GOVERNANCE AND THE EFFICIENCY
OF ECONOMIC SYSTEMS
GESY

Discussion Paper No. 524

Delegation and Communication

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September 2015

Financial support from the Deutsche Forschungsgemeinschaft through SFB/TR 15 is gratefully acknowledged.

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Delegation and Communication*

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September 21, 2015

Abstract

This paper analyzes delegation and joint decision making in an environment with private information and partially aligned preferences. We compare the benefits of these two decision making procedures as well as the interaction between them. We give a condition under which delegation is preferred to ex post joint decision making and we show how the interaction between delegation and ex post joint decision making always crowds out delegation. Finally, we analyze how the availability of the principal at the communication stage affects our results.

JEL Classification: D23, D82, L23

*We thank Jeanne Hagenbach, Daniel Krähmer, Anne-Katrin Roessler and Hidenori Takahashi as well as seminar participants at the University of Mannheim and conference participants at ASSET 2014, Geilo 2015 conference and EEA 2015 for helpful comments.

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1 Introduction

Effective decision making often relies on obtaining valuable information from informed parties. In many cases, the informed party might not have the same preferences as the decision maker and needs to be given the right incentives to reveal this information. Contract theory has shown how to optimally solve this adverse selection problem using a menu of contracts containing information-contingent decisions and payments.[1] However, payments that depend on the information (or "state" as the literature calls it) are not always feasible. For instance, a manager's wage might depend on the overall profit and state of the firm, but it is never directly contingent on the outcome of all the smaller decisions that the manager is involved in.

In this paper we analyse two different decision making procedures without state-contingent transfers as well as their interaction. Namely, we focus on how delegation and ex post joint decision making give incentives to reveal private information and how the outcome in terms of actually decision is chosen under these schemes. Following the literature on delegation (Holmström, 1977, 1984; Melumad and Shibano, 1991) we show that when ex post joint decision making is not possible, the standard delegation results apply. That is, it is optimal to allow the agent to take any decision below a certain threshold but restrict the upper end of admissible decisions.

Furthermore, we show how the possibility of ex post joint decision making always leads to information revelation when delegation is not an option and the standard unraveling result (Grossman, 1981; Milgrom, 1981) also holds in our environment. However, because the final decision is a joint decision, it will be not be equal to the principal's preferred decision but it will be somewhere between the principal and the agent's preferred decisions. We compare the benefits of these two decision-making procedures and give a condition under which delegation is preferred by the principal. In particular, our condition implies that even though ex post joint decision making always leads to information revelation, there are settings in which delegation does better. This is for instance the case when the agent has a lot of bargaining power at the decision making stage.

We also consider the interaction of delegation and ex post joint decision making. Importantly, we show how the possibility of ex post joint decision making always crowds out delegation as it gives incentives to deviate from the delegation set for a range of information realizations and thus in equilibrium delegation breaks down. This happens regardless of whether delegation is preferred to joint decision making or not. Therefore,

[1] See for instance Laffont and Martimort (2002).

in some case not being able to fully commit to delegation can be detrimental for the principal.

In practice, a principal has a lot of responsibilities and tasks that might impose restrictions on her availability for communication with other agents. Any web search including the words "delegation" and "managers" will give you a bunch of articles[2] which talk about the manager's limited time. We incorporate this into our framework and show how the availability of the principal at the communication stage affects our results. In particular, we show that the more likely it is that the principal is available for communication the smaller is the delegation set. Furthermore the principal's expected utility is convex in the probability of being available for communication. Thus, in a stylized model with two ex ante symmetric agents, we show that it is optimal for the principal to treat the agents asymmetrically; always communicate with one agent and thus credibly commit to full delegation to the other agent.

We are motivated by the burgeoning literature on delegation pioneered by Holmström (1977, 1984). As in Holmström (1977, 1984) we focus on interval delegation. However Melumad and Shibano (1991) characterize the optimum among all delegation sets and show that it is an interval.[3] Although Alonso and Matouscheck (2008) generalize the optimal delegation problem to a generalized quadratic loss function, we stick to the "simple" quadratic loss function with a constant bias and weight. Again, this is to be able to better focus on communication without introducing too many other issues. Our contribution to this literature is twofold. First we compare the benefits of delegation to joint decision making. Second, we add communication via ex post joint decision making into a delegation environment and thereby focus on this positive aspect rather than the more normative aspect of the optimality of delegation that this literature studies. [4]

We are also related to the literature on limited attention in that we assume that the principal might not always be available for communication and ex post decision making. In line with Matějka and McKay (2012), Persson (2013) and Gabaix (2014) we assume

[2]For instance "Smart Leadership: Delegate, Prioritize and Simplify" Business News Daily (October 25, 2013) and "Tips for the Overworked Manager" (http://work.chron.com/tips-overworked-manager-5247.html).

[3]Martimort and Semenov (2006) provide a more general condition on the distribution of the state under which the optimal mechanism is indeed continuous as under interval delegation.

[4]Other contributions to the delegation literature include Shin and Strausz (2014) who show how delegation, by generating additional private information, can improve dynamic incentives under limited commitment. Other papers have focused on the allocation of formal decision rights (Aghion and Tirole, 1997), project choice rather than project size (Armstrong and Vickers, 2010), relational delegation (Alonso and Matouscheck, 2007), delegation in moral hazard environment (Bester and Krähmer, 2008), etc.

that the principal has limited attention and cannot always deal with everything.

The model is presented in Section 2. In Section 3, we characterize the outcome of delegation and in Section 4 that of ex post joint decision making. Section 5 compares the two outcomes whereas Section 6 analyze the interaction between delegation and joint decision making. Section 7 provides a robustness check and discusses how the principal's time constraint affects delegation and the probability that the principal actually is available for joint decision making at the ex post stage. Section 8 briefly concludes.

2 The model

We use the workhorse model of the delegation and communication literature in which a principal has the authority to make a decision but an agent has acces to decision relevant information. We study the principal's optimal decision and the agent's incentives to reveal his private information. As in the afore-mentionned literature, contracts with contingent transfers are not allowed, but the principal can commit to transferring the decision rights to the agent (delegation). We allow for this transfer to be incomplete, in that the principal can constrain the agent's decision making. We also allow the principal and the agent to make a joint decision at the ex post stage. Whereas the principal has decision power at the initial stage, this joint decision making should be thought of as capturing ex post hold-up issues within the organization.

Preferences: The principal's and the agent's utilities depend on the implemented decision, the state of the world and are partially aligned. The decision is represented by $y \in Y \subset \mathfrak{R}$ and the state is denoted $\theta \in \Theta = [0,1]$. Both the principal and the agent have von Neumann-Morgenstern utility functions that take the form of quadratic loss functions where the principal's preferred decision is $y_P(\theta) = \theta$ and the agent's preferred decision is $y_A(\theta) = \theta + b$, where $0 < b < \frac{1}{2}$[5]. Formally, the principal's utility is

$$u_P(y, \theta) = -(y - \theta)^2 .$$

The agent's utility is

$$u_A(y, \theta, b) = -(y - \theta - b)^2 .$$

Information: Before the decision y is made, the agent observes the value of θ, but the principal does not. It is common knowledge that θ is distributed according to the

[5]This assumption simply means that the difference in preferences between the principal and the agent is not too large so that there is (some) scope for non-trivial delegation and communication.

uniform distribution on $\Theta = [0, 1]$.

Decision making: We allow for two different ways of reaching a decision; Delegation and Joint Decision Making. First we study them separately and compare the outcomes. Then we allow the interaction of the two procedures.

- **Delegation:** We adopt an incomplete contracting approach where the principal cannot rely upon a court-enforced contracts with contingent transfers. However, she can commit to allowing the agent to take whatever decision he finds appropriate within a set of pre-specified decisions. Like Holmström (1977, 1984) we focus on interval delegation in which the principal specifies a set $D = [\underline{d}, \bar{d}]$ from which the agent is free to choose any decision he wants.

- **Joint Decision Making:** After the agent has observed the private information θ, he can choose to reveal this to the principal. Revealing this information does not only allow the agent to transfer this knowledge to the principal, it also allows him to participate in the final decision making. Formally we assume that following communication of θ, a decision is reached jointly by the principal and the agent so as to maxmize a weighted sum of the two players' utility. The relative weight on the agent's utility is denoted by $a \in [0, 1]$.[6]

Therefore, when considering the interaction between delegation and joint decision making, if the decision has been delegated to the agent, he either makes a decision within the permissible delegation set or communicates information to the principal and they reach the final decision together. If no delegation has been implemented, then either the agent does not reveal any information about θ and the principal alone takes the final decision or the agent reveals information about θ and they reach the final decision together.

Timing: The timing is as follows. First, the principal chooses a delegation set in which the agent is free to take any decision he wants. Second, the agent obtains a perfect signal $s \in S = \Theta$. Having observed this signal, if the decision has been delegated the agent can choose any decision within the initial delegation set or he can communicate θ to the principal and enter the joint-decision-making stage. If the decision has not been delegated, the agent can choose whether to reveal the signal to the principal or not and the principal has to make a decision. Finally, the decision is implemented and payoffs are realized. The timing is illustrated below.

[6]The relative weight on the principal's utility is thus $1 - a$.

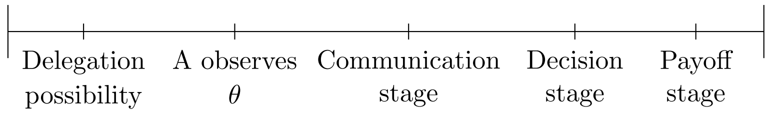

Figure 1: Timeline

3 Delegation

In this section we study the outcome of delegation in an environment where ex post joint decision making is not possible. That is, the principal commits to allowing the agent to choose any decision within a pre-specified set and no other decisions (or communication) can take place between the players after this delegation decision has been made. When no deviation from the initial delegation set is possible, then the principal's problem is to find a delegation set that maximizes her expected utility. To this aim, it is useful to first characterize the agent's behavior when faced with a given delegation set.

Lemma 1. *Consider a given delegation set* $D = [\underline{d}, \bar{d}]$.

- *If* $\theta < \max\{0, \underline{d} - b\}$, *then the agent will choose* $y_d = \underline{d}$.
- *If* $\theta \in [\max\{0, \underline{d} - b\}, \max\{0, \bar{d} - b\}]$, *then the agent will choose* $y_d = \theta + b$.
- *If* $\theta > \bar{d} - b$, *the agent will choose* $y_d = \bar{d}$.

The proof of this lemma is straightforward (and therefore omitted). But the intuition goes as follows: If θ is in the interval $[\max\{0, \underline{d} - b\}, \max\{0, \bar{d} - b\}]$, then the agent's preferred decision is within the admissible set D and the agent will opt for this decision as it maximizes his utility. If θ is higher or lower than this set, the agent's preferred decision is not permissible and the agent is constrained to pick the decision that is closest to his preferred decision among all permissible decisions (i.e., D). The preferred decision is increasing in θ so that for low values of θ, this means choosing $y^d = \underline{d}$ and for high values $y^d = \bar{d}$. This is illustrated in the following figure.

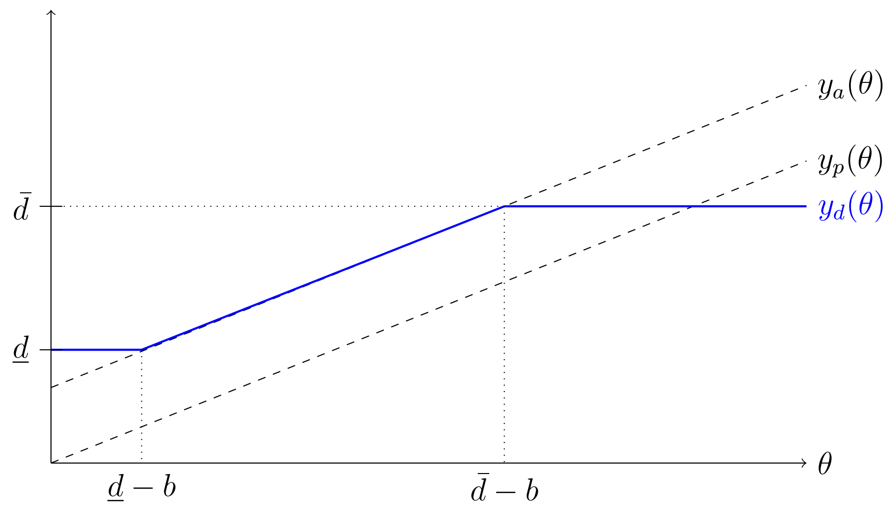

Figure 2: The agent's decision for a given delegation set

Using this lemma, it is easy to write the principal's expected utility as the sum of the utility of the three intervals identified above. Formally this can be written as

$$\max_{\underline{d},\bar{d}} - \int_0^{\max\{0,\underline{d}-b\}} (\underline{d}-\theta)^2 d\theta - \int_{\max\{0,\underline{d}-b\}}^{\min\{1,\max\{0,\bar{d}-b\}\}} b^2 d\theta - \int_{\min\{1,\max\{0,\bar{d}-b\}\}}^{1} (\bar{d}-\theta)^2 d\theta \quad (1)$$

subject to $0 \leq \underline{d} \leq \bar{d} \leq 1$.

The optimal delegation set that solves (1) follows from the literature (Melumad and Shibano, 1991) and is summarized in the following proposition.[7]

Proposition 1. *When ex-post communication is not possible, then the optimal delegation set is* $D^o = [0, 1-b]$.

In this setting the interval contract D^o is an optimal contract and is illustrated below. It is such that the agent can make any decision below a certain threshold $\bar{d}$. The value of the upper bound $\bar{d}$ depends on the agent's bias. When preferences become less aligned (b goes to θ), delegation becomes less valuable because the agent always wants to make too "high" decisions. Conversely, when preferences becomes more aligned (b is close to 0), then full delegation becomes optimal since the agent's optimal decision is the same as the principal's optimal decision.

[7] In fact, there exists many optimal delegation sets (See Alonso and Matouscheck (2008) page 267). Decisions between 0 and b will never be chosen by the agent, but for notational simplicity and to get a more coherent representations, we allow them in our characterisation. Any other admissible decisions that are never chosen by the agent are excluded from what we call "the" optimal delegation set.

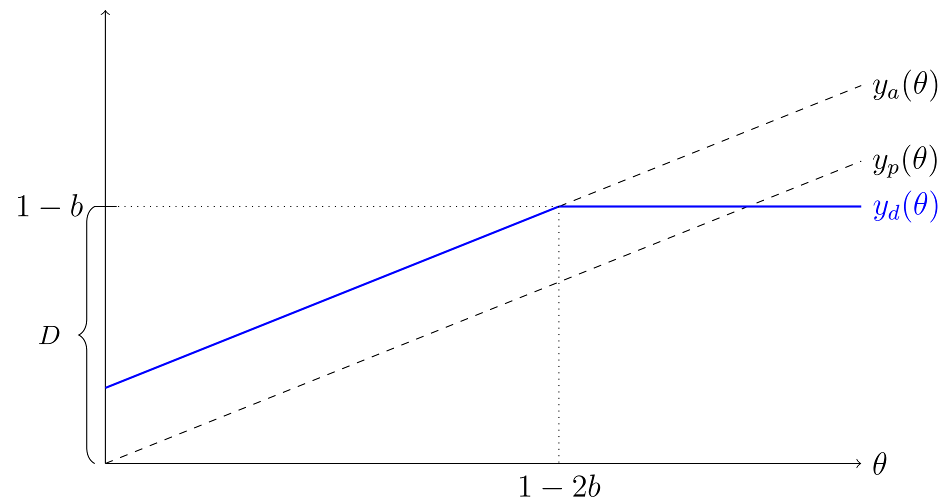

Figure 3: Optimal delegation without communication

The principal's expected surplus from the optimal delegation set can easily be calculated and is given in the next lemma.

Lemma 2. *The principal's expected surplus from D^o is equal to*

$$W^D = -b^2(1 - \frac{4}{3}b). \tag{2}$$

The lower the agent's bias, the higher is the principal's expected surplus. This is of course a direct consequence of preferences become more aligned as b decreases.

The expected surplus reported in Lemma 2 only takes into account the gains for the specific task y. However, delegation has other benefits as well. In particular it frees up time for the principal so that she can engage in other (more important?) tasks. This is often a reason for the task being delegated in the first place. However, these benefits are not taken into account here. In Section 7 we attempt to at least partially include such benefits.

4 Joint Decision Making

In this section we analyze the pure joint decision making game and do not allow the principal to delegate the decision. That is, the principal cannot commit to a delegation set. However, the agent can communicate information at the communication stage and the final decision is taken after that. Our focus is thus on the outcome of the communication and decision stage in the absence of delegation. In this environment, the appropriate equilibrium concept is that of Perfect Bayesian Equilibrium and we proceed by backward induction.

Whenever the agent reveal his private information and the principal and the agent enter (jointly) into the decision stage, the outcome of the joint decision making is given in the next lemma.

Lemma 3. *Joint decision making over y yields the following outcome:*

$$y_b(\theta) = \theta + ab. \tag{3}$$

The outcome of the joint-decision-making stage is somewhere between the principal and the agent's preferred decision. The higher the agent's relative weight (a), the closer the outcome will be to the agent's preferred decision. While a lower a leads to the joint decision making being closer to the principal's preferred decision. This is illustrated in Figure 4.

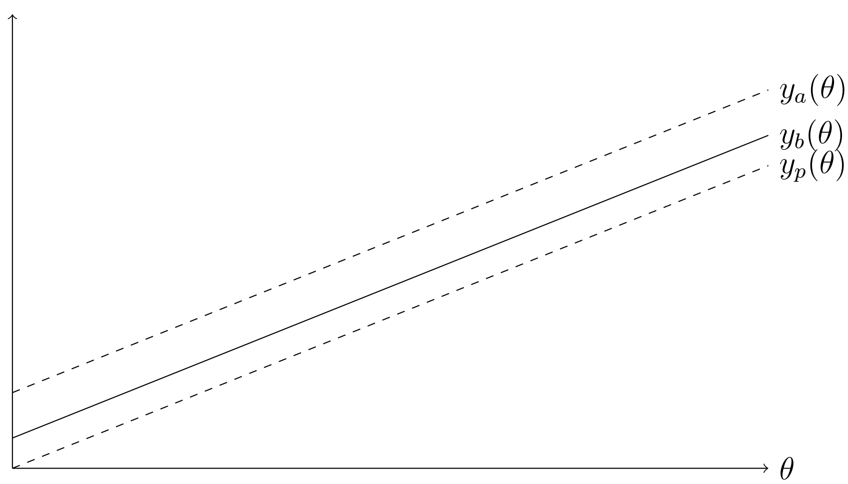

Figure 4: Decision under joint decision making

As the next proposition shows, even in this environment where, because of ex post joint decision making, the principal is not allowed to impose her preferred decision, the usual unraveling result (Grossman, 1981; Milgrom, 1981) applies.

Proposition 2. *The agent always have an incentive to reveal his private information and engage in ex post joint decision making. The final decision is given by Lemma 3.*

We can easily calculate the principal's expected utility.

Lemma 4. *The principal's expected utility is*

$$W^B = -(ab)^2. \tag{4}$$

5 Delegation vs. Joint Decision Making

We are now ready to compare the outcome under delegation to that under joint decision making setting and give conditions under which the principal prefers one procedure over the other.

Proposition 3. *Joint decision making is preferred to delegation if and only if*

$$1 - \frac{4}{3}b \geq a^2. \tag{5}$$

This proposition states that joint decision making is preferred to delegation if and only if the agent's relative weight in the joint decision making process is sufficiently small. When the agent's relative weight is not too high, the gains from using all the information available in the environment (at the small loss from achieving a slightly suboptimal decision) outweighs the expected gains from delegation. We have thus shown that the unraveling (Milgrom, 1981; Grossman, 1981)[8] and subsequent decision-making is beneficial to the principal even beyond the case where the agent has no say in the ex post decision making. However, it does not extend indefinitely and Proposition 3 characterizes this limit.

[8]Hagenbach et al. (2014) prove the unravelling result in a more general setting.

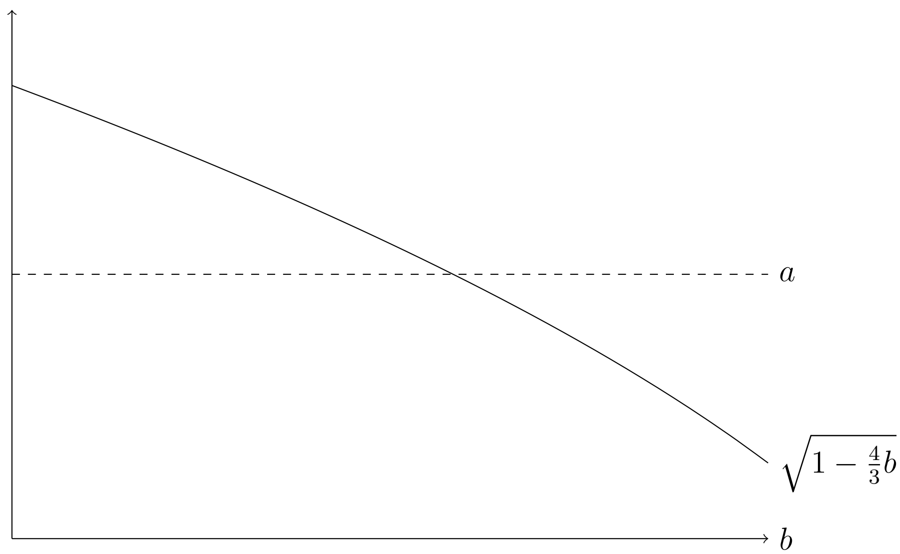

Figure 5: Illustration of (5)

The agent's bias influences both the expected surplus under delegation and joint decision making. However, for high values of b, it is preferred to engage in delegation rather than joint decision making. The intuition for this is that under joint decision making the principal cannot control the loss associated with a high b as the decision is always ab above the principal's preferred decision. However, under delegation, the principal can control the losses associated with having a biased agent through the upper bound of the delegation set. When b is sufficiently large, controling this upper bound becomes more valuable.

6 Delegation and joint decision making

The previous section compared delegation and joint decision making. In this section we allow the two to coexist and show how joint decision making always crowds out delegation. In this environment, it is as if the joint decision making game as described in Section 4 is augmented by a commitment to a delegation set before the game is played. Although with an initial delegation decision, the default decision maker at the final decision making stage is now the agent[9] and this might change incentives to engage in communication, the timing described in Section 2 and 4 is otherwise unchanged. The perfect Bayesian equilibrium concept still applies.

[9] At least when the principal chooses to delegate the decision to the agent.

We solve the model by backward induction and start by characterizing the outcome of the decision stage. If the signal s is disclosed at the communication stage, then joint decision making leads to $y = s + ab$. If no communication took place, then the agent chooses his preferred decision within the initial delegation set D. In the case where no delegation has taken place, we know from Section 4 that the usual unraveling result applies.

Consider a given delegation interval $D = [\underline{d}, \bar{d}]$. At the communication stage, the agent's best response is to withhold s and stick to the initial delegation set (which is legally binding) if and only if $u_A(s, s) < \max_{y \in D} u(y, s)$. This is formally stated below.

Lemma 5. *Given an initial delegation set $D = [\underline{d}, \bar{d}]$, the agent communicates s if and only if $s = \theta$ is such that*

$$s > \bar{d} - ab \text{ or } s < \underline{d} - (2 - a)b.$$

Figure 6 illustrates the result in Lemma 5 and shows that for values of θ included in or close to the initial delegation set information is withheld while it is disclosed for more extreme values of θ.

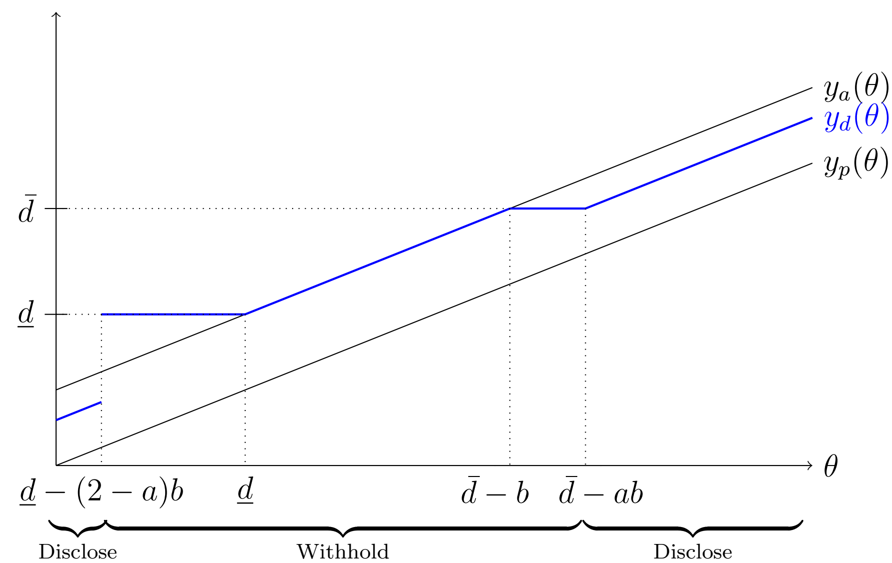

Figure 6: The agent's communication decision and associated outcome y

The agent has an incentive to reveal extreme information because any decision that might be taken when witholding the information is considered to be worse for the agent than the decision that the principal will take when she knows the state of the world. Or equivalently, for extreme values of θ any $d \in D$ is worse than the weighted average of the principal and the agent's preferred decisions.

It can easily be shown that in the presence of joint decision making, the equilibrium delegation set collapses to a single point, i.e., a degenerate delegation set. This is the purpose of the next lemma.

Lemma 6. *An equilibrium delegation set is a point:*

$$\underline{d}^* = \bar{d}^*.$$

Using this lemma, we can show that the possibility of ex post communication always crowds out delegation, i.e., the only possible equilibria are such that either no delegation takes place or a degenerate delegation set which is such that communication always takes place is optimal.

Proposition 4. *In equilibrium, the only admissible delegation sets are $D = \{d\}$ where $d \leq ab$. These delegation sets and no delegation lead to communication for all values of θ and are payoff equivalent for the principal.*

The intuition behind this result is somewhat reminiscent of Szalay (2005) who shows that to give the agents incentives to acquire information, the principal should exclude options that are interesting for uninformed agents. In our setting, the principal gives incentives to communicate through a similar mechanism where the no-communication (status quo) option is generally very bad for the agent.

The result in Proposition 4 is linked to the literature on veto-based mechanisms[10] in which the principal delegates a decision to the agent, but retains the right to veto any decision made by the agent. Mylovanov (2008) establishes the "veto-power principle", according to which the principal can implement an optimal outcome through veto-based delegation with a properly chosen default decision. In our model everything happens *as if* the action $y = 0$ is the default option and this gives the agent incentives to communicate his information to improve upon the default option.

As mentioned previously, Proposition 4 shows that communication always crowds out delegation because the equilibrium delegation set is degenerate and always leads to communication. This is the case even delegation alone performs better than communication (see condition 5). Our result can therefore be interpreted as showing that communication is a powerful tool and beneficial tool whenever (5) holds. However, in situations where

[10] Dessein (2002), Marino (2007) and Mylovanov (2008).

(pure) delegation is preferred (condition 5 is violated), communication is very bad in that it prevents the principal from credibly committing to a non-degenerate delegation set. In the next section we extend our results to the case where ex post communication is not always possible and discuss informally how the principal may commit to avoid ex post communication in a credible way.

7 Availability

As mentioned previously, our model does not take into account the *external* benefits from delegation such as freeing up time, etc. However, this is often seen as the major benefit from delegation. In this section we extend our model and show that our results are robust to the introduction of these additional benefits from delegation. To keep things simple, we focus on the situation where the principal has all the bargaining power at the communication stage.[11]

7.1 Exogenous availability

We do not directly model these additional gains from delegation, but allow for them to indirectly influence our results in the following way: We assume that at the communication stage, the principal is only available to communicate with probability $\alpha \in [0, 1]$. A large α means that the gains from delegation in freed up time is low and the principal is available for communication with a high probability. If α is low, the gains from freed up time are large and the principal is very likely to be engaged in work elsewhere and is thus prevented from engaging in communication with the agent.

Proposition 5. *When the principal is available for ex post communication with probability $\alpha \in (0, 1)$, the optimal delegation set is $D^\alpha = [0, \bar{d}^\alpha]$, where*

$$\bar{d}^\alpha = \begin{cases} 1 - \frac{b}{\sqrt{1-\alpha}} & \text{if } b \leq \frac{\sqrt{1-\alpha}}{1+\sqrt{1-\alpha}} \\ \frac{\sqrt{1-\alpha}}{1+\sqrt{1-\alpha}} & \text{otherwise.} \end{cases}$$

Before commenting on this proposition it is worth noticing that when α is equal to one, communication is always possible, we get a degenrate delegation set found in Proposition 4. In that case, there is minimal delegation. This means that the default option within

[11] This is equivalent to the setting where the agent's relative weight in the joint decision making is equal to 0.

the initial contract is so low that there are very strong incentives to communicate θ to the contracting authority. In fact, this allows the principal to obtain her preferred decision in all states of the world.

The more general result in Proposition 5 paints a more nuanced picture. The principal will still reduce the delegation set to give incentives to communicate, but she will not reduce it as much as in Proposition 4.

Clearly the upper bound of the delegation set D^α is between 0 and $1-b$. The principal therefore gives more incentives to communicate than in the delegation set in Proposition 1, but less than when she is always available to adapt the initial decision set after receiving the new information via communication. In fact, the size of the delegation set now trades off the gains from incentivising the agents to communicate and gains from a better-informed decision when the principal is too busy to communicate. This is illustrated in the following figure which plots the agent's preferred decision in D under no communication, full communication and communication with probability α.

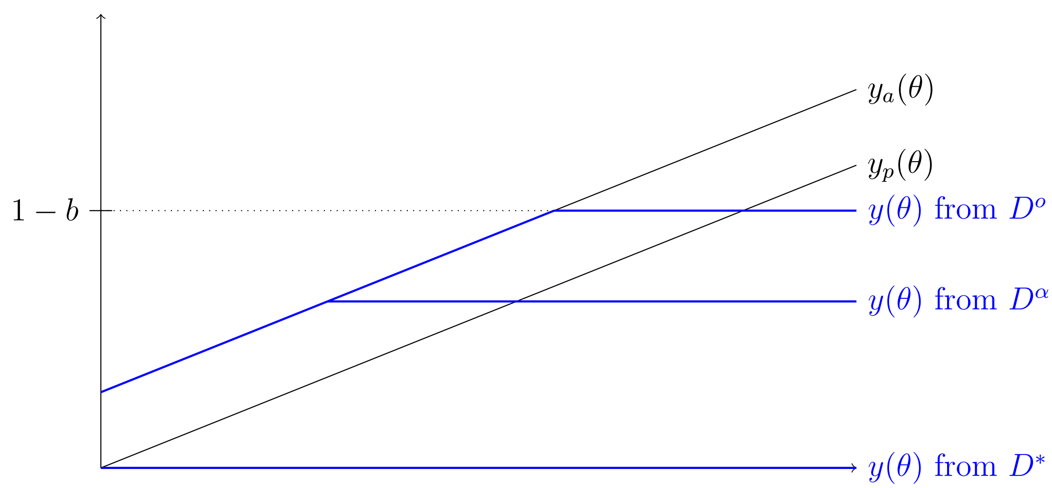

Figure 7: Decisions allowed within the initial delegation sets

Furthermore, taking the derivative of $\bar{d}^\alpha$ with respect to α shows that this upper bound is decreasing and concave in α. Intuitively, the more likely it is that the principal is available for communication, the smaller is the initial delegation set since this gives stronger incentives to communicate at the ex post stage. The following corollary states that the principal's expected utility is everywhere increasing and convex in α. Thus being more likely to be available for communication is good for the principal in this environment since it allows her to benefit from ex post communication and better decisions.

Corollary 1. *The principal's expected utility is increasing and convex in α.*

If the principal could directly influence α, it is immediate that the optimal solution would be to choose $\alpha = 1$ if condition 5 holds and $\alpha = 0$ otherwise. Our model suggests that if by making her time scarce, the principal indirectly commits to not communicating with the agent and when condition 5 is violated, this increases the principal's expected utility.

Finally, the convexity of the principal's expected utility in the one-agent environment suggests that in a more complex environment with multiple agents, these should be treated asymmetrically even when they are ex ante identical. This is the purpose of the next subsection.

7.2 Endogenous availability

In the previous section, we considered the interaction between a principal and a single agent. However, often a principal (contracting officer or manager) is facing several agents. We therefore introduce two agents, 1 and 2, into our framework. This allows us to endogenize the probability that the principal can communicate with an agent (denoted by α).

To do so we extend the model presented in Section 2 to include two agents; agent 1 and agent 2. We denote a decision related to agent i by $y_i \in Y_i \subset \mathcal{R}$ and the associated state by $\theta_i \in \Theta = [0, 1]$. Each agent has the same bias $0 < b < \frac{1}{2}$.[12] θ_1 and θ_2 are independently distributed.[13].

Absent *limited attention*, our previous results still hold (in particular Proposition 4). This suggests that strategically using delegation allows the principal to maximize the incentives to engage in communication and allows her to always implement her preferred decision. This would lead to a policy recommendation in which all agreements should be designed to lead to communication. However, in practice principals are busy and cannot spend time communicating with everyone all the time.

We therefore assume that the principal can at most communicate with one agent. We assume that, upon learning about a possibility of communication the principal does not instantly learn about its value, but this is only learnt through the actual communication.[14]

[12]We make the agents as similar as possible, and show that even in this case an asymmetric solution is preferred.

[13]For instance, different projects take different things into account in the decision-making process or the agents work on two different projects.

[14]This is what Dewatripont and Tirole (2005) coin *executive decision making*.

Therefore, the principal chooses with which agent to communicate without knowing the exact values of these communication opportunities.

The timing of this two-agent model is as follows. First, the contracting authority chooses a delegation set for each of the agents. The only enforceable delegation sets are the ones that freely allow the agent to choose whatever action he wants within a pre-specified set.[15] Second, the agents obtain a perfect signal $s_i \in \Theta$. Having observed this signal, each agent independently either chooses a decision within the initial delegation set or he can see if the principal wants to communicate. If the principal does not want to communicate with agent i, agent i has to choose a decision within his initial delegation set. If the principal wants to communicate with agent i, the agent reveals his information to the principal and the principal uses this information to allow for a decision outside the initial delegation set. For simplicity, we assume that the principal has all the bargaining power.

Let $V(D)$ be the expected value from communicating with an agent who has a delegation set D. Using Lemma 5, $V(D)$ can be written as

$$V(D) = \int_0^{\max\{0,\underline{d}-2b\}} (\underline{d}-\theta)^2 d\theta + \int_{\overline{d}}^1 \left(\overline{d}-\theta\right)^2 d\theta.$$

If there is only one agent who suggests ex post communication, then the principal engages in communication with this agent and chooses her preferred decision. If both agents propose communication, then the principal chooses to communicate with agent 1 if and only if

$$V(D_1) \geq V(D_2),$$

We allow for the principal to randomize in his choice of communication partner, but he cannot commit to this ex ante and the probability with which she randomizes has to be an equilibrium outcome.

The principal's problem thus becomes one of designing the two initial delegation sets D_1 and D_2 so as to maximize her ex ante expected utility, given how she will behave when/if the agents propose communication and a joint decision has to be made.

The result of this maximization is given in the next Proposition and its proof is related to Corollary 1 and the convexity of the principal's expected utility in α. In

[15] This assumption excludes delegation sets that are contingent on whether the other agent wants to communicate or not.

fact, in Proposition 6 we show that it is optimal for the principal to treat the agents asymmetrically. This result is in fact a direct consequence of Jensen's inequality.

Proposition 6. *The optimal delegation strategy for the principal is the "extreme asymmetry" situation where $D_i = D^*$ and $D_j = D^o$, $(i,j) \in \{1,2\}^2$, $i \neq j$. At the communication stage the principal always communicates with agent i.*

This proposition illustrates how even in an ex ante symmetric environment, the principal may prefer an asymmetric solution. Because of limited attention, treating the agents symmetrically either leads to too high losses due to lost communication opportunities or too much loss of control in the sense that the agents get to choose their preferred option too often. By treating the agents asymmetrically, the principal acknowledges that time is scarce and allows one of the agents some liberty in his option but strictly controls the decision of the other agent.

In fact, the convexity of the principal's expected utility with respect to the probability of being available for communication with one agent (see Corollary 1) implies that always communicating with one agent and never with the other yields a higher expected utility than communicating with both agent with equal probability.

A note of caution: Clearly this result depends on the assumptions of a quadratic loss function used in the delegation and communication literature. However, as long as the principal's utility is convex, the results should still go in this direction.

8 Discussion and conclusion

This paper has analysed two important decision making procedures; delegation and joint decision making. We have compared the benefits of these decision making procedures as well as the interaction between them. We have given a condition under which delegation is preferred to ex post joint decision making. Finally we have shown how the interaction between delegation and ex post joint decision making, always crowds out delegation.

Although some aspects of this paper are very stylized, the topic that is addressed is of tantamount importance. As our results indicate, introducing communication may change the delegation decisions drastically and therefore understanding this issue is indeed important.

Appendix

Proof of Proposition 1: From (1), the first-order condition with respect to $\underline{d}$ yields

$$2\int_0^{\max\{0,\underline{d}-b\}}(\underline{d}-\theta)d\theta = 0.$$

Any $\underline{d} \in [0, b]$ solves this. However, they are all payoff equivalent to $\underline{d} = 0$ since the agent never wants to choose $y \in [0, b)$.[16]

The first-order condition with respect to $\bar{d}$ is

$$2\int_{\min\{1,\max\{0,\bar{d}-b\}\}}^{1}(\bar{d}-\theta)d\theta = 0.$$

This is equivalent to

$$(1-\bar{d})^2 = (\bar{d} - \min\{1, \max\{0, \bar{d}-b\}\})^2.$$

If $\bar{d} \leq b$ then $\min\{1, \max\{0, \bar{d} - b\}\} = 0$. Then the first-order condition yields $\bar{d} = \frac{1}{2}$ which is a contradiction (since $b < \frac{1}{4}$). The two possible candidates are thus $\bar{d} = 1$ and $\bar{d} = 1 - b$.

Comparing the expected utility from these two candidates, allows us to conclude that $D^o = [1 - b]$.[17] ■

Proof of Proposition 2: The proof of Proposition 2 consists in showing that if an agent doesn't reveal θ, then it is optimal for the principal to choose $y = 0$.

Assume that if no information is revealed, then the principal chooses $\hat{y} > 0$. Any agent with $\theta > \hat{y} - ab$ has an incentive to reveal his information since this yields a benefit of $-(1-a)^2b^2$ whereas keeping quiet yields $-(\hat{y} - \theta - b)^2$. The latter is less than the former for all $\theta > \hat{y} - ab$.

Denote by $m(\theta)$ the distribution of θ for which the agent remains silent. We know from above that no agent with $\theta > \hat{y} - ab$ remains silent. Thus the support of m is included in $[0, \hat{y} - ab)$.

Without further information on θ it is optimal for the principal to choose y so as to

[16] To be complete, this also requires that $\bar{d} > b$. However, the next paragraph shows that this is not possible.

[17] These computations are straightforward. The expected utility when $\bar{d} = 1$ is $-b^2$ and when $\bar{d} = 1 - b$ it is $-b^2 + \frac{4}{3}b^3 > -b^2$.

maximize

$$\int_0^{\hat{y}-ab} (y-\theta)^2 \, m(\theta) d\theta.$$

This is equivalent to choosing y to the expected value of θ. However, because these θs are distributed on $[0, \hat{y} - ab)$, clearly the mean is less than $\hat{y}$. This contradict $\hat{y}$ being optimal and the principal should deviate to a lower y.

Therefore, the only possible equilibrium decision for the principal that faces a silent agent is to choose $y = 0$.

For all values of θ, choosing to reveal information and implementing $y = \theta + ab$ yields a higher utility than remaining silent and $y = 0$. Formally:

$$-((1-a)b)^2 > -(0-\theta-b)^2,$$

which is equivalent to

$$-ab < \theta.$$

This latter inequality is always true. ■

Proof of Proposition 3: Comparing W^D and W^B from Lemma 2 and 4 directly yields the result. ■

Proof of Lemma 5: An agent reveals information if and only if revealing θ allows him to get a higher utility then the best decision in D. That is, an agent with information θ reveals θ if and only if

$$-(1-a)^2 b^2 \geq \max_{d \in D} -(d-\theta-b)^2.$$

Since the agent's preferred decision is monotone in θ this is equivalent to $-(1-a)^2b^2 \geq -(\underline{d}-\theta-b)^2$ or $-(1-a)^2b^2 \geq -(\bar{d}-\theta-b)^2$.

These to inequalities are the same as $\theta \leq \underline{d} - (2-a)b$ or $\theta \geq \bar{d} - ab$. ■

Proof of Lemma 6: Denote the lower bound of the optimal delegation set D^* by $\underline{d}^*$. Assume that $\underline{d}^* < \bar{d}^*$. The proof consists of showing that a slight increase in this lower bound always (weakly) improves the expected utility of the principal compared to what D^* yields.

If $\underline{d}^* \in (0, b]$, then replacing $\underline{d}^*$ by $\underline{d}^* + \epsilon$, where $\epsilon > 0$ is sufficiently small, doesn't change the agent's behaviour since he never chooses an action in $(0, \underline{d}^*]$ when the action $\underline{d}^* + \epsilon$ is available. Therefore this change doesn't affect the principal's expected payoff.

I.e., in this case we can without changing anyone's behavior increase the lower bound of the optimal delegation set slightly. Thus, $\underline{d}^*$ cannot be optimal.

If $\underline{d}^* > b$, then by replacing $\underline{d}^*$ by $\underline{d}^* + \epsilon$, where $\epsilon > 0$ is sufficiently small, we obtain a profitable deviation from D^*. In fact, the value of this change of lower bound (as measured by the difference in the principal's expected utility) is

$$\begin{aligned}
\Delta U_P &= U_P(\underline{d}^* + \epsilon) - U_P(\underline{d}^*) \\
&= -\int_0^{\underline{d}^*+\epsilon-(2-a)b} (ab)^2 d\theta - \int_{\underline{d}^*+\epsilon-(2-a)b}^{\underline{d}^*+\epsilon} (\underline{d}^* + \epsilon - \theta)^2 d\theta - \int_{\underline{d}^*+\epsilon}^{\bar{d}^*-b} b^2 d\theta \\
&\quad + \int_0^{\underline{d}^*-(2-a)b} (ab)^2 d\theta + \int_{\underline{d}^*-(2-a)b}^{\underline{d}^*} (\underline{d}^* - \theta)^2 d\theta + \int_{\underline{d}^*}^{\bar{d}^*-b} b^2 d\theta.
\end{aligned}$$

This can be rewritten as

$$\begin{aligned}
\Delta U_P &= -\int_{\underline{d}^*-(2-a)b}^{\underline{d}^*+\epsilon-(2-a)b} (ab)^2 d\theta + \int_{\underline{d}^*-(2-a)b}^{\underline{d}^*+\epsilon-(2-a)b} (\underline{d}^* - \theta)^2 d\theta \\
&\quad + \int_{\underline{d}^*+\epsilon-(2-a)b}^{\underline{d}^*+\epsilon} \left[(\underline{d}^* - \theta)^2 - (\underline{d}^* + \epsilon - \theta)^2\right] d\theta - \int_{\underline{d}^*}^{\underline{d}^*+\epsilon} (\underline{d} + \epsilon - \theta)^2 d\theta \\
&\quad + \int_{\underline{d}^*}^{\underline{d}^*+\epsilon} b^2 d\theta \\
&= -\int_{\underline{d}^*+\epsilon-(2-a)b}^{\underline{d}^*} (2(\underline{d}^* - \theta) + \epsilon)\epsilon d\theta + \int_{\underline{d}^*}^{\underline{d}^*+\epsilon} \left[b^2 - (\underline{d}^* + \epsilon - \theta)^2\right] d\theta \\
&\quad + \int_{\underline{d}^*-(2-a)b}^{\underline{d}^*+\epsilon-(2-a)b} \left[(\underline{d}^* - \theta)^2 - (ab)^2\right] d\theta
\end{aligned}$$

For ϵ small enough, the first integral is negative (because $\underline{d}^* \leq \theta$ on this domain). Furthermore, for ϵ small enough, the second integral is positive since on this domain the loss from $\underline{d}^* + \epsilon$ is less than the loss from full delegation. It remains to show that the last integral is postive to conclude that $\Delta U_P \geq 0$.

On the domain $[\underline{d}^* - (2-a)b, \underline{d}^* + \epsilon - (2-a)b]$, we know that $\underline{d}^* - \theta \geq -\epsilon + (2-a)b$. Therefore, we have

$$\int_{\underline{d}^*-(2-a)b}^{\underline{d}^*+\epsilon-(2-a)b} \left[(\underline{d}^* - \theta)^2 - (ab)^2\right] d\theta \geq \int_{\underline{d}^*-(2-a)b}^{\underline{d}^*+\epsilon-(2-a)b} \left[(-\epsilon + (2-a)b)^2 - (ab)^2\right] d\theta.$$

Since $-\epsilon + (2-a)b()^2 \geq ((2-a)b)^2$, we have the following inequality

$$\int_{\underline{d}^*-(2-a)b}^{\underline{d}^*+\epsilon-(2-a)b} \left[(\underline{d}^*-\theta)^2-(ab)^2\right] d\theta \geq \int_{\underline{d}^*-(2-a)b}^{\underline{d}^*+\epsilon-(2-a)b} \left[b^2((2-a))^2-(a)^2\right] d\theta$$
$$\geq \int_{\underline{d}^*-(2-a)b}^{\underline{d}^*+\epsilon-(2-a)b} \left[b^2((2-a))^2-(a)^2\right] d\theta$$

Since $a \in [0,1]$, we can conclude that

$$\int_{\underline{d}^*-(2-a)b}^{\underline{d}^*+\epsilon-(2-a)b} \left[(\underline{d}^*-\theta)^2-(ab)^2\right] d\theta \geq \int_{\underline{d}^*-(2-a)b}^{\underline{d}^*+\epsilon-(2-a)b} \left[b^2(1-(a)^2\right] d\theta \geq 0$$

Thus $\Delta_P \geq 0$ and we have found our contradiction. ∎

Proof of Proposition 4: From Lemma 6 we know that an optimal delegation set is a point. Denote this point d.

The principal's objective is to maximize his expected utility.

$$\max_d - \int_0^{\max\{0,d-(2-a)b\}} (ab)^2 d\theta - \int_{\max\{0,d-(2-a)b\}}^{\min\{1,\max\{0,d-(2-a)b\}\}} b^2 d\theta - \int_{\min\{1,\max\{0,d-(2-a)b\}\}}^{1} (ab)^2 d\theta.$$

Define $M(d) = \max\{0, d-(2-a)b\}$ and $m(d) = \min\{1, M(d)\}$. The principal's objective reduces to

$$\max_d -b^2\left[a^2\left(1-m(d)+M(d)\right)+m(d)-M(d)\right].$$

Before proving Proposition 4 it is useful to show that we cannot at the same time have $d-(2-a)b<0$ and $d-ab>1$. In fact, this follows directly from studying the distance between the two and recalling that $a \in [0,1]$ and $b \in (0, \frac{1}{4})$. Formally,

$$d-(2-a)b-d+ab = -2(1-a)b \in [-\frac{1}{2}, 0].$$

If the distance (in absolute terms) is less than $\frac{1}{2}$, then one value cannot be below one and the other above one at the same time. The five possible configurations of $d-(2-a)b<0$ and $d-ab>1$ are as follows:

1. $d-(2-a)b \leq d-ab \leq 0$. This yields an expected utility $U_1 = -(ab)^2$
2. $d-(2-a)b \leq 0 \leq d-ab \leq 1$. This yields an expected utility $U_2 = -(ab)^2 - 2b^2(1-$

$a^2)(d-ab)$. This is less than U_1 and is therefore not optimal.

3. $0 < d-(2-a)b \leq d-ab \leq 1$. This yields an expected utility $U_3 = -(ab)^2 - 2b^3(1-a^2)(1-a)$. This is less than U_1 and is therefore not optimal.

4. $0 < d-(2-a)b < 1 \leq d-ab$. This yields an expected utility $U_4 = -(ab)^2 - b^2(1-a^2)(1-d+(2-a)b)$. Since by assumption $1-d+(2-a)b \in (0,1)$, this is less than U_1 and is therefore not optimal.

5. $1 \leq d-(2-a)b \leq d-ab$. This yields an expected utility $U_5 = -(ab)^2$. However, this candidate involves choosing $d > 1$ which is not admissible (recall that d is required to be in $[0,1]$).

It is thus optimal to choose d such that $d-ab \leq 0$ (and this is payoff equivalent to $d=0$).
■

Proof of Proposition 5: To prove Proposition 5 the following lemma is useful.

Lemma 7. *When $\underline{d}$ is such that $\underline{d}-2b > 0$, then a delegation set $D = [\underline{d}, \bar{d}]$ yields the same expected utility to the principal as $D' = [\underline{d}-\epsilon, \bar{d}-\epsilon]$.*

Proof.

$$\begin{aligned}
U_P(D') - U_P(D) = &- \int_0^{\underline{d}-2b-\epsilon} (ab)^2 d\theta - \int_{\underline{d}-2b-\epsilon}^{\underline{d}-\epsilon} (\underline{d}-\epsilon-\theta)^2 d\theta - \int_{\underline{d}-\epsilon}^{\bar{d}-b-\epsilon} b^2 d\theta \\
&- \int_{\bar{d}-b-\epsilon}^{\bar{d}-\epsilon} (\bar{d}-\epsilon-\theta)^2 d\theta - \int_{\bar{d}-\epsilon}^{1} (ab)^2 d\theta \\
&+ \int_0^{\underline{d}-2b} (ab)^2 d\theta + \int_{\underline{d}-2b}^{\underline{d}} (\underline{d}-\theta)^2 d\theta + \int_{\underline{d}}^{\bar{d}-b} b^2 d\theta \\
&+ \int_{\bar{d}-b}^{\bar{d}} (\bar{d}-\theta)^2 d\theta + \int_{\bar{d}}^{1} (ab)^2 d\theta.
\end{aligned}$$

Long and tedious (but straightforward) computations show that $U_P(D') - U_P(D) = 0$. □

We can thus limit the attention to delegation sets that have $\underline{d} \leq 2b$.

When the principal can engage in ex post communication with probability α, the principal's optimization problem can be written as

$$\max_{\bar{d}} - \int_0^{\max\{0,\bar{d}-b\}} b^2 d\theta - \int_{\max\{0,\bar{d}-b\}}^{\bar{d}} (\bar{d}-\theta)^2 d\theta - (1-\alpha) \int_{\bar{d}}^{1} (\bar{d}-\theta)^2 d\theta$$

subject to $0 \leq \bar{d} \leq 1$.

If $\bar{d}-b>0$, the first-order condition yields $\bar{d}=1-\frac{b}{\sqrt{1-\alpha}}$. This is such that $\bar{d}-b>0$ if and only if $\frac{\sqrt{1-\alpha}}{1+\sqrt{1-\alpha}}>b$.

If $\bar{d}-b\leq 0$, the first-order condition yields $\bar{d}=\frac{\sqrt{1-\alpha}}{1+\sqrt{1-\alpha}}$. This is such that $\bar{d}-b\leq 0$ whenever $\frac{\sqrt{1-\alpha}}{1+\sqrt{1-\alpha}}\leq b$. ■

Proof of Corollary 1: From Proposition 5 we obtain the following values for the first- and second derivative of $\bar{d}^{\alpha}$:

$$\frac{d\bar{d}^{\alpha}}{d\alpha}=\begin{cases}\frac{-b}{2(1-\alpha)^{3/2}} & \text{if } b\leq\frac{\sqrt{1-\alpha}}{1+\sqrt{1-\alpha}}\\ \frac{-1}{2(1+\sqrt{1-\alpha})^{2}\sqrt{1-\alpha}} & \text{otherwise,}\end{cases}$$

and

$$\frac{d^{2}\bar{d}^{\alpha}}{d\alpha^{2}}=\begin{cases}\frac{-3b}{4(1-\alpha)^{5/2}} & \text{if } b\leq\frac{\sqrt{1-\alpha}}{1+\sqrt{1-\alpha}}\\ \frac{-[1+3\sqrt{1-\alpha}]}{4(1+\sqrt{1-\alpha})^{3}(1-\alpha)^{3/2}} & \text{otherwise,}\end{cases}$$

Therefore, we know that $\bar{d}^{\alpha}$ is decreasing and concave in α. The principal's expected utility for a given α is

$$\begin{aligned}E[u_p] &= -\int_0^{\bar{d}^{\alpha}-b} b^2 d\theta-\int_{\bar{d}^{\alpha}-b}^{\bar{d}^{\alpha}}(\bar{d}^{\alpha}-\theta)^2 d\theta\\ &= \frac{2}{3}b^3-b^2\bar{d}^{\alpha}.\end{aligned}$$

It is thus straightforward to conclude that $E[u_p]$ is increasing and convex in α. ■

Proof of Proposition 6: Before proving the final result of Proposition 6, we present two partial results.

The first partial result states that when communication with one agent is more valuable than communication with the other agent, then the principal prefers *extreme asymmetric* delegation.

Lemma 8. *If there exists a solution to the principal's problem such that $V(D_i)>V(D_j)$, then $D_i=D^*$ and $D_j=D^o$, $(i,j)\in\{1,2\}^2$, $i\neq j$.*

Proof. If $V(D_1)>V(D_2)$, then at the communication stage, the principal prefers communication with agent 1.

If there is communication with agent i, then the principal can choose her preferred decision d_i and her utility from this is 0. In the case where both agents want to communicate, the principal chooses to communicate with agent 1 and ignore agent 2 (thereby forcing him to remain within his initial delegation set). The principal's objective is therefore to minimize her loss from decisions within the authorized delegation sets as well as losses from "missed" communication opportunities.

Furthermore, the same argument that was used to prove Lemma 6 can be used to show that $\underline{d}_1 = \underline{d}_2 = 0$.

The principal's optimization problem can therefore be written as

$$\begin{aligned}\max_{\bar{d}_1,\bar{d}_2} \int_0^1 & \left\{ -\int_0^{\bar{d}_1-b} b^2 d\theta - \int_{\bar{d}_1-b_1}^{\bar{d}_1} (\bar{d}_1-\theta)^2 d\theta \right\} d\tilde{\theta} \\ &+ \int_0^{\bar{d}_1} \left\{ -\int_0^{\bar{d}_2-b} b^2 d\theta - \int_{\bar{d}_2-b}^{\bar{d}_2} (\bar{d}_2-\theta)^2 d\theta \right\} d\tilde{\theta} \\ &+ \int_{\bar{d}_1}^1 \left\{ -\int_0^{\bar{d}_2-b} b^2 d\theta - \int_{\bar{d}_2-b}^{1} (\bar{d}_2-\theta)^2 d\theta \right\} d\tilde{\theta}.\end{aligned}$$

This simplifies to

$$\max_{\bar{d}_1,\bar{d}_2} \sum_{i=1,2} \left[-\int_0^{\bar{d}_i-b} b^2 d\theta - \int_{\bar{d}_i-b}^{\bar{d}_i} (\bar{d}_i-\theta)^2 d\theta \right] - (1-\bar{d}_1) \int_{\bar{d}_2}^1 (\bar{d}_2-\theta)^2 d\theta.$$

The derivative of this objective function with respect to $\bar{d}_1$ is constant and equal to $-b^2 + \int_{\bar{d}_2}^1 (\bar{d}_2-\theta)^2 d\theta$.

If this is negative, then we can conclude that $\bar{d}_1 = 0$ and, from the first-order condition with respect to $\bar{d}_2$, that $\bar{d}_2 = 1-b$. Plugging these values into $-b^2+\int_{\bar{d}_2}^1 (\bar{d}_2-\theta)^2 d\theta$ confirms that this value is indeed negative.

If $-b^2 + \int_{\bar{d}_2}^1 (\bar{d}_2-\theta)^2 d\theta$ is positive, then $\bar{d}_1 = 1$ and the derivative of the objective function with respect to $\bar{d}_2$ is negative so that $\bar{d}_2 = 0$. However, this cannot be the solution as it violates $V(D_1) > V(D_2)$.

Finally, if $-b^2 + \int_{\bar{d}_2}^1 (\bar{d}_2-\theta)^2 d\theta = 0$, it must be that $\bar{d}_2 = 1 - \sqrt[3]{3b^2}$. From the first-order condition with respect to $\bar{d}_2$, we also know that $\bar{d}_2 = 1 - \sqrt[3]{\frac{b^2}{1-\bar{d}_1}}$. Therefore, this can only be a solution if $\bar{d}_1 = \frac{2}{3}$. This is a candidate only if $b < \frac{1}{9}$ (otherwise $V(D_1)$ is not greater than $V(D_2)$). Furthermore, straightforward computations of the principal's expected utility shows that this candidate is dominated by $\bar{d}_1 = 0$ and $\bar{d}_2 = 1-b$.

The proof is the same when $V(D_2) > V(D_1)$. $\square$

Lemma 9. *If an interior solution exists such that $V(D_1) = V(D_2) \equiv V$, then at the* *communication stage the principal chooses to communicate with each agent with equal probability, the delegation sets are equal to* $D^s = [0, \bar{d}]$ *where*

$$\bar{d} = \begin{cases} 1 - \sqrt[3]{\frac{3}{2}b^2} & \textit{if } b \leq 0.38949 \\ 0.38949 & \textit{otherwise.} \end{cases}$$

Proof. The principal is maximizing her expected utility. Namely,

$$\max_{\underline{d}_1, \bar{d}_1, \underline{d}_2, \bar{d}_2} \sum_{i=1}^{2} \left[-\int_{\max\{0, \underline{d}_i - 2b\}}^{\max\{0, \underline{d}_i - b\}} (\underline{d}_i - \theta)^2 d\theta - \int_{\max\{0, \underline{d}_i - b\}}^{\bar{d}_i - b} b^2 d\theta - \int_{\bar{d}_i - b}^{\bar{d}_i} (\bar{d}_i - \theta)^2 d\theta \right]$$
$$\alpha \left(1 - \bar{d}_1 + \max\{0, \underline{d}_1 - 2b\}\right) \left[-\int_0^{\max\{0, \underline{d}_2 - 2b\}} (\underline{d}_2 - \theta)^2 d\theta - \int_{\bar{d}_2}^{1} (\bar{d}_2 - \theta)^2 d\theta \right]$$
$$(1 - \alpha) \left(1 - \bar{d}_2 + \max\{0, \underline{d}_2 - 2b\}\right) \left[-\int_0^{\max\{0, \underline{d}_1 - 2b\}} (\underline{d}_1 - \theta)^2 d\theta - \int_{\bar{d}_1}^{1} (\bar{d}_1 - \theta)^2 d\theta \right],$$

where $\alpha \in [0, 1]$ is the (endogenous) probability that the principal engages in communication with agent 1 in the communication sub-game where both agents wants to communicate.

The first step of the proof is to show that $\underline{d}_1 = \underline{d}_2 = 0$. Assume, on the contrary, that the optimal delegation set is such that $\underline{d}_1 = 0$. We will now proceed by showing that this is impossible.[18]

If $\underline{d}_1 \leq 2b$, then D_1 is payoff equivalent to $D_1 \bigcup [0, \underline{d}_1]$ and we can consider that the optimal delegation set has $\underline{d}_1 = 0$.

The proof is more involved when $\underline{d}_1 > 2b$. In this case we have an interior solution for $\underline{d}_1$ and therefore it must satisfy the first-order condition

$$b^2 - \alpha U_2 - (1 - \alpha) \left(1 - \bar{d}_2 + \max\{0, \underline{d}_2 - 2b\}\right) \underline{d}_1^2 = 0,$$

where $U_2 = \int_0^{\max\{0, \underline{d}_2 - 2b\}} (\underline{d}_2 - \theta)^2 d\theta + \int_{\bar{d}_2}^{1} (\bar{d}_2 - \theta)^2 d\theta$ is independent of D_1. Note that the first-order condition with respect to $\bar{d}_1$ looks very similar

$$b^2 - \alpha U_2 - (1 - \alpha) \left(1 - \bar{d}_2 + \max\{0, \underline{d}_2 - 2b\}\right) (1 - \bar{d}_1)^2 = 0.$$

We can therefore conclude that in this scenario $\underline{d}_1 = 1 - \bar{d}_1$.

[18] The same argument can be used to show that $\underline{d}_2 = 0$.

Now, consider the delegation set $D_1' = [\underline{d}_1 - \epsilon, \bar{d}_1 - \epsilon]$. For $\epsilon = 0$ this is the optimal delegation set and for $\epsilon > 0$ this is a delegation set of the same length as the optimal delegation set, but shifted to the left (to lower values of the decision parameter). The principal's expected utility from (D_1', D_2) is

$$\begin{aligned}
U_P^e = &- \int_{1-\bar{d}_1-2b-\epsilon}^{1-\bar{d}_1-b-\epsilon} (1-\bar{d}_1-\epsilon-\theta)^2 d\theta - \int_{1-\bar{d}_1-b-\epsilon}^{\bar{d}_1-b-\epsilon} b^2 d\theta - \int_{\bar{d}_1-b-\epsilon}^{\bar{d}_1-\epsilon} (\bar{d}_1-\epsilon-\theta)^2 d\theta \\
&- 2\alpha(1-\bar{d}_1-\epsilon-b)U_2 \\
&- (1-\alpha)(1-\bar{d}_2+\max\{0,\underline{d}_2-2b\}) \left[\int_0^{1-\bar{d}_1-2b-\epsilon} (1-\bar{d}_1-\epsilon-\theta)^2 d\theta + \int_{\bar{d}_1-\epsilon-\epsilon}^{1} (\bar{d}_1-\epsilon-\theta)^2 d\theta \right].
\end{aligned}$$

The derivative of this expression with respect to ϵ is

$$\begin{aligned}
\frac{dU_P^e}{d\epsilon} =& 2\alpha U_2 + (1-\alpha)(1-\bar{d}_2+\max\{0,\underline{d}_2-2b\}) \left[(1-\bar{d}_1-\epsilon)^2 - (1-\bar{d}_1+\epsilon)^2 \right] \\
=& 2\alpha U_2 - 4\epsilon(1-\alpha)(1-\bar{d}_2+\max\{0,\underline{d}_2-2b\})(1-\bar{d}_1).
\end{aligned}$$

For small enough ϵ, this derivative is positive (the first term dominates) and we can therefore conclude that for small values of ε the expected utility of the principal is higher than with the optimal delegation set. Hence the optimal delegation set is not optimal and we can conclude that $\underline{d}_1 = 0$.

When $V(D_1) = V(D_2)$, the principal is indifferent between communication with the two agents. However, since $\underline{d}_1 = \underline{d}_2 = 0$, this is equivalent to $\int_{\bar{d}_1}^1 \left(\bar{d}_1 - \theta\right)^2 d\theta = \int_{\bar{d}_2}^1 \left(\bar{d}_2 - \theta\right)^2 d\theta$. This can only be true if $\bar{d}_1 = \bar{d}_2$.

The principal's objective can be written as

$$\begin{aligned}
\max_{\bar{d}_1,\bar{d}_2} &- \int_0^{\max\{0,\bar{d}_1-b\}} b^2 d\theta - \int_0^{\max\{0,\bar{d}_2-b\}} b^2 d\theta - \int_{\max\{0,\bar{d}_1-b\}}^{\bar{d}_1} (\bar{d}_1-\theta)^2 d\theta - \int_{\max\{0,\bar{d}_2-b\}}^{\bar{d}_2} (\bar{d}_2-\theta)^2 d\theta \\
&- \alpha(1-\bar{d}_1) \int_{\bar{d}_2}^1 (\bar{d}_2-\theta)^2 d\theta - (1-\alpha)(1-\bar{d}_2) \int_{\bar{d}_1}^1 (\bar{d}_1-\theta)^2 d\theta.
\end{aligned}$$

If $\alpha \in \{0,1\}$, we get the same delegation sets as in Lemma 3. However, this violates $V(D_1) = V(D_2)$.

The rest of the proof therefore focuses on $\alpha \in (0,1)$. When $\bar{d}_i - b \geq 0$ the associated first-order conditions for an interior solution are

$$
\begin{aligned}
&- b^2 + \alpha \int_{\bar{d}_2}^{1} (\bar{d}_2 - \theta)^2 d\theta + (1-\alpha)(1-\bar{d}_2)(\bar{d}_1 - 1)^2 = 0,\\
&- b^2 + (1-\alpha) \int_{\bar{d}_1}^{1} (\bar{d}_1 - \theta)^2 d\theta + \alpha(1-\bar{d}_1)(\bar{d}_2 - 1)^2 = 0.
\end{aligned}
$$

Since $V(D_1) = V(D_2)$ implies that $\bar{d}_1 = \bar{d}_2$, we can only have a solution for $\alpha = \frac{1}{2}$. Using the first-order conditions yields $\bar{d}_1 = \bar{d}_2 = 1 - \sqrt[3]{\frac{3}{2}b^2}$. This satisfies $\bar{d}_i - b \geq 0$ if and only if $b \leq 0.38949$.

When $\bar{d}_i - b < 0$, the associated first-order conditions for an interior solution are

$$
\begin{aligned}
&- \bar{d}_1^2 + \alpha \int_{\bar{d}_2}^{1} (\bar{d}_2 - \theta)^2 d\theta + (1-\alpha)(1-\bar{d}_2)(\bar{d}_1 - 1)^2 = 0,\\
&- \bar{d}_2^2 + (1-\alpha) \int_{\bar{d}_1}^{1} (\bar{d}_1 - \theta)^2 d\theta + \alpha(1-\bar{d}_1)(\bar{d}_2 - 1)^2 = 0.
\end{aligned}
$$

Using the same arguments as above. $\bar{d}_1 = \bar{d}_2 \equiv \bar{d}$ and $\alpha = \frac{1}{2}$. Therefore the first-order condition can be rewritten as

$$
-\bar{d}^2 + \frac{2}{3}(1-\bar{d})^3 = 0.
$$

the solution is $\bar{d} = 0.38949$. □

Finally, by comparing the outcome of all the possible delegation scenarios, we obtain the main result in the multi-agent case. In fact, there the five possible candidates are given in Lemma 8, 9 and by the possible symmetric corner solutions ($\underline{d} = \bar{d} = 0$, $\underline{d} = \bar{d} = 1$ and ($\underline{d} = 0, \bar{d} = 1$)). Comparing the principal's expected payoff in these cases yields the result:

1. For $\underline{d}_1 = \underline{d}_2 = 0$, $\bar{d}_i = 0$ and $\bar{d}_j = 1 - b$, the principal's expected payoff is $W_1 = -(1-2b)b^2 - \frac{2}{3}b^3$.

2. If the solution is given by Lemma 4, the principal's expected payoff is

$$
W_2 = \begin{cases} -2(1-b)b^2 - \frac{2}{3}b^3 + \frac{3}{2}b^2\sqrt[3]{\frac{3}{2}b^2} & \text{if } b \leq 0.38949 \\ -0.085698405 & \text{otherwise.} \end{cases}
$$

3. $\underline{d}_1 = \underline{d}_2 = \bar{d}_1 = \bar{d}_2 = 0$: The principal's expected payoff is $W_3 = -\frac{1}{3}$.

4. $\underline{d}_1 = \underline{d}_2 = 0$ and $\bar{d}_1 = \bar{d}_2 = 1$: The principal's expected payoff is $W_4 = -2(1-b)b^2 - \frac{2}{3}b^3$.

5. $\underline{d}_1 = \underline{d}_2 = \bar{d}_1 = \bar{d}_2 = 1$: The principal's expected payoff is $W_5 = -\frac{2}{3}[b+(2b)^3(1-b)]$.

Comparisons of $W_1 - W_4$ show that W_1 is largest for all $b \in (0, \frac{1}{2})$. Thus case 4 cannot characterize the optimal delegation set.

It is easy to show that the case 4 is (in the eyes of the principal) dominated by the extreme asymmetry candidate from Lemma 3. In fact, $W_1 - W_4 = b^2 > 0$.

For the case 3, it is also easy to show that the principal prefers the solution candidate from Lemma 3.

$$W_1 - W_3 = -(1-2b)b^2 - \frac{2}{3}b^3 + \frac{1}{3}.$$

Since $(1-2b) > 0$ and $b < \frac{1}{2}$, we have

$$W_1 - W_3 > -(1-2b)\frac{1}{4} - \frac{1}{12} + \frac{1}{3}.$$

Finally, because $b > 0$, we have

$$W_1 - W_3 > -\frac{1}{4} - \frac{1}{12} + \frac{1}{3} = 0.$$

Furthermore, case 5 is dominated by case 4. In fact

$$\begin{aligned} W_4 - W_5 =& \frac{2}{3} - 2b^2 + \frac{b^3}{3}[20 - 16b] \\ >& \frac{2}{3} - \frac{2}{4} + 12\frac{b^3}{3} > 0, \end{aligned}$$

where the first inequality follows from $b < \frac{1}{2}$.

It only remains to show when the extreme asymmetry solution in Lemma 3 is preferred to the symmetric interior solution in Lemma 4.

When $b \leq 0.38949$ a direct comparison of payoffs yields

$$\frac{1}{b^2}(W_1 - W_2) = 1 - \frac{3}{2}\sqrt[3]{\frac{3}{2}b^2}$$

This is positive (and thus the extreme asymmetry solution is preferred) if and only if $b \leq \frac{4}{9}$. Since $b \leq 0.38949$, we can conclude that the extreme asymmetric solution always dominates the interior solution for $b \leq 0.38949$.

Note that $\frac{dW_1}{db} = -2b(1-2b) < 0$. Therefore when $b > 0.38949$, we have $W_1 - W_2 > W_1|_{b=\frac{1}{2}} - W_2 = -0.083333333 + 0.085698405 = 0,002365072 > 0$. ■

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