GOVERNANCE AND THE EFFICIENCY
OF ECONOMIC SYSTEMS
GESY

Discussion Paper No. 8

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June 2004

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Financial support from the Deutsche Forschungsgemeinschaft through SFB/TR 15 is gratefully acknowledged.

Banks without Parachutes — Competitive Effects of Government Bail-out Policies†

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Abstract: The explicit or implicit protection of banks through government bail-out policies is a universal phenomenon. We analyze the competitive effects of such policies in two models with different degrees of transparency in the banking sector. Our main result is that the bail-out policy unambiguously leads to higher risk-taking at those banks that do not enjoy a bail-out guarantee. The reason is that the prospect of a bail-out induces the protected bank to expand, thereby intensifying competition in the deposit market and depressing other banks' margins. In contrast, the effects on the protected bank's risk taking and on welfare depend on the transparency of the banking sector.

Keywords: Government bail-out, banking competition, transparency, opacity, "too big to fail", financial stability.

JEL-Classification: G21, G28, L11.

First Version: November 12, 2003.

This Version: June 16, 2004.

†We thank Jürgen Eichberger, Martin Hellwig and Dmitri Vinogradov and conference participants at the "Workshop on Efficiency, Competition, and Regulation in Banking" in Sulzbach as well as seminar participants at the University of Frankfurt and at the Max Planck Institute for Research on Collective Goods for helpful comments and suggestions. Financial support from Deutsche Forschungsgemeinschaft is gratefully acknowledged.

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1 Introduction

In most countries, part of the banking sector is protected through implicit or explicit government guarantees. These come in two forms: Public banks directly enjoy the backing by the government, while very large banks are often subject to implicit guarantees because they are "too big to fail". Both the political and academic discussion focus on the detrimental effects of such guarantees on the risk-taking behavior of the protected banks. In contrast, the reactions of the *remaining* banks in the banking system have not been dealt with in the literature. We close this gap by analyzing the competitive effects of government bail-out policies on those banks that do *not* enjoy a public guarantee. An understanding of other banks' reaction is crucial for the judgment of overall welfare effects of public bail-out policies.

The relevance of such competitive effects can be illustrated with an example from Japan. Since the 1990s, Japanese private banks' profitability has been compromised by thin interest margins. These have been attributed to the competition from government financial institutions as well as from (mostly large) banks receiving disguised subsidies.[1] In particular, private banks face strong competition from Japan's postal savings system, the biggest deposit taker in the world, which benefits from an explicit government guarantee and tax exemptions and is subject to limited prudential supervision. The extent of welfare losses arising from this type of "unfair competition" (Fukao, p. 25, 2003b) depends essentially on how smaller private banks adjust their risk-taking in reaction to shrinking profitability due to the subsidization of public and larger banks.

The relationship between banks' profit margins and their risk taking is one of the central themes in the literature on competition and stability in the banking sector. The basic idea is that competition tends to reduce the rents of banks, who react by increasing their asset risk because of the well-known risk-shifting problem described by Jensen and Meckling (1976). If one applies this idea to a setting with public bail-out guarantees, one may conjecture that there will be a similar risk-shifting problem at those banks that are not expected to be bailed out. In fact, this effect will be the driving force in our model.

Our starting point is a model by Allen and Gale (2000, chapter 8.3) who analyze the tradeoff between competition and stability in a static agency model.[2] We extend that model by introducing an asymmetric government bail-out policy where some bank is bailed out with a higher probability than the others. Since there is no full insurance of deposits as in Allen and Gale, depositors care about the risk of banks' assets and demand default premia to be compensated for expected losses from bank insolvencies. Like Allen and Gale, we model competition on the liabilities side of

[1] See Fukao (2003a,b) and Kashyap (2002) for an extensive overview of these problems. See also the diagnosis in the Annual Report of the Bank for International Settlements (2002, pp. 133).

[2] See also Allen and Gale (2004).

banks' balance sheets in a Cournot fashion. We consider two time structures with different patterns of information revelation: In the first model, banks are *opaque* in the sense that depositors cannot observe risk before setting deposit rates. Hence, default premia are set before deposit volumes and risk choices are determined.[3] In the second model, we reverse the timing. Depositors can observe their bank's risk choice and the level of deposits before setting default premia. We call banks *transparent* in this case.

Our main result is that the government bail-out policy unambiguously leads to higher risk-taking at banks that do not enjoy a government guarantee. The reason is that the subsidization induces the protected bank to expand its deposit volume, no matter whether banks are opaque or transparent. Since deposit volumes are strategic substitutes in our model, the other banks react by decreasing their deposit volumes. However, the overall effect on aggregate deposits is positive such that there is an increase in the deposit rate, depressing smaller banks' margins and inducing them to take higher risks.

Contrary to conventional wisdom, the effect of the guarantee on the *protected* bank appears to be ambiguous. In the model with opaque banks, the protected bank may have lower incentives to take risks, because the subsidy increases the bank's rents.[4] With transparent banks, risk-taking unambiguously increases in the bail-out probability, as suggested by public discussions of government bail-out policies. Here, the argument is similar to the literature discussing excessive risk-taking in the context of unfairly priced deposit insurance.

The overall effect on the stability of the banking system is ambiguous. With opaque banks, a government bail-out policy may even increase the stability of the system ex ante, while stability is clearly reduced in the model with transparent banks. Hence welfare effects depend on the information structure in the banking sector.

The paper proceeds as follows: Section 2 contains a brief review of the related literature. In section 3, we derive the competitive effects of an asymmetric bail-out policy for the cases of opaque and transparent banks. In both models, we start by analyzing the monopoly case before modelling the situation with several banks with different bail-out probabilities. Welfare implications are analyzed for each model. Section 4 summarizes our major findings and discusses some extensions to our model.

2 Related literature

Our paper is related to two strands of literature: first, to the extensive literature on competition and stability in the banking sector, and second, to the literature on the effects of public bail-out guarantees.

[3]This is also the time structure chosen by Allen and Gale (2000, chapter 8.3).

[4]This effect is comparable to that of Keeley (1990) and Allen and Gale (2000, chapter 8).

The work by Keeley (1990) was the first of a large number of papers to establish the trade-off between competition and stability in the banking sector. In a simple model, Keeley shows that the reduction of rents through competition exacerbates the risk-shifting problem at banks caused by limited liability and/or unfairly priced deposit insurance.[5] Hence, the creation of "charter value" (i. e. the present discounted value of future rents) through restrictions on competition can induce banks to refrain from overly risky behavior if the expected loss of the charter value is larger than the gains from increased risk-taking.

The work by Keeley has been extended in a number of ways, with differing conclusions about the existence of the presumed tradeoff. Allen and Gale (2000, chapter 8.3) generalize Keeley's results in a static agency model, confirming the negative relationship between competition and stability.[6] While the tradeoff appears to be robust to the introduction of product differentiation,[7] it typically breaks down in the presence of competition in loan markets (and not just deposit markets).[8] Dynamic models yield contradictory results.[9]

Similar to the theoretical literature, the empirical literature yields ambiguous results as to the trade-off between competition and stability. Keeley (1990) presents some evidence for the view that the surge in bank failures in the 1980s in the United States may be explained by the disappearance of monopoly rents in banking due to financial deregulation. Similarly, the accumulation of systemic banking crises in developed and developing countries in the past two decades has been attributed to financial liberalization, which has also been shown to be accompanied by declining charter values in banking (see Demirguc-Kunt and Detragiache, 1999).[10] In contrast, a recent cross-country study by Beck, Demirguc-Kunt, and Levine (2003) shows that systemic banking crises are less likely in countries with more concentrated banking sectors, but more likely in countries with tighter restrictions on entry and banking activities. These findings are inconsistent with the "charter value hypothesis", according to which crises should be less likely in the latter case as well.

The second strand of literature related to our paper concerns the effects of public bail-out guarantees. With respect to public banks, the literature is scarce. The most important empirical findings are that government ownership of banks is pervasive

[5]This literature review is restricted to the papers most closely related to ours. For more detailed surveys on the relationship between competition and stability in banking, see Canoy, van Dijk, Lemmen, de Mooij, and Weigand (2001) and Carletti and Hartmann (2003). Allen and Gale (2004) provide a useful overview of what type of models tend to yield what type of results as to the sign of the relationship between competition and stability.

[6]Due to its simplicity and clarity, we chose this framework as the basis of our analysis.

[7]See Matutes and Vives (2000) and Cordella and Yeyati (2002).

[8]See Koskela and Stenbacka (2000), Caminal and Matutes (2002), and Boyd and De Nicolò (2003).

[9]See Hellman, Murdoch, and Stiglitz (2000), Perotti and Suarez (2002), and Allen and Gale (2000, chapter 8.4).

[10]In a similar vein, Demirguc-Kunt, Laeven, and Levine (2004) find that banks' interest margins are higher in countries with tighter restrictions on competition in banking.

all over the world and that it tends to be associated with more poorly operating financial systems and slower growth performance.[11]

In contrast, there exists a fairly large number of papers on the so-called "*too-big-to-fail*" (hereafter TBTF) problem. Large banks may be subject to an incentive problem because the public authorities cannot credibly commit to not supporting these banks in case of impending failure. The theoretical effects of a TBTF policy are similar to those of an unfairly priced deposit insurance, namely the complete insurance of all uninsured deposits and other liabilities at zero costs. Since Merton (1977), it is well-known that unfair deposit insurance entails a risk-shifting problem, similar to the problem arising from limited liability.[12] Hence one may expect that a more concentrated banking sector with TBTF banks entails higher risk-taking at the largest banks, and thus higher fragility. Since a higher concentration implies less competition, this result is just the opposite of what would be predicted by the "charter value literature" described above. Our paper aims at resolving this apparent contradiction.

The TBTF problem also seems to be an empirically relevant phenomenon. Boyd and Gertler (1994) document a TBTF problem at the largest commercial banks in the United States in the 1980s. Schnabel (2003, 2004) describes a similar phenomenon at the so-called "great banks" in Germany at the time of the Great Depression. The episode studied most intensively is the near-failure of Continental Illinois in 1984 and the consequent public announcement by regulators that the 11 largest US banks were too big to be allowed fail. In an event study, O'Hara and Shaw (1990) find significant positive abnormal returns for the TBTF bank after the announcement, which is consistent with the existence of a positive subsidy to TBTF banks. Studies using bond market data tend to confirm the existence of conjectural government guarantees.[13]

Another strand of the empirical literature looks at the question whether the prospect of becoming TBTF is a motivation for bank mergers.[14] While Benston, Hunter, and Wall (1995) reject this hypothesis for the years 1981 to 1986, the evidence for the 1990s in Kane (2000) and Penas and Unal (2001) is consistent with the hypothesis.

One striking gap in the literature concerns the impact of a TBTF policy on smaller banks. O'Hara and Shaw (1990) find negative effects on banks not included in the list of banks deemed to be "too big to fail" and attribute this finding to the self-financing character of the deposit insurance system. Apart from this finding, we were not able to find any theoretical or empirical paper dealing with the banks that are "too small to be saved". Our paper contributes to closing this gap.

[11]See Barth, Caprio, and Levine (1999) and Porta, de Silanes, and Shleifer (2002). For an analysis of public banks in Germany, see Sinn (1999).

[12]Empirical evidence for the adverse effects of deposit insurance on banking stability has been presented by Demirguc-Kunt and Detragiache (2002).

[13]See Flannery and Sorescu (1996) and Morgan and Stiroh (2002).

[14]This argument is similar to the dynamic arguments in the literature on competition and stability. It was first stated by Hunter and Wall (1989) and Boyd and Graham (1991).

3 The model

The basic setup of our model is the same as in Allen and Gale (2000, chapter 8.3). We consider an economy with n chartered banks, indexed $i = 1, \dots, n$. Banks collect deposits and invest their funds in risky projects. They can choose the "risk level" of their investment by fixing some target return y_i per unit invested. Projects yield the return y_i with probability $p(y_i)$, otherwise they return zero. The success probability decreases in the target return, i. e., $p'(y_i) < 0$.[15] The volume of deposits collected by bank i is denoted by d_i. The aggregate amount of deposits in the economy is named $D = \sum_{i=1}^{n} d_i$. Depositors demand an expected return $R(D)$, which increases in the aggregate volume of deposits, i. e., $R'(D) > 0$. Banks and depositors are assumed to be risk neutral.

Up to now, the model is identical to Allen and Gale (2000, chapter 8.3). However, instead of assuming that deposits are *fully* insured, bank i is bailed out by the government with probability $\beta_i \in (0; 1)$ in the case of failure.[16] The government can commit itself to the exogenous probability β_i.[17] Given the bail-out policy β_i, depositors are repaid with probability $p(y_i) + \beta_i\,(1 - p(y_i))$. With probability $(1 - \beta_i)\,(1 - p(y_i))$, they receive nothing. In order to obtain an expected return of $R(D)$, they demand a nominal return of $\rho_i\, R(D)$, where the "default premium" ρ_i depends on both β_i and y_i.

The expected profit of bank i is then a function of four endogenous variables, namely its risk level y_i, the default premium ρ_i, its deposit volume d_i, and the competitors' deposit volume, $D_{-i} = \sum_{k \neq i} d_k = D - d_i$,

$$\Pi_i(y_i, \rho_i, d_i, D_{-i}) = p(y_i)\,[y_i\, d_i - \rho_i\, R(d_i + D_{-i})\, d_i] = p(y_i)\, d_i\,[y_i - \rho_i\, R(D)].$$

Within this setting, we define two games characterized by different degrees of transparency in the banking sector, modeled through varying time patterns of actions and information revelation. In each game, we first discuss the monopoly case with $n = 1$. This yields insights into the banks' incentives to take risks and expand volume, abstracting from competitive effects. These insights prove to be useful in the subsequent analysis of the oligopoly case.

3.1 Opaque banks

In the first model, banks are *opaque* in the sense that depositors must set the default premium ρ *before* banks choose their deposit volumes d and their target returns y.

[15] Hence a bank's probability of default $1 - p(y_i)$ is an increasing function with y_i.

[16] Because of the risk neutrality of depositors, β_i can just as well be interpreted as the *fraction* of deposits that the government refunds in the case of bank failure.

[17] A relaxation of the exogeneity assumption is discussed in section 4.

Figure 1: Time structure when banks are opaque

- For each bank i, the government fixes a bail-out probability β_i
- Investors (knowing β_i and anticipating d_i and y_i) set a default premium ρ_i
- Banks choose d_i (anticipating y_i), $R(D)$ is determined in the deposit market
- Banks choose y_i and invest
- Projects mature and return y_i with probability $p(y_i)$. Banks pay $\rho_i\, R(D)$ to their depositors if possible. Otherwise, the government pays $\rho_i\, R(D)$ with probability β_i

Depositors cannot exert any *market discipline* because they cannot react to the risk-taking of banks. This time structure (see figure 1) generates a moral hazard problem between depositors and banks, known as risk-shifting or asset substitution.[18]

For tractability reasons, we make the following technical assumption throughout subsection 3.1.[19]

Assumption 1 *$R(d)$ and $p(y)$ do not bend too much in a neighborhood of the equilibrium, i. e., $R''(d) \approx 0$ and $p''(y) \approx 0$.*

3.1.1 The monopoly case

To abstract from competitive effects, we first look at the case with only one bank ($n = 1$), such that $D = d_1$. For readability, we omit all indices. As usual, we analyze

[18]To give the market for deposits a micro foundation, assume that we have an auctioneer who first asks each infinitesimal depositor j to report her individual supply function $r^j(D)$ for deposits, and the default premium ρ_i^j she would demand from each bank i. We assume that depositors are homogenous and name identical functions $r^j(D)$ and values ρ_i^j. The auctioneer aggregates the supply, $R(D) = \int r^j(D)\, dj$, and communicates supply and default premia to the banks. Now each bank i chooses a volume d_i and communicates it to the auctioneer. The auctioneer now determines the aggregate deposit volume $D = \sum_i d_i$ and the risk-free market rate $R(D)$, and he fixes the nominal deposit rate $\rho_i\, R(D)$ for each bank.

[19]Here and elsewhere, $\approx$ denotes an approximation for vanishing second derivatives, i. e., $R''(d) \to 0$ and $p''(y) \to 0$.

the problem backwards. First, we determine y for given β, d and ρ,[20]

$$\Pi = p(y)\, d\, [y - \rho\, R(d)], \tag{1}$$

$$\frac{\partial \Pi}{\partial y} = d\, \big[p(y) + [y - \rho\, R(d)]\, p'(y)\big] = 0,$$

$$0 = p(y) + [y - \rho\, R(d)]\, p'(y). \tag{2}$$

This maximization yields a function $y(d, \rho)$. The relationship between the optimal y and d can be derived from the implicit equation (2) for y, making use of the implicit function theorem,

$$0 = y_d\, p'(y) + [y_d - \rho\, R'(d)]\, p'(y) + [y - \rho\, R(d)]\, y_d\, p''(y),$$

$$y_d = \frac{\rho\, \overset{-}{p'(y)}\, \overset{+}{R'(d)}}{\underbrace{2\, p'(y) + [y - \rho\, R(d)]\, p''(y)}_{-}} > 0.$$

The intuition for the positive relationship between risk and deposit volume is straightforward. Expected profits Π of the bank consist of three factors, the probability of success $p(y)$, the deposit volume d, and some "margin" given by the difference between y and the nominal repayment $\rho\, R(d)$. If d rises, $\rho\, R(d)$ goes up, and the bank compensates the shrinking margin by increasing y. If $p''(y)$ is small, then $y_d \approx \frac{1}{2}\, \rho\, R'(d)$. Hence, the increase in y dampens the original decrease in the margin by 50 percent.

An analogous procedure can be used to derive y_ρ,

$$0 = y_\rho\, p'(y) + [y_\rho - R(d)]\, p'(y) + [y - \rho\, R(d)]\, y_\rho\, p''(y),$$

$$y_\rho = \frac{\overset{-}{p'(y)}\, \overset{+}{R(d)}}{\underbrace{2\, p'(y) + [y - \rho\, R(d)]\, p''(y)}_{-}} > 0.$$

An increase in ρ reduces the margin, which is compensated by raising the risk level y. If $p''(y)$ is small, $y_\rho \approx \frac{1}{2} R(d)$. Again, the increase in y dampens the original increase in the margin by 50 percent.[21]

We now turn to the determination of the deposit volume. Banks choose d given the default premium ρ and anticipating y. Incorporating the dependence of y on d and ρ into the profit function (1), the bank's expected profits are given by

$$\Pi = p(y(d, \rho))\, d\, [y(d, \rho) - \rho\, R(d)]. \tag{3}$$

[20] We assume that there exists an optimal level of risk, which maximizes the expected return $y_i\, p(y_i)$. Therefore, the second derivative of the expected return must be weakly negative, i. e. $\partial^2\, (y_i\, p(y_i))/\partial {y_i}^2 = 2\, p'(y_i) + y_i\, p''(y_i) \leq 0$. Under this assumption, the second-order condition of the profit maximization will always be satisfied.

[21] Note that both results are driven by the same mechanism, namely a decrease in the margin that translates into an increase in risk. This mechanism is central to the "charter value literature".

The first-order condition to this maximization problem yields an implicit relation between d and ρ, which we denote by $d(\rho)$. Making use of the implicit function theorem, one can derive the following lemma.[22]

Lemma 1 (Optimal d for given ρ) *The optimal deposit volume decreases in the default premium, i. e., $d'(\rho) < 0$.*

Again an increasing default premium reduces the margin $y - \rho\, R(d)$. The bank can react by either increasing y or decreasing d. It is not immediately clear why reaching the new optimum could not possibly be based on a strong increase in y, accompanied by a weak increase in d. Lemma 1 implies that d decreases in any case, at least if $R(d)$ and $p(y)$ do not bend too much.

Finally, we turn to the determination of the default premium ρ. Anticipating y and d, depositors set a fair default premium, such that they expect a return of $R(d)$,

$$R(d) = p(y)\, \rho\, R(d) + \beta\, (1 - p(y))\, \rho\, R(d),$$

$$\rho = \frac{1}{p(y) + \beta\, (1 - p(y))}. \tag{4}$$

The default premium ρ does not directly depend on d. However, there is an indirect dependence through y. The resulting function $\rho(d)$ is characterized by lemma 2.

Lemma 2 (Fair ρ for given d) *The fair default premium increases in the deposit volume, i. e., $\rho'(d) > 0$.*

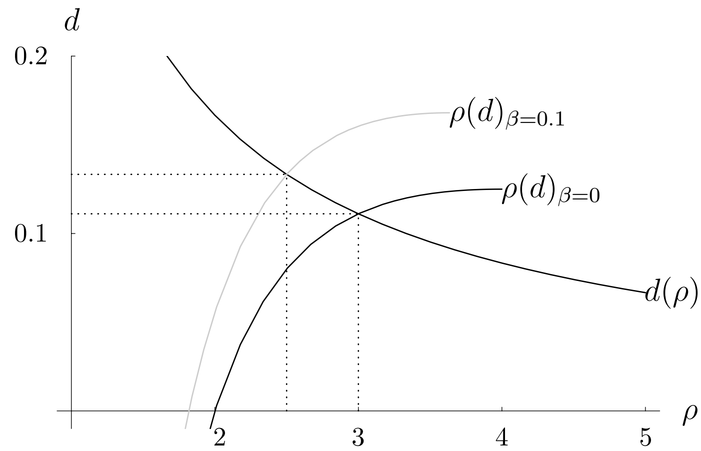
The intuition for this result is as follows: An increase in d reduces the margin $y - \rho\, R(d)$. The bank compensates this reduction by increasing risk, which must be taken into account by the depositors by demanding a higher default premium. The increase in ρ induces further risk shifting, leading to a multiplier effect, reinforcing the initial effect. Hence, the overall effect on the default premium is positive.

In equilibrium, the default premium must be fair, given the anticipated deposit volume, and the deposit volume has to be optimal, given the default premium. Therefore, we can determine the equilibrium by looking at the intersection of the two curves $d(\rho)$ and $\rho(d)$. Figure 2 displays such equilibria for two different bail-out policies β, taking into account the results from lemmata 1 and 2. The following proposition characterizes the effects of different bail-out policies on the equilibrium.

Proposition 1 (Effects of bail-out policy in monopoly with opaque banks) *In an opaque monopolistic banking system, an increase in the bail-out probability induces depositors to demand a lower default premium, $\partial\rho/\partial\beta < 0$. The bank reacts by choosing a higher deposit volume, $\partial d/\partial\beta > 0$. It chooses a higher risk level ($\partial y/\partial\beta > 0$) if and only if the supply of deposits is inelastic.*

[22] All proofs can be found in the appendix.

Figure 2: Default premium and deposit volume for varying β in the monopoly



This example is based on the functions $p(y) = 1 - y$ and $R(d) = d$. For comparability, the same functions are used throughout the paper.

Figure 2 illustrates this proposition. An increase in β implies that depositors are compensated with a higher probability (or to a higher degree) in the case of bank failure, which reduces the fair default premium for a given d; hence, the function $\rho(d)$ is shifted to the left. In contrast, $d(\rho)$ does not depend on β for a given ρ. From the graph, it is clear that the bank always expands in reaction to an increasing β, and that the default premium falls. The effect on risk-taking is not obvious, because the effects of d and ρ on y go into opposite directions. Proposition 1 states that the overall effect on risk-taking depends on the elasticity of the supply of deposits. If the supply of deposits is highly elastic, the inverse function $R(d)$ is highly inelastic. Therefore, the expansion of the bank has little effects on the deposit rate $R(d)$. As a result, the indirect incentive for risk shifting is negligible. If, on the other hand, the supply of deposits is highly inelastic, then $R(d)$ is elastic, and an expansion of d leads to a sharp increase in the deposit rate $R(d)$. In this case, the indirect incentive to shift risk outweighs the direct incentives for a safer investment strategy. This ambiguous result is interesting because it contradicts the conventional wisdom according to which a higher bail-out probability *always* leads to an increase in risk-taking.

Summing up, a more generous bail-out policy leads to an expansion of the deposit volume and a decrease in the default premium. It may lead to an increase or decrease in risk-taking, depending on the elasticity of the supply of deposits.

3.1.2 The oligopoly case

Assume now that n banks have been chartered instead of just one. We are interested in how the market as a whole reacts when the government changes the bail-out policy for one bank. Without loss of generality, assume that the government raises the bail-out probability β_1 of bank 1. This bank could be thought of as either a large or a public bank.

Assume for the moment that the deposit volume D_{-1} of competitor banks is given. Then proposition 1 implies that, just as in the monopoly case, the increase in β_1 leads to a fall in the default premium ρ_1. This induces bank 1 to increase its volume d_1.

The question then is how bank 1's behavior affects the remaining banking sector. In our model, an interaction between banks takes place only in the deposit market, namely through the deposit rate $R(D)$. In equilibrium, the deposit volume of each bank must be an optimal reaction to the volumes of all competitors. Lemma 3 summarizes the strategic interactions in the deposit market.

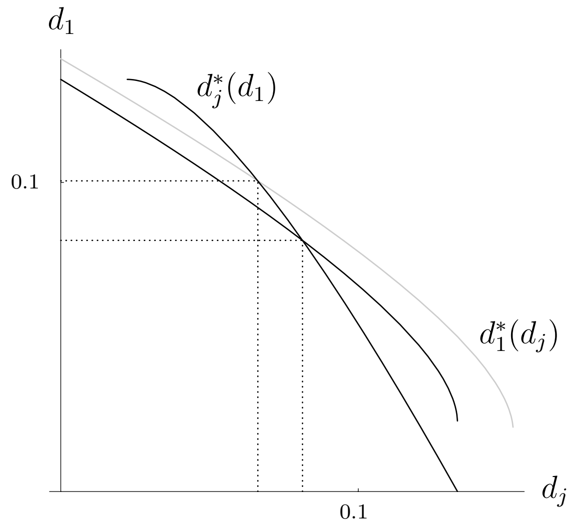
Lemma 3 (Strategic interactions in the deposit market) *The reaction function of any bank i, $d_i(d_j)$, is a strictly decreasing function. Starting from an equilibrium with $d_1 > 0$ and $d_j > 0$, an outward shift of bank 1's reaction function leads to an increase in bank 1's and a decrease in bank j's deposit volume. Since the former effect dominates the latter, aggregate deposits D increase.*

The first part of lemma 3 implies that deposit volumes are strategic substitutes in our model. Figure 3 plots the reaction functions for a numerical example. From proposition 1, we know that the reaction function of bank 1 shifts outward as β_1 increases, while the reaction functions of the competitors remain unchanged. The second part of lemma 3 implies that an increase in β_1 leads to an expansion of deposits at the subsidized bank and a contraction of deposits at the remaining banks. Finally, the overall effect is an expansion of the deposit volume.

This last point is crucial: It implies that if one bank expands its deposit volume, the aggregate volume D increases, and so does the market rate $R(D) = R(d_1 + D_{-1})$. Shrinking margins $y_j - \rho_j R(D)$ imply higher risk-taking by the competitor banks. In turn, the decrease in the competitors' deposit volumes induces bank 1 to raise d_1, causing competitor banks to lower their volumes even further. This multiplier effect continues until a new equilibrium is reached. In the process, the volumes of some competitors possibly go down to zero, i. e., the banks close.

The following proposition sums up actions and reactions of bank 1 and its competitors.

Figure 3: Reaction functions in the deposit market for varying β_1



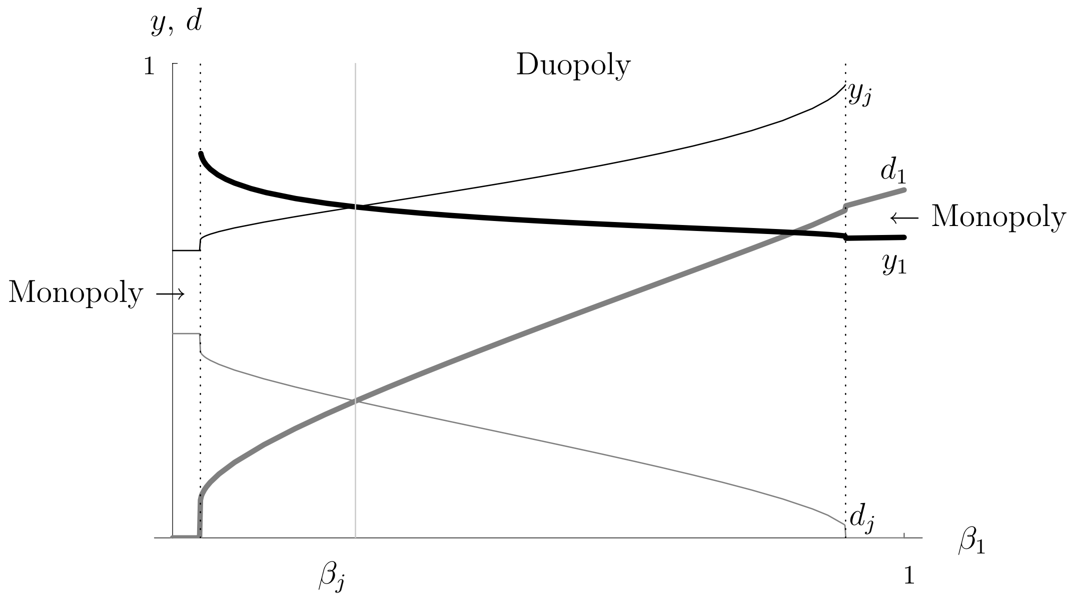
Black lines stand for $\beta_1 = \beta_j = 0.10$, the gray line for $\beta_1 = 0.13$. Equilibria are indicated by the dotted lines. Note that the increase in d_1 is larger than the decrease in d_j such that aggregate deposits increase.

Proposition 2 (Competitive effects of bail-out policy with opaque banks) *In an opaque banking system, an increase in the bail-out probability β_1 leads to*

1. *an expansion of deposits at bank 1 and a contraction of deposits at its competitor banks $j \neq 1$, $\partial d_1/\partial\beta_1 > 0$ and $\partial d_j/\partial\beta_1 < 0$;*
2. *a reduction in risk at bank 1 if the supply of deposits is not too inelastic (if $\varepsilon \not\ll 1$). In any case, the default premium falls, $\partial y_1/\partial\beta_1 < 0$ and $\partial\rho_1/\partial\beta_1 < 0$;*
3. *an increase in risk at the competitor banks j, accompanied by higher default premia, $\partial y_j/\partial\beta_1 > 0$ and $\partial\rho_j/\partial\beta_1 > 0$.*

Proposition 2 is illustrated in figure 4 for the case of two banks. As the bail-out probability β_1 rises, d_1 rises and bank 1 grows because of this subsidy. The subsidy leaves bank 1 with low nominal deposit rates $\rho_1 R(D)$, therefore bank 1 reduces its riskiness y_1. Due to fiercer competition, bank j is crowded out with d_j falling. For banks j, the nominal rate $\rho_j R(D)$ rises, leading to an increased risk-shifting problem, implying an increase in y_j. If bail-out policies become too asymmetric ($\beta_1 \gg \beta_j$), bank j's incentives to take risk become overwhelming, inducing depositors to demand ever higher default premia, which in turn fuel risk-taking, such that the process reaches no new equilibrium. Then y_j jumps to the maximum, $p(y_j)$ drops to zero, and as a result, d_j drops to zero, and bank j closes. Bank 1 is left with a monopoly, as described in section 3.1.1. If $\beta_1 \ll \beta_j$, the effect is reversed.

Figure 4: Effects of an asymmetric bail-out policy



Black lines denote risk choices y, gray lines deposit volumes d. Thick lines stand for bank 1, thin lines for a competitor bank j. In this example, $\beta_j = 1/4$, so for $\beta_1 = 1/4$, both banks are symmetric (light gray vertical line). With rising β_1, risk-taking of bank 1 decreases, whereas bank j takes on more risk. Bank 1 grows, whereas bank j shrinks. If banks become too asymmetric, one bank may quit the market because of bone-crushing competition (vertical dotted lines). It then leaves behind a monopoly. At the same time, the volume of the competitor bank jumps up, whereas risk-taking drops.

3.1.3 Welfare analysis

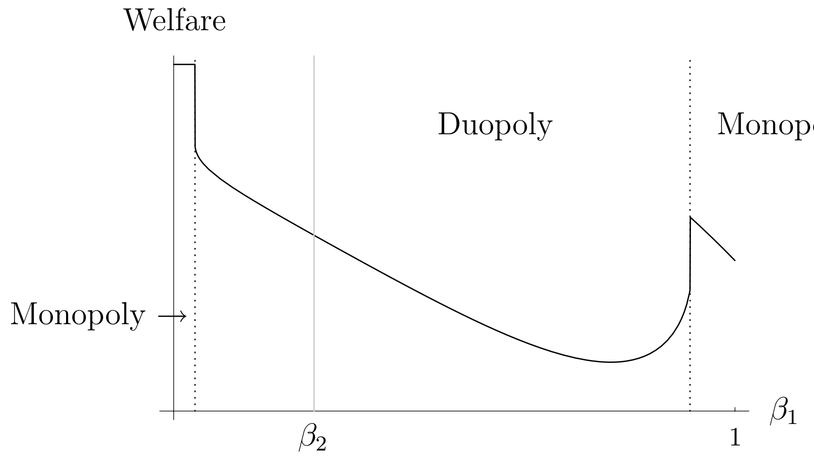
We now turn to the overall welfare effects of a bail-out policy. These are not obvious: On the one hand, the protected bank may become safer, which enhances welfare. On the other hand, competition in the deposit market intensifies, and the competitor banks react by taking on more risk. In addition, the aggregate deposit volume expands. This increases welfare, as more depositors can invest at an interest rate that beats the opportunity investment which yields $R(D)$. Finally, the bank that is getting safer grows, whereas the banks that become more risky shrink. This constitutes a second-order effect in the direction of increasing welfare.

The aggregate welfare generated by the banking system is given by

$$W = \sum_{i=1}^{n} d_i \, p(y_i) \, y_i - \int_0^D R(\Delta) \, \mathrm{d}\Delta. \tag{5}$$

Projects have expected returns of $d_i \, p(y_i) \, y_i$. We have to subtract the depositors' opportunity costs, given by the integral over returns of opportunity investments.

Figure 5: Aggregate welfare with opaque banks



In this example, $\beta_2 = 0.1$. For $\beta_1 = 0.1$, both banks are symmetric. For very asymmetric bail-out policies, one bank quits the market, leaving behind a monopoly (exactly as in figure 4).

Banks' expected repayments to depositors are $p(y_i)\, d_i\, \rho_i\, R(D)$, but these payments are welfare-neutral. Similarly, the payments from the government are welfare-neutral, so they do not appear in (5).

In the monopoly case, the protection of the bank tends to increase welfare. The reason is that, if the supply elasticity of deposits is moderate, then risk-taking is scarcely affected by the protection. Still, the deposit volume expands, tending towards the competitive level. This leads to an increase in welfare. However, for extremely inelastic supply, one can find numerical examples where welfare decreases if the bail-out probability becomes too large because the welfare loss due to excessive risk-taking exceeds the welfare gain because of deposit volume expansion.

Now let us to look at the oligopoly case. Figure 5 displays aggregate welfare as a function of the bail-out probability β_1 for one specific numerical example with two banks. We see that welfare increases when the asymmetry in β increases. With a large degree of asymmetry, the less subsidized banks becomes very weak, leaving the highly subsidized bank in a rather strong competitive position. The result is a near-monopoly for the highly subsidized bank. The large rents induce the strong bank to choose a low risk level, which increases welfare, just as in the monopoly case. While this result is not general, it illustrates nicely that welfare may be both increasing and decreasing in the bail-out probability of the subsidized bank.

If one took the results from this analysis literally, the economic policy conclusions would be clear: With unrestricted resources, the best a benevolent government could do is to bail-out all banks in case of default. With restricted resources, an asymmetric bail-out policy may be optimal, i. e., the welfare-maximizing government

Figure 6: Time structure when banks are transparent

- For each bank i, the government fixes a bail-out probability β_i
- Banks choose y_i (anticipating d_i and ρ_i)
- Banks choose d_i (knowing β_i and y_i), $R(D)$ is determined in the deposit market
- Investors observe y_i and set a default premium ρ_i, banks invest
- Projects mature and return y_i with probability $p(y_i)$. Banks pay $\rho_i\, R(D)$ to their depositors if possible. Otherwise, the government pays $\rho_i\, R(D)$ with probability β_i

should fix $\beta = 1$ for as many banks as it can afford, and $\beta = 0$ for the rest. This result is not surprising. It is just another way of saying that restrictions of competition may enhance welfare, as has also been claimed in the literature on competition and banking stability.[23]

3.2 Transparent banks

Our second model deals with *transparent* banks. Here the time structure is reversed (see figure 6), such that depositors can observe their bank's risk choice and the level of deposits before setting default premia. Therefore, depositors can (and do) exert *market discipline*. In making their risk choices, banks must take into account that they are directly punished for excessive risk-taking by increased nominal deposit rates because of higher default premia. This reduces risk shifting. If $\beta = 0$ (the bank is never bailed out), there is no risk shifting at all. If $\beta > 0$, there is again a risk-shifting problem, this time arising from the problem that the implicit government guarantee is cost free.[24]

To keep our proofs tractable, we again have to make a technical assumption, valid throughout subsection 3.2:

Assumption 2 *$\beta_i > 0$ is not too large for all banks i, i. e., $\beta_i \approx 0$.*

[23] See the literature review in section 2.

[24] As an alternative time structure, one may assume that banks can observe their mutual risk choices when fixing volumes in the deposit market. In this case, banks will tend to choose higher risk levels for strategic reasons: Since a bank's optimal deposit volume rises in its exposure to risk, competitor banks interpret increased risk-taking as a commitment to choose a high volume afterwards in the deposit market. Numerical calculations suggest that our main results remain valid in this case.

3.2.1 The monopoly case

Again we start with the monopoly case to abstract from competitive effects. First, we determine the default premium ρ, given the level of deposits d and the bank's risk choice y. The expression for ρ looks exactly as in (4),

$$\rho = \frac{1}{p(y) + \beta\,(1 - p(y))}. \tag{6}$$

ρ increases in y and does not depend directly on d. Furthermore, it decreases in β. In the extreme case with $\beta = 1$, there is no default premium, independent of the chosen risk level, i. e., $\rho = 1$. We can directly incorporate the fair default premium into the expected profit function (1),

$$\Pi = p(y)\, d \left(y - \frac{R(d)}{p(y) + \beta\,(1 - p(y))} \right). \tag{7}$$

Now there are only two endogenous variables left, y and d. First, we determine the optimal level of deposits d for a given β and y,[25]

$$\begin{aligned} \frac{\partial \Pi}{\partial d} &= p(y)[y - \rho(y)\, R(d)] - p(y)\, d\, \rho(y)\, R'(d) \overset{!}{=} 0, \\ 0 &= y - \rho(y)\, [R(\overset{+}{d}) + d\, R'(\overset{+}{d})]. \end{aligned} \tag{8}$$

This first-order condition implicitly defines a function $d(y)$. We see that an increase in d has two effects: First, it increases profits at a given margin; second, it decreases the margin due to an increase in the deposit rate $R(d)$. At the optimum, these two effects just balance.

In order to determine the effect of y on the choice of d, we make use of the implicit function theorem,

$$\begin{aligned} 0 &= 1 - \rho'(y)\, [R(d) + d\, R'(d)] + \rho(y)\, d_y\, [2\, R'(d) + d\, R''(d)], \\ d_y &= \frac{1 - \overset{+}{\rho'(y)}\, \overset{+}{[R(d) + d\, R'(d)]}}{\underset{+}{\rho(y)}\, \underset{+}{[2\, R'(d) + d\, R''(d)]}}. \end{aligned} \tag{9}$$

One can distinguish a direct effect and an indirect effect working through ρ. The direct effect is positive: An increase in y directly increases the target return and, hence, the margin, inducing the bank to expand its deposit volume. However, a rise in y also affects the default premium ρ positively, and the corresponding decrease in the margin leads to a countervailing effect on d. Lemma 4 states that the direct effect dominates the indirect effect, at least for small β.

[25] Again we assume that there always exists a solution to the profit maximization problem, i. e., that the second-order condition is always satisfied. A sufficient condition is $\left[2\, R'(d) + d\, R''(d)\right] \geq 0$.

Lemma 4 (Optimal d for given y) *In the neighborhood of the optimal y, $d_y > 0$.*

The example $\beta = 0$ conveys much of the intuition. For any β, the bank will choose the highest deposit volume when its expected return from the project is maximal. If the bail-out probability is zero, the bank's optimal risk choice coincides with the first-best level of risk. Hence, the function $d(y)$ will have a maximum at the first-best y, such that $d_y = 0$ at this point; the direct and the indirect effect on d just cancel at this point. For $\beta > 0$ and y close to the first-best, the direct effect on d is stronger than the indirect effect since the increase in ρ does not reflect the full increase in risk. Hence, the volume-maximizing y will be larger than the first-best, and the function $d(y)$ will be strictly increasing at the first-best. Figure 7 shows the function $d(y)$ for different choices of β.

In the final step, we determine the bank's optimal risk choice. Anticipating d and ρ, banks choose an optimal risk level y. By deriving the profit function with respect to y, we get the following implicit relation,[26]

$$\begin{aligned}
\Pi &= p(y)\, d\, \left[y - \rho(y)\, R(d)\right], \\
\frac{\partial \Pi}{\partial y} &= p'(y)\, d\, \left[y - \rho(y)\, R(d)\right] + p(y)\, d\, \left[1 - \rho'(y)\, R(d)\right] = 0, \\
0 &= p'(y)\, \left[y - \rho(y)\, R(d)\right] + p(y)\, \left[1 - \rho'(y)\, R(d)\right].
\end{aligned} \tag{10}$$

An increase in y has two effects: First, it decreases profits through the success probability; second, it increases profits through a rising margin. Again, these two effects just balance at the optimum.

Using the usual procedure, one can derive the relationship between y and d,

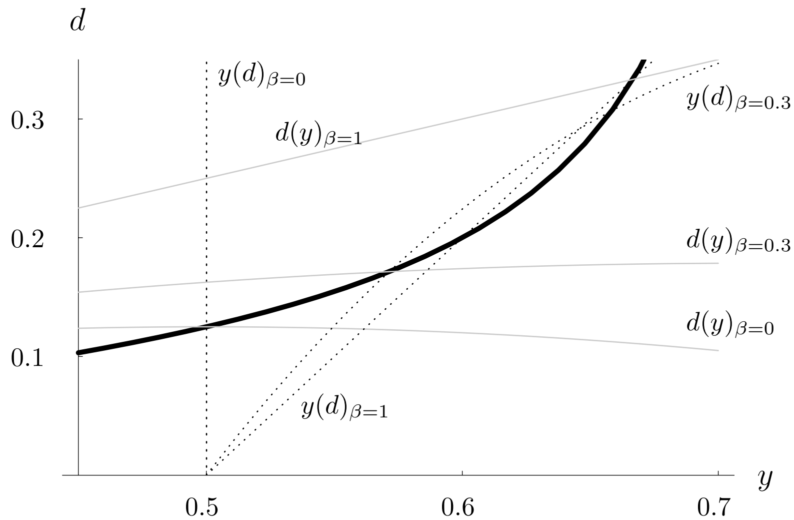
$$y_d = \frac{\overset{+}{R'}(d)\, [\overset{-}{p'}(y)\, \rho(y) + p(y)\, \overset{+}{\rho'}(y)]}{p''(y)\, [y - \rho(y)\, R(d)] + 2\, p'(y)\, [1 - \rho'(y)\, R(d)] - p(y)\, \rho''(y)\, R(d)}. \tag{11}$$

The denominator of this equation is negative if we assume that a solution to the optimization problem exists. In the numerator, we can distinguish two countervailing effects: First, a rise in d mitigates the effect of y on the success probability, inducing the bank to raise y. Second, an increasing d also mitigates the effect of y on the margin, reducing the bank's incentive to raise y. The following lemma states that the first effect always dominates the second.

Lemma 5 (Optimal y for given d) *$y_d \geq 0$, and $y_d = 0$ if and only if $\beta = 0$.*

[26] Usually, one would first plug in the function $d(y)$ and then derive the profit function with respect to y. Due to the envelope theorem, the two procedures are equivalent.

Figure 7: Risk level and deposit volume for varying β in the monopoly



Dotted lines refer to $y(d)$, gray lines to $d(y)$ for $\beta = 0$, 0.3 and 1. Equilibria (i.e., intersections of curves pertaining to the same β) are marked by the solid black curve. As predicted in lemma 4, $d(y)$ rises with y near the equilibrium; it is locally constant only for $\beta = 0$. $y(d)$ rises with d, it is constant even globally if $\beta = 0$. Furthermore, the curve $d(y)$ moves up when β rises. The curve $y(d)$ moves to the right for rising β if β is small, but bends back to the left for large d. Still, the curve describing the equilibria is monotonous, i.e., $y_\beta > 0$ and $d_\beta > 0$.

Intuitively, for $\beta = 0$ the two effects always add up to zero because any increase in y is accompanied by a "fair" increase in ρ. The bank therefore always opts for the first-best risk level, which is constant, hence $y_d = 0$ in this case. For $\beta > 0$, the increase in ρ is less than fair, and the bank has an incentive to increase y above the first-best level even if part of the increase in returns is eaten up by the resulting rise in the default premium $\rho(y)$. Figure 7 displays the function $y(d)$ for different choices of β.

The equilibria are given by the intersections of the curves pertaining to the same β. d must be optimal given y and vice versa. We are interested in the reaction of the optimal y and d to a change in β.

First, we examine how the curve $d(y)$ moves when β rises. At a given risk level y, an increase in β leads to a decrease in ρ. This induces the bank to expand its deposit volume d. Hence, the curve $d(y)$ shifts upwards. The movement of $y(d)$ is somewhat more complicated: On the one hand, an increase in β lowers ρ and thus reinforces the effect working through the success probability. This induces the bank to reduce y. On the other hand, an increase in β lowers the influence of y on the default premium ρ, i.e., $\rho'(y)$ falls. Therefore, raising risk is less costly for the bank, giving the bank an incentive to raise y. Hence, the movement of the curve is not monotonous.[27] One can show, however, that the second effect dominates the first one for small β.

The following proposition characterizes the effect of an increase in the bail-out probability on the equilibrium choices of y and d.

Proposition 3 (Effects of bail-out policy in monopoly with transparent banks) *In a transparent monopolistic banking system, an increase in the bail-out probability induces the bank to raise its deposit volume d and choose a higher risk y. The default premium ρ decreases if $R(d)$ is small enough.*

The monopolist bank reacts to the subsidization by the government by increasing risk and expanding its deposit volume because part of the potential losses can be shifted to the government. This implies that, just as in the model with opaque banks, conditions on the deposit market tighten, i. e., $R(d)$ will rise. The effect on ρ is not immediately clear because there are two countervailing effects: the rise in β and the rise in y. Proposition 3 states that the effect of the rising β dominates, at least for small β.

3.2.2 The oligopoly case

Now we turn to the oligopoly case with n banks. Banks first announce y_i simultaneously; then they simultaneously take in deposits d_i. Analytically, we first have to solve the system $(\partial\Pi_i/\partial d_i = 0)_i$, and then in a second step the system $(\partial\Pi_i/\partial y_i = 0)_i$. As in section 3.1.2, assume without loss of generality that the government raises β_1.

The chain of reactions is almost identical to the one in section 3.1.2. First, there is a direct effect on bank 1, taking the deposit volume of competitors as given: The rise in β leads to an increase in bank 1's risk and deposit volume (see proposition 3).

In a second step, we analyze the reactions of competitor banks $j \neq 1$ to the increase in d_1, neglecting for the moment further reactions of bank 1. Lemma 6 summarizes the reactions of d_j and y_j.[28]

Lemma 6 (Reactions of competitors) *An increase in D_{-j} leads to*

1. *a decrease in bank j's deposit volume, i. e., $\partial d_j/\partial D_{-j} < 0$, and*

[27] This can also be seen in the example in figure 7.

[28] Note that, in addition to assumption 2, we have to make assumption 1 from subsection 3.1 for the proofs of lemma 6 and proposition 4.

2. *an increase in bank j's risk, i. e.,* $\partial y_j/\partial D_{-j} > 0$.

The effects of an increase in D_{-j} (or d_1 in our case) work through a rising deposit rate $R(D)$. As before, this induces bank j to raise its risk level y_j and to lower its deposit volume d_j. Just as in the case with opaque banks, deposit volumes are strategic substitutes. In fact, banks' reactions functions look very similar as in figure 3 with slightly different slopes.[29]

Finally, we must take into account that the subsidized bank's deposit volume rises in reaction to the competitor banks' contraction of deposits, in turn inducing a further decrease in deposits at the competitor banks. This multiplier effect continues until a new equilibrium is reached. The effect on total deposits will again be positive, as in figure 3. As a result, the market rate rises, and competitors' risk levels increase accordingly.

Proposition 4 summarizes the reactions of bank 1 and its competitors.

Proposition 4 (Competitive effects of bail-out policy with transparent banks) *In a transparent banking system, an increase in the bail-out probability* β_1 *leads to*

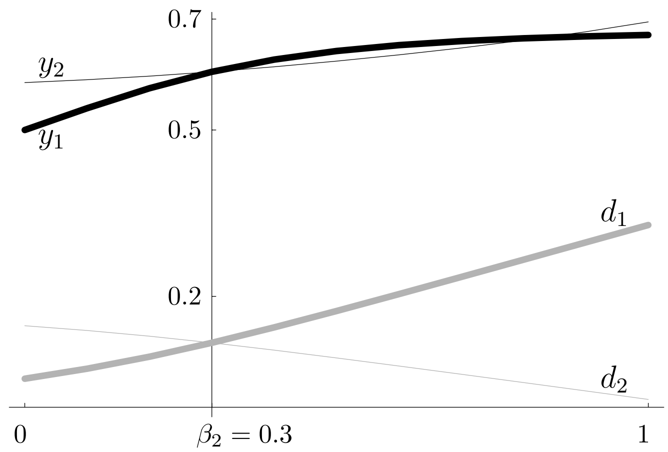
1. *an expansion of deposits at bank 1 and a contraction of deposits at its competitor banks* $j \neq 1$*, i. e.,* $\partial d_1/\partial \beta_1 > 0$ *and* $\partial d_j/\partial \beta_1 < 0$*;*
2. *an increase in risk at bank 1, but a decrease in the default premium (if* $R(D)$ *is not too large), i. e.,* $\partial y_1/\partial \beta_1 > 0$ *and* $\partial \rho_1/\partial \beta_1 < 0$*;*
3. *an increase in risk at the competitor banks j, accompanied by higher default premia, i. e.,* $\partial y_j/\partial \beta_1 > 0$ *and* $\partial \rho_j/\partial \beta_1 > 0$.

The proposition is illustrated in figure 8 for the case of two banks. As the bail-out probability β_1 rises, bank 1 grows, while bank j is crowded out. Since the increase in d_1 is larger than the decrease in d_j, the aggregate deposit volume increases, leading to a higher market rate $R(D)$. As a result, the risk-shifting problem at bank j is exacerbated. Opposite to proposition 2, y_1 also increases.

The most important result is that the competitive effects of the bail-out policy on the remaining banking sector are independent of the time and information structure of the model: In both of our models, the subsidized bank expands, causing a rise in the market rate, which aggravates the risk-shifting problems at competitor banks.

[29] In the special case of $R''(d) = 0$, reaction functions are linear.

Figure 8: Effects of an asymmetric bail-out policy



Black lines denote risk choices y, gray lines deposit volumes d. Thick lines stand for bank 1, thin lines for a competitor bank j. In this example, $\beta_j = 0.3$, so for $\beta_1 = 0.3$, both banks are symmetric. With rising β_1, risk and deposit volume of bank 1 increase. Due to fiercer competition in the deposit market, bank j reacts by reducing its deposit volume d_j and and increasing its risk y_2. The aggregate level of deposits increases. Interestingly, the effect of competition on risk-taking may be even stronger than the direct effect: The curves of y_1 and y_j intersect when β_1 becomes large.

3.2.3 Welfare analysis

When banks are transparent, the welfare effects of an asymmetric bail-out policy are straightforward. With β positive for all banks, we start from a situation where risk-taking is above the first-best level. In proposition 4, we have shown that an increase in the bail-out probability of some bank leads to a further increase in risk-taking at all banks. Therefore, a policy increasing the bail-out probability β for *any* bank is always welfare decreasing.

4 Conclusion

We started from the question of how government bail-out policies affect competition in the banking sector. While the existing literature has focused mainly on the effects of bail-out policies on the bank that enjoys the public guarantee, we are mainly interested in the competitive effects of such policies on the remaining banking sector.

We have presented two types of models, differing only with respect to their time and information structures. In the first model with *opaque* banks, risk-taking is unobservable by depositors, such that there is no market discipline. Therefore, a bank's risk choice does not affect its refinancing costs directly. In the second model with *transparent* banks, investments are perfectly observable, and depositors exert market discipline. As a consequence, deposit rates react promptly to a bank's risk choice.

Our main contribution is to show that an increase in the bail-out probability of one bank unambiguously leads to an *increase* in the risk-taking of the *competitor banks*. At the same time, competitor banks are crowded out. In contrast, the effect on the protected bank's risk-taking depends on the degree of transparency in the banking system. If banks are opaque, the protected bank *may* take less risk, while it always assumes more risk in a transparent environment. This qualifies the existing literature that suggests that an increase in the bail-out probability always leads to higher risk-taking at the protected bank. As a direct consequence, the welfare effects of raising the bail-out probability are ambiguous in an opaque setting: welfare may, but does not necessarily increase. In contrast, welfare clearly decreases in the case of transparent banks.

Note that the competitive effects are particularly strong in the opaque setting. The reason is a multiplier effect whereby the original effect from risk-taking on the default premium is reinforced through the feedback from the default premium to risk-taking. Therefore, the observation that protected banks do not take higher risks would be anything but reassuring.

There is a number of interesting extensions to our paper. One of the most important issues is the determination of the bail-out probability. In our model, this probability is determined outside of the model. In the case of public banks, one can reasonably argue that the bail-out probability is exogenous. In contrast, the bail-out probability should depend on the size of banks in the case of a "too-big-to-fail" policy. By allowing the bail-out probability to depend on size, we would get an additional strategic effect. Since high bail-out probabilities are beneficial for banks, a strategic tendency towards increased volume would develop. This raises the deposit rate, exacerbating the risk-shifting problem.

Another extension concerns the chosen market structure. In our model, the only market where banks interact is the market for deposits, and this interaction is modelled in a Cournot fashion. Therefore, one may change the market structure of our model in two ways. First, competition in the market for deposits may be modelled as price competition with product differentiation or transportation costs.[30] We believe that our central result remains valid, such that competitor banks are still pushed towards higher risk-taking. Suppose, for example, that banks are located on a Salop circle and that the bail-out probability of one bank increases. The direct effect is a fall in the default premium of the protected bank, leading to cheaper refinance opportunities. In reaction, the protected bank will expand. Neighboring banks find

[30] This has been done, e. g., by Matutes and Vives (2000) and Cordella and Yeyati (2002), however without considering government bail-outs.

themselves threatened by the competition from the protected bank and may react by increasing their deposit rates to regain some "territory" and move away from the protected banks. This effect spills over to the neighbors of the neighbors and so forth. In equilibrium, all competitor banks have lost some territory and have increased their deposit rates, accompanied by higher risk-taking.

Second, competition may take place in the market for loans, and not only in the deposit market. In this case, it is less clear that our main result is still valid. In fact, the paper by Boyd and De Nicolò (2003) has shown that the main effect driving the "charter-value" literature – and our model – disappears, once one introduces competition in deposit and loan markets simultaneously. However, there are a number of different ways how to introduce competition in the loan market. We conjecture that the results would depend on the exact model specification.

As a normative implication of our model, governments should refrain from bail-out policies, especially in transparent banking markets. Regulatory initiatives towards greater transparency should be accompanied by a "zero bail-out policy", if such a policy is at all credible. The welfare effects of bail-out policies on the competitor banks are always detrimental. Only the subsidized bank may profit, at the cost of an increased instability of the remaining banking sector. Market transparency and government intervention are substitutes, they should never prevail at the same time.

A Appendix

A.1 Proofs for opaque banks

A.1.1 The monopoly case

Proof of lemma 1: Because of the monopoly, $D = d$. Assume that $p''(y)$ and $R''(d)$ are small in the neighborhood of the equilibrium (for a given β). Then there are good Taylor approximations,

$$p(y) \approx p(y^*) + (y - y^*)\, p'(y^*),$$
$$R(d) \approx R(d^*) + (d - d^*)\, R'(d^*).$$

For readability, we define

$$p_0 := p(y^*) - y^*\, p'(y^*), \quad p_1 := -p'(y^*),$$
$$R_0 := R(d^*) - d^*\, R'(d^*), \quad R_1 := R'(d^*),$$

which implies

$$p(y) \approx p_0 - p_1\, y, \tag{12}$$
$$R(d) \approx R_0 + R_1\, d. \tag{13}$$

Then

$$\begin{aligned} \Pi &= d\,(p_0 - p_1\,y)\,(y - \rho\,(R_0 + R_1\,d)), \\ \frac{\partial \Pi}{\partial y} &= d\,(p_0 - p_1\,y) + d\,p_1\,(y - \rho\,(R_0 + R_1\,d)) \stackrel{!}{=} 0, \\ y(d,\rho) &= \frac{p_0 + p_1\,(R_0 + R_1\,d)\,\rho}{2\,p_1}, \qquad (14) \\ \Pi &= \frac{d\,(p_0 - p_1\,(R_0 + R_1\,d)\,\rho)^2}{4\,p_1}. \end{aligned}$$

We get two determining equations for d and ρ,

$$\frac{\partial \Pi}{\partial d} = \frac{(p_0 - p_1\,(R_0 + d\,R_1)\,\rho)\,(p_0 - p_1\,(R_0 + 3\,d\,R_1)\,\rho)}{4\,p_1} \stackrel{!}{=} 0 \quad \text{and} \qquad (15)$$

$$\rho = \frac{2}{2\,\beta + (1-\beta)\,[p_0 - p_1\,(R_0 + d\,R_1)\,\rho]}. \qquad (16)$$

There is one meaningful solution,

$$d = \frac{(\Box - \sqrt{\Box^2 - 12\,(1-\beta)\,p_1\,R_0})\,p_0 - 6\,p_1\,R_0}{18\,p_1\,R_1} \quad \text{and} \qquad (17)$$

$$\rho = \frac{6}{\Box - \sqrt{\Box^2 - 12\,(1-\beta)\,p_1\,R_0}} \quad \text{with} \qquad (18)$$

$$\Box = 3\,\beta + (1-\beta)\,p_0.$$

Three problems may occur: First, d may become zero if β is small. The moral hazard problem is then so large that the depositor prefers not to lend at all,

$$d \geq 0 \quad \Longleftrightarrow \quad \beta \leq \frac{p_0\,(3-p_0) + 6\,(p_1\,R_0 + \sqrt{p_1\,R_0\,(3 - p_0 + p_1\,R_0)})}{(3-p_0)^2}.$$

Second, if β is small, there may be no ρ that compensates investors for the risk. A higher ρ leads to higher risk shifting y, which again induces a higher ρ. For small β, this vicious circle may have no fixed point (the term in the square root becomes negative),

$$\Box^2 - 12\,(1-\beta)\,p_1\,R_0 \geq 0 \quad \Longleftrightarrow \quad \beta \geq \frac{p_1\,R_0}{p_0}.$$

However, if

$$R_0 \geq \frac{p_0^2}{(3+p_0)\,p_1},$$

the effect of d going to zero is larger. Third, y may be so small that $p(y)$ becomes larger than one. We have

$$p(y) \leq 1 \quad \Longleftrightarrow \quad p_0 \leq 3 + p_1\,R_0. \qquad (19)$$

This problem is of purely algebraical nature, because we have assumed that $p(y) = p_0 - y\,p_1$ is a good approximation to an actual, possibly nonlinear probability distribution in a region around the choice of y. If this is the case, (19) gives no further constraint.

Now let us come back to the discussion of the solution of equations (15). Making use of the implicit function theorem,

$$\begin{aligned}
\frac{\partial^2 \tilde\Pi}{\partial d^2} &= -\frac{1}{2}\,(2\,p_0 - p_1\,(2\,R_0 + 3\,d\,R_1)\,\rho) \\
&= -\frac{3\,R_1\left(p_0\,(\Box + \sqrt{\Box^2 - (1-\beta)\,12\,p_1\,R_0}) - 6\,p_1\,R_0\right)}{\left(\Box + \sqrt{\Box^2 - (1-\beta)\,12\,p_1\,R_0}\right)^2}, \\
\frac{\partial^2 \tilde\Pi}{\partial d\,\partial \rho} &= -\frac{1}{2}\,(p_0\,(R_0 + 2\,d\,R_1) - p_1\,(R_0 + d\,R_1)(R_0 + 3\,d\,R_1)\,\rho) \\
&= -\frac{p_0\left(p_0\,(\Box + \sqrt{\Box^2 - (1-\beta)\,12\,p_1\,R_0}) - 6\,p_1\,R_0\right)}{36\,p_1}, \\
\frac{\partial d}{\partial \rho} &= -\frac{\partial^2\tilde\Pi/\partial d\,\partial\rho}{\partial^2\tilde\Pi/\partial d^2} \\
&= -\frac{p_0\,\left(\Box + \sqrt{\Box^2 - (1-\beta)\,12\,p_1\,R_0}\right)^2}{108\,p_1\,R_1},
\end{aligned} \tag{20}$$

which is clearly negative. ■

Proof of lemma 2: The equation that determines ρ is easier to discuss,

$$\rho = \frac{1}{p(y)} \qquad \Longleftrightarrow \qquad \rho\,p(y) - 1 = 0. \tag{21}$$

In the case of a deposit insurance of β (in the case of bankruptcy, depositors still receive a fraction β of the face value), (21) becomes

$$\rho = \frac{1}{p(y) + \beta\,(1 - p(y))} \qquad \Longleftrightarrow \qquad \rho\,[p(y) + \beta\,(1 - p(y))] - 1 = 0. \tag{22}$$

The default premium ρ clearly does not depend directly on deposit volume d. However, there are two indirect effects. First, an increase of d leads to a rising $R(d)$, which induces the bank to raise y. This reduces $p(y)$, the depositors ask a higher ρ. Second, this increased ρ brings the bank to increase y even further, which leads to a further increase of ρ. Luckily, both effects point into the same direction. Mathematically, derive (22) with respect to d, considering that $y = y(d, \rho(d))$,

$$
\begin{aligned}
0 &= \rho'(d)\,[p(y) + \beta\,(1 - p(y))] + \rho\, y_\rho\, \rho'(d)\, p'(y)\,(1-\beta) + \rho\, y_d\, p'(y)\,(1-\beta) \\
&= \frac{\rho'(d)}{\rho} + \rho\, p'(y)\,(1-\beta)\,[y_\rho\, \rho'(d) + y_d],
\end{aligned}
$$

$$
\rho'(d) = -\frac{(1-\beta)\,\rho^2\, \overset{-}{p'(y)}\, y_d}{1 + (1-\beta)\,\rho^2\, \underset{-}{p'(y)}\, y_\rho}. \tag{23}
$$

The effects described above are identifiable: $(1-\beta)\,\rho^2\,(-p'(y))\,y_d$ is the first effect, the multiplier effect is embodied in the factor $1/[1 + (1-\beta)\,\rho^2\,p'(y)\,y_\rho]$. Because the denominator is smaller than 1, the whole fraction is greater than the numerator alone. If the denominator converges to 0, this implies that risk becomes so large that it cannot be compensated by a finite ρ any longer. Hence in the area of finite ρ, we have $\rho'(d) > 0$. ■

Proof of proposition 1: In lemma 1, we have already shown that $\rho(d)$ is an increasing function. Lemma 2 proves that $d(\rho)$ is decreasing (if $p''(y) \approx 0$ and $R''(d) \approx 0$). Clearly, $d(\rho)$ does not depend from β. We now show that $\rho(d)$ moves to the left when β increases,

$$
0 = \rho\,[1 - \beta\,(1 - p(y))] - 1, \tag{24}
$$

$$
0 = \rho'(\beta)\,[1 - \beta\,(1 - p(y))] + \rho\,(1 - p(y)),
$$

$$
\frac{\partial \rho}{\partial \beta} = -\rho\,\frac{1 - p(y)}{1 - \beta\,(1 - p(y))} = -\frac{1 - p(y)}{(1 - \beta\,(1 - p(y)))^2} < 0. \tag{25}
$$

This implies a leftward shift of the function $\rho(d)$. We have shown that $\partial\rho/\partial\beta < 0$ and if $p(y)$ and $R(d)$ do not bend too much, $\partial d/\partial\beta > 0$. The effect on y is not immediately clear, because the increase of d leads to a higher y whereas the decrease of ρ reduces y. Intuition tells that because the decrease of ρ is a direct consequence of a rising β, the risk level y should fall. We look at y from (14) and substitute d and ρ as given in (17) and (18) and get

$$
\begin{aligned}
y &= \frac{5\,\square - 12\,\beta + \sqrt{\square^2 - 12\,(1-\beta)\,p_1\,R_0}}{6\,p_1\,(1-\beta)}, \\
\frac{\partial y}{\partial \beta} &= \frac{2\,p_1\,R_0\,(1-\beta) - \square + \sqrt{\square^2 - 12\,(1-\beta)\,p_1\,R_0}}{2\,p_1\,(1-\beta)^2\,\sqrt{\square^2 - 12\,(1-\beta)\,p_1\,R_0}}.
\end{aligned}
$$

This derivative swaps sign in two cases: for $p_0 = 3 + p_1\,R_0$ and for $R_0 = 0$. Because of (19), we know that $p_0 < 3 + p_1\,R_0$ as long as $p(y) < 1$, hence we need to look only at the case $R_0 = 0$. For $R_0 < 0$, the sign of $\partial y/\partial\beta$ is positive, whereas for $R_0 > 0$, it is negative. The remaining task is to interpret the sign of R_0 in terms of elasticities. We have

$$
\begin{aligned}
R(D) = R_0 + R_1\,D \quad &\Longleftrightarrow \quad D(R) = (R - R_0)/R_1, \\
\varepsilon = \frac{\partial D}{\partial R}\,\frac{R}{D} &= \frac{1}{R_1}\,\frac{R\,R_1}{R - R_0} = \frac{R}{R - R_0}.
\end{aligned}
$$

Clearly, the elasticity ε is smaller than one if R_0 is positive, and vice versa. ■

A.1.2 The oligopoly case

Proof of lemma 3: We first show that a banks deposit volume shrinks and risk rises as competitors' deposit volume expands. Assume that β_1 is changed. As in the proof of lemma 2, we use the linear approximations (12) and (13). Expected profits for a competitor bank $j \neq 1$ are then

$$\begin{aligned} \Pi_j &= d_j\,(p_0 - p_1\,y_i)\,(y_i - \rho_i\,(R_0 + R_1\,D)), \\ y_j(D, \rho_j) &= \frac{p_0 + p_1\,(R_0 + R_1\,D)\,\rho_j}{2\,p_1}, \\ \tilde{\Pi}_j &= \frac{d_j\,(p_0 - p_1\,(R_0 + R_1\,D)\,\rho_j)^2}{4\,p_1}. \end{aligned}$$

Analogously to (15) and (16), one can use $\partial\tilde{\Pi}_i/\partial d_i = 0$ and $\rho_i = 1/[p(y_i) + \beta\,(1 - p(y_i))]$ to calculate d_i and ρ_i,

$$\begin{aligned} d_i &= \frac{(\square - \sqrt{\square^2 - 12\,(1-\beta_i)\,p_1\,(R_0 + D_{-i}\,R_1)})\,p_0 - 6\,p_1\,(R_0 + D_{-i}\,R_1)}{18\,p_1\,R_1} \quad \text{and} \\ \rho_i &= \frac{6}{\square - \sqrt{\square^2 - 12\,(1-\beta_i)\,p_1\,(R_0 + D_{-i}\,R_1)}} \quad \text{with} \\ \square &= 3\,\beta_i + (1-\beta_i)\,p_0. \end{aligned}$$

Apparently, if D_{-j} grows, both curves $d_j(\rho_j)$ and $\rho_j(d_j)$ move, as in figure 9. The slope of the curves is the same as in figure 2 in the monopoly case.

It remains to check where the intersection point moves when D_{-j} rises. From figure 9, at least $\partial d_j/\partial D_{-j} < 0$ seems evident (both curves move downwards). Formally,

$$\frac{\partial d_j}{\partial D_{-j}} = -\frac{36\,p_1\,R_1}{\Delta\,(\square + \Delta)^2} < 0 \qquad \text{with} \tag{26}$$

$$\Delta = \sqrt{\square^2 - (1-\beta_j)\,12\,p_1\,(R_0 + D_{-j}\,R_1)}$$

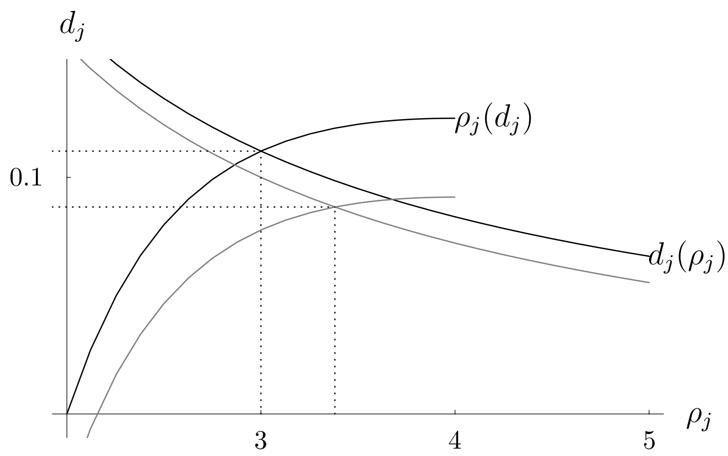
$$\frac{\partial \rho_j}{\partial D_{-j}} = -\frac{\square - 3\,\beta_j + \Delta}{3\,\Delta} < 0, \tag{27}$$

$$\frac{\partial y_j}{\partial D_{-j}} = \frac{R_1}{\Delta} > 0, \tag{28}$$

which proves our first assertion.

We now show that a competitor's increase in volume is compensated by a bank, but still the aggregate effect on volume remains positive. Be $d_1^*, \ldots, d_n^*$ the initial

Figure 9: Relation between Volume and default premium, Varying D_{-j}



Black lines stand for $D_{-i} = 0$, gray lines for $D_{-i} = 1/30$.

equilibrium levels of deposit volumes. Then β_1 rises marginally, and equilibrium levels adjust to $d_1^{**}, \dots, d_n^{**}$. We have a closer look at the adjustment process. We have already shown that the direct effect of the rise of β_1 is an expansion of d_1. We have also shown that this leads to a contraction of $d_2, \dots, d_n$. However, and this is the crucial argument, the D expands, because when $d_2, \dots, d_n$ contract so much that the original D^* is reached again, for each bank $j \neq 1$ the choice d_j^* is again optimal, only bank 1 chooses a $d_1 > d_1^*$.

As shown above, the contraction of $d_2, \dots, d_n$ leads to a further expansion of d_1, which again entails a further contraction of $d_2, \dots, d_n$. Eventually, this convergence process comes to an end. Possibly, $d_i = 0$ for some $i \in \{2, \dots, n\}$. The above argument that the contraction of $d_2, \dots, d_n$ never overcompensates the expansion of d_1 holds true, hence in the new equilibrium

$$d_1^{**} > d_1^*, \quad d_j^{**} > d_j^* \text{ for all } j \in \{2 \dots, n\}, \quad \text{and} \quad D^{**} > D^*,$$

which shows our second assertion. ■

Proof of proposition 2: Not much remains to be shown. The direct effects on the affected bank 1 are as in the monopoly case: ρ_1 falls and d_1 rises. As in the monopoly case, the effect on risk taking y_1 depends on the supply elasticity of deposits. However, the elasticity of individual supply is larger than that of aggregate supply. To see this in the case of moderate aggregate elasticity ($\varepsilon \approx 1$), determine the individual elasticity if bank 1 as

$$\varepsilon_1 = \frac{\partial d_1}{\partial R(D)} \frac{R(D)}{d_1} \approx \frac{1}{1 + \partial D_{-1}/\partial d_1} \frac{d_1 + D_{-1}}{d_1} > 1.$$

Hence the protected bank increases risk only if the supply elasticity of deposits is considerably lower than one.

The indirect effect is a fall of $d_2, \ldots, d_n$, which leads to a further rise of d_1 and a further fall of y_1 and ρ_1. The effect on the other banks j stems from the rise of their competitors' deposit volume D_{-j}: d_i falls, y_j rises and ρ_j falls (cf. (26), (27), and (28)). ■

A.2 Proofs for transparent banks

A.2.1 The monopoly case

Proof of lemma 4: Substituting $\rho(y) = 1/[p(y) + \beta\,(1 - p(y))]$, we get

$$d_y \approx \frac{\beta + (1-\beta)\,[p(y) + y\,p'(y)]}{2\,R'(d) + d\,R''(d)}.$$

Now if $\beta \approx 0$, then $p'(y) + y\,p(y) \approx 0$. Substituting this into (11) yields

$$d_y \approx \frac{\beta}{2\,R'(d) + d\,R''(d)} > 0,$$

which was to be shown. ■

Proof of lemma 5:

In order to find out y_d, we derive (7) with regard to d and get

$$\begin{aligned}
\frac{\partial \Pi}{\partial y} &= d\left(p'(y)\left[y - \frac{R(d)}{p(y) + \beta\,(1-p(y))}\right] + p(y)\left[1 + \frac{(1-\beta)\,p'(y)\,R(d)}{(p(y) + \beta\,(1-p(y)))^2}\right]\right) = 0,\\
0 &= [p(y) + y\,p'(y)]\,[p(y) + \beta\,(1-p(y))]^2 - \beta\,p'(y)\,R(d), \qquad (29)\\
0 &= y_d\,[2\,p'(y) + y\,p''(y)]\,[p(y) + \beta\,(1-p(y))]^2\\
&\quad + [p(y) + y\,p'(y)]\,2\,(1-\beta)\,y_d\,p'(y)\,[p(y) + \beta\,(1-p(y))]\\
&\quad - \beta\,\big(y_d\,p''(y)\,R(d) + p'(y)\,R'(d)\big). \qquad (30)
\end{aligned}$$

If we assume that β is not too large and hence $p'(y) + y\,p(y) \approx 0$ in (30), solving for y_d we get

$$y_d = \frac{\beta\,p(y)\,R'(d)}{\beta\,y\,\underset{-}{p''(y)}\,\underset{+}{R(d)} - [y\,\underset{-}{(2\,p'(y) + y\,p''(y))}]\,\underset{+}{[p(y) + \beta\,(1-p(y))]^2}}.$$

Again, if β is small, then $\beta\,y\,p''(y)$ becomes small and the denominator is positive. Because $R'(d) \neq 0$, the derivative y_d can vanish only if $\beta = 0$. ■

Proof of proposition 3: We first show that c. p., volume d increases as β increases. The equation determining d implicitly is

$$\begin{aligned} 0 &= y\,(p(y) + \beta\,(1 - p(y))) + R(d) + d\,R'(d) \\ 0 &= y\,(1 - p(y)) + d_\beta\,(2\,R'(d) + R''(d)) \\ d_\beta &= \frac{y\,(1 - p(y))}{2\,R'(d) + d\,R''(d)} > 0. \end{aligned}$$

Clearly, $d_\beta > 0$.

Next, we show that also y rises as β rises. The equation determining y implicitly is (29),

$$\begin{aligned} 0 &= [p(y) + y\,p'(y)]\,[p(y) + \beta\,(1 - p(y))]^2 - \beta\,p'(y)\,R(d), \\ 0 &= y_\beta\,[2\,p'(y) + y\,p''(y)]\,[p(y) + \beta\,(1 - p(y))]^2 \\ &\quad + [p(y) + y\,p'(y)]\,2\,[p(y) + \beta\,(1 - p(y))]\,[1 - p(y) + y_\beta\,(1 - \beta)\,p'(y)] \\ &\quad - R(d)\,p'(y). \end{aligned} \tag{31}$$

If β is not too large, then $p'(y) + y\,p(y) \approx 0$, and (31) simplifies to

$$\begin{aligned} 0 &= y_\beta\,[y^2\,p''(y) - 2\,p(y)]\,[p(y) + \beta\,(1 - p(y))] - [y\,\beta\,y_\beta\,p''(y) - p(y)]\,R(d) \\ y_\beta &= -\frac{p(y)\,R(d)}{\underbrace{[y^2\,p''(y) - 2\,p(y)]}_{-}\,\underbrace{[p(y) + \beta\,(1 - p(y))]}_{+} - y\,\beta\,\underbrace{p''(y)}_{-}\,R(d)}. \end{aligned} \tag{32}$$

Here, $\partial^2(y\,p(y))/\partial y^2 = 2\,p'(y) + y\,p''(y) < 0$, otherwise there is no solution to the first best problem. Therefore, $-2\,p(y)/y + y\,p''(y) < 0$, and hence $y^2\,p''(y) - 2\,p(y) < 0$. Because of the assumption that β is small, the first addend in the denominator dominates the second. As a result, $y_\beta > 0$.

Summing up, the increase of β leads directly to a rise of y and d. Because both y_d and d_y are positive (cf. lemma 4 and 5), there is a multiplier effect into the same direction. Now look at the effect on ρ. Clearly, ρ falls if $p(y) + \beta\,(1 - p(y))$ rises. Therefore, examine

$$\begin{aligned} \frac{\partial(p(y) + \beta\,(1 - p(y)))}{\partial\beta} &= y_\beta\,p'(y) + (1 - p(y)) - \beta\,y_\beta\,p'(y) \\ &= 1 - p(y) + (1 - \beta)\,y_\beta\,p'(y). \end{aligned}$$

Now incorporate (32) and consider $p(y) + y\,p'(y) \approx 0$, then the term becomes

$$\begin{aligned} &1 - p(y) + (1 - \beta)\,\frac{p(y)^2\,R(d)}{[y^2\,p''(y) - 2\,p(y)]\,[p(y) + \beta\,(1 - p(y))] - y\,\beta\,p''(y)\,R(d)} \\ &\overset{\beta\approx 0}{\approx} 1 - p(y) + \frac{R(d)}{y}\,\underbrace{(y^2\,p''(y) - 2\,p(y))}_{-}. \end{aligned}$$

This is positive whenever $R(d)$ is small enough,

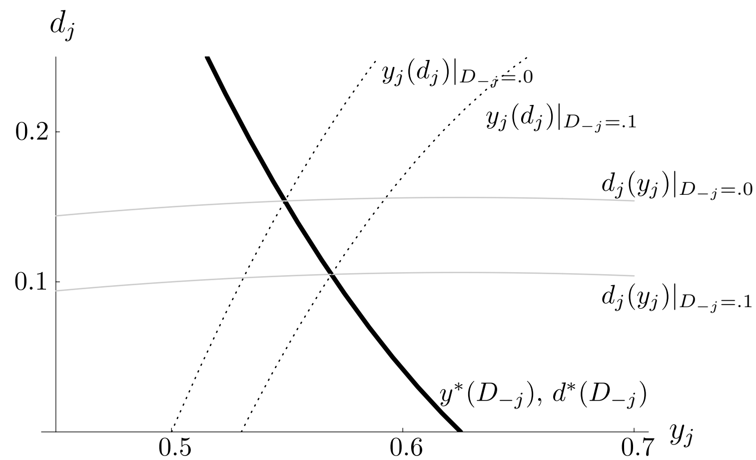
$$R(d) < y\,(1 - p(y))\,(2\,p(y) - y^2\,p''(y)). \tag{33}$$

Note that this is the condition for the case that β is small, hence that y is close to the first best case. As β becomes larger, (33) may be overly strict. I numerical calculations (e. g., in the example $p(y) = 1-y$ and $R(d) = d$), ρ rises with β globally, even when (33) does not hold. ■

A.2.2 The oligopoly case

Proof of lemma 6: The proof falls into three steps, as becomes clear from figure 10. We first show that if D_{-j} rises, the curve $d_j(y_j)$ goes down. Second, under somewhat stricter conditions, the curve $y_j(d_j)$ moves up. Finally, we look at the contemporaneous reaction of the optimal y and d.

Figure 10: Relation between Volume and Risk, Varying D_{-j}



This figure is the analogy to figure 9. Dashed lines show the optimal $d_j(y_j)$, gray lines show $y_j(d_j)$ for $\beta_j = 0.2$ and $D_{-j} = 0$ and 0.1. Equilibria (i. e., intersections of curves pertaining to the same d_1) are marked by the solid black curve. This example is based on the functions $p(y) = 1 - y$ and $R(d) = d$.

Claim If the competitors deposit volume D_{-j} rises, the curve defining the optimal deposit volume d_j of bank j given the risk choice y_j goes down ($\partial d_j(y_j)/\partial D_{-j} < 0$).

Proof of the claim: The equation that implicitly determines the optimal d_j given y_j and D_{-j} is

$$\frac{\partial \Pi_j}{\partial d_j} = 0 = y_j\,\big(p(y_j) + \beta_j\,(1 - p(y_j))\big) + R(d_j + D_{-j}) + d_j\,R'(d_j + D_{-j}),. \tag{34}$$

Taking the derivative with regard to D_{-j}, regarding that d_j is a function of D_{-j} and then solving for $\partial d_j(y_j)/\partial D_{-j}$ yields

$$\begin{aligned} 0 &= \Big(1 + 2\,\frac{\partial d_j}{\partial D_{-j}}\Big)\, R'(D) + \Big(1 + \frac{\partial d_j}{\partial D_{-j}}\Big)\, d_j\, R''(D) \\ \frac{\partial d_i}{\partial D_{-j}} &= -\frac{R'(D) + d_j\, R''(D)}{2\, R'(D) + d_j\, R''(D)}. \end{aligned}$$

Numerator and denominator are positive, hence the derivative $\partial d_j(y_j)/\partial D_{-j}$ is negative. $\square$

Claim Assume that the bail-out probability of bank j is not too large ($\beta_j \approx 0$). Then if the competitors deposit volume D_{-j} rises, the curve defining the optimal level of risk y_j of bank j given the risk choice y_j goes up ($\partial y_j(d_j)/\partial D_{-j} > 0$).

Proof of the claim: The equation that implicitly determines the optimal y_j given d_j and D_{-j} is

$$\frac{\partial \Pi_j}{\partial y_j} = 0 = \big(p(y_j) + y_j\, p'(y_j)\big)\, \big(p(y_j) + \beta_j\,(1 - p(y_j))\big)^2 - \beta_j\, p'(y_j)\, R(d_j + D_{-j}). \tag{35}$$

Taking the derivative with regard to D_{-j}, regarding that y_j is a function of D_{-j} and then solving for $\partial y_j(d_j)/\partial D_{-j}$ and using $p'(y_j) \approx -p(y_j)/y_j$ yields

$$\begin{aligned} \frac{\partial y_j(d_j)}{\partial D_{-j}} &= \frac{\beta_j\, p'(y_j)\, R'(D)}{[p(y_j) + \beta_j\,(1 - p(y_j))]^2\, [2\, p'(y_j) + y_j\, p''(y_j)] - \beta_j\, p''(y_j)\, R'(D)} \\ &\overset{\beta_j \approx 0}{\approx} \beta_j\, \frac{\overset{-}{p'(y_j)}\, \overset{+}{R'(D)}}{p(y_j)^2\, [2\, p'(y_j) + y_j\, p''(y_j)]}. \end{aligned}$$

The whole fraction is positive whenever $2\, p'(y_j) + y_j\, p''(y_j)$ is negative. Now $p'(y_j) + y_j\, p''(y_j) < 0$ for the second order condition (global assumption), and $p'(y_j) < 0$ per assumption. Therefore, the derivative $\partial y_j(d_j)/\partial D_{-j}$ is positive. $\square$

Finally, we now have to look at the equations (34) and (35) defining y_j and d_j simultaneously. Generally, if we have two equations $F(y, d, \epsilon = 0)$ and $\tilde{F}(y, d, \epsilon) = 0$ that implicitly define a dependence $y(\epsilon)$ and $d(\epsilon)$, then the implicit function theorem implies that

$$\begin{aligned} \frac{\partial y}{\partial \epsilon} &= \frac{F_d\, \tilde{F}_\epsilon - F_\epsilon\, \tilde{F}_d}{F_y\, \tilde{F}_d - F_d\, \tilde{F}_y} \quad \text{and} \\ \frac{\partial d}{\partial \epsilon} &= \frac{F_y\, \tilde{F}_\epsilon - F_\epsilon\, \tilde{F}_y}{F_d\, \tilde{F}_y - F_y\, \tilde{F}_d}. \end{aligned}$$

Applied to equations (34) and (35), we get the following. For clearness, we have already assumed that $p''(y) \approx 0$ and $R''(d) \approx 0$.

$$\frac{\partial y_j}{\partial D_{-j}} = \frac{\beta_j \, R'(D)}{(1-\beta_j)\big([11\beta_j + 8(1-\beta_j)p(y_j)]p(y_j) + [3\beta_j + 4(1-\beta_j)\,p(y_j)]y_j p'(y_j)\big) + 3\beta_j^2}$$

$$\frac{\partial d_j}{\partial D_{-j}} = -\frac{(1-\beta_j)\big([5\beta_j + 4(1-\beta_j)p(y_j)]p(y_j) + [\beta_j + 2(1-\beta_j)p(y_j)]y_j p'(y_j)\big) + \beta_j^2}{(1-\beta_j)\big([11\beta_j + 8(1-\beta_j)p(y_j)]p(y_j) + [3\beta_j + 4(1-\beta_j)p(y_j)]y_j p'(y_j)\big) + 3\beta_j^2}.$$

Considering that $p(y) + y\,p'(y) \approx 0$, we get

$$\frac{\partial y_j}{\partial D_{-j}} = \frac{\beta_j \, R'(D)}{4\,(1-\beta_j)\,[2\,\beta_j + (1-\beta_j)\,p(y_j)]\,p(y_j) + 3\,\beta_j^2} \quad \text{and}$$

$$\frac{\partial d_j}{\partial D_{-j}} = -\frac{2\,(1-\beta_j)\,[2\,\beta_j + (1-\beta_j)\,p(y_j)]\,p(y_j) + \beta_j^2}{4\,(1-\beta_j\,[2\,\beta_j + (1-\beta_j)\,p(y_j)]\,p(y_j) + 3\,\beta_j^2}.$$

Because assumedly $\beta \approx 0$, we finally get

$$\frac{\partial y_j}{\partial D_{-j}} = \beta_j \, \frac{R(D)}{4\,p(y_j)^2} > 0 \quad \text{and}$$

$$\frac{\partial d_j}{\partial D_{-j}} = -\frac{2\,p(y_j)^2}{4\,p(y_j)^2} = -\frac{1}{2} < 0. \tag{36}$$

This was to be shown. ■

Proof of proposition 4: The proof draws heavily on proposition 3 and lemma 6. Because of (36), competitors react on an expansion of d_1 with contracting their d_j by half the expansion of d_1. A new equilibrium emerges with increased d_1, decreased d_j for all j and increased D. Hence from the view of bank 1, the interest rate on the deposit market is described by a function $\tilde{R}(d_1)$ instead of $R(D)$. $\tilde{R}(d_1)$ still has a positive slope. As a result, the statements about bank 1 from proposition 4 follow directly from proposition 3 by replacing $R(d)$ with $\tilde{R}(d)$ in the monopoly case.

From the view of the competitor banks, an increase of β_1 is equivalent to a rise of volume of one of their competitor banks. Therefore, the reaction of d_j and y_j are already described by lemma 6. The reaction of ρ_j is obvious. Because β_j remains constant, the risk level y_j rises, the default premium ρ_j must also rise. ■

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