GOVERNANCE AND THE EFFICIENCY
OF ECONOMIC SYSTEMS
GESY

Discussion Paper No. 47

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May 2005

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Financial support from the Deutsche Forschungsgemeinschaft through SFB/TR 15 is gratefully acknowledged.

Rent seeking in sequential group contests[*]

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Abstract

In this paper, a group contest is analyzed, where the groups are allowed to determine their sharing rules either sequentially or simultaneously. It is found that in case the more numerous group determines its sharing rule prior to the smaller group, rent dissipation in the group contest is higher than in an individual contest. However, if the order of moves is endogenized, the smaller group will always act prior to the bigger group. Competition between the groups is in this way weakened and the groups are able to save on expenditures.

Key Words: Group contest, rent seeking, sequential choices, sharing rule

JEL classification: D 72

[*] I would like to thank Matthias Kräkel for helpful comments. Financial support by the Deutsche Forschungsgemeinschaft (DFG), SFB-TR 15 (Governance and the efficiency of economic systems), is gratefully acknowledged.

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1. Introduction

In practice, there are many examples of rent seeking contests: Different cities are in competition to host the Olympic Games. Thereby, they spend a lot of resources during a nomination process in order to increase the probability of being selected. For the reconstruction of the World Trade Centre, as another example, eight international groups of architects presented suggestions, how to design the new building. Finally, the proposal of an architect group from Berlin was selected. Again, all the architect groups were in competition for a given prize (here the fame and monetary gain from designing the new World Trade Centre) and spent resources while developing their proposals. Moreover, one can also think of an election campaign as a rent seeking contest. There are several parties, with each investing resources to support its candidate.

In the literature, some contest success function, assigning to each individual or group a winning-probability for given resource expenditures, is used to model a rent seeking contest. In group competition some sharing rule is additionally needed, determining the share in the prize that each member of a group receives in case his group wins the contest. Nitzan (1991a, b) firstly introduced such a sharing rule. He thought of a rule that rewards the groups` members partly on an egalitarian basis, and partly according to relative outlays. This kind of sharing rule was adopted in most of the literature on group rent seeking contests (see, e.g., Lee (1995), Baik and Lee (1997), Lee and Kang (1998) or Gürtler (forthcoming)).

Comparing the outcomes of the individual and the group contest, there is a very interesting and surprising result: Although the groups are allowed to "overpay" relative effort, i.e., although they are allowed to choose a more outlay-based incentive scheme than in the individual contest, the group contest will never lead to higher rent-seeking activities than the individual contest.[1]

[1] See again Gürtler (forthcoming) for a formal proof.

Noticeable is, that this result is derived under the assumption of simultaneous actions of the players. In the individual contest, the players choose simultaneously their outlays. In the group contest, the groups determine simultaneously their sharing rules, and, thereafter, the groups` members again choose simultaneously their resource expenditures. The assumption of simultaneous actions in the literature on contests was justified using the following argument: Each contestant would have an incentive to be the first mover in a sequential contest. Competition for this first-mover advantage would then lead to simultaneity. However, one could also think that the actions in a rent seeking contest are undertaken sequentially. Leininger (1993) and Morgan (2003) allowed for sequential actions. In both papers, a contest between two individuals is considered. The individuals are able to choose between two dates to make their outlays. If they choose the same date, there will be a simultaneous contest. If they choose different dates, there will be a sequential contest. Hence, sequential play is not exogenously assumed, it only may occur endogenously. The results are very interesting: Leininger found out that, in case of asymmetric individuals, i.e., individuals with different valuations for the prize, there is a unique subgame perfect equilibrium, where the individuals act sequentially. Assuming symmetry, the individuals are indifferent between sequential and simultaneous play. Consequently, either sequential or simultaneous actions may arise in equilibrium. The intuition behind these results is very simple. In the case of asymmetric players, competition will decline, if the weaker player acts in first place. Hence, it is optimal for the individuals to choose sequential play with the weaker player acting first in order to save on resource expenditures. If the players are symmetric, this cost advantage of sequential play disappears and, therefore, both kinds of play are equally good. While in Leininger both individuals are informed about their own valuation for the prize and the valuation of their opponent when they decide about the date at which they make their outlays, in Morgan, this decision is characterized by ex ante symmetry. However, his results are even stronger in favor for sequential contests. There does not exist any equilibrium with simultaneous actions. It is always better for the individuals to act sequentially.

In this paper, the possibility of sequential actions is extended to a group contest. Elements of the papers of Nitzan, Leininger and Morgan are combined in order to determine the importance of sequential play in group competition. To keep things as simple as possible and to enable a comparison of total rent seeking in group and individual contests, it is assumed that the groups may choose their sharing rules either simultaneously or sequentially, but that outlays are chosen simultaneously. This new approach reveals some interesting findings. In an asymmetric situation (here measured by the size of the groups), total rent seeking in the group contest will be higher than total rent seeking in an individual contest, if the more numerous group determines its sharing rule in first place. However, if the timing of the contest is chosen endogenously, there will exist an unique subgame perfect equilibrium with the smaller group determining its sharing rule prior to the bigger group.

The paper is organized as follows: The model assumptions are introduced in section 2. As mentioned before, total rent seeking in the individual contest shall be compared to total rent seeking in the group contest. Hence, total rent seeking in the individual contest has to be determined. This is briefly done in section 3. Section 4 contains the solution to the group contest and the main results of the paper. Concluding remarks are offered in section 5.

2. The model

I consider a contest between two groups, a and b. The groups compete for a fixed and divisible rent or prize $S > 0$. Group i (i= a, b) consists of $n_i \geq 2$ identical and risk-neutral members, with $n_a + n_b = N$. There is no further restriction on n_a and n_b, i.e., there may either be a symmetric situation with $n_a = n_b$ or an asymmetric situation with $n_a \neq n_b$. Moreover, it is assumed that the individuals are not able to switch from one group to the other one or to compete as an individual in the contest. Yet, they can decide not to participate in the contest at all. If individual k of group i chooses to take part in the contest, it invests observable outlay in the amount of x_{ki} for its group. As usual in the contest literature, the winning-probability of group i, Π_i, is given by the ratio of its total outlays $X_i = \sum_{k=1}^{n_i} x_{ki}$ relative to the aggregate outlays of the two groups $X = X_a + X_b$ (see, e.g., Tullock (1980) or for an axiomatic approach Skaperdas (1996) and Clark and Riis (1998)):

(1) $\Pi_i = X_i / X$.

The expected utility of individual k of group i, V_{ki}, can therefore be written as:

(2) $V_{ki} = \Pi_i \cdot (f_{ki} \cdot S) - x_{ki}$.

In this context, the variable f_{ki} denotes the sharing rule used in group i. In case group i wins the contest, member k of this group receives the fraction f_{ki} of the rent S. As firstly introduced by Nitzan (1991 a,b), this sharing rule is assumed to be given by:

(3) $f_{ki} = \alpha_i \cdot 1/n_i + (1-\alpha_i) \cdot x_{ki}/X_i$.[2]

The "payment" of a group member consists of two components. A fraction α_i of the rent is distributed on an egalitarian basis, while the rest is distributed according to relative outlays within the group. Noticeable is that there is no restriction on α_i, hence, the groups are able to penalize relative outlay, i.e. to choose $\alpha_i > 1$, or to overpay it, i.e. to choose $\alpha_i < 0$. A group chooses its sharing rule such that the aggregate expected utility of its members is maximized.

The timing of the contest game is as follows: At date zero, both groups announce, when to determine their sharing-rule parameters. Thereby, they are allowed to determine it either at date one or at date two, respectively. For simplicity, there is no discounting. If both groups announce the same date for determining their sharing rule, there will be a simultaneous game

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[2] This sharing rule is not as restrictive as it may seem. Under the weak condition $n_a, n_b > 2$, it can replicate every Nash equilibrium outcome that can be achieved with the most general sharing rule. This is shown in the Appendix.

at date one or two. Moreover, if the groups announce different dates for determining their sharing rules, there will be a sequential game with one group determining its sharing rule at date one and the other determining it at date two. A date three, the individuals decide whether or not to take part in the contest. Finally, at date four, the individuals taking part in the contest simultaneously choose their optimal outlays. It is assumed that the groups` sharing rules are perfectly observable. That is, at date three and four every individual knows the two sharing rules and in the sequential contest, the group acting at date two is able to observe the sharing rule of the prior acting group.[3]

3. An individual contest as a benchmark case

Before the solution to the group contest is provided, briefly a contest between N individuals is considered. If the individuals simultaneously determine their optimal rent seeking expenditures, and if an individual`s winning probability again is given by the ratio of its outlay relative to the aggregate outlay in the contest, individual i will choose its outlay in order to maximize the function

(4) $$V_i = \frac{x_i}{\sum_{j=1}^{N} x_j} \cdot S - x_i .$$

The first-order condition is given by:

(5) $$\frac{\partial V_i}{\partial x_i} = \frac{\sum_{j=1}^{N} x_j - x_i}{\left(\sum_{j=1}^{N} x_j\right)^2} \cdot S = 1 .$$

[3] This assumption is crucial to the results to be derived in this paper. In determining the sharing rules, the two groups (strategically) commit to a certain behavior. Choosing its sharing-rule parameter very low, a group credibly demonstrates that it will act rather aggressively, i.e., that its members will choose high outlays. However, as Bagwell (1995) and Maggi (1999) have shown, in games, where the first mover has no private information, this kind of commitment will only be possible, if all actions are perfectly observable by the late-moving players.

The second-order condition is satisfied. The right-hand-side of equation (5) is the same for every other contestant. Hence, the left-hand-side also has to be the same and, consequently, there exists an unique symmetric equilibrium. Aggregate rent seeking in this symmetric equilibrium is given by $X = \frac{N-1}{N} \cdot S$. It is an increasing function in the number of contestants and the size of the rent.

4. Solution to the model

Using backwards induction, our aim is to find a subgame perfect equilibrium of the contest game described in section 2. First, the decisions at date four, i.e. the outlay choices for given sharing rules, have to be determined. Individual k of group i chooses that outlay that maximizes the subsequent function (6):

$$(6) \qquad V_{ki} = \frac{X_i}{X} \cdot \left(\alpha_i \cdot 1/n_i + (1-\alpha_i) \cdot x_{ki}/X_i\right) \cdot S - x_{ki}.$$

Comparing the target functions in (4) and (6), it can immediately be seen that rent seeking in the individual contest will be at least as high as in the group contest, if the restriction $\alpha_i \geq 0$ is imposed, i.e. if overpaying relative outlay is forbidden. For $\alpha_a = \alpha_b = 0$, both types of contest are equivalent, and, therefore, the total amount of rent seeking is the same in both. Clearly, if α_a or α_b increases, the incentive scheme in the group contest will become less outlay-based and rent seeking will decrease. Hence, only in case overpaying relative outlay is possible, rent seeking in the group contest might be higher than in the individual contest.

Deriving (6) with respect to x_{ki}, yields the subsequent first-order condition:

$$(7) \qquad \frac{\partial V_{ki}}{\partial x_{ki}} = \left(\frac{X_j}{X^2} \cdot \frac{\alpha_i}{n_i} + \frac{X - x_{ki}}{X^2} \cdot (1-\alpha_i)\right) \cdot S = 1, \text{ where } j \neq i.$$

The second-order condition will be satisfied, if the condition $-X_j \cdot \frac{\alpha_i}{n_i} - (X - x_{ki}) + \alpha_i \cdot (X - x_{ki}) < 0$ holds, i.e., if the target function in (6) is concave. It is easy to see that this condition is fulfilled for all $\alpha_i \in [0,1]$. Moreover, since $X - x_{ki} \geq X_j$, it is also fulfilled for $\alpha_i < 0$. However, if $\alpha_i > 1$ the second-order condition might be violated. The intuition behind this result is clear: An individual initially contributing an outlay of zero has to consider several effects when deciding about its rent seeking activity. On the one hand, if it increases its outlay, its group will be more likely to win the contest. On the other hand, it has costs since it expends resources, and, additionally, it will be penalized by the sharing rule. Due to the two negative effects, an marginal increase of outlay may lower its expected utility, in other words, the target function may be convex and an inner-solution would describe an expected utility minimum. We will see later, that the groups never choose $\alpha_i > 1$. The second-order condition will therefore be satisfied in all what follows.

Summation of equation (7) over all members of group i, yields the subsequent condition:

(8) $$\frac{X_i}{X} = \alpha_i + (1-\alpha_i) \cdot n_i - n_i \cdot \frac{X}{S}.$$

Simultaneous solution of condition (8) for both groups, gives:

(9) $$X_a = \frac{S \cdot Q}{N^2} \cdot \left(n_b \cdot \alpha_a - n_a \cdot \alpha_b + n_a \cdot n_b \cdot (\alpha_b - \alpha_a) + n_a\right),$$

(10) $$X_b = \frac{S \cdot Q}{N^2} \cdot \left(n_a \cdot \alpha_b - n_b \cdot \alpha_a + n_a \cdot n_b \cdot (\alpha_a - \alpha_b) + n_b\right),$$

(11) $$X = \frac{S \cdot Q}{N}, \text{ with } Q = \left(\alpha_a + \alpha_b + n_a \cdot (1-\alpha_a) + n_b \cdot (1-\alpha_b) - 1\right).$$

The decisions at date four are now analyzed. In order to find a subgame perfect equilibrium of the contest model, all possible outcomes at date one and two have to be determined.[4] Doing

[4] It will be seen in sections 4.1 and 4.2 that each group receives a positive expected rent. Since the members of a group are symmetric, they all receive a positive expected utility from participating in the contest. Hence, at date three, all individuals decide to take part in the contest.

this, both situations, the situation of simultaneous play (both groups at date zero announce the same date for determining their sharing rule) and the situation of sequential play (one group announces date one and the other date two) have to be analyzed separately. I start with the former one:

4.1 Simultaneous play

If both groups announce the same date for determining their sharing rule, i.e., if either both groups announce date one or both groups announce date two, the maximization problems of group a and group b will be as follows:

(12a) $$\underset{\alpha_a}{\text{Max}}\ V_a := \sum_{k=1}^{n_a} V_{ka} = \frac{X_a}{X} \cdot S - X_a \, ,$$

(12b) $$\underset{\alpha_b}{\text{Max}}\ V_b := \sum_{k=1}^{n_b} V_{kb} = \frac{X_b}{X} \cdot S - X_b \, .$$

The first-order conditions are given by (13a) and (13b), respectively:

(13a)

$$\frac{\partial V_a}{\partial \alpha_a} = (n_b - n_a \cdot n_b) - \frac{(1-n_a)\cdot(n_a \cdot n_b \cdot (\alpha_b - \alpha_a) + n_b \cdot \alpha_a - n_a \cdot \alpha_b + n_a) + Q \cdot (n_b - n_a \cdot n_b)}{N} = 0,$$

(13b)

$$\frac{\partial V_b}{\partial \alpha_b} = (n_a - n_a \cdot n_b) - \frac{(1-n_b)\cdot(n_a \cdot n_b \cdot (\alpha_a - \alpha_b) - n_b \cdot \alpha_a + n_a \cdot \alpha_b + n_b) + Q \cdot (n_a - n_a \cdot n_b)}{N} = 0.$$

Solving these first-order conditions for the equilibrium sharing-rule parameters, we get:

(14) $$\alpha_{a,si} = \frac{n_a - n_b}{N \cdot (n_a - 1)} \, ; \ \alpha_{b,si} = \frac{n_b - n_a}{N \cdot (n_b - 1)} \, .$$[5]

In the case of simultaneous actions and groups of unequal size, the more numerous group always installs a less outlay-based incentive scheme than the less numerous group. A group consisting of fewer members than the other, has a size-advantage since it suffers less from free-riding. However, it also has a size-disadvantage since less members contribute rent-

[5] The index si stands for simultaneous play.

seeking activities. In the optimum, the smaller group strengthens its advantage due to free-riding by installing a more outlay-based incentive scheme than the more numerous group. Moreover, the smaller group chooses a negative sharing-rule parameter, while the more numerous group chooses a positive one. At first sight, it might therefore be possible that rent is more dissipated in the group contest than in the individual contest. However, from the literature on rent seeking contests we know that this is not true. Inserting $\alpha_{a,si}$ and $\alpha_{b,si}$ into equation (11), we see that aggregate rent seeking equals $\frac{(N-1)\cdot S}{N}$, hence, total rent seeking in the group contest and the individual contest is the same, although the incentive structure is quite different. If the two groups are of equal size, the solution will be $\alpha_{a,si} = \alpha_{b,si} = 0$. As mentioned before, the group contest is then equivalent to the individual contest and rent seeking again is $\frac{(N-1)\cdot S}{N}$. Proposition 1 summarizes these results:

Proposition 1: If the groups simultaneously determine their sharing rules, total rent seeking in the group contest equals total rent seeking in the individual contest.

Lastly, the groups` aggregate utilities have to be calculated. Using the optimal sharing-rule parameters in (14), we get:

(15) $$V_a = n_b \cdot \frac{S}{N^2}\,;\; V_b = n_a \cdot \frac{S}{N^2}.$$

The aggregate utility of the smaller group always exceeds the aggregate utility of the more numerous group. It increases in the size of the rent, while it decreases in the size of the more numerous group as well as in its own size. The aggregate utility of the more numerous group increases in the size of the rent and the size of the opponent group, and decreases in the own size. The group size effects on the groups` utilities are mainly due to changes in the incentive schemes. If the size of the smaller group increases, the bigger group will choose a more outlay-based incentive scheme, while the smaller group decides to install a more egalitarian remuneration of its members. As a result, the more numerous group is better off, while the smaller group suffers from the new situation. Similarly, if the size of the bigger group increases, the smaller group will increase the incentives of its members, while there will be no clear effect on the sharing rule of the bigger group. However, in this case, both groups suffer from the new situation, since the aggregate utilities of both groups get lower.

4.2 Sequential play

If one group announces to determine its sharing rule at date one, while the other announces to determine it at date two, there will be a sequential game, starting at date one with one group deciding before the other. This (sub)game again has to be solved by backwards induction. For expositional purposes, the group acting first is denoted as group 1 and the group acting in second place is denoted as group 2. It is assumed that group 2 is able to observe the chosen sharing rule of the other group 1. Hence, it has to solve the subsequent maximization problem:

$$(16) \qquad \underset{\alpha_2}{\text{Max}}\ V_2 := \sum_{k=1}^{n_2} V_{k2} = \frac{X_2}{X} \cdot S - X_2 \,.$$

The first-order condition is given by:

(17)

$$\frac{\partial V_2}{\partial \alpha_2} = \left(n_1 - n_1 \cdot n_2\right) - \frac{\left(1 - n_2\right) \cdot \left(n_1 \cdot n_2 \cdot \left(\alpha_1 - \alpha_2\right) - n_2 \cdot \alpha_1 + n_1 \cdot \alpha_2 + n_1\right) + Q \cdot \left(n_1 - n_1 \cdot n_2\right)}{N} = 0.$$

From this first-order condition, the reaction function of group 2 can be determined as follows:

$$(18) \qquad \alpha_2 = \alpha_1 \cdot \frac{\left(n_2 - n_1\right) \cdot \left(n_1 - 1\right)}{2 \cdot n_1 \cdot \left(n_2 - 1\right)} + \frac{n_2 - n_1}{2 \cdot n_1 \cdot \left(n_2 - 1\right)} \,.$$

Group 1 has to consider this reaction function, when it decides about its sharing rule. Hence, its maximization problem is given by:

$$
(19)\qquad \begin{aligned} &\underset{\alpha_1}{\text{Max}}\ V_1 := \sum_{k=1}^{n_1} V_{k1} = \frac{X_1}{X}\cdot S - X_1 \\ &\text{s.t.}\quad \alpha_2 = \alpha_1\cdot\frac{(n_2-n_1)\cdot(n_1-1)}{2\cdot n_1\cdot(n_2-1)} + \frac{n_2-n_1}{2\cdot n_1\cdot(n_2-1)}. \end{aligned}
$$

The solution to this maximization problem is:

$$
(20)\qquad \alpha_1 = 0.
$$

We therefore get:

$$
(21)\qquad \alpha_2 = \frac{n_2-n_1}{2\cdot n_1\cdot(n_2-1)}.
$$

From (21), it is obvious that $\alpha_2 < 0$ holds if and only if $n_1 > n_2$. As in the case of simultaneous actions, the less numerous group chooses a smaller sharing-rule parameter, and, therefore, a more outlay-based incentive scheme than its opponent. Implications about total rent seeking are given in the following proposition:

Proposition 2: In case both groups determine their optimal sharing rules sequentially, the degree of rent seeking depends on which team moves first:

- *(i)* *If the less numerous team firstly determines its sharing rule, less resources will be dissipated in the group contest than in the individual contest.*
- *(ii)* *If the more numerous group firstly determines its sharing rule, the group contest will lead to higher rent seeking activities than the individual contest.*
- *(iii)* *If both groups are of the same size, total rent dissipation will be the same in both kinds of contest.*

Similarly to the results in Leininger, the order of moves affects the intensity of competition between the two groups. If the more numerous group acts in first place, it will determine its sharing-rule parameter extremely low in order to show its opponent that it is willing to win the contest no matter what it costs. The smaller group would then choose its best response according to condition (18) and, therefore, it would "accept the challenge". There would be some kind of rat race, in the sense of Akerlof (1976), since competition between the two groups would escalate significantly. As a result, both groups would choose extremely outlay-based incentive schemes and total rent seeking would be very high. As described in proposition 2, it would even exceed total rent seeking in the individual contest. The result that rent seeking in the group contest is never higher than in the individual contest therefore only holds for simultaneous determinations of the sharing rules, for sequential choices it is not true. However, in case the smaller group firstly determines its sharing rule, competition is extremely weakened. The smaller group realizes that if it would choose a very little sharing-rule parameter, its opponent would react by also choosing an extremely outlay-based incentive scheme. Hence, it chooses a rather moderate sharing rule in order to commit to a less intensified competition. As a result, there will be a kind of implicit collusion between the two groups.[6]

Again, the groups` aggregate utilities need to be determined. Using the optimal sharing rules, they can be written as:

(22) $$V_1 = \frac{S}{4 \cdot n_1}; \; V_2 = \frac{S}{4 \cdot n_1}.$$

Both groups, the group acting at date one and the group acting at date two, receive the same aggregate utility. This utility depends on which group determines its sharing rule first. As can be seen from (22) and as is intuitively clear, the groups` aggregate utilities are higher if the smaller group determines its sharing rule prior to the more numerous group. In this case, competition is weakened and the groups can save on expenditures.

4.3 The optimal decision at date zero

[6] See, e.g., Dye (1984) on collusion in contests.

The reduced game at date zero is a simultaneous two player game. Each player, i.e. each group has two possible actions: It can announce either to determine its sharing rule at date one or at date two. Hence, the game at date zero can be presented appropriately by using the matrix in figure 1:

		Group b	
		Date one	Date two
Group a	Date one	$n_b \cdot \frac{S}{N^2}$, $n_a \cdot \frac{S}{N^2}$	$\frac{S}{4 \cdot n_a}$, $\frac{S}{4 \cdot n_a}$
	Date two	$\frac{S}{4 \cdot n_b}$, $\frac{S}{4 \cdot n_b}$	$n_b \cdot \frac{S}{N^2}$, $n_a \cdot \frac{S}{N^2}$

Figure 1. Reduced Game at date zero

From figure 1, it can be seen that the reduced game at date zero will have an unique equilibrium, if the two groups are of unequal size. In this case, the smaller group announces to determine its sharing rule at date one, while the more numerous group announces date two for determining its sharing rule. If both groups are of equal size, all payoffs in the matrix will be identical. Consequently, each group is always indifferent between acting at date one or at date two (and any probability distribution over these two actions). Hence, in any subgame perfect equilibrium, there can either be simultaneous or sequential play. Proposition 3 considers the degree of rent seeking in the group contest:

Proposition 3:

(i) *If both groups are of different size, there will be a unique subgame perfect equilibrium to the group contest with the smaller group determining its sharing*

rule prior to the more numerous one. Rent seeking activities will be lower than in the individual contest.

(ii) *If both groups are of equal size, both groups are always indifferent between acting at date one or date two. In equilibrium, there can either be sequential or simultaneous play. Rent dissipation will always be the same as in the individual contest.*

The results derived in this section are similar to the results that Leininger derived for the contest between individuals. In the case of asymmetric groups, there exists a unique subgame perfect equilibrium, in which the smaller (or weaker) group acts prior to the more numerous (or stronger) group. Using this order of moves, the two groups are able to weaken competition and, therefore, to save on resource expenditures. As a consequence, in the unique subgame perfect equilibrium of the asymmetric group contest, rent seeking is lower than in the individual contest. In the case of equal numerous groups, the two groups do not care any more about the order of moves. In this case, rent-seeking activities in the group contest and in the individual contest are the same.

4.4 Preemptive behavior

In section 4.2, the possibility of preemptive behavior was not considered. However, the group determining its sharing rule prior to the other might be interested in choosing such an aggressive sharing rule that the other group drops out of the contest, i.e. that the other group determines its sharing rule such that its members contribute zero outlays. This corner solution is analyzed in this section. Using the reaction function (18) together with one of the conditions (9) or (10), one can see that the group acting at date two will drop out of the contest, if the condition $\alpha_1 \leq \dfrac{1-n_2}{n_1 \cdot n_2 - N + 1}$ holds. Notice that the right-hand-side of this condition is negative. Group 1 therefore has to choose an extremely aggressive, i.e. an extremely outlay-based incentive scheme to achieve a resignation of group 2. If preemptive behavior is optimal, group 1 will choose $\alpha_1 = \frac{1 - n_2}{n_1 \cdot n_2 - N + 1}$ and we will get $\alpha_2 = 0$, $X_2 = 0$ and $X_1 = S$. Group 1 has to spend extremely high resources in order to keep group 2 off the contest. The total rent S would be dissipated and the aggregate utility of group 1 would equal zero. Clearly, this behavior cannot be optimal. Hence, we get the following corollary:

Corollary 1: In the group rent seeking contest, there will never be preemptive behavior by the first acting group.

Group 1 never chooses such an aggressive incentive scheme that group 2 decides not to enter the contest. It prefers a sensible degree of competition and "allows" group 2 to enter the contest. As a consequence, the results derived in section 4.1 to 4.3 remain unchanged.

5. Conclusion

In this paper, a group contest was considered. Extending the literature on these contests, the groups were allowed to determine their sharing rules either simultaneously or sequentially. Several interesting results were derived. If the more numerous group chooses its sharing rule prior to the smaller group, the group contest will lead to higher rent seeking activities than an individual contest. As known from the literature on contests, this would never happen, if play had been restricted to occur simultaneously. However, if the timing of the contest is endogenized, the smaller group will always act first. In this case, the group contest leads to less rent dissipation than the individual contest. Further, it was seen that preemptive behavior is never optimal. In order to prevent its opponent from entering the contest, the first acting group had to spend resources equal to the rent the two groups compete for. Hence, the first moving group prefers a more moderate strategy. It accepts that the other group enters the contest. The disadvantage of eventually not receiving the prize is outbalanced by economizing on resource expenditures.

Appendix

In this Appendix, it is shown that the sharing rule described in (3) is able to replicate every Nash-equilibrium outcome that can be achieved with the most general sharing rule when $n_a, n_b > 2$.

Think that group i uses some (general) sharing rule, that is, $f_{ki} = f_{ki}\left(x_{1i}, x_{2i}, \ldots, x_{n_i i}\right)$, where $\sum_{k=1}^{n_i} f_{ki} = 1$. The share in the rent that member k of group i receives, (somehow) depends on the outlays of all the group's members.

Individual k of group i chooses that outlay that maximizes:

$$(23) \qquad V_{ki} = \frac{X_i}{X} \cdot f_{ki} \cdot S - x_{ki}.$$

Deriving (23) with respect to x_{ki}, yields the subsequent first-order condition:

$$(24) \qquad \frac{\partial V_{ki}}{\partial x_{ki}} = \left(\frac{X_j}{X^2} \cdot f_{ki} + \frac{X_i}{X} \cdot \frac{\partial f_{ki}}{\partial x_{ki}} \right) \cdot S = 1.$$

Summation of equation (24) over all members of group i, yields the subsequent condition:

$$(25) \qquad \left(\frac{X_j}{X^2} + \frac{X_i}{X} \cdot \sum_{k=1}^{n_i} \frac{\partial f_{ki}}{\partial x_{ki}} \right) \cdot S = n_i \Leftrightarrow \sum_{k=1}^{n_i} \frac{\partial f_{ki}}{\partial x_{ki}} = \frac{n_i \cdot X^2}{S \cdot \left(X_j + X_i \cdot X_j + \left(X_i \right)^2 \right)}.$$

Condition (25) characterizes the Nash-equilibrium of the subgame beginning at date four. Since the rent has to be completely distributed within the winning group, a group can influence the total outlay of its members (and of the other group's members) only by adjusting $\sum_{k=1}^{n_i} \frac{\partial f_{ki}}{\partial x_{ki}}$.

We now have to show, that the sharing rule in (3) is able to replicate every general Nash-equilibrium outcome. In particular, we have to show that for this sharing rule, $\sum_{k=1}^{n_i} \frac{\partial f_{ki}}{\partial x_{ki}}$ may adopt every possible value by appropriately setting α_i.

Given the sharing rule in (3), it is straightforward to show that $\sum_{k=1}^{n_i} \frac{\partial f_{ki}}{\partial x_{ki}} = (1-\alpha_i)\cdot\frac{n_i - 1}{X_i}$.

From (9) and (18), we know that $X_i = \frac{S}{N^2}\cdot\left(\alpha_i + \alpha_j + n_i\cdot(1-\alpha_i) + n_j\cdot(1-\alpha_j) - 1\right)\cdot\left(n_j\cdot\alpha_i - n_i\cdot\alpha_j + n_i\cdot n_j\cdot(\alpha_j - \alpha_i) + n_i\right)$ and $\alpha_j = \frac{(n_j - n_i)\cdot(n_i - 1)}{2\cdot(n_j - 1)\cdot n_i}\cdot\alpha_i + \frac{(n_j - n_i)}{2\cdot(n_j - 1)\cdot n_i}$. With these expressions we get $\sum_{k=1}^{n_i} \frac{\partial f_{ki}}{\partial x_{ki}}(\alpha_i) = \frac{4\cdot n_i\cdot(n_i - 1)\cdot(1-\alpha_i)}{S\cdot(-2\cdot n_i + \alpha_i\cdot(n_i - 1) + 1)\cdot(\alpha_i\cdot(n_i - 1) - 1)}$.

This function has exactly one null at $\alpha_i = 1$. Further, we can show that $\left.\partial\sum_{k=1}^{n_i} \frac{\partial f_{ki}}{\partial x_{ki}}(\alpha_i)\middle/\partial\alpha_1\right|_{\alpha_1 = 1} > 0$, for $n_i > 2$. On that account, for $n_i > 2$ there is no maximum or minimum at $\alpha_i = 1$. Moreover, there are two points, where the function is not defined, namely $\hat{\alpha}_i = \frac{2\cdot n_i - 1}{n_i - 1}$ and $\tilde{\alpha}_i = \frac{1}{n_i - 1}$. Note that $\tilde{\alpha}_1 < \hat{\alpha}_1$. Between $\tilde{\alpha}_1$ and $\hat{\alpha}_1$, the function $\sum_{k=1}^{n_i} \frac{\partial f_{ki}}{\partial x_{ki}}(\alpha_i)$ is continuous. Since $\left|\lim_{\varepsilon\to 0}\sum_{k=1}^{n_i} \frac{\partial f_{ki}}{\partial x_{ki}}(\hat{\alpha}_i + \varepsilon)\right| \to \infty$ and $\left|\lim_{\varepsilon\to 0}\sum_{k=1}^{n_i} \frac{\partial f_{ki}}{\partial x_{ki}}(\tilde{\alpha}_i + \varepsilon)\right| \to \infty$, the sharing rule could replicate every general equilibrium outcome if $\tilde{\alpha}_i < 1$ and $\hat{\alpha}_i > 1$. One easily see that $\hat{\alpha}_i > 1$ always holds. Further, one can see that $\tilde{\alpha}_i < 1 \Leftrightarrow n_i > 2$. Hence, if group i consists of more than two members, it will be able to determine the parameter α_i to achieve the same outcome as if it would use some general sharing rule. The same argumentation could be done for group j. Consequently, when $n_a, n_b > 2$, the sharing rule in (3) is able to replicate every Nash-equilibrium outcome.

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