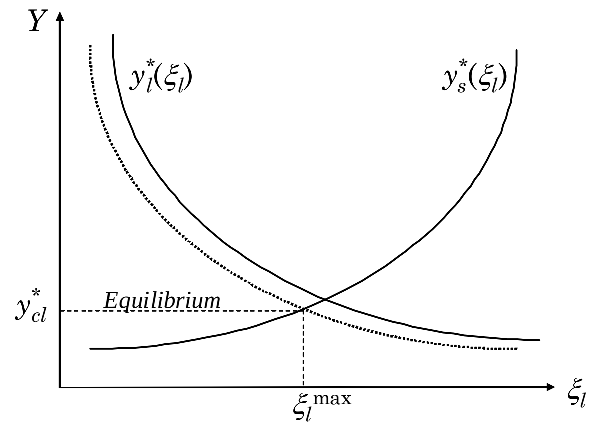

Figure 5. Reaction functions with multiple bidders.

The figure plots the reaction functions of the acquirer and the target, as a function of the bidder's ownership share. The increasing line represents the target's strategy. The solid (dashed) decreasing line represents the large firm bidder's strategy when the losing bidder is weak (strong). The crossing points of the reaction functions represent the option exercise equilibria of Proposition 8.

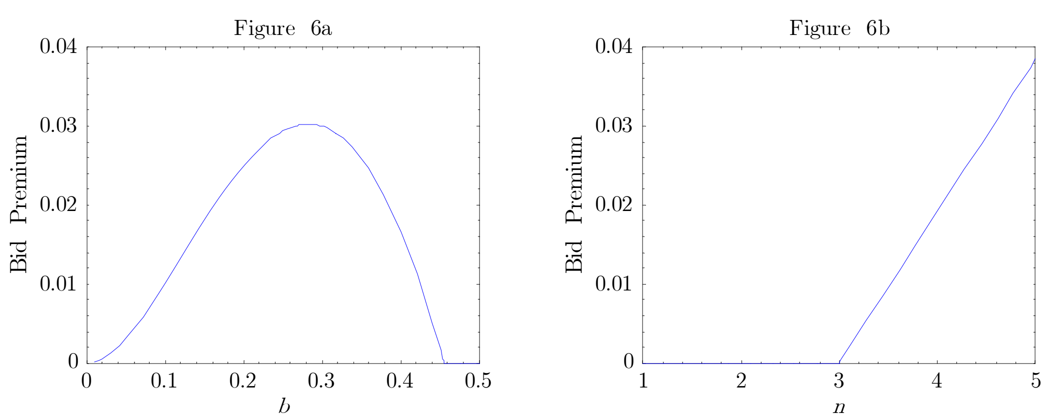

Figure 6. Bidder competition and industry competition.

The figure presents the bid premium $\left(\xi_l^* - \bar{\xi}_l^{\max}\right) / (1 - \xi_l^*)$ as a function of the number of large firms n and the price sensitivity of demand b. Parameter values are $a = 100$, $b = 0.4$, $K = 1$, $k = 0.2$, $n = 4$, $r = 8\%$, $X_l = 20$, $X_s = 1$, $\mu = 1\%$, and $\sigma = 20\%$.