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The Merger-Paradox: A Tournament-Based Solution

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Abstract

According to the well-known "merger paradox", in a Cournot market game mergers are generally unprofitable unless most firms merge. The present paper proposes an optimal merger mechanism. With this mechanism mergers are never unprofitable, more profitable than in other known mechanism, and in many cases welfare increasing. The proposed mechanism assumes that merged firms continue to operate as independent subsidiaries that are rewarded according to a simple and commonly observed relative performance measure.

KEYWORDS: Mergers, multi-divisional firms, tournaments, industrial organization.
JEL CLASSIFICATIONS: L00, D4.

1 Introduction

According to the well-known "merger paradox", in a Cournot market game mergers are generally unprofitable unless almost all firms merge. In fact, if firms are symmetric and demand and cost functions are linear, it has been claimed that a merger can only be profitable if at least 80% of all firms merge (see Salant, Switzer, and Reynolds, 1983).[1]

While this finding has been welcomed by some as an explanation of the fact that the majority of mergers leads to losses and ends up in "divorce",[2] economists generally find it hard to believe that firms engage in activities that are predictably unprofitable.

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[1] In the linear model, 80% is sufficient if the number of firms is equal to 5 and the required percentage is even higher for all $n \neq 5$. If one replaces linear by concave inverse demand, that percentage becomes smaller, but not smaller than 50% (see Faulí-Oller, 1997).

[2] See, for example, Ravenscraft and Scherer (1989); Andrade, Mitchell, and Stafford (2001).

The subsequent literature mitigated the merger paradox, and emphasized that mergers may be profitable if firms are sufficiently different, cost functions are sufficiently convex, mergers are subject to significant synergies or Cournot is replaced by Bertrand competition.[3]

An important change in perspective was introduced by Creane and Davidson (2004) and Huck, Konrad, and Müller (2004). They emphasized that merged firms typically become independently managed subsidiaries of a holding company. Mergers facilitate the information exchange between subsidiaries which in turn can be used for "... setting up an internal game in which the divisions compete against one another"(Creane and Davidson, 2004, p. 953). Both papers propose a particular mechanism in which the merged firm "staggers" the output decisions and instructs its subsidiaries to move sequentially. That mechanism improves the profitability of mergers, although it neither assures that all mergers are profitable nor does it realize all possible gains from merger.

The present paper proposes a more profitable merger mechanism that assures that mergers are never unprofitable. That mechanism induces the subsidiaries of the merged firm to choose an output profile that cannot be improved while satisfying the conditions for an equilibrium of the simultaneous moves game between the merged and the non-merged firms. Therefore, the proposed mechanism is optimal.

The paper proceeds as follows: Section 2 motivates the analysis with a linear example, assuming bilateral mergers. Section 3 proves the optimality of the proposed mechanism in a general framework, allowing for mergers of arbitrary size, nonlinear demand, and nonlinear cost functions. Section 4 addresses the commitment problem and shows that the proposed mechanism can also be interpreted as the result of an exchange of non-voting shares between the owners of the firms that merge.

2 Motivating example

Consider a simple Cournot oligopoly with $n \geq 3$ identical firms, linear inverse demand and linear cost, with unit cost normalized to zero. Denote firms' outputs by q_i, inverse demand by $P(Q) := 1 - Q, Q := \sum_i q_i$, and firms' payoff functions by $\pi_i(q_i, q_{-i}) := P(Q)q_i$.

Suppose two firms merge, say firms 1 and 2.

If the merger is in the form of a "fusion" (f), as is implicitly assumed in the "merger paradox", the two firms are completely absorbed in the merged firm that maximizes its profit. In that case the merger reduces the number of firms from n to $n-1$ and the merged firm is simply one of $n-1$ firms that play a simultaneous moves Cournot market game. Firms' equilibrium strategies and profits before and after the merger are $q_0 = 1/(n+1), \pi_0 = 1/(n+1)^2$, respectively $q_f = 1/n, \pi_f = 1/n^2$. The gain for those who merge is equal to $G_f := \pi_f - 2\pi_0 = (1-n(n-2))/(n^2(n+1)^2) < 0$. Hence, the merger is unprofitable for those who merge and benefits only those who do not merge.

[3]For the impact of asymmetries and synergies, convex cost functions, and the assumed market game see Farrell and Shapiro (1990) and Ding, Fan, and Wolfstetter (2013), Perry and Porter (1985), and Deneckere and Davidson (1985).

As an alternative to fusions, we propose the following merger mechanism: 1) The two firms that merge continue to operate as independent subsidiaries. 2) The headquarter of the merged firm rewards the managers of the subsidiaries according to their relative performance. Each manager is paid the salary:

$$S_i := \pi_i - \alpha \pi_j - t, \quad i \neq j, i, j \in \{1,2\} \tag{1}$$

where t is a lump-sum "tax", and α, t are set optimally to maximize the headquarter's profit. That reward scheme is made known to all firms. 3) After the merger the subsidiaries 1 and 2 and the $n-2$ non-merged firms $3,\ldots,n$ play a simultaneous moves Cournot market game. Note that such a reward scheme is not accessible to non-merged firms, because they cannot observe each other's profits.

This mechanism implements the outcome of a hypothetical Stackelberg game in which the merged firm is a multi-plant Stackelberg leader who operates two plants and chooses a uniform output per plant, q_L, and the $n-2$ non-merged firms are followers who choose their outputs, q_F, simultaneously, after having observed the leader's output per plant, q_L.

The equilibrium of that hypothetical Stackelberg game is $(q_L^*, q_F(q_L)) = (1/4, (1-2q_L)/(n-1))$, which leads to the equilibrium output profile

$$(q_L^*, q_F(q_L^*)) = \left(\frac{1}{4}, \frac{1}{2(n-1)}\right). \tag{2}$$

Now consider the proposed mechanism for given (α, t). As a working hypothesis, suppose the equilibrium is symmetric in the sense that each subsidiary plays the strategy q_M and each non-merged firm plays q_N. The equilibrium must solve the following requirements:

$$\begin{aligned} q_M &= \arg\max_q \left(1 - q - q_M - (n-2)q_N\right)(q - \alpha q_M) - t \\ q_N &= \arg\max_q \left(1 - 2q_M - q - (n-3)q_N\right) q, \end{aligned}$$

which yields the equilibrium solution as a function of α: $q_M(\alpha) = 1/(n+1-(n-1)\alpha)$, $q_N(\alpha) = (1-\alpha)/(n+1-(n-1)\alpha)$.

The headquarter sets (α, t) to maximize its profit. This is achieved by setting

$$\alpha^* = \frac{n-3}{n-1}, \quad t^* = \frac{1}{4(n-1)^2}, \tag{3}$$

because this induces the same equilibrium outputs as the hypothetical Stackelberg equilibrium,

$$q_M(\alpha^*) = q_L^*, \quad q_N(\alpha^*) = q_F(q_L^*), \tag{4}$$

and allows the headquarter to extract the entire profits of the subsidiaries:[4]

Obviously, this mechanism induces the most profitable equilibrium output profile of the subsidiaries, and hence is optimal.

[4]The choice of t assumes, for simplicity, that managers' opportunity cost is equal to zero.

The resulting gain from merger, G, is equal to

$$G := (1-2q_M(\alpha^*)-(n-2)q_N(\alpha^*))\,2q_M(\alpha^*)-2\pi_0 = \frac{(n-3)^2}{4(n^2-1)(n+1)}. \tag{5}$$

G is positive for all $n>3$ and equal to zero for $n=3$. Hence the merger is never unprofitable and profitable for all $n>3$, and non-merged firms are never better-off and worse-off for all $n \geq 4$.

Moreover, the merger changes the equilibrium aggregate output from $Q_0 = n/(n+1)$ to $Q = 1 - 1/2(n-1)$ which implies $Q - Q_0 = (n-3)/2(n^2-1)$. Therefore, the merger never reduces welfare and increases it whenever the merger is profitable.

Finally, we compare the profitability of the proposed mechanism with that of the "staggered competition" mechanism by Creane and Davidson (2004) and Huck, Konrad, and Müller (2004), where the subsidiaries are instructed to move sequentially. There, one subsidiary moves first and informs the other, but not the $n-2$ non-merged firms, of its output choice.

If this mechanism (indicated by the subscript s) is employed, the gain from merger is equal to

$$G_s = \frac{n(n-2)-5}{(n+1)^2(n+2)^2}. \tag{6}$$

G_s is negative for $n=3$, yet positive for all $n \geq 4$.

This mechanism is far less profitable than the proposed mechanism. Indeed,

$$G - G_s = \frac{(n-4)^2}{4(n-1)(n+2)^2}, \tag{7}$$

is non-negative, positive for all $n \neq 4$, and strictly increasing in n for all $n>4$. For $n=8$, switching from "staggered competition" to the proposed mechanism already more than doubles the profitability of the merger.

One reason why the "staggered competition" mechanism is less profitable, and even entails losses if $n=3$, is that it induces the first mover to raise its profit at the expense of the second mover.

3 Generalization

We now generalize and allow for mergers of arbitrary size, nonlinear inverse demand, $P(Q)$, and non-linear cost functions, $C(q)$. We assume that P and C are twice continuously differentiable, with $P'(Q)<0, C'(q)>0$, and $P''(Q) \leq 0, C''(q) \geq 0$.

Consider mergers of $k+1$ firms, where $k \in \{1,\ldots,n-1\}$. This allows for mergers of all possible sizes, ranging from the merger of two firms ($k=1$) to the merger of all firms ($k=n-1$). The number k indicates how many independent firms leave the market due to the merger and either vanish as independent firms (in the case of a fusion) or continue to operate as subsidiaries of the merged firm. The manager of each subsidiary i is paid a salary equal to $S_i := \pi_i - \alpha\bar{\pi}_{-i} - t$, where $\bar{\pi}_{-i}$ denotes the average profit of all subsidiaries other than subsidiary i.

To prepare our general result, we first state some properties of the hypothetical Stackelberg game in which one firm is a "multi-plant" Stackelberg leader who operates $k+1$ plants and chooses a uniform output per plant, q_L, and $(n-k-1)$ firms are followers who simultaneously choose their outputs, q_F, after having observed the leader's output per plant, q_L. The equilibrium strategies of that hypothetical game, $(q_L^*, q_F(q_L))$, must satisfy the following equilibrium requirements:

$$q_F = \arg\max_q P\big((k+1)q_L + q + (n-k-2)q_F\big)q - C(q), \quad \forall q_L$$
$$q_L^* = \arg\max_q P\big((k+1)q + (n-k-1)q_F(q)\big)q - C(q).$$

Therefore, $(q_L^*, q_F(q_L))$ solves the conditions:

$$P(Q) - C'(q_F(q_L)) + P'(Q)q_F(q_L) = 0, \quad \forall q_L \tag{8}$$
$$P(Q^*) - C'(q_L^*) + P'(Q^*)q_L^*\big(k+1+(n-k-1)q_F'(q_L^*)\big) = 0 \tag{9}$$
$$Q = (k+1)q_L + (n-k-1)q_F(q_L),\ Q^* = (k+1)q_L^* + (n-k-1)q_F(q_L^*). \tag{10}$$

Now return to the proposed mechanism and denote the equilibrium outputs of the $k+1$ subsidiaries of the merged firm by q_M and the equilibrium outputs of the non-merged firms by q_N. We find:

Proposition 1. *The proposed mechanism with optimally chosen (α^*, t^*) implements the equilibrium outcome of the above hypothetical Stackelberg game, i.e.,*

$$q_M = q_L^*, \quad q_N = q_F(q_L^*). \tag{11}$$

Therefore, the merged firm earns the same equilibrium profit as the leader in that hypothetical Stackelberg game. Mergers are generally profitable and never unprofitable.

Proof. For given (α, t), the equilibrium outputs, (q_M, q_N), must satisfy the following equilibrium requirements:

$$q_M = \arg\max_q P\big(q + kq_M + (n-k-1)q_N\big)(q - \alpha q_M) - C(q) + \alpha C(q_M) - t$$
$$q_N = \arg\max_q P\big((k+1)q_M + q + (n-k-2)q_N\big)q - C(q).$$

Therefore, (q_M, q_N) solve the conditions:

$$P(Q) - C'(q_M) + P'(Q)(1-\alpha)q_M = 0 \tag{12}$$
$$P(Q) - C'(q_N) + P'(Q)q_N = 0 \tag{13}$$
$$Q = (k+1)q_M + (n-k-1)q_N. \tag{14}$$

Comparing conditions (13)-(14) with (8)-(10), we confirm that

$$(q_M, q_N) = (q_L^*, q_F(q_L^*)) \iff \alpha = \alpha^* := -(n-k-1)q_F'(q_L^*) - k, \tag{15}$$

and the headquarter extracts the entire profit if and only if:[5]

$$t = t^* := \Big(P(Q^*)q_L^* - C(q_L^*)\Big)(1-\alpha^*). \tag{16}$$

[5]This choice of t assumes that the opportunity cost of managers is equal to zero.

It follows that by setting $(\alpha,t) = (\alpha^*,t^*)$ the merged firm earns the same profit as the multi-plant Stackelberg leader in the hypothetical Stackelberg game.

The proposed mechanism is optimal because one cannot find a more profitable output profile of the merged firm that satisfies the conditions for an equilibrium of the simultaneous moves game between the subsidiaries and the non-merged firms. $\square$

We close with the special case of arbitrary size mergers in the linear model. There, we find: $\alpha^* = {}^{(n-2k-1)}/_{(n-k)}$, $q_M = {}^{1}/_{(2(k+1))}$, and $q_N = {}^{1}/_{(2(n-k))}$. Therefore, the gain from merger of size $k+1$ is equal to:

$$G(k) = \frac{(n-2k-1)^2}{4(n-k)(n+1)^2}. \tag{17}$$

$G(k)$ is non-negative, strictly convex, and has a global minimum. Ignoring, for a moment, that k must be an integer, the minimum is reached at $k = {}^{(n-1)}/_{2}$, which represents the worst size merger. Therefore, every size merger is profitable if n is an even number, whereas, if n is an odd number, the merger breaks even for $k = {}^{(n-1)}/_{2}$ and is profitable for all other k.

The merger also increases welfare (social surplus) if and only if $k < {}^{(n-1)}/_{2}$. This follows from the fact that the merger changes the aggregate equilibrium output from Q_0 to $Q = 1 - {}^{1}/_{(2(n-k))}$ and $Q - Q_0 = {}^{(n-2k-1)}/_{(2(n+1)(n-k))} \gtreqless 0 \Leftrightarrow k \lesseqgtr {}^{(n-1)}/_{2}$. This suggests that antitrust authorities should be permissive and prohibit only large mergers that include more than half the number of firms.

4 Alternative interpretation

The proposed mechanism assumes delegation and requires the ability to commit to a reward scheme for managers. However, it can also be interpreted as the result of an exchange of non-voting shares among the original owners of the firms that merge.

In that interpretation the original owners of the firms that merge remain residual claimants of the respective subsidiaries. For simplicity consider a merger of two firms. The mechanism requires each of them to short-sell non-voting shares of the other firm to each other. Specifically, firm i must short-sell shares in firm j to such an extent that firm j acquires a contingent claim to $\beta\pi_j$ in exchange for a fixed payment equal to $\beta\pi_j^*$ (where π_j^* denotes j's equilibrium profit).

As a result of this exchange of financial assets, the merged firms' payoff function becomes $\Pi_i = \pi_i - \beta(\pi_j - \pi_j^*) + \beta(\pi_i - \pi_i^*)$, $i \neq j$, $i,j \in \{1,2\}$. The maximizer of Π_i is the same as that of ${}^{\Pi_i}/_{(1+\beta)}$ (as long as $\beta \neq -1$). Therefore, merged firms can be viewed as maximizing $\pi_i - \alpha\pi_j - t$, $i \neq j$, $i,j \in \{1,2\}$, where t is a constant equal to $\alpha(\pi_i^* - \pi_j^*) = 0$ and $\alpha := {}^{\beta}/_{(1+\beta)}$. The only difference is that no delegation to managers is used, the original owners of the subsidiaries maximize their profit, and the headquarter enables the information exchange between the subsidiaries but does not extract their profits.

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