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Size Matters - "Over"investments in a Relational Contracting Setting

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Abstract

The corporate finance literature documents that managers tend to overinvest into physical assets. A number of theoretical contributions have aimed to explain this stylized fact, most of them focussing on a fundamental agency problem between shareholders and managers. The present paper shows that overinvestments are not necessarily the (negative) consequence of agency problems between shareholders and managers, but instead might be a second-best optimal response if the scope of court-enforceable contracts is limited. In such an environment a firm has to rely on relational contracts in order to manage the agency relationship with its workforce. The paper shows that investments into physical productive assets enhance the enforceability of relational contracts and hence investments optimally are "too high".

JEL Codes: C73, D21, D86, G32

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1 Introduction

A prominent and well established stylized fact in corporate finance is that managers tend to overinvest, or, as Stein (2003) puts it, that they "...have an excessive taste for running large firms, as opposed to simply profitable ones" (p. 119). Numerous theories have been developed to explain this pattern: managers' taste for empire building; see Williamson (1964), Jensen (1986), Jensen (1993); short-termism of managers who focus on activities the market can easily observe; see Stein (1989), Bebchuk and Stole (1993); managerial overconfidence into their own abilities; see Roll (1986), Heaton (2002); or asymmetric information with respect to new investment opportunities; see Inderst and Klein (2007). All these theories share the perception that overinvestments are caused by agency problems between a firm and its management. Hence, mechanisms to reduce free cash-flow – and consequently the management's ability to start projects with a negative net-present-value – have been suggested as optimal responses to this perceived fundamental agency problem. However, there is no clear evidence that reducing a firm's free cash-flow and restricting a manager's investment opportunities increases firm value - on the contrary, investors often assess capital investments positively; see McConnell and Muscarella (1985), Myers (2003).

This paper shows that overinvestments – investments where marginal costs exceed (direct) marginal benefits – can have a positive impact on firm value, namely by making it easier to motivate a firm's workforce: If formal, court-enforceable incentive contracts are limited in their scope – e.g., because of a lack of good verifiable performance measures – managing a firm's employer-worker relationships in a volatile environment becomes difficult. In this case, implicit agreements (so-called relational contracts) are needed to motivate a firm's workforce. If the firm's commitment in these relational contracts is limited, though, a firm's employment might be restricted to a level that is inefficiently low because the firm cannot make credible promises to all agents it would like to motivate. Moreover, if it is unable to motivate the employee, it is not optimal to employ him in the first place. Similar problems can arise if firms are liquidity constrained in some states of the world which means that they have to defer payments into the future, exacerbating commitment problems.

Then, managers optimally overinvest in order to improve the enforceability of relational contracts with their employees as capital investments ceteris paribus increase future revenues, whereas the associated investment costs are sunk. Thereby, larger investments A) strengthen the firm's commitment by raising future profits, and B) increase its financial flexibility by generating additional cash flow. Hence, overinvestments partially mitigate contracting frictions, and are not necessarily an inefficient manifestation of intra-firm agency problems.

The starting point of this paper is the moral hazard principal agent literature, which focuses on unobservable effort choice as a determinant of firm profitability (or productivity). An improved solution to the moral hazard problem will, ceteris paribus, increase productivity. While there exists a large literature, building on Holmstrom (1979) and Grossman and Hart (1983), focusing on explicit contracts that reward the agent based on verifiable performance measures, there has been an increased interest in implicit contracts as a way to mitigate the moral hazard problem; see, e.g. Bull (1987), MacLeod and Malcomson (1989), Levin (2003). In a more recent contribution, Gibbons and Henderson (2013) argue that different aspects of relational contracts are responsible for observed persistent performance differences among seemingly similar enterprises that also exist within industrialized countries.

Relational (or implicit) contracts employ repeated game logic to use observable but unverifiable information and do not rely on explicit, court enforceable, performance measures to motivate workers. In the recent past there have been a number of papers investigating richer dynamics and the effect of stochastic shocks on the efficiency and stability of implicit contracts – see, e.g., Li and Matouschek (2013), Englmaier and Segal (2011) – to which the present paper relates. Finally, we relate to some recent papers linking a firm's financing conditions and decisions to the enforceability of relational contracts. Contreras (2013) analyzes how relational contracts formed between a firm and its supplier interact with the quality of financial markets. Fahn, Merlo, and Wamser (2014) show how equity financing helps to enforce relational contracts. If debt is too high, a firm is able to share some of the costs of reneging on relational contracts with its creditors.

We develop a model where a firm's inputs are physical capital and labor, and the time horizon is infinite. The firm can employ an arbitrary number of agents and output in every period is increasing in the firm's workforce, as long as the employed agents choose high effort. Moreover, capital investments are made at the beginning of the game and increase output as well.

Different from the corporate finance literature – where overinvestments generally are the consequence of agency problems between shareholders (who might have financed the initial investment into capital) and a firm's management – we assume that both parties' preferences are perfectly aligned. However, there are two features of the environment that can restrict the firm's behavior and consequently reduce efficiency and profits:

First, neither an employees's effort nor the resulting output are verifiable.[1] As the firm and its management want to induce agents to exert high effort and agents have to be sufficiently incentivized, relational contracts where the firm implicitly promises to reward performance have to be employed. In case the firm does not honour its promise of rewarding performance,

[1]To motivate this assumption, assume that there are components of an employee's effort where verification is too difficult or too costly.

this will induce a loss of trust of the firm's workforce, which will subsequently refrain from exerting high effort. Furthermore, if the firm reneges on one of its promises, it is optimal to renege on all of its promises, and hiring additional workers not only increases equilibrium profits but also the temptation to deviate. All this implies that the firm only has an incentive to honor its promises if the future value of the firm – determined by the net present value of its future profits – is sufficiently large. This constraint on credible promises concerning future rewards for high effort will limit the number of agents the firm can motivate (and hence profitably employ) through such promises. The firm may be restricted to an employment level that is below the (unconstrained) profit-maximizing value. Hence, frictions in the enforcement of formal contracts can restrict firm size, a result also derived by Powell (2013).

Second, we explicitly model the consequences of a volatile environment. In particular, we assume that the firm may face varying market conditions: demand may be either high or low. We assume that the firm faces a liquidity constraint and can only use generated cash-flow – funds that have been earned by selling its output – to compensate employees.[2] Under this assumption an employee's compensation will have to vary with the company's earnings. Then, a major part of total payments is paid out to agents whenever demand is high. However, the maximum amount the firm is willing to pay out instead of reneging and shutting down is determined by its expected future profits, independent of the difference between available liquidity across states of the world. A larger difference between the situation in good and bad states and hence higher payment obligations in the good state increases the firm's reneging temptation – and will limit the firm's employment to an inefficiently low level even if its discount factor is arbitrarily close to (but still bounded away from) 1.

The firm can mitigate its problems with motivating and managing its workforce by (seemingly excessive) investments into physical capital. Since investment costs are sunk, they are not considered by the firm whenever it faces the decision whether to keep its promises in the relational contract or not. Due to sunk investment costs and as an increased capital base positively affects future profits, investments into capital improve the enforceability of relational contracts. In this context, overinvestments – i.e., capital levels where the marginal investment costs exceed the direct marginal benefits – are possible. Overinvestments also relax employment restrictions induced by low liquidity: additional investments raise the output in all states and hence increase the available cash-flow also in bad states of the world, i.e., when demand is low.

[2] In a robustness exercise we show that the ability to accumulate and hold cash reserves does not solve this problem: Holding cash reserves is associated with present costs - these funds cannot be consumed today - in exchange for future profits, since higher employment can only be enforced in later periods. Hence, it will not be optimal to accumulate cash reserves that are sufficient to enforce efficient employment.

This result is interesting in comparison to the classic literature where overinvestments are interpreted as a consequence of agency problems between shareholders and management: Means to *reduce* free cash-flow – like issuing debt – are proposed as (second-best) solutions to the problem of overinvestments; see, e.g., Hart and Moore (1995), Zwiebel (1996). We show that the additional cash-flow generated by overinvestments can be used to *increase* the firm's financial flexibility and *mitigate* the agency problem between the firm and its workforce.

Moreover, our results are in line with several empirical studies comparing the behavior of private and publicly listed firms. The former are generally regarded to attach more importance to long-term relationships, which in our model translates into having a larger discount factor. We predict that the use of overinvestments generally is more pronounced if the firm's discount factor is small. This relates to Bargeron, Schlingemann, Stulz, and Zutter (2008), who show that publicly listed US firms pay substantially more for acquisitions than privately held firms. According to our model, such investments are more valuable for publicly listed than for privately-held firms – since the former also use them to increase commitment in relational contracts with their employees. Hence, the willingness to pay for acquisitions of publicly listed firms should ceteris paribus be higher. In addition, the "constrained-liquidity+limited-commitment" mechanism is most likely to restrict employment for intermediate discount factors. This also implies that residual profits (more precisely, freely available cash flows) vary more at firms with a higher discount factor, because constrained liquidity forces many of them to pass all their revenues on to employees if demand is low. Firms with a low discount factor, on the other hand, can also keep some cash in low-demand states. Given we expect firms to pay higher dividends to their shareholders in states where they have more free cash flow, we predict dividend payments of firms with a higher discount factor to vary more than of firms with a lower discount factor. This prediction is consistent with data documented by Michaely and Roberts (2012): privately held firms (having higher discount factors) smooth dividends significantly less than firms whose shares are publicly traded.

The paper proceeds as follows. After introducing the basic model setup we show how the firm's liquidity constraint can restrict firm size. If the volatility in the market, i.e., the difference between the firm's earning in the good and bad state, is sufficiently large, employment always is inefficiently low, irrespective of the total expected surplus that can be generated. In the next part, we show that investments into physical assets relax the firm's constraints, inducing overinvestments whenever employment is restricted. Finally, we show that our results still hold when the firm is able to accumulate and hold cash reserves. The Appendix collects proofs and details of the derivations.

2 Model Setup

We first describe the basic model. In the next subsection we will describe the informational structure of the game. There is one firm ("principal"), inputs for production are capital and labor. Time is discrete, the time horizon infinite, and all players are risk-neutral and share a common discount factor $\delta \in (0,1)$. In the first period of the game, $t = 0$, the firm makes capital investments $k \in \mathbb{R}^+$. Capital investments are associated with marginal investment costs of 1. They can either be funded by the firm itself or raised from outside investors. To sharpen our argument, we abstract from any agency conflicts between the principal and outside investors. Hence, the following analysis does not depend on the sources of funds and we do not further pursue this issue. Furthermore, we assume that capital investments are relationship specific and for simplicity set the outside value of invested capital to zero. Our qualitative results do not hinge on this stark assumption and would still hold as long as the outside value was sufficiently smaller than the inside value or if a liquidation decision was irreversible.

The labor market consists of a mass N of homogenous prospective employees ("agents"). In every period $t = 1, 2, ...$, the firm makes a short-term employment offer to $\underline{n}_t \geq 0$ agents; if $\underline{n}_t = 0$, the firm consumes its outside option $\overline{\pi} \geq 0$. Agents receiving an offer are indexed $i \in [0, \underline{n}_t]$. N – the size of the labor market – is sufficiently large for the firm to not be exogenously bounded when choosing $\underline{n}_t$. Each offer consists of a fixed wage $w_{it} \geq 0$ and the promise to make a contingent bonus payment $b_{it} \geq 0$.[3] This bonus promise provides the agents with incentives and is paid at the principal's discretion.

An agent's decision whether to accept an offer or not is captured by $d_{it} \in \{0,1\}$, where $d_{it} = 1$ describes an acceptance and $d_{it} = 0$ a rejection. The number of agents who accept an offer is n_t, i.e., $n_t = \int_0^{\underline{n}_t} d_{it} di$. After accepting an offer, all n_t employed agents then make their effort choice. Effort is binary, $e_{it} \in \{0,1\}$, and associated with private costs $c(e_{it})$, where $c(0) = 0$ and $c(1) = c > 0$. All agents who rejected an offer as well as those who did not receive one consume their outside options $\overline{u} \geq 0$ in the respective period.

The firm's output in period t is $y_t = f\left(\int_0^{n_t} e_{it} di,\, k\right)$. Note that if all of the n_t employed agents choose $e_t = 1$, then $y_t = f(n_t,\, k)$. $f(\cdot,\, k)$ is a continuous function in both arguments, with $f_1, f_2 > 0$ and $f_{11}, f_{22} < 0$. We do not place restrictions on the sign of f_{12} but assume that the second order condition in the maximization problem below is always satisfied.

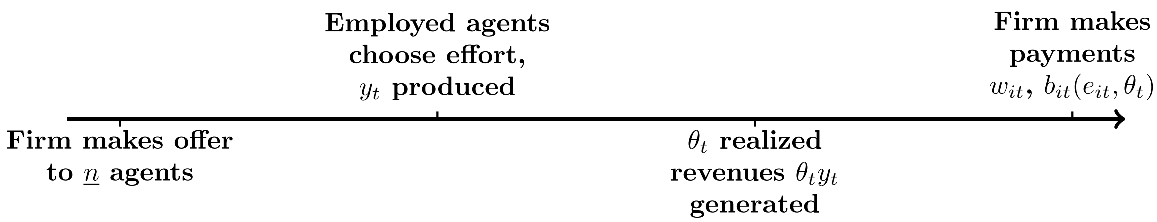
After producing, the firm sells the output and generates revenues $\theta_t y_t$. $\theta_t \in \{\theta^l, \theta^h\}$ is a parameter specifying the demand conditions – think "price" – for the firm's output - with

[3] Note that the restriction of payments to non-negative values is without loss of generality in our setting.

$0 < \theta^l < \theta^h$ - and is realized after the output has been produced. High demand is realized with probability p, low demand with $1-p$. These probabilities are independent over time, i.e., there is no persistency in demand conditions. After the sale of output, payments w_{it} and b_{it} are made. Hence, the bonus can be contingent on the realization of θ (in addition to chosen effort), i.e., $b_{it}(e_{it}, \theta_t)$, whereas w_{it} is fixed by assumption.

We assume the firm to be liquidity constrained: all funds used to compensate employees must be earned via the sale of its products. This implicitly assumes that the firm does not retain profits earned in earlier periods and has no access to credit markets. We relax the first assumption in Section 6 and show that it does not have any impact on our qualitative results if retaining profits is costly. The second assumption is without loss of generality if firms can not use their physical assets to collateralize a loan (e.g. because they have zero value to outsiders). In this case borrowing would only shift the commitment problem from the firm's relationship with its employees to the relationship with its creditors.

The timing within every period t is summarized in the following graph:



Information, Payoffs, Strategies and Equilibrium

As discussed above, an employed agent's effort, e_{it}, is not verifiable and hence cannot be used in formal, court-enforceable, contracts. Moreover, we abstract from explicit incentive contracts and assume output y_t and revenues $\theta_t y_t$ to be not verifiable as well.[4]

Effort levels, $\underline{n}_t$, acceptance decisions d_{it}, and wage and bonus payments can be observed by the principal as well as by all other agents. Thus, multilateral relational contracts can be used to motivate employees; see Levin (2002). Informally speaking, this implies that all agents as well as the labor market can detect any deviation from equilibrium behavior. In particular, any deviation by the principal can be punished by all agents, increasing the principal's commitment power.

[4] Note that this assumption is without loss of generality since output cannot be used to detect an individual agent's effort level in this multi-agent production setting.

Finally, every realization of the demand parameter θ is observable to all parties – it might reflect the general state of the economy or the specific industry where the firm is active.[5] Hence there are no informational asymmetries, and agency problems only arise due to the non-verifiability of effort.

The firm's payoff in any period $t \geq 1$ is

$$\Pi_t = \mathrm{E}\left\{\sum_{\tau=t}^{\infty}\delta^{\tau-t}\left[\overline{\pi} + \mathbf{1}_{\{\underline{n}_\tau>0\}}\left(\theta_\tau f\left(\int_0^{n_\tau} e_{i\tau}di,\, k\right) - \int_0^{n_\tau}\left(w_{i\tau} + b_{i\tau}(e_{it},\theta_\tau)\right)di - \overline{\pi}\right)\right]\right\}$$

where the indicator function describes whether the principal made offers (and hence operates the firm) in period t, and expectation is over the realizations of θ. Furthermore,

$$\Pi_0 = -k + \delta\Pi_1.$$

For an agent i who received an offer, the expected discounted payoff stream equals

$$U_{it} = \mathrm{E}\left\{\sum_{\tau=t}^{\infty}\delta^{\tau-t}\left[\overline{u} + d_{i\tau}\left(w_{i\tau} + b_{i\tau}(e_{it},\theta_\tau) - c(e_{it}) - \overline{u}\right)\right]\right\}.$$

We only consider pure strategies, and define a multilateral relational contract as a *Subgame Perfect Equilibrium (SPE)* where after every history strategies determine a Nash Equilibrium. More precisely, we are interested in a SPE that maximizes the firm's expected profits at the beginning of the game, i.e., Π_0; we think this is a natural choice given that the labor market is assumed to be competitive, which allows firms to make take-it-or-leave-it offers to agents.

Furthermore, we can omit the index i. Since agents are identical and multilateral relational contracts can – and should – be used, all employees will be treated the same in equilibrium. In addition, we focus on equilibria where employment offers are accepted by employees ($d_{it} = 1$ for all $i \in [0, \underline{n}_t]$), and where all employed agents choose $e = 1$ (i.e., $\int_0^{n_t} e_{it}di = n_t$). This clearly is optimal, since making an offer which is rejected and employing an agent who does not exert effort is dominated by not making an offer to any of such agents.

In Appendix A, in Lemma A1, we show that we can focus on stationary contracts that are independent of calendar time, as well as past realizations of demand shocks. Hence, employment as well as w_t and b_t are constant over time, allowing us to omit time subscripts t. This is driven by shocks being distributed i.i.d., by employment being chosen *before* the

[5] See Englmaier and Segal (2011) or Li and Matouschek (2013) for an analysis of situations where shocks to the firm are not observable to the workforce.

state of the world is realized, and by the principal - the party facing the liquidity constraint - being able to reap the whole surplus. Hence, only equilibrium bonus payments might vary over time – depending on the respective realization of θ. There, b^h is the equilibrium bonus given θ^h is observed, and b^l the bonus for θ^l. Finally, we set the outside options $\overline{\pi} = \overline{u} = 0$. This assumption does not affect our results qualitatively, but simplifies our analysis given the firm's liquidity constraint.

3 Maximization Problem and Constraints

Our objective is to find levels of capital k, (stationary) employment n, as well as a compensation package (w, b^l, b^h) to maximize

$$\Pi_0 = -k + \delta\Pi,$$

where

$$\Pi = \frac{p(\theta^h f(n,k) - nb^h) + (1-p)(\theta^l f(n,k) - nb^l) - nw}{1-\delta}$$

is the firm's expected discounted equilibrium payoff stream in any period $t \geq 1$.

In addition, the following constraints have to be satisfied to enforce a stationary SPE. First, it must be optimal for an agent who receives an offer to accept it. This is captured by an individual rationality (IR) constraint,

$$U \geq 0, \tag{IR}$$

where $U = w + pb^h + (1-p)b^l - c + \delta U$ is an employed agent's expected discounted equilibrium payoff stream and where we assume that an agent who rejects an offer does not receive another one in any subsequent period; this latter assumption will turn out to be without loss of generality.

Since employed agents are supposed to exert high effort, an incentive compatibility (IC) constraint must hold for each agent. Given promised bonus payments b^l (after $\theta = \theta^l$) and b^h (after $\theta = \theta^h$) are made, we have

$$pb^h + (1-p)b^l - c + \delta U \geq 0. \tag{IC}$$

We assume that an agent is fired (and never re-hired) after selecting $e = 0$[6]. Note that

[6]This is based on the assumption that once an agent deviated, the principal assumes he will not exert effort in the future as well. The analysis would be identical, though, if the principal believed that an agent's

given $w \geq 0$, (IR) is automatically implied by the (IC) constraint, and we can omit the former in the following.

It must be in the interest of the principal to actually pay out b^l and b^h to each agent, which is characterized by dynamic enforcement (DE) constraints. There, if she fails to make a promised payment (which never happens in equilibrium), we assume a reversion to the static Nash equilibrium.[7] This is characterized by no bonuses paid to any agent, who in return all choose $e = 0$ given they are hired. The principal then is indifferent between shutting down or running the firm without profits, in any case her off-equilibrium payoff stream is zero. Informally speaking, the firm's failure to keep promises made to one agent leads to a loss of trust of the whole (current and prospective new) workforce.[8] Therefore, the firm either rewards all agents or none.

The two dynamic enforcement (DE) constraints, one for b^l and one for b^h, are

$$-nb^l + \delta\Pi \geq 0 \tag{DEl}$$

and

$$-nb^h + \delta\Pi \geq 0. \tag{DEh}$$

In addition, running the firm (and making employment offers) has to be optimal for the firm in every period, i.e. $\Pi \geq 0$. Because of stationarity and given that bonus payments are non-negative, this condition is automatically implied by the firm's (DE) constraints. Since the right hand sides of (DEl) and (DEh) are identical, only one of them has to be considered, depending on whether b^l or b^h is larger.

Finally, payments must not violate the firm's liquidity (L) constraints, which state that payments in any period cannot exceed respective revenues:

$$n\left(w + b^l\right) \leq \theta^l f(n,k) \tag{Ll}$$

and

$$n\left(w + b^h\right) \leq \theta^h f(n,k). \tag{Lh}$$

deviation was a singular event, which is driven by agents not receiving a rent in any profit-maximizing equilibrium (derived below).

[7]Following Abreu (1988): The static Nash Equilibrium determines the lower bound on the principal's profits and should hence constitute her punishment following observable deviations.

[8]Note that even if prospective agents were *not* able to observe the principal's previous actions, merely receiving an offer could serve as a signal of an earlier deviation of the principal.

4 Equilibrium Employment n^*

In this section, we derive (profit-maximizing) equilibrium employment – denoted n^* – and how it is affected by the firm's liquidity constraints. We show that the latter induce risk-aversion-like behavior: For a given level of expected profits, the firm would always (weakly) prefer a lower volatility.

As a first step, we can show that (IC) constraints bind in any profit-maximizing equilibrium, and that it is further optimal to set $w = 0$ (see Lemma A1 in Appendix A). These results follow from the observability of effort and the excess supply of agents in the labor market, allowing the firm to reap the entire surplus from its employment relationship in any profit-maximizin equilibrium. Hence, the firm will try to maximize the total surplus subject to the constraints derived above.

Benchmark – First-Best-Employment As a benchmark we derive the value of n that maximizes the total surplus. In the following, we denote this efficient – or first-best – employment level n^{FB}. For a given capacity level k , n^{FB} is characterized by

$$\left(p\theta^h + (1-p)\theta^l\right) f_1(\cdot,\cdot) - c = 0. \tag{FB}$$

To keep the analysis interesting, we impose Assumption 1, implying that running the firm is strictly better than not running it. Given the concavity of the production function, this is also the case if first-best effort is not enforceable.

Assumption 1: $f(n^{FB},k)\left(p\theta^h + (1-p)\theta^l\right) - n^{FB}c > 0$ for all levels of $k \geq 0$.

Furthermore, we potentially want to allow the firm's liquidity constraints to have some bite, hence we impose

Assumption 2: $n^{FB}c > \theta^l f(n^{FB},k)$

If Assumption 2 was violated, the firm would never be restricted by constrained liquidity and could always set $b^h = b^l = c$.

Benchmark – Verifiable Effort As another benchmark, we consider the situation where effort is verifiable but the liquidity constraints are in force. In this case, employment will in any case be at its efficient level. Hence, the firm's constrained liquidity alone does not yield any restrictions with respect to enforceable employment, only in interaction with the non-verifiability of effort.

Lemma 1 *Assume that effort is verifiable. Then, equilibrium employment equals n^{FB}.*

Proof: See Appendix B.

The intuition is straightforward: Given that the firm has no commitment problems, it can choose to pay the agent in whatever state of the world it wants to. This allows the firm to pay mostly in those states of the world where it is not liquidity constraint (i.e. after high profits) and liquidity constraints in individual states of the world are never a problem.

Result I – Restricted Employment If effort is not verifiable, employment can be inefficiently low, namely when one of the remaining constraint binds. More precisely, we have

Proposition 1 *For a given capital level k, the firm chooses equilibrium employment n^* to maximize $\Pi = \frac{\left(p\theta^h+(1-p)\theta^l\right)f(n,k)-nc}{(1-\delta)}$, subject to*

$$n^*c \leq \frac{\delta p^2}{1-\delta+\delta p}\theta^h f(n^*,k) + (1-p)\theta^l f(n^*,k), \tag{DE-L}$$

and

$$n^*c \leq \delta f(n^*,k)\left(p\theta^h + (1-p)\theta^l\right). \tag{DE}$$

There are discount factors $\underline{\delta} = \frac{\theta^l}{\left(p\theta^h+(1-p)\theta^l\right)}$ and $\overline{\delta}$, with $\underline{\delta} < \overline{\delta} < 1$, such that

- *$n^* = n^{FB}$ for $\delta \geq \overline{\delta}$*
- *$n^* < n^{FB}$ for $\underline{\delta} \leq \delta < \overline{\delta}$, and n^* is determined by the binding* (DE-L) *constraint*
- *$n^* < n^{FB}$ for $\delta < \underline{\delta}$, and n^* is determined by the binding* (DE) *constraint.*

Proof: See Appendix B.

If the principal is very patient, i.e. $\delta \geq \overline{\delta}$, there is no commitment problem and the first-best can be enforced. Because of Assumption 2, bonus payments vary across states, and $b^h > b^l$. However, the discounted future value of the relationship is so large that the firm credibility is sufficient to promise large payments. Note that $\overline{\delta}$ not only depends on the surplus of the relationship (as $\underline{\delta}$ does), but also on the volatility of earnings. This aspect will be made more precise below, when describing result 2.

If δ is between $\underline{\delta}$ and $\overline{\delta}$, the firm's faces some commitment problem, which however is not solely created by its discounted future surplus being too low (if the firm was not liquidity constrained, the first-best could still be enforced in this range). Since constrained liquidity restricts the firm's ability to compensate agents in low-demand states, it will still be optimal to have $b^h > b^l$, i.e. total bonus payments are higher in the good than in the bad state. However, the principal's willingness to reward agents is determined by the *expected* discounted future surplus, not by current profits. Therefore, the firm's reneging temptation is higher in the good than in the bad state, which restricts employment given that the expected discounted future surplus is not high enough, i.e. if $\delta < \overline{\delta}$. Hence, the lower δ, the lower are enforceable payments in the good state. However, at $\delta = \underline{\delta}$ we reach the point where b^h reaches b^l and a further reduction of δ reduces enforceable payments in both states. Then, constrained liquidity is not an issue and employment is only restricted by the future surplus being too small. In this case $b^h = b^l$, and the total compensation received by the workforce does not vary over time.

To sum up the results so far, $\underline{\delta} = \frac{\theta^l}{\left(p\theta^h+(1-p)\theta^l\right)}$ determines whether (DE-L) or (DE) is the relevant constraint. Fixing θ^l, $\underline{\delta}$ decreases in p and θ^h. Thus, higher values of p or θ^h make it more likely that the firm is affected by (DE-L), i.e. its employment might also be restricted by a cash shortage and not only by its future returns being too low.

Result II – Volatility Matters In a next step, we demonstrate that it is not only players' impatience (i.e. a low δ) that limits commitment if the firm faces a liquidity constraint. This is different from "standard" contributions like MacLeod and Malcomson (1989) or Levin (2003), where the enforceability of relational is solely determined by the discount factor. We can show that the liquidity constraint will bind – and hence the firm's employment level is restricted to inefficiently low levels – even if the principal is arbitrarily patient. This is the case if the firms' cash-flow is very uncertain in the sense that there is a small probability that the firm makes very high profits.

Proposition 2 *Assume $\delta \geq \underline{\delta}$, i.e., the (DE) constraint* (DE) *can be omitted. Then, for fixed employment $\hat{n} > \frac{\theta^l f(n,k)}{c}$, θ^l and c, as well as for a fixed per-period surplus $f(n)\left(p\theta^h + (1-p)\theta^l\right) - nc$, there exists a $\underline{p}$ such that for $p < \underline{p}$, constraint* (DE-L) *does not hold for $\hat{n}$.*

Proof: See Appendix B.

A reduction of p, accompanied by an increase of θ^h in order to keep the surplus for a given employment level fixed will eventually lead to a violation of (DE-L).[9] This result is driven by the combination of liquidity and dynamic enforcement constraints. Absent (Ll), the enforceability of an employment level n would – for a given discount factor δ – only depend

[9]Note that given employment, θ^l, c and surplus stay constant, also $\underline{\delta}$ is unaffected.

on the future surplus, independent of the exact specification of p and θ. If a large surplus is accompanied by a larger variability of income, the principal's commitment is still restricted. The reason is that when constrained liquidity has bite, larger shares of the compensation package must be shifted to high-demand states, ceteris paribus increasing the temptation to renege. Hence, the (DE-L) constraint is more likely to bind if the expected surplus generated in the relationship is high, and if the firm operates in a high-risk/high return environment.

In addition, we can show that given $\delta \geq \underline{\delta}$ and given the surplus is held constant, enforceable employment increases in p. Define $\tilde{n}$ as the employment level where constraint (DE-L) binds, i.e., $\tilde{n}$ is characterized by

$$\tilde{n}c = \frac{p^2\delta}{1-\delta+p\delta}\theta^h f(\tilde{n}) + (1-p)\theta^l f(\tilde{n}). \tag{1}$$

Due to the concavity of $f(n)$ in n, $\tilde{n}$ exists, (DE-L) is satisfied for $n \leq \tilde{n}$, and violated for $n > \tilde{n}$. Hence, for $\tilde{n} < n^{FB}$ only an inefficiently low employment level can be enforced. Then, we can establish

Lemma 2 *Fix θ^l and the total per-period surplus (given positive effort),*

$$f(n)\left(p\theta^h + (1-p)\theta^l\right) - nc.$$

Then, $\tilde{n}$ is strictly increasing in p.

Proof of Lemma 2: This follows from the fact that the right hand side of (1) is increasing in p, given the surplus is held constant (see the proof to Proposition 1). ■

To sum up, this section establishes the potential enforcement problem induced by the liquidity constraint, assuming a fixed capacity level. Even if the firm is very patient, its commitment in relational contracts with its employees is limited if its earnings are volatile: Bonus payments must vary across states, whereas the maximum amount of bonuses that the principal can commit to pay in any state of the world is determined by the expected future surplus. This implies that the highest equilibrium bonus level determines whether high effort is enforceable or not. If the likelihood of the good state is too low, the necessary bonus to imply effort becomes so high that it is not optimal for the principal to honor his promises anymore, consequently leading to inefficiently low employment.

5 Optimal Capacity and the Scope for Overinvestments

We now derive the firm's optimal capital choice. Because capital relaxes both enforceability constraints, overinvestments will occur whenever either constraint (DE) or (DE-L) binds.

At the beginning of the game, the firm determines its optimal capital investments k^* to maximize $\Pi_0 = -k + \delta\Pi$, taking into account that the constraints

$$n^* c \leq \delta f(n^*, k^*) \left(p\theta^h + (1-p)\theta^l\right) \tag{DE}$$

and

$$n^* c \leq \frac{\delta p^2}{1-\delta+\delta p}\theta^h f(n^*, k^*) + (1-p)\theta^l f(n^*, k^*) \tag{DE-L}$$

have to be satisfied in all later periods.

As a benchmark, the optimal capicity level absent the constraints is implicitly defined by

$$\frac{\partial \Pi_0}{\partial k} = -1 + \frac{\delta}{1-\delta}\left(p\theta^h + (1-p)\theta^l\right) f_2 = 0. \tag{2}$$

If one of the constraints binds, though, the firm chooses capital investments that are higher than specified by (2).

Proposition 3 *Optimal capital investments are characterized by* (2) *if and only if efficient employment* n^{FB} *is enforceable, i.e., if* $\delta \geq \overline{\delta}$ *(as defined in Proposition 1). If* $\delta < \overline{\delta}$*, i.e., if either (DE) or (DE-L) binds, overinvestments into capital are optimal in the sense that marginal costs exceed its direct marginal returns, i.e.* k^* *exceeds the level specified by* (2)*.*

Proof: See Appendix B.

The intuition underlying Proposition 3 is straightforward. If $\delta < \underline{\delta}$, i.e., the (DE) constraint binds, ex-ante overinvestments increase revenues in all future periods. Since investment costs are sunk, the difference between profits in and out-of equilibrium increases, and promises made to the firm's workforce become more credible.[10] If $\delta \geq \underline{\delta}$ (but below $\overline{\delta}$) and the (DE-L) constraint binds, overinvestments are optimal because a higher value of k increases output and thereby the available cash-flow in each state of the world. Then a larger share of total compensation can be shifted to the low state, thereby relaxing the firm's (DEh) constraint.

[10] A similar argument is used by Halac (2014) to show that the hold-up problem (generating underinvestments into relationship-specific assets) can be less severe in a relationship where relational contracts have to be used ex-post.

While in the classic corporate finance literature, a reduction of free cash-flow is regarded as a potential remedy to overcome overinvestment problems, we show that the very lack of free cash-flow in some states makes overinvestments ex-ante optimal.

6 Allowing the Firm to Save

In this section, we show that overinvestments can still be optimal if the firm is able to save revenues, using them to relax the liquidity constraint in later periods. Assume that firms are able to save cash revenues generated in high-demand periods and use them to compensate agents in subsequent low-demand periods. This allows the firm to temporarily increase its employment level. Holding cash reserves might or might not be optimal when the firm's liquidity constraint binds, but can never make it irrelevant. Hence, overinvestments will continue to be part of the firm's optimal investment decisions whenever employment is restricted.

To simplify the analysis, we assume that accumulated cash stays within the firm and does not generate interest payments. Our results remain valid as long as interest payments are not so high that they render deferring consumption fully to future periods optimal.

If the firm decides to save, accumulated cash can be used in two ways. On the one hand, employment can be increased until the next realization of θ^l. On the other hand, a given employment level can be sustained longer, i.e. for more than just one subsequent realization of θ^l. We delegate a general characterization of the firm's optimal behavior to Appendix C and only present the main result in this section.

Proposition 4 *Savings are optimal if and only if in the situation absent savings, the (DE-L) constraint binds (i.e., if δ is between $\underline{\delta}$ and $\overline{\delta}$, as defined above in Proposition 1)* ***and*** *the following condition is satisfied in equilibrium:*

$$\left(\frac{\delta\,(1-p)}{[1-\delta p]}p\theta^h + (1-p)\theta^l\right) f_1 - c > 0. \tag{3}$$

If savings are optimal, maximum employment is characterized by

$$\left(\frac{\delta\,(1-p)}{[1-\delta p]}p\theta^h + (1-p)\theta^l\right) f_1 - c = 0. \tag{4}$$

Proof: See Appendix C.

Holding cash reserves is never optimal if only dynamic enforcement constraints bind. However, a binding liquidity constraint alone is only a necessary and not a sufficient condition for savings to be optimal. Instead, the costs of saving – delayed consumption – might still be too high compared to its benefits, which is the case if condition (3) does not hold.

In Appendix C, we further show that after a cash stock has been built up and the firm is hit by a number of negative shocks, employment is decreased gradually. Then, each low-demand period triggers an employment reduction until all savings have been used up. This parallels results in Li and Matouschek (2013) where implemented effort levels gradually decrease with every adverse shock hitting a firm.

Here, we do not solve for the actual amount of equilibrium savings in high-demand periods, since it is not required for our purpose. Determining savings would be straightforward, though. After solving for optimal employment for each history of demand-realizations (what we do), the respective binding (DE-L) constraints give us the total savings needed in order to maintain respective employment levels.

However, it is important to note that the possibility to accumulate cash reserves does not eliminate overinvestments.

Proposition 5 *Assume savings are possible. Then overinvestments are still observed when either (DE) or (DE-L) constraints bind.*

Proof of Proposition 5: When only (DE) constraints bind, savings are not optimal and the situation is equivalent to above – overinvestments are an optimal response to relax the constraint. If savings are optimal due to a binding (DE-L) constraint, employment never exceeds the level characterized by (4) – hence is below the first best. The rest follows from the proof to Proposition 3. ■

7 Discussion and Conclusion

The present paper shows that observed overinvestments are not necessarily the (negative) consequence of agency problems between shareholders and managers. Instead, they might actually be a second-best-optimal response to contracting frictions: If firms cannot fully rely on court-enforceable contracts to motivate their workforce but have to use relational contracts, when they furthermore face volatile market conditions and hence varying cash-flow streams, and when they are facing liquidity constraints, they have to rely on "excessive" capital investments to increase their cash-flow base.

Our argument is robust to granting firms access to an external capital market. The "quality" of this external capital market, determined by the interest rate and to what extent credits can be pledged against the firm's assets, then affects the firm's credibility in relational contracts formed with its workforce.

The predictions of our model can be matched to empirically documented regularities. Proposition 1 also implies that on average, firms with a higher discount factor have a larger variation in residual profits and hence free cash flow: Given δ is between $\underline{\delta}$ and $\overline{\delta}$, then $b^l n = \theta^l f(n, k)$, and the firm has to use all of its revenues to compensate agents if demand is low. On the other hand, if $\delta < \underline{\delta}$, the liquidity constraint does not bind, and free cash flow is also available in the low-demand state. If we assume that dividend payments (or more generally the distribution of profits to owners/shareholders) are larger in states when more cash-flow is available, we should expect firms with higher discount factor to have a larger variability in dividend payments. This matches the finding in Michaely and Roberts (2012) that privately held firms in the UK smooth dividends significantly less than their publicly listed counterparts, and respond more to transitory earnings shocks. Michaely and Roberts (2012) conjecture that this might be due to agency problems, which are more prominent in publicly held firms. We offer an alternative but related explanation: Privately held firms are often assumed to focus more on long-term goals – in particular with respect to their employment relationships – compared to publicly listed firms. Take family firms, where for example a study by Price Waterhouse Coopers (2012) identifies a larger commitment to jobs, which leads "family-run businesses ... to have more loyalty toward their staff – people are not just a number" (p. 6). In terms of our model, privately-held firms hence have a larger discount factor, making it more likely that their dynamic enforcement constraint (DE-L) is the relevant constraint, implying more variation in dividend payments.

In a related vein, a lower discount factor makes overinvestments (which are optimal given $\delta < \overline{\delta}$) generally more likely in our model. Bargeron, Schlingemann, Stulz, and Zutter (2008) show that publicly listed firms in the US pay substantially more for acquisitions than privately owned firms and argue that this might be due to larger agency problems in publicly-traded firms. Our model again can provide a complementary explanation: Given publicly held firms have a lower discount factor, they are more likely to use overinvestments in order to increase their commitment in relational contracts with their workforce.[11] This implies that – ceteris paribus – their willingness to pay for a given acquisition target and henceforth investment should be higher.

Finally, Lemma 2 has a testable prediction: If θ^h is very high, but the prospects of it

[11]Note that this result would still hold absent a liquidity constraint.

being realized are low, enforceable employment is smaller than with a lower θ^h but higher probability p. We should thus observe that industries which ceteris paribus are more volatile have lower employment levels than industries where earnings are more balanced. We are not aware of a study that has looked into this, but the prediction appears readily testable.

A Maximization Problem, Constraints, and Proof of Stationarity

Note that it is sufficient to regard equilibrium employment as well as compensation as a function of the history of past shocks. The reason is our focus on pure strategies. Denote the history at the beginning of period t as $\theta^{t-1} = \{\theta_1, \theta_2, ..., \theta_{t-1}\}$, with $\theta_t \in \{\theta^l, \theta^h\}$, and $\theta^0 = \emptyset$. Then, expected payoff streams can be written as

$$\begin{aligned}\Pi(\theta^{t-1}) =& p\left[\theta^h f\left(n(\theta^{t-1}), k\right) - n(\theta^{t-1}) b^h(\theta^{t-1})\right] \\ &+ (1-p)\left[\theta^l f\left(n(\theta^{t-1}), k\right) - n(\theta^{t-1}) b^l(\theta^{t-1})\right] \\ &- n(\theta^{t-1}) w(\theta^{t-1}) + \delta\left[p\Pi\left(\theta^{t-1}, \theta^h\right) + (1-p)\Pi\left(\theta^{t-1}, \theta^l\right)\right]\end{aligned}$$

and

$$\begin{aligned}U(\theta^{t-1}) =& w(\theta^{t-1}) + pb^h(\theta^{t-1}) + (1-p)b^l(\theta^{t-1}) - c \\ &+ \delta\left[pU\left(\theta^{t-1}, \theta^h\right) + (1-p)U\left(\theta^{t-1}, \theta^l\right)\right],\end{aligned}$$

where $b^h(\theta^{t-1})$ is the bonus paid for history θ^{t-1} given a high shock is realized in period t. Equivalent definitions hold for $b^l(\theta^{t-1})$, $\Pi\left(\theta^{t-1}, \theta_t\right)$ and $U\left(\theta^{t-1}, \theta_t\right)$.

Then, the firm's objective function is to choose k as well as $n(\theta^{t-1})$, $w(\theta^{t-1})$, $b^h(\theta^{t-1})$ and $b^l(\theta^{t-1})$ to maximizue

$$\Pi_0 = -k + \delta\Pi(\theta^0),$$

subject to the following constraints, which must be satisfied for every history θ^{t-1}:

$$U(\theta^{t-1}) \geq 0 \tag{IRA}$$

$$pb^h(\theta^{t-1}) + (1-p)b^l(\theta^{t-1}) - c + \delta\left[pU\left(\theta^{t-1}, \theta^h\right) + (1-p)U\left(\theta^{t-1}, \theta^l\right)\right] \geq 0 \tag{IC}$$

$$n(\theta^{t-1}) b^l(\theta^{t-1}) \leq \delta\Pi\left(\theta^{t-1}, \theta^l\right) \tag{DEl}$$

$$n(\theta^{t-1}) b^h(\theta^{t-1}) \leq \delta\Pi\left(\theta^{t-1}, \theta^h\right) \tag{DEh}$$

Given both (DE) constraints, the firm's individual rationality constraint, $\Pi(\theta^{t-1}) \geq 0$, is automatically satisfied.

$$n(\theta^{t-1})\left(w(\theta^{t-1})+b^l(\theta^{t-1})\right)\leq\theta^l f(n(\theta^{t-1})) \tag{Ll}$$

$$n(\theta^{t-1})\left(w(\theta^{t-1})+b^h(\theta^{t-1})\right)\leq\theta^h f(n(\theta^{t-1})) \tag{Lh}$$

Lemma 3 *(IC) constraints bind, and $w(\theta^{t-1})=0$ for every history θ^{t-1}.*

Proof of Lemma 3: To the contrary, assume there is a history $\tilde{\theta}^{t-1}$ where (IC) does not bind. At this point, reduce $b^h(\tilde{\theta}^{t-1})$ as well as $b^l(\tilde{\theta}^{t-1})$ by a small $\varepsilon>0$ such that (IC) for history $\tilde{\theta}^{t-1}$ is still satisfied. Furthermore, increase $w(\tilde{\theta}^{t-1})$ by ε and leave everything else unchanged. This has no impact on Π_0, as well as $\Pi(\theta^{t-1})$ and $U(\theta^{t-1})$ for *any* history θ^{t-1} , hence does not affect any (IRA) constraint. Furthermore, all (Ll) and (Lh) constraints remain unchanged. Finally (DEh) and (DEl) for history $\tilde{\theta}^{t-1}$ are relaxed and unaffected for any other history.

Plugging this result into $U(\theta^{t-1})$ gives $U(\theta^{t-1})=w(\theta^{t-1})$; hence, setting $w(\theta^{t-1})=0$ for all histories θ^{t-1} is optimal, since it relaxes the firm's constraints and increases Π_0. ■

Using the results of Lemma A1 gives

$$\begin{aligned}\Pi(\theta^{t-1})=&p\theta^h f\left(n(\theta^{t-1}),k\right)+(1-p)\theta^l f\left(n(\theta^{t-1}),k\right)-n(\theta^{t-1})c\\&+\delta\left[p\Pi\left(\theta^{t-1},\theta^h\right)+(1-p)\Pi\left(\theta^{t-1},\theta^l\right)\right]\end{aligned}$$

and $b^h(\theta^{t-1})=\frac{c-(1-p)b^l(\theta^{t-1})}{p}$.

Furthermore, the remaining constraints are

$$n(\theta^{t-1})b^l(\theta^{t-1})\leq\delta\Pi\left(\theta^{t-1},\theta^l\right) \tag{DEl}$$

$$n(\theta^{t-1})\frac{c-(1-p)b^l(\theta^{t-1})}{p})\leq\delta\Pi\left(\theta^{t-1},\theta^h\right) \tag{DEh}$$

$$n(\theta^{t-1})b^l(\theta^{t-1})\leq\theta^l f\left(n(\theta^{t-1}),k\right) \tag{Ll}$$

$$n(\theta^{t-1})\frac{c-(1-p)b^l(\theta^{t-1})}{p}\leq\theta^h f\left(n(\theta^{t-1}),k\right) \tag{Lh}$$

This allows us to prove

Lemma 4 *Contracts are stationary in a sense that employment as well as bonus payments are independent of the history of shocks θ^{t-1}.*

Proof of Lemma 4: First of all, note that none of the employment levels can optimally be above n^{FB} and that profits are increasing in $n(\theta^{t-1})$ for any possible history as long as employment there is inefficiently low. Now, take any equilibrium employment level $n(\theta^{t-2})$ and assume that $n(\theta^{t-2}, \theta^h) \neq n(\theta^{t-2}, \theta^l)$. If $n(\theta^{t-2}, \theta^h) > n(\theta^{t-2}, \theta^l)$, replacing $n(\theta^{t-2}, \theta^l)$ with $n(\theta^{t-2}, \theta^h)$ would violate no constraint and increase profits. If $n(\theta^{t-2}, \theta^l) > n(\theta^{t-2}, \theta^h)$, on the other hand, replacing $n(\theta^{t-2}, \theta^h)$ with $n(\theta^{t-2}, \theta^l)$ would violate no constraint and increase profits. Hence employment in any period is independent of previous shock realizations and might only be history-dependent based on the number of observed shocks, i.e. on timing. There, however, note that the structure of the game is stationary. This implies that the highest employment level that is enforceable in any period of the game can be enforced in all other periods as well. Since the principal is able to collect the whole surplus of the game, it is optimal to choose the maximum feasible employment (subject to $n \leq n^{FB}$) in every period. ■

B Omitted Proofs

Proof of Lemma 1: If effort is verifiable, the firm's dynamic enforcement constraints can be omitted. We just have to make sure that the firm's expected profits are positive.

Assume the following contract: Employment equals n^{FB}, each agent who chooses positive effort receives expected bonus payments $pb^h + (1-p)b^l = c$ and gets fired otherwise. If $n^{FB}c \leq \theta^l f(n^{FB}, k)$, then $b^l = b^h = c$, satisfying all constraints and obviously being optimal for the principal due to Assumption 1.

Otherwise, i.e., if $n^{FB}c > \theta^l f(n^{FB}, k)$, the firm sets $b^l = \theta^l f(n^{FB}, k)/n^{FB}$, and $b^h = \left(c - (1-p)\theta^l f(n^{FB}, k)/n^{FB}\right)/p$. Plugging this value into (Lh) gives

$0 \leq p\theta^h f(n^{FB}, k) + (1-p)\theta^l f(n^{FB}, k) - n^{FB}c$, which is always satisfied due to Assumption 1. ■

Proof of Proposition 1. Recall that the firm's objective is to maximize $\Pi_0 = -k + \delta\Pi$, subject to the relevant constraints. For a given value of k, however, n is chosen to maximize Π. Furthermore, we show in Appendix A that (IC) and (IRA) constraints bind. Hence, we can use $w = 0$ and $pb^h + (1-p)b^l - c = 0$, giving the problem

$$\max_n \Pi = \frac{p\theta^h f(n,k) + (1-p)\theta^l f(n,k) - nc}{1-\delta},$$

subject to

$$nb^l \leq \delta\Pi \qquad \text{(DEl)}$$

$$n\frac{c-(1-p)b^l}{p} \leq \delta\Pi \tag{DEh}$$

$$nb^l \leq \theta^l f(n,k) \tag{Ll}$$

$$n\frac{c-(1-p)b^l}{p} \leq \theta^h f(n,k) \tag{Lh}$$

It follows that (DEl) and (Ll), as well as (DEh) and (Lh) generally do not bind at the same time. In a next step, we show that (Lh) cannot bind in equilibrium. To the contrary assume it binds. Then, either (Ll) or (DEl) must bind as well because otherwise, b^l could be increased and (Lh) relaxed without violating any constraint. First, assume that (Ll) binds together with (Lh). This, however, would imply that $\Pi = 0$, which is not possible in an equilibrium with positive employment. Now, assume that (DEl) binds together with (Lh). From section 3, we know that setting $b^h \geq b^l$ is optimal. Hence, (DEh) has to bind as well, implying $b^h = b^l$. Then, $\theta^h f(n,k) = nb^h = nb^l = \theta^l f(n,k)$, which - due to $\theta^h > \theta^l$ - is not possible for $n > 0$.

Now, consider all employment levels with $\theta^l f(n,k) \geq \delta\Pi$. In this case, (Ll) is automatically satisfied given (DEl). Adding (DEh) and (DEl) proves the necessity of (DE-L). Sufficiency immediately follows: Assuming (DE-L) holds, there always exists a $b^l \geq 0$ such that (DEh) and (DEl) are satisfied.

For employment levels $\theta^l f(n,k) < \delta\Pi$ (DEl) is automatically satisfied given (Ll). In this case, necessity and sufficiency of (DE) are obtained equivalently as for (DE-L).

To prove that $n^* \leq n^{FB}$, we set up the Lagrange function,

$$\begin{aligned} L =& \frac{p\theta^h f(n,k) + (1-p)\theta^l f(n,k) - nc}{1-\delta} + \lambda_{DE}\left[\delta f(n,k)\left(p\theta^h + (1-p)\theta^l\right) - nc\right] \\ &+ \lambda_{DELL}\left[\frac{\delta p^2}{1-\delta+\delta p}\theta^h f(n,k) + (1-p)\theta^l f(n,k) - nc\right], \end{aligned}$$

giving the first order condition

$$\frac{\partial L}{\partial n} = \left(p\theta^h f_1 + (1-p)\theta^l f_1 - c\right)\left(\tfrac{1}{1-\delta} + \delta\lambda_{DE} + \lambda_{DELL}\right)$$
$$-\lambda_{DE}c(1-\delta) - \lambda_{DELL}p\theta^h f_1 \tfrac{1-\delta}{1-\delta+\delta p} = 0.$$

Hence, if either (DE-L) or (DE) binds $\left(p\theta^h f_1 + (1-p)\theta^l f_1 - c\right) > 0$, and n^* is inefficiently small.

To determine $\underline{\delta}$ and $\overline{\delta}$, fix employment n. Furthermore, (DE-L) is the relevant constraint when the right hand side of (DE) is smaller than (DE-L) for a given employment level, or if

$$\delta\left(p\theta^h + (1-p)\theta^l\right) \geq \theta^l \tag{5}$$

Condition (5) holding as an equility gives the value $\underline{\delta} = \theta^l / \left(p\theta^h + (1-p)\theta^l\right)$. To show that n^{FB} cannot be enforced at $\underline{\delta}$, plug $\underline{\delta}$ into each of the constraints. This gives $nc \leq \theta^l f(n,k)$ which – by Assumption 2 – does not hold for n^{FB}.

To prove the rest of the proposition, and in particular the existence of $\overline{\delta}$, we need monotonicity of the constraints in δ, and that first-best employment can be enforced for $\delta \to 1$. The latter follows from Assumption 1 ((DE-L) and (DE) both converge to $n^*c \leq p\theta^h f(n^*,k) + (1-p)\theta^l f(n^*,k)$ for $\delta \to 1$). To prove monotoncity in δ, we obtain the derivatives of the right hand sides of the (DE) and (DE-L) constraints with respect to δ, $f(n,k)\left(p\theta^h + (1-p)\theta^l\right)$ and $\frac{p^2}{(1-\delta+\delta p)^2}\theta^h f(n,k)$, respectively. Since both expression are positive, both constraints are relaxed by larger values of δ. ■

Proof of Proposition 2. As we fix the surplus, as well as θ^l and n, a decrease in p has to be compensated by an appropriate increase in θ^h. More precisely, taking the total derivative of the per-period surplus, $f(n,k)\left(dp\theta^h + pd\theta^h - dp\theta^l\right) = 0$, implies $\frac{d\theta^h}{dp} = -\frac{(\theta^h-\theta^l)}{p}$.

Take an arbitrary high-state probability $\overline{p}$ where constraint (DE-L) is satisfied. For any probability $p^* < \overline{p}$, always counterbalanced by an increase of θ^h that keeps the surplus constant, the right hand side of (DE-L) equals

$$\frac{\delta}{1-\delta+p^*\delta}(p^*)^2\left(\theta^h + d\theta^h\right) f(n,k) + (1-p^*)\theta^l f(n,k)$$

$$= \frac{\delta}{1-\delta+p^*\delta}(p^*)^2\left(\theta^h - (\theta^h-\theta^l)\int_{p^*}^{\overline{p}}\frac{1}{p}dp\right) f(n,k) + (1-p^*)\theta^l f(n,k)$$

$$= \frac{\delta}{1-\delta+p^*\delta}(p^*)^2\left(\theta^h - (\theta^h-\theta^l)\ln\overline{p} + (\theta^h-\theta^l)\ln p^* + C\right) f(n,k) + (1-p^*)\theta^l f(n,k)$$

For $p^* \to 0$, the last expression becomes

$$\frac{\delta}{1-\delta(1-p^*)}(\theta^h-\theta^l)\frac{\ln p^*}{\frac{1}{(p^*)^2}} f(n,k) + \theta^l f(n,k)$$

$$= \frac{\delta}{1-\delta}(\theta^h-\theta^l)\frac{(p^*)^2}{-2} f(n,k) + \theta^l f(n,k) = \theta^l f(n,k).$$

Since $\theta^l f(n,k) < nc$, the employment level n will eventually not be enforceable anymore.

Finally, we have to show that the right hand side of (DE-L) is increasing in p.

$$\frac{d}{dp} = \delta\frac{2p(1-\delta+p\delta)-p^2\delta}{(1-\delta+p\delta)^2}\theta^h f(n,k) + \frac{p^2\delta}{1-\delta+p\delta}\frac{d\theta^h}{dp} f(n,k) - \theta^l f(n,k)$$

$$= \frac{(1-\delta)}{(1-\delta+p\delta)^2}\left[\delta\left(\theta^h f(n,k)p + (1-p)\theta^l f(n,k)\right) - \theta^l f(n,k)\right].$$

Since $\theta^l f(n,k) < cn$, this expression is positive as long as n is enforceable if the liquidity constraint is absent. However, if n is not enforceable without the principal's liquidity constraint, (DE-L) is violated anyway.

■

Proof of Proposition 4. The Lagrange function in this case equals

$$
\begin{aligned}
L = & -k + \frac{\delta}{1-\delta}\left[\left(p\theta^h + (1-p)\theta^l\right) f(n,k) - nc\right] \\
& + \lambda_{DE}\left[\delta f(n^*,k)\left(p\theta^h + (1-p)\theta^l\right) - n^* c\right] \\
& + \lambda_{DELL}\left[\frac{\delta p^2}{1-\delta+\delta p}\theta^h f(n^*,k) + (1-p)\theta^l f(n^*,k) - n^* c\right],
\end{aligned}
$$

and the first order condition with respect to k is

$$
\begin{aligned}
\frac{\partial L}{\partial k} = & -1 + \frac{\delta}{1-\delta}\left(p\theta^h + (1-p)\theta^l\right) f_2 + \lambda_{DE}\delta f_2\left(p\theta^h + (1-p)\theta^l\right) \\
& + \lambda_{DELL}\left[\frac{\delta p^2}{1-\delta+\delta p}\theta^h f_2 + (1-p)\theta^l f_2\right] = 0.
\end{aligned}
$$

Hence, if either (DE) or (DE-L) binds, k^* will satisfy$-1 + \frac{\delta}{1-\delta}\left(p\theta^h + (1-p)\theta^l\right) f_2 < 0$, implying a k^* that is larger than the one characterized by (2). ■

C Optimal Firm Behavior Given Savings are Possible

Assume that whenever demand conditions are high, the firm can save some of its revenues. These cash reserves are used to increase employment from the next period on and compensate some of the additional agents. After a number of subsequent low-demand periods, the cash reserves are used up and employment is down at its original level again - until the next high-demand period.

Define $m \geq 1$ as the number of periods a higher employment level can at least be enforced, i.e., m is the subsequent number of low-demand periods after which all cash reserves are used up. Furthermore, define the amount saved as s_m, and the employment level in the first period after s_m has been accumulated as n_m.

Now, assume that s_m has been saved and employment raised to n_m in the next period. If the firm faces a low-demand shock in this period, all n_m agents are compensated accordingly. However, some of the savings are needed, and available funds go down, to s_{m-1}. Furthermore, employment in the next period will be n_{m-1}. This process is continued until either all cash reserves are used (and employment is at n_0) or a high-demand shock allows to fill up cash reserves and increase employment to n_m again (which will always be optimal due to the stationarity of the problem). To avoid unnecessary complications, we assume that income in a high-demand state is sufficiently high to save the optimal amount s_m, i.e., we impose

Assumption A1: Assume savings are possible. Then, (Lh) does not bind in a profit-maximizing equilibrium.

Assumption A1 implies that only one high-demand state is needed in order to fill up the firm's desired cash stock.

Profits

We write payoffs as functions of the remaining subsequent low-demand shocks before all cash reserves are used up. Profits given savings are at its maximum level are denoted $\tilde{\Pi}(m)$, since m has been defined as the number of periods a higher employment level can at least be enforced. Hence,

$$\Pi(m) = p\left[\theta^h f(n_m, k) - n_m b_m^h + \delta\Pi(m)\right] + (1-p)\left[\theta^l f(n_m, k) - n_m(1-p)b_m^l + (s_m - s_{m-1}) + \delta\Pi(m-1)\right].$$

Note that bonus payments are the amounts actually paid out to agents. Hence, if in a low-demand state part of the payments are handed over to agents, this enters the principal's profits positively.

Furthermore, profits after $j < m$ subsequent low-demand periods are

$$\Pi(m-j) = p\left[\theta^h f(n_{m-j}, k) - n_{m-j}b_{m-j}^h - (s_m - s_{m-j}) + \delta\Pi(m)\right] + (1-p)\left[\theta^l f(n_{m-j}, k) - n_{m-j}(1-p)b_{m-j}^l + (s_{m-j} - s_{m-j-1}) + \delta\Pi(m-j-1)\right].$$

After $m-1$ subsequent low-demand period, a higher employment can only be enforced for maximum one more low-demand period. Then,

$$\Pi(1) = p\left[\theta^h f(n_1, k) - n_1 b_1^h - (s_m - s_1) + \delta\Pi(m)\right] + (1-p)\left[\theta^l f(n_1, k) - n_1(1-p)b_1^l + s_1 + \delta\Pi(0)\right].$$

Finally,

$$\Pi(0) = p\left(f(n,k)\theta^h - n_0 b^h - s_m + \delta\Pi(m)\right) + (1-p)\left(f(n,k)\theta^l - n_0 b^l + \delta\Pi(0)\right)$$

are profits given the firm has used all its cash.

Objective

The objective is to find the levels of $m \geq 0$, n_{m-j} ($j \leq m$) and the respective amounts of cash that maximize $\Pi(0)$, given the constraints derived below. Hence, we maximize profits given no cash is initially available. It will turn out, though, that the respective strategy also maximizes $-s_m + \delta\Pi(m)$, the firm's objective given savings could also be raised at the beginning of the game (when capital k is raised).

Constraints

The following constraints have to be satisfied in any equilibrium. For all $j \in [0, m]$, dynamic enforcement constraints for low- and high-demand states must hold:

$$n_{m-j}b^l_{m-j} \leq \delta\Pi(m-j) - s_{m-j} \tag{DEl(j)}$$

$$n_{m-j}b^h_{m-j} \leq \delta\Pi(m) - s_m. \tag{DEh(j)}$$

Savings enter the above constraints since if the principal reneges on payments promised to agents, it will also be optimal to consume accumulated savings. Furthermore, note that $s_0 = 0$

Since (Lh) is satisfied by assumption, liquidity constraints must only hold for low-demand states.

For all $j \in [0, m-1]$, we have

$$n_{m-j}b^l_{m-j} \leq \theta^l f(n_{m-j}, k) + (s_{m-j} - s_{m-j-1}), \tag{Ll(j)}$$

where also $s_{-1} = 0$ in LLl(0).

In addition (IC) constraints must hold (where we already take into account that agents receive no rent), namely

$$pb^h_{m-j} + (1-p)b^l_{m-j} - c \geq 0$$

for all $j \in [0, m]$.

For the same reasons as before, (IC) constraints will bind, giving $b^l_{m-j} = \frac{c - pb^h_{m-j}}{1-p}$.

Results

In this section, we first assume that it is optimal to accumulate strictly positive cash reserves (which implies $m \geq 1$) and derive properties of a profit-maximizing equilibrium. Then, we work out conditions under which it is actually optimal to save.

First, we show that given savings are optimal, (DEl) constraints can be omitted.

Lemma 5 *Assume $m \geq 1$. Then, all (DEl) are automatically implied by the respective (Ll) constraints.*

Proof of Lemma 5: First, we plug $b^l_{m-j} = \frac{c - pb^h_{m-j}}{1-p}$ into profits, giving the set of constraints

$$n_{m-j}\frac{c - pb^h_{m-j}}{1-p} \leq \delta\Pi(m-j) - s_{m-j} \tag{DEl(j)}$$

$$n_{m-j}b^h_{m-j} \leq \delta\Pi(m) - s_m \tag{DEh(j)}$$

$$n_{m-j}\frac{c-pb^h_{m-j}}{1-p}\leq\theta^l f(n_{m-j},k)+(s_{m-j}-s_{m-j-1}) \tag{Ll(j)}$$

Hence, for each j, DEl(j) and LLl(j) constraints can generally not bind simultaneously, but only one of them - together with the respective DEh(j) constraint.

Adding (DElj) and (DEhj) for each j gives a set of necessary constraints:

$$n_{m-j}c\leq\delta p\tilde{\Pi}(m)-ps_m+(1-p)\left(\delta\tilde{\Pi}(m-j)-s_{m-j}\right). \tag{DE(j)}$$

Note that any DE(j) constraint can only bind if the respective DEl(j) constraint binds as well. Now, assume there is a profit maximizing equilibrium with a $j^*\geq 1$ where DE(j^*) binds but LLl(j^*) is slack.

To see that this cannot be optimal, reduce all s_{m-j} with $j\leq j^*$ by ε and leave everything else unchanged. This tightens LLl(j^*) which however will still hold for ε sufficiently small. LLl(j) constraints for all j besides j^* remain unaffected.

Furthermore, profits change by $\Delta\Pi(0)=p\varepsilon\frac{1-(\delta(1-p))^{j^*+1}}{1-\delta(1-p)}$, hence go up. Furthermore, we can show that no constraint gets violated. To see that, note that

$$\Delta\Pi(m)=-\varepsilon\left(\delta(1-p)\right)^{j^*}(1-p),$$

$$\Delta\Pi(m-j^*)=-\frac{\varepsilon\,(1-p)}{1-\delta(1-p)}\left(1-\delta+\delta p\left(\delta(1-p)\right)^{j^*}\right)$$

and - for any $1<k<m$,

$$\Delta\Pi(m-(j^*-k))=\frac{\delta p\Delta\Pi(m)-\varepsilon\left(\delta(1-p)\right)^k[1-p](1-\delta)}{1-\delta(1-p)}$$

and

$$\Delta\Pi(m-(j^*+k))=\frac{p\varepsilon\left(1-\left(\delta(1-p)\right)^{j^*+1}\right)}{1-\delta(1-p)}.$$

These considerations help us to show that by reducing all s_{m-j} with $j\leq j^*$ by ε relaxes all constraints:

DE(0) is relaxed because its right hand side changes by

$p\varepsilon\left(1-\left(\delta(1-p)\right)^{j^*+1}\right)+\delta(1-p)\Delta\Pi(0)>0$. DE($j^*$) is relaxed, because its right hand side changes by

$\varepsilon\left[\frac{(1-\delta)\left(1-\delta(1-p)^2\right)+p\left(\delta-\left(\delta(1-p)\right)^{j^*+1}\right)}{1-\delta(1-p)}\right]>0$. DE($j^*+k$) is relaxed because its right hand side changes by

$\varepsilon\frac{(1-\delta)\left(1-(1-p)(\delta(1-p))^{k+1}\right)+p\left(\delta-(\delta(1-p))^{j^*+1}\right)}{1-\delta(1-p)} > 0$. Finally, DE($j^* - k$) is relaxed because its right hand side changes by

$p\left(\delta\Delta\tilde{\Pi}(m)+\varepsilon\right)+(1-p)\left(\delta\Delta\tilde{\Pi}(m-(j^*+k))\right) > 0.$

Hence, DE(j) – and consequently DEl(j) constraints can be omitted for all $j \geq 1$. Finally, we have to show that this also holds for $j = 0$.

Adding DEh(j) and LLl(j) for all $j \in [1, m]$ gives constraints that are necessary and sufficient for the respective employment and savings levels to be enforceable:

$$\delta p\tilde{\Pi}(m) - ps_m + (1-p)\theta^l f(n_{m-j}, k) - n_{m-j}c + (1-p)\left(s_{m-j} - s_{m-j-1}\right) \geq 0 \qquad \text{(DE-L(j))}$$

For $j = 0$, we have the necessary constraints

$$\delta p\Pi(m) - ps_m + (1-p)\theta^l f(n, k) - nc \geq 0 \qquad \text{(DE-L(0))}$$

and

$$\delta p\Pi(m) - ps_m + \delta(1-p)\Pi(0) - nc \geq 0. \qquad \text{(DE(0))}$$

To see that DE(0) constraints can also be omitted as long as savings are positive, note that using the previously established result that DE-L(j) are stricter than DE(j) constraints for $j \geq 1$ and in particular for $j = m-1$ gives us

$\delta(1-p)\left(p\theta^h f(n_1,k)+(1-p)\theta^l f(n_1,k)\right)$
$-\left(\delta(1-p)\right)^2\left(p\theta^h f(n_1,k)+(1-p)\theta^l f(n_1,k)-n_1c\right)$
$+\delta^2(1-p)^2\left[pf(n_0,k)\theta^h+(1-p)f(n_0,k)\theta^l-nc\right]$
$-(1-p)\left(1-\delta(1-p)\right)s_1(1-\delta)$
$\geq (1-p)\theta^l f(n_1,k)+(1-p)s_1.$

Since $n_1 \geq n_0$ (and given that employment cannot be inefficiently high), we also know that $\left(p\theta^h f(n_1,k)+(1-p)\theta^l f(n_1,k)-n_1c\right)-\left(pf(n_0,k)\theta^h+(1-p)f(n_0,k)\theta^l-n_0c\right) \geq 0$.

Therefore,

$\delta(1-p)\left(p\theta^h f(n_1,k)+(1-p)\theta^l f(n_1,k)\right)$
$\geq (1-p)\theta^l f(n_1,k)+(1-p)s_1+(1-p)\left(1-\delta(1-p)\right)s_1(1-\delta)$and - since $s_1 > 0$ -

$$\delta\left(p\theta^h+(1-p)\theta^l\right) \geq \theta^l, \qquad (6)$$

if savings are optimal.

Now, rewrite DE(0) constraints as

$p\left(\delta\Pi(m) - s_m\right) + \delta(1-p)\left(pf(n_0,k)\theta^h + (1-p)f(n_0,k)\theta^l\right) - n_0c \geq 0$. This condition is implied by DE-L(0) if

$p\left(\delta\Pi(m) - s_m\right) + \delta(1-p)\left(pf(n_0,k)\theta^h + (1-p)f(n_0,k)\theta^l\right) - n_0c$
$\geq \delta p\Pi(m) - ps_m + (1-p)\theta^l f(n_0,k) - n_0c$, or if

$$\delta\left(p\theta^h + (1-p)\theta^l\right) \geq \theta^l,$$

which is equivalent to (6). ■

Hence, all DEl constraints can be omitted given savings are optimal. In a next step, we simplify the problem by adding DEh(j) and LLl(j) constraints for each j, which gives a set of necessary and sufficient constraints (sufficiency follows from the same reasoning as in the situation without savings). For each $j \in [0,m]$, we have

$$\delta p\tilde{\Pi}(m) - ps_m + (1-p)\theta^l f(n_{m-j},k) - n_{m-j}c + (1-p)\left(s_{m-j} - s_{m-j-1}\right) \geq 0, \quad \text{(DE-L(j))}$$

where $s_0 = s_{-1} = 0$.

In a next step, we show that given savings are optimal, all DE-L constraints must bind:

Lemma 6 *If $s_m > 0$, DE-L(j) constraints bind for each $j \in [0,m]$.*

Proof of Lemma 6: If there was one j where DE-L(j) did not bind, the respective employment level could be raised without violating any constraint, thereby increasing profits. ■

This result allows us to use $\delta p\Pi(m) = n_0c - (1-p)\theta^l f(n_0,k) + ps_m$ as well as $(s_{m-j} - s_{m-j-1}) = \frac{n_{m-j}c - \delta p\Pi(m) - (1-p)\theta^l f(n_{m-j},k) + ps_m}{(1-p)}$ to rewrite profits:

$$\Pi(0) = \frac{p}{1-\delta(1-p)} f(n_0,k)\theta^h$$

$$\Pi(m-j) = p\theta^h f(n_{m-j},k) + ps_{m-j} + \delta(1-p)\Pi(m-j-1),$$

where $s_0 = 0$. Finally,

$$\Pi(m) = \sum_{i=0}^{m-1} \left(\delta(1-p)\right)^i \left(p\theta^h f(n_{m-i},k) + ps_{m-i}\right) + \left(\delta(1-p)\right)^m \frac{p}{1-\delta(1-p)} f(n_0,k)\theta^h$$

In a next step, we can show that employment is gradually going down once cash reserves are used in low-demand periods:

Lemma 7 $n_{m-j+1} > n_{m-j}$ *for all* $j \in [1, m]$

Proof of Lemma 7: First, we show that $n_{m-j+1} \geq n_{m-j}$. To the contrary, assume there is a $\tilde{j}$ with $n_{m-\tilde{j}+1} < n_{m-\tilde{j}}$. Consider the following change: Replace both employment levels and reduce $s_{m-\tilde{j}}$ to keep DE-L($\tilde{j}+1$) unaffected, i.e., $\Delta s_{m-\tilde{j}} = \theta^l f(n_{m-\tilde{j}}, k) - \frac{n_{m-\tilde{j}}c}{(1-p)} - \left(\theta^l f(n_{m-\tilde{j}+1}, k) - \frac{n_{m-\tilde{j}+1}c}{(1-p)}\right)$. Note that this leaves DE-L($\tilde{j}$) as well as other constraints unaffected. However, this change increases $\Pi(m)$ and thereby $\Pi(0)$:

$$\begin{aligned}\frac{\Delta\Pi(m)}{(\delta(1-p))^{\tilde{j}-1}} &= p\theta^h\left(f(n_{m-j}, k) - f(n_{m-j+1}, k)\right) + \delta(1-p)p\theta^h\left(f(n_{m-j+1}, k) - f(n_{m-j}, k)\right)\\ &+ p\left[\theta^l f(n_{m-j}, k) - \frac{n_{m-j}c}{(1-p)} - \left(\theta^l f(n_{m-j+1}, k) - \frac{n_{m-j+1}c}{(1-p)}\right)\right]\\ &\geq p\theta^h\left(f(n_{m-j}, k) - f(n_{m-j+1}, k)\right) + (1-p)p\theta^h\left(f(n_{m-j+1}, k) - f(n_{m-j}, k)\right)\\ &+ p\left[\theta^l f(n_{m-j}, k) - \frac{n_{m-j}c}{(1-p)} - \left(\theta^l f(n_{m-j+1}, k) - \frac{n_{m-j+1}c}{(1-p)}\right)\right]\\ &= p\left(p\theta^h f(n_{m-j}, k) + (1-p)\theta^l f(n_{m-j}, k) - n_{m-j}c\right)\\ &- p\left(p\theta^h f(n_{m-j+1}, k) + (1-p)\theta^l f(n_{m-j+1}, k) - n_{m-j+1}c\right) > 0,\end{aligned}$$

where the last inequality follows from $n_{m-j} > n_{m-j+1}$, and from both employment levels being inefficiently low.

To complete the proof, it remains to show that $n_{m-j+1} = n_{m-j}$ is not possible. To the contrary, assume there is a $\hat{j}$ with $n_{m-\hat{j}+1} = n_{m-\hat{j}}$. Now, marginally increase $n_{m-\hat{j}+1}$ and marginally decrease $n_{m-\hat{j}}$. Furthermore, have $ds_{m-\hat{j}} = \theta^l \frac{\partial f(n_{m-\hat{j}+1},k)}{\partial n} - \frac{c}{1-p}$ to keep DE-L($\hat{j}$+1) and DE-L($\hat{j}$) unaffected. Then,

$$\begin{aligned}\frac{d\Pi(m)}{(\delta(1-p))^{\hat{j}-1}} &= p\theta^h\frac{\partial f(n_{m-\hat{j}+1}, k)}{\partial n} + \delta(1-p)\left(-p\theta^h\frac{\partial f(n_{m-\hat{j}+1}, k)}{\partial n} + p\left(\theta^l\frac{\partial f(n_{m-\hat{j}+1}, k)}{\partial n} - \frac{c}{1-p}\right)\right)\\ &\geq p\left(p\theta^h\frac{\partial f(n_{m-\hat{j}+1}, k)}{\partial n} + (1-p)\theta^l\frac{\partial f(n_{m-\hat{j}+1}, k)}{\partial n} - c\right) > 0.\end{aligned}$$

Therefore, $n_{m-j+1} > n_{m-j}$ because otherwise profits were not at its maximum. ■

Finally, we can show that maximum employment n_m is independent of m.

Lemma 8 *Maximum employment* n_m *is characterized by*

$$\frac{\delta(1-p)}{(1-\delta p)}p\theta^h\frac{\partial f(n_m, k)}{\partial n_m} + (1-p)\theta^l\frac{\partial f(n_m, k)}{\partial n_m} - c = 0.$$

Proof of Lemma 8: Note that if savings are optimal, all DE-L(j) constraints bind for a given m, which implies that $\delta p\Pi(m) - ps_m + (1-p)\theta^l f(n_{m-j}, k) - n_{m-j}c + (1-p)(s_{m-j} - s_{m-j-1}) = 0$ can be used to obtain the necessary savings for all levels of n_{m-j}. Then, the objective is to maximize $\Pi(0) = \frac{p}{1-\delta(1-p)} f(n_0, k)\theta^h$. Thus, employment levels n_{m-j} are chosen in order to maximize n_0. Savings can only be optimal if DE-L(0) binds, in which case n_0 is determined by $\delta p\Pi(m) - ps_m + (1-p)\theta^l f(n_0, k) - n_0 c = 0$. Hence, employment levels n_{m-j} are optimally chosen to maximize $\delta\Pi(m) - s_m$ and determined by setting $\frac{\partial(\delta\Pi(m)-s_m)}{\partial n_{m-j}} = 0$. This substantially simplifies the analysis, since generally all savings and employment levels are interconnected.

Furthermore, $\frac{\partial s_{m-\tilde{j}}}{\partial n_{m-\hat{j}}} = 0$ for $\tilde{j} > \hat{j}$, i.e. in particular $\frac{\partial s_{m-j}}{\partial n_m} = 0$ for $j > 0$. This is implied by solving DE-L(m-j) for savings level s_{m-j}:

$s_{m-j} = -\sum_{i=0}^{j} \frac{\delta p\Pi(m) - ps_m + (1-p)\theta^l f(n_{m-i}, k) - n_{m-i}c}{1-p}$,

and using $\frac{\partial(\delta\Pi(m)-s_m)}{\partial n_{m-j}} = 0$.

Hence, solving for DE-L(m) for s_m and plugging the value into $\delta\Pi(m) - s_m$ gives

$\frac{\partial(\delta\Pi(m)-s_m)}{\partial n_m} = \delta p\theta^h \frac{\partial f(n_m,k)}{\partial n_m} - \frac{\delta p^2\theta^h \frac{\partial f(n_m,k)}{\partial n_m} + (1-p)\theta^l \frac{\partial f(n_m,k)}{\partial n_m} - c}{-(1-2p+\delta p^2)}(1-\delta p)$, and n_m is determined by

$$\frac{\delta(1-p)}{(1-\delta p)} p\theta^h \frac{\partial f(n_m, k)}{\partial n_m} + (1-p)\theta^l \frac{\partial f(n_m, k)}{\partial n_m} - c = 0.$$

■

This result shows that a larger value of m does not increase maximum employment, but rather "smoothes" the process of layoffs after negative demand shocks.

Finally, we can prove Proposition 3:

Proof of Proposition 3: Lemma (8) gives (4) for maximum employment. Furthermore, note that if savings are optimal, then $n_m > n_{m-1} > ... > n$. Hence, if maximum enforceable employment without savings satisfies $\left(\frac{\delta(1-p)}{[1-\delta p]} p\theta^h + (1-p)\theta^l\right) f_1 - c \leq 0$, savings cannot be optimal. ■

Here, we do not aim for solving for the optimal m, i.e. the maximum number of subsequent negative shocks until savings are used up. The optimal level of m would again be determined by maximizing $\delta\Pi(m) - s_m$. To get around integer problems, we would first treat m as a continuous variable, set $\frac{\partial(\delta\Pi(m)-s_m)}{\partial m} = 0$ and get the optimal m as the largest integer for which $\frac{\partial(\delta\Pi(m)-s_m)}{\partial m} \geq 0$.

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