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# Two-sided reputation in certification markets*

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**Abstract**

We consider a market where privately informed sellers resort to certification to overcome adverse selection. There is uncertainty about the certifier's ability to generate accurate information. The profit of a monopolistic certifier is an inverted U-shaped function of his reputation for accuracy: being perceived as more precise allows to attract more good sellers but a high expected precision also deters bad sellers. Since the certifier tries to reach a balanced reputation to attract both types, reputation has a disciplining effect when the certifier is perceived as insufficiently accurate, but gives incentives to decrease precision when he is perceived as "too" accurate. The impact of competition depends on whether sellers "multihome" or "singlehome". Under singlehoming, certifiers compete to attract good sellers, which makes higher reputation more valuable. Multihoming makes higher reputations less desirable because the competitor exerts a negative externality by providing extra information. Therefore, singlehoming attenuates bad reputation effects, while multihoming exacerbates inefficiencies.

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# 1 Introduction

In a seminal contribution, Akerlof (1970) shows how asymmetric information may result in market breakdown. In markets plagued by adverse selection, certification mechanisms play a critical role: by providing a third-party opinion, certifiers breach the informational gap between buyers and sellers and contribute to restore market functioning. Some markets could actually not exist absent certifiers. In January 2011, the final report of the US Financial Crisis Inquiry Commission (2011) emphasized that "without the active participation of the rating agencies, the market for mortgage-related securities could not have been what it became." The central role of certifiers is reinforced by regulations that rely on their seal of approval.[1] However, because certifiers are themselves subject to incentive problems, they are not a panacea for adverse selection problems. The unfolding of the financial crisis from 2008 suggests that, far from improving market efficiency, rating agencies have been instrumental in a massive misallocation of capital.[2] While the well-known conflict of interests that issuer-paid rating agencies face has been under heavy fire since 2009, a report from the Security and Exchange Commission in September 2011 still casts doubt on their incentives to provide unbiased information.[3]

In this paper, we investigate a central incentive mechanism for certifiers: reputation. Reputational concerns have been a central defense of rating agencies against accusations of conflict of interest and misaligned incentives. In the words of Thomas McGuire, former executive vice-president of Moody's, "what's driving us is primarily the issue of preserving our track record. That's our bread and butter."[4] However, as Mark Froeba, former senior vice-president of Moody's, suggests, rating agencies have striven

[1] For instance, under the Basel II regulation, banks can use credit ratings from approved agencies in the derivation of their capital requirements. The SEC also uses ratings for the regulation of broker-dealers.

[2] The same report from the FCIC states: "We conclude the failures of credit rating agencies were essential cogs in the wheel of financial destruction. The three credit rating agencies were key enablers of the financial meltdown."

[3] See for instance: "SEC critical of rating agency's controls," Financial Times, September 30, 2011. See also Cornaggia, Cornaggia, and Hund (2011), which provides empirical evidence suggesting a systematic bias of rating agencies towards issuers that generate a higher turnover.

[4] Quoted by Becker and Milbourn (2011).

in the same breath to develop a reputation for being business-friendly as well: "This was a systematic and aggressive strategy to replace a (...) getting-the-rating-right kind of culture with a culture that was supposed to be "business-friendly", but was consistently less likely to assign a rating that was tougher than our competitors."[5] The essence of the business of credit ratings agencies is therefore to humour both parties, by displaying leniency to issuers without jeopardizing the confidence of investors. Of course, in a rational model, it is not possible to run with the hare and hunt with the hounds. Certifiers must accordingly find a compromise between these two goals, and this is precisely what we aim at describing here.

We formalize the idea that reputation is essentially two-sided for certifiers and examine incentives to build up a reputation in this context. We first develop a static model in which the certifier's profit is maximum when he his perceived as neither too accurate nor too inaccurate: being perceived as more accurate allows to attract more good sellers, who have nothing to hide and prefer certification to be as credible as possible; but being perceived as "too accurate" in the same time discourages bad sellers, who are less likely to obtain certification. As a result, the certifier would ideally like to achieve a 'balanced" reputation for accuracy.

In this context, we show that the direction of reputational incentives depends on the current reputation of the certifier: when perceived as insufficiently accurate, the certifier tries to build a reputation for more accuracy, and increases the precision of his signal accordingly; otherwise, a certifier with a high reputation is concerned with being perceived as too precise and hence decreases the precision of the information he provides to signal he is (bad) seller-friendly. In terms of welfare, reputation can therefore be welfare-increasing ("discipline"), by sometimes inducing the certifier to be more precise than in the static case; but it is welfare-decreasing otherwise ("bad reputation"), as it then provides incentives to decrease the quality of the information provided to the market.

Finally, we examine the impact of competition, which we model as entry threat: the incumbent monopolist faces the threat of entry of a second certifier with unknown

[5] "How Moody's sold its ratings - and sold out investors", McClatchy Newspapers, Oct. 19, 2009.

reputation. We first focus on the case where the seller can only be certified by a single certifier ("singlehoming").In this market structure, only the more reputable certifier is active. Intuitively, even though a lower reputation for accuracy sometimes increases the popularity of a certifier vis-à-vis bad sellers, a certifier faces no demand unless he attracts good sellers. Since the latter have an unambiguous preference for accuracy, they flock to the more reputable certifier. As a result, a certifier is all the less likely to face entry as his reputation is high and the incumbent's profit is maximized for a bliss reputation higher than in the monopoly case. It follows from this shift in the profit function that competition attenuates bad reputation effects: a monopolist certifier who decreases precision for reputational motives would provide more precise information if he were under the threat of entry. Second, we analyze "multihoming," that is, the possibility for the seller to solicit certification from more than one certifier. This practice is commonplace for credit ratings: Chen, Lookman, Schürhoff, and Seppi (2009) report that the overwhelming majority of large corporate bond issues have at least two ratings.[6] We abstract from the issue of rating shopping and assume that applications for certification are publicly observed.[7] We first show that, provided that the cost of an extra certifier is sufficiently small, the seller asks for two ratings or none in equilibrium. The presence of multihoming impacts certifiers' preferences over their own reputations in a dramatically different way from singlehoming: each certifier's bliss reputation is lower under multihoming than under monopoly. Intuitively, the size of the total market for certification is maximal when the certification process is neither too precise nor too imprecise; the presence of a second certifier who produces an independent signal increases the precision of the process everything else equal. A certifier with an ideal reputation under monopoly is now too accurate, because of the externality exerted by the competitor. To compensate for this additional information, each certifier would like to be perceived as less accurate than in the monopoly case (and less so the more reputable the other certifier). A consequence is that multihoming exacerbates bad rep-

[6] In their sample of 8,767 bonds taken from the Barclays Capital Bond Index, 99.5% of bond issues are rated by S&P and Moody's and 70% are rated by Fitch.

[7] Rating shopping refers to the possibility for issuers to secretly apply for several ratings and pick the most favorable one. See among others Skreta and Veldkamp (2009) and **?**.

utation effects: a certifier who would decrease precision for reputational motives if it were a monopoly provides even less precise information under the threat of entry.

While rating agencies constitute a natural illustration of our framework and an example we use repeatedly in the paper, the analysis applies to any certification market in which certifiers care about the size of their customer base and issuers may hold certifications from several certifiers at the same time. Examples of such market include financial audit, technical standards (e.g., ISO, CEN), school accreditations (e.g., EQUIS, AACSB) or individual proficiency tests (e.g., GMAT, GRE, TOEIC). Hence, this paper belongs to the literature on the reputation and credibility of experts.

This paper belongs to the literature on the reputation and credibility of experts. After Sobel (1985), Benabou and Laroque (1992) and Mathis, McAndrews, and Rochet (2009) have shown that reputation has a disciplining effect but is not sufficient to ensure truthful information transmission. As most papers on reputation in the literature, these papers are based on a trade-off between short-term incentives to manipulate information in order to inflate short-term profits and long-term incentives to build up a reputation. By contrast, we show that, even in the absence of an immediate reward from information manipulation, reputation itself can lead a certifier to decrease the quality of information. Therefore, reputation can be "bad", i.e. welfare-reducing, while it is welfare-enhancing in those two papers.[8] Another stream of papers investigate conditions under which reputation may have an adverse effect on welfare (Morris, 2001; Ely and Välimäki, 2003; Ely, Fudenberg, and Levine, 2008). In these papers, some types try to separate themselves from other types with an intrinsic motivation to misbehave, which cripples their incentives to behave. On the contrary, in our model, the certifier distorts the quality of information because he wants to develop a reputation for being what would be the "bad type" in these models. Finally, our paper is related to models of reputation with multiple audiences. In particular, Frenkel (2010) studies a model where a rating agency tries to develop two reputations, one public and one private. Bar-Isaac

[8] Another difference with Benabou and Laroque (1992) and Mathis, McAndrews, and Rochet (2009) is that our model features adverse selection in the product market, while they assume that sellers do not have any informational advantage over buyers.

and Deb (2012) study a general framework where an agent tries to develop a reputation vis-à-vis several audiences.[9] We differ from their paper by focussing especially on the issue of information transmission in certification markets. This allows us to derive qualitative insights on the impact of competition (singlehoming and multihoming) in these markets.

# 2 The model

## 2.1 The market

We consider a setup with three categories of risk neutral players: a seller, buyers and a certifier. The seller owns a product of quality $\theta \in \{\theta_g, \theta_b\}$, where $Pr(\theta = \theta_g) = \beta$, and $\theta$ is private information of his. There is a continuum of competitive buyers with valuation 1 for a high-quality product ($\theta = \theta_g$) and 0 for a low-quality product ($\theta = \theta_b$).

The seller has a reserve price $\lambda$ for a good product, where $\lambda$ is a continuous random variable with density $f$ and a log-concave cumulative distribution $F$ on $\mathbb{R}_+$.[10] Therefore, there are gains from trade for realizations of $\lambda$ smaller than 1. When product quality is public information, only high-quality products are traded. However, since the seller is privately informed about his good, there is adverse selection. For simplicity, we focus on the extreme case where adverse selection precludes any trade in the absence of additional information. Specifically, we assume:

**Assumption 1.** $\beta < \beta_0 \equiv \min\limits_{P \in [0,1]} \frac{P}{P+(1-P)F(P)}$.[11]

Consider a candidate price $P$ at which the seller could sell his good to buyers. A high-quality seller is willing to sell at price $P$ if and only if $\lambda \leq P$. Hence, conditional on the seller being of high quality, the probability that he sells is $F(P)$. Bad sellers are willing to sell at any price $P \geq 0$. Buyers are willing to pay at most the

[9]See also Gertner, Gibbons, and Scharfstein (1988) and Austen-Smith and Fryer (2005) on signalling to multiple audiences.

[10]We assume that the realization of $\lambda$ is privately observed by the seller as well.

[11]Note that we need to impose that $F$ is differentiable at 0 to get that $\frac{P}{P+(1-P)F(P)}$ is bounded away from 0 for $P \in [0, 1]$.

expected value of the product conditional on the seller being willing to sell, that is, $\frac{\beta F(P)}{\beta F(P)+(1-\beta)}$. Assumption 1 ensures that this expected value is strictly smaller than $P$ for any $P \in (0,1]$, which implies that the market does not have a positive equilibrium price. Intuitively, when the probability of a low-quality seller is sufficiently high ($\beta$ is low), adverse selection drives all high-quality buyers out of the market, which then collapses. We focus on certification as a way to alleviate adverse selection. Our setting is therefore meant to capture any market where sellers resort to certification agencies or standard-setting organizations (Lerner and Tirole, 2006; Farhi, Lerner, and Tirole, 2008). A possible application is the bond market: a bank willing to securitize assets for diversification or liquidity motives issues bonds backed by mortgages, while these bonds are bought by mutual or pension funds who seek exposure the the real estate market.[12] In this market, credit rating agencies play a fundamental role by providing information on the issuer's credit risk.

## 2.2 The certification process

The certifier is endowed with a technology which produces a signal $\sigma \in \{\varnothing, b\}$ on the quality of the product, with conditional distributions $\Pr(\sigma = \varnothing | \theta = \theta_g) = 1$ and $\Pr(\sigma = b | \theta = \theta_b) = \alpha + e$. The precision of the certifier's signal depends both on an enduring technological parameter $\alpha \in \{\alpha_L, \alpha_H\}$ and on some unobservable effort $e \in [-\varepsilon, \varepsilon]$ that the certifier exerts, where $\varepsilon < \min\{\alpha_L, 1-\alpha_H\}$. Effort $e$ is allowed to be negative and involves a cost $\frac{1}{2}ce^2$: while increasing the precision of the signal takes extra effort and resources, decreasing the precision might require destroying or falsifying the information that enters the signal-generating process, or might expose the certifier *ex post* to the risk of lawsuits or regulatory sanctions. This specification allows to capture the idea that the certifier may be willing to pay to decrease his precision for reputational motives, as will be shown below.[13] Notice that a signal $\sigma = b$

[12] Interpreting sellers as financial institutions is consistent with them being "sophisticated" market participants, having superior information about the quality of their product.

[13] We assume here costly negative effort, but could alternatively assume an intrinsic preference for truthtelling which induces the certifier to exert positive effort even in the absence of reputational concerns. We would then compare the level of effort with reputational concerns to this benchmark effort level, while, in our specification, effort always equals 0 in the benchmark case of no-reputation.

provides perfect evidence that the product is of bad quality, while perfect evidence of high quality is never available.[14] We interpret $\sigma$ as the certification outcome: if $\sigma = \varnothing$, the product is said to be certified; when $\sigma = b$, the seller's application is rejected.

While effort is private information of the certifier, the signal $\sigma$ is publicly observed. Therefore, the certifier affects the *ex ante* precision of the signal through costly effort, but cannot manipulate the signal *ex post*. This specification of information production is in the spirit of Rayo and Segal (2010) and Kamenica and Gentzkow (2011), who model a a game of persuasion in which a sender commits to disclose the signal he produces but can *ex ante* choose the distribution of the signal. Note however that in these models, the structure of the signal can be adjusted at no cost and is public information, while adjustments are costly and unobservable in our model.[15] This captures the idea that (costly) manipulations of the technology are more difficult to detect than manipulations of the signal produced itself.[16]

Finally, we assume that certification involves a fixed cost $\phi$ for the seller. This cost consists of a fixed and upfront fee paid to the certifier, $z\phi$, and of additional costs, $(1 - z)\phi$, related to information production and product design.[17] For instance, financial claims may have to be repackaged and distributed to institutional investors, which requires the services of a range of financial intermediaries. Importantly, since the certifier is paid upfront, he has no direct incentive to make any (positive or negative) effort to change his precision, while a report-contingent payment would create incentives to lower effort to increase fees, even in the one-shot game.[18] Hence, effort is here

[14]This asymmetry in the distribution of the signal greatly simplifies the analysis but is not essential. What ultimately matters for our results is that the probability that a bad seller obtains certification decreases with the certifier's (expected) precision.

[15]The assumption that effort is costly is not essential, but makes the results cleaner, in that we would obtain multiple equilibria if altering the precision of the signal was costless for the certifier.

[16]This is consistent with reports on how credit rating agencies have been adjusting the information they provide to markets: rather than directly manipulating the outcome of their credit risk models -the rating itself- they adjust their models or the type of information inputed into these models (see, e.g., SEC (2008)).

[17]$z$ is irrelevant in the monopoly case and can be though as being equal to 1, but will prove useful once we introduce multiple certifiers. See section **??**.

[18]One of the sharpest criticisms following the subprime crisis was that part of rating agencies' fees were indeed contingent on a favourable rating. During the Summer 2008, an agreement was found between the New York State General Attorney Andrew Cuomo and the three main credit rating agencies, which required that rating fees be charged upfront.

purely driven by long-term reputational concerns. Note also that we take the fee, $z\phi$, to be exogenous. The question of the optimal structure of the market for certification services has been extensively studied, for instance in Faure-Grimaud, Peyrache, and Quesada (2009) or Bolton, Freixas, and Shapiro (2012). These papers have shown that the pricing structure of certification, as well as the identity the party that purchases certification services influence both the composition of the market for certification and certifiers' incentives to manipulate information. Incorporating these effects into our model would compromise its tractability, while our objective is to insulate the impact of reputation on the ability of the certifier to attract sellers. Implicitly, we therefore assume that certification fees are somewhat rigid and cannot be adjusted to changes in the certifier's reputation. A consequence is that the certifier's profit is proportional to the total demand for certification.

## 2.3 Timeline

We conclude this description of the model with the timing of the game. There are two periods; the seller and the buyers only live one period, while the certifier is long-lived with a discount factor normalized to 1. Within each period $t$, the game unfolds as follows:

a. The seller observes the quality $\theta_t$ of his product and decides whether to solicit certification.

b. The certifier exerts effort $e_t$ and produces a signal $\sigma_t$.

c. Buyers observe $\sigma_t$ and independently submit bids for the product in a second-price auction.

We assume that the certifier does not have private information on $\alpha$. In the beginning of period 1, all players share the common belief that $\Pr(\alpha = \alpha_H) = \rho_1$. If certification takes place in period 1, all the players present in period 2 observe both the certification outcome $\sigma_1$ and the quality of the product $\theta_1$ in the previous period. We denote $\rho_2 = \Pr(\alpha = \alpha_H | \sigma_1, \theta_1)$ and will henceforth refer to $\rho_t$ as the certifier's reputation in period $t$. A feature of our game is that no realization of $(\sigma_1, \theta_1)$ is ever out of the equilibrium path, which results in all players sharing the same beliefs all along the game.

# 3 Two-sided reputation: the monopoly case

## 3.1 The costs and benefits of reputation

We first analyze the final period ($t = 2$), in which there are no reputation-building concerns. At stage 2b., the certifier exerts zero effort and the precision of its signal is fully determined by the technology $\alpha$. The expected probability at $t = 2$ that a bad-type seller $\theta_b$ obtains a favourable rating ($\sigma_2 = \varnothing$) given $\rho_2$ is

$$q_2 \equiv \Pr(\sigma = \varnothing | \theta = \theta_b) = 1 - [\rho_2 \alpha_H + (1 - \rho_2)\alpha_L]$$

$q_2$ measures the certifier's perceived precision at $t = 2$; it decreases from $1 - \alpha_L$ to $1 - \alpha_H$ as $\rho_2$ increases from 0 to 1. We first characterize the period 2 equilibrium as a function of $q_2$, and then derive the expression of the certifier's profit as a function of $\rho_2$.

A market equilibrium features a cutoff type $\overline{\lambda}_2 \in [0, 1]$ such that a good seller with reservation value $\lambda$ solicits certification if and only if $\lambda \leq \overline{\lambda}_2$, a probability $\gamma_2$ that a bad seller solicits certification, and a price $P_2$ that the seller obtains following certification, i.e. a report $\sigma = \varnothing$.[19] Before we characterize the equilibrium with certification, note that there always exists a no-certification equilibrium, $(\overline{\lambda}_2 = \gamma_2 = 0, P_2 \leq \phi)$. This equilibrium is sustained by any distribution of buyers' out-of-equilibrium beliefs which results in a price smaller than $\phi$. We rule out this equilibrium and focus instead on the equilibrium where trade occurs with positive probability. Consider a candidate equilibrium $(\overline{\lambda}_2, \gamma_2, P_2)$. Since buyers are competitive, risk neutral, and have identical

[19]Since buyers are competitive, risk neutral and hold the same beliefs in equilibrium, they bid up to the expected value of the good. Therefore, it is indifferent to consider the transaction price $P_2$ or Bayesian posterior beliefs as a component of the Bayesian equilibrium. Note also that there is no transaction following a bad report $\sigma_2 = b$, as it perfectly reveals that the product is of bad quality.

information, the transaction price $P_2$ when the product is certified ($\sigma_2 = \varnothing$) is equal to the expected value of the project:

$$P_2 \equiv \Pr(\theta = \theta_g | \sigma_2 = \varnothing) = \frac{\beta F(\overline{\lambda}_2)}{\beta F(\overline{\lambda}_2) + (1-\beta)\gamma_2 q_2} \in [0,1].$$

A good seller with reservation value $\lambda$ solicits certification iff $P_2 - \phi \geq \lambda$. Since $P_2 \leq 1$, a seller with value $\lambda = 1$ never solicits certification. If $P_2 \leq \phi$, no seller ever solicits certification (no-certification equilibrium). We assume that $P_2 > \phi$ and will check that it is true *ex post*.

Consider some $q_2 \in [1-\alpha_H, 1-\alpha_L]$. If $P_2 > \phi$, $(\overline{\lambda}_2, \gamma_2, P_2)$ must satisfy

$$P_2 - \overline{\lambda}_2 = \phi \tag{1}$$

$$\gamma_2 \in \underset{\tilde{\gamma} \in [0,1]}{\operatorname{argmax}}\ \tilde{\gamma}(q_2 P_2 - \phi) \tag{2}$$

$$P_2 = \frac{\beta F(\overline{\lambda}_2)}{\beta F(\overline{\lambda}_2) + (1-\beta)\gamma_2 q_2} \tag{3}$$

We immediately see that we indeed have $P_2 > \phi$ : if $\gamma_2 = 0$, then $P_2 = 1 > \phi$. If $\gamma_2 \geq 0$, we must have $P_2 \geq \frac{\phi}{q_2} > \phi$.

Notice also that, since $P_2$ is a decreasing function of $\gamma_2$, the solution to (2) must be unique.

We restrict attention to the case where $\gamma_2 \in (0,1)$. This assumption is not essential for our results but simplifies the analysis, as it ensures that the certifier's profit function is differentiable everywhere. Necessary conditions to obtain an interior solution are

**Assumption 2.** $\phi < 1 - \alpha_H$ *and* $\beta < \beta_1 \equiv \frac{1}{1 + \max_{q_2 \in [1-\alpha_H, 1-\alpha_L]} \frac{1}{\phi}(1-\frac{\phi}{q_2})F(\frac{1-q_2}{q_2}\phi)}$

The first inequality states that the cost of certification for the seller is smaller than the minimal probability for a bad seller to obtain certification. Therefore, the bad seller is willing to solicit certification with positive probability for all $q_2$. This also implies that $P < 1$. The second inequality ensures that there are *ex ante* too many bad types to sustain an equilibrium in which a bad seller always solicits certification (see the proof of Lemma 1 to check how it kicks in as a necessary condition). Note that this condition imposes the same type of constraint as Assumption 1, namely that the adverse selection problem is severe. Under assumption 2, solving for the system ((1),(2),(3)) yields:

**Lemma 1.** *For all* $q_2 \in [1-\alpha_H, 1-\alpha_L]$, *the period-2 equilibrium is such that:*

$$\begin{cases} \overline{\lambda}_2 &= \frac{1-q_2}{q_2}\phi \\ \gamma_2 &= \frac{\beta}{1-\beta}(1-\frac{\phi}{q_2})\frac{1}{\phi}F(\frac{1-q_2}{q_2}\phi) \\ P_2 &= \frac{\phi}{q_2} \end{cases}$$

*Proof.* Assuming $\gamma_2$ interior, (2) implies $P_2 = \frac{\phi}{q_2}$, which allows to derive $\overline{\lambda}_2$ and $\gamma_2$. Assumption 2 ensures that $0 < \gamma_2 < 1$. Since we have established uniqueness of the equilibrium, this is the only possible solution of the system involving certification. □

Since the certifier charges a fixed fee $z\phi$, his expected profit in period 2 is given by

$$\pi_2 \equiv [\beta F(\overline{\lambda}_2) + (1-\beta)\gamma_2]z\phi = \beta z(1 - \frac{1-q_2}{q_2}\phi)F(\frac{1-q_2}{q_2}\phi). \tag{4}$$

Note that $\frac{1-q_2}{q_2}$ is a measure of the certifier's (perceived) precision at the beginning of period 2, $\rho_2$. It is then apparent from the expression in (4) that the impact of a change in precision on the certifier's profit $\pi_2$ is potentially ambiguous. Intuitively, a higher expected precision has two effects: (a) the probability of obtaining certification decreases for bad sellers, while it is unchanged for good sellers ($q_2$ decreases); (b) conditional on certification, buyers are willing to pay a higher price ($P_2$ increases). Hence, a higher reputation for transparency unambiguously raises participation of good sellers ($\overline{\lambda}_2$ increases).[20] As for bad sellers, the impact of precision is unclear. On the one hand, the price conditional on certification is higher both because the signal is more precise, and because good types are more likely to participate. This tends to increase the participation of bad types. But on the other hand, the probability of certification is lower, which decreases their incentive to participate. Which of these effects dominate

[20]Note that if a higher precision was also increasing the probability that good sellers are certified, this effect would be magnified.

depends on the shape of $F$ and of the parameters of the model. We make the following assumption:

**Assumption 3.** $F(\frac{\alpha_L}{1-\alpha_L}\phi) \leq (1-\frac{\alpha_L}{1-\alpha_L}\phi)f(\frac{\alpha_L}{1-\alpha_L}\phi)$ *and* $F(\frac{\alpha_H}{1-\alpha_H}\phi) \geq (1-\frac{\alpha_H}{1-\alpha_H}\phi)f(\frac{\alpha_H}{1-\alpha_H}\phi)$

Rewriting $\pi_2$ as a function of $\rho_2$ :

$$\pi_2(\rho_2) = \beta z(1 - \frac{\rho_2\alpha_H + (1-\rho_2)\alpha_L}{1-[\rho_2\alpha_H + (1-\rho_2)\alpha_L]}\phi)F(\frac{\rho_2\alpha_H + (1-\rho_2)\alpha_L}{1-[\rho_2\alpha_H + (1-\rho_2)\alpha_L]}\phi),$$

we derive from Assumption 3 the following proposition:

**Proposition 1.** $\exists \rho_2^* \in (0,1), \pi_2'(\rho_2) \geq 0$ *on* $[0, \rho_2^*]$ *and* $\pi_2'(\rho_2) \leq 0$ *on* $[\rho_2^*, 1]$

*Proof.* In the Appendix. □

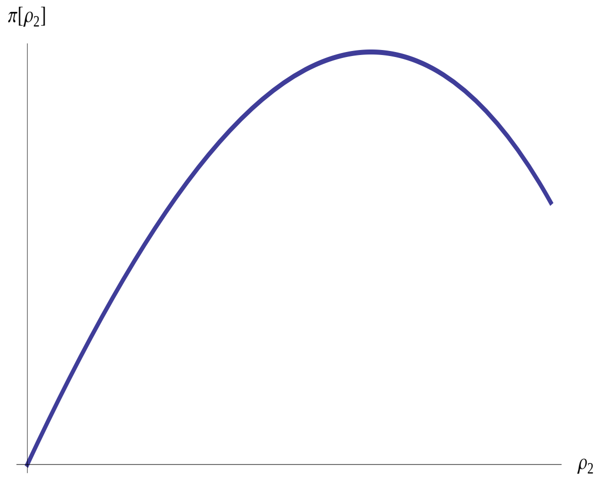


Figure 1: The certifier's profit in period 2

For low levels of precision (low $\rho_2$, i.e. high $q_2$), a higher reputation for accuracy increases profit. Participation of good types increases, while bad types may participate more or less. But, in any case, the probability that the bad seller solicits certification decreases slower than the participation of the high type increases. However, for high reputations, the probability for a bad type to be certified decreases and the bad seller's participation $\gamma_2$ rate must drop. Assumption 3 states that this drop is too important to be outweighed by the increase in good seller's participation. Consequently, the overall profit of the certifier decreases beyond a certain level of expected precision.

Proposition 1 tells us that, under fairly simple conditions, reputation for accuracy is essentially two-sided: while a good seller always prefers a more accurate certifier, a bad seller would like the certifier to be neither too accurate nor too imprecise. This results in total demand being maximized for a level of expected accuracy which is not the maximal one: a certifier can be "too accurate". In terms of reputational incentives, the certifier would then like to develop a reputation for being more accurate when perceived precision is low ($\rho_2 \to 0$). Conversely, a certifier with a high expected precision ($\rho_2 \to 1$), should aim at developing a reputation for being less accurate. Between this two extremes, the model captures the two-sidedness of reputation. Therefore, the direction of reputation incentives is essentially ambiguous and critically depends on the current reputation of the certifier.

We close this subsection with a few comments on some features of the model. Notice first that if assumption 3 was not verified, the certifier's profit would be monotonic in its reputation. Hence the analysis of reputational incentives would be essentially identical to previous contributions on this topic (e.g., Benabou and Laroque, 1992 or Mathis, McAndrews, and Rochet, 2009). Since the market for certification is essentially two-sided, the analysis of reputation-building when reputational incentives are ambiguous is warranted. Second, since the probability that good sellers are willing to participate in the market is increasing in the precision of the certifier's signal, total welfare is also an increasing function of his reputation $\rho_2$ (only good-quality products generate gains from trade). The certifier does not internalize total welfare, and as a result, his profits are maximized for intermediate values of his reputation. In the model, this is driven by the assumption of a fixed price set *ex-ante* which prevents the certifier from screening out bad types, for instance by offering menus of contracts and contingent payments. However, any mechanism by which the certifier could extract rents from bad sellers without jeopardizing too much his attractiveness to good sellers would qualitatively generate the same effects.

## 3.2 Reputation building

In period 2, given that the certifier has no reputational concerns, he picks the cost-minimizing level of effort $e_2^* = 0$. However, the certifier has an incentive to try and build a reputation in period 1 because the accuracy of his report conveys information about his type $\alpha$ : following a bad report, the updated belief $\rho_2$ that the certifier is the accurate type $\alpha_H$ is given by

$$\rho^+(e_1) \equiv \frac{\rho_1(\alpha_H + e_1)}{\rho_1\alpha_H + (1-\rho_1)\alpha_L + e_1} = \rho_1 + \frac{\rho_1(1-\rho_1)(\alpha_H - \alpha_L)}{\rho_1\alpha_H + (1-\rho_1)\alpha_L + e_1},$$

where $e_1$ is the anticipated level of effort.[21] Conversely, a good report on a bad quality product triggers an updating from $\rho_1$ down to

$$\rho^-(e_1) \equiv \frac{\rho_1(1-\alpha_H - e_1)}{1-\rho_1\alpha_H - (1-\rho_1)\alpha_L - e_1} = \rho_1 - \frac{\rho_1(1-\rho_1)(\alpha_H - \alpha_L)}{1-\rho_1\alpha_H - (1-\rho_1)\alpha_L - e_1}.$$

Note that since good-quality products are certified with probability 1 regardless of the certifier's accuracy, observing a good product certified in period 1 is uninformative and $\rho_2 = \rho_1$. However, conditional on the product being of bad quality, the certifier can affect the probability that $\rho_2 = \rho^+$,

$$[\rho_1\alpha_H + (1-\rho_1)\alpha_L] + e_1 \equiv 1 - q_1(e_1)$$

by adjusting his effort $e_1$. Finally, the seller in period 1 decides about soliciting a rating based on his anticipation of the effort $e_1$. As in period 2, an equilibrium in period 1 features a cutoff type $\overline{\lambda}_1(e_1)$, the probability for a bad seller to solicit certification $\gamma_1(e_1)$, and a transaction price $P_1(e_1)$.

$$\overline{\lambda}_1(e_1) = \frac{1-q_1(e_1)}{q_1(e_1)}\phi,$$

$$\gamma_1(e_1) \in \underset{\tilde{\gamma}\in[0,1]}{\operatorname{argmax}}\ \tilde{\gamma}\left[q_1(e_1)P_1(e_1) - \phi\right],$$

[21] In order to avoid heavy notation, we do not explicitly write the dependence of the posterior $\rho_2$ on the prior $\rho_1$, except in the Appendix.

$$P_1(e_1) = \frac{\beta F[\overline{\lambda}_1(e_1)]}{\beta F[\overline{\lambda}_1(e_1)] + (1-\beta)\gamma_1(e_1) q_1(e_1)}.$$

Furthermore, buyers and sellers' anticipation of $e_1$ is correct in equilibrium and the certifier chooses the effort level that maximizes his expected profit in period 2:

$$e_1^* \in \underset{e_1}{\operatorname{argmax}} \frac{(1-\beta)\gamma_1(e_1^*)}{\beta F\left[\overline{\lambda}_1(e_1^*)\right] + (1-\beta)\gamma_1(e_1^*)} \left\{[1-q(e_1)]\pi_2[\rho^+(e_1^*)] + q(e_1)\pi_2[\rho^-(e_1^*)]\right\} - c\frac{e_1^2}{2} \tag{5}$$

Note that the seller's decision in period 1 feeds back into the effort decision of the certifier as it affects the probability that an application comes from a bad seller, hence the probability that effort impacts reputation.

**Proposition 2.** *For $c$ sufficiently large, there is a unique equilibrium level of effort given by a function $e_1^*(\rho_1)$ and a threshold $\overline{\rho} \in (0,1)$ such that:*

- *$e_1^*(\rho_1)$ is continuous in $\rho_1$,*
- *$e_1^*(0) = e_1^*(1) = e_1^*(\overline{\rho}) = 0$,*
- *$e_1^*(\rho_1) > 0$ for $\rho_1 \in (0; \overline{\rho})$,*
- *$e_1^*(\rho_1) < 0$ for $\rho_1 \in (\overline{\rho}; 1)$.*

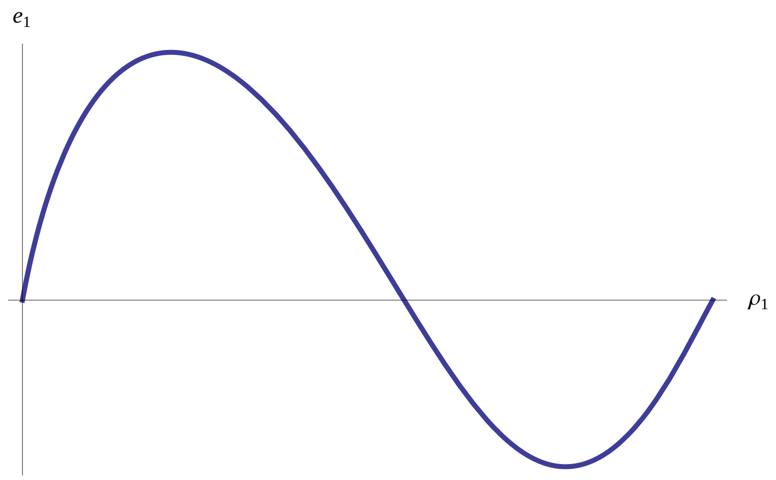


Figure 2: Equilibrium effort in period 1

*Proof.* In the Appendix. □

The assumption that $c$ is large enough allows to ensure uniqueness of the equilibrium. A formal expression of the lower bound on $c$ is given in the Appendix. Proposition 2 tells us that reputation can provide both good or bad incentives to produce information: as compared to no-reputation case, reputational concerns result in a higher precision in period 1 when $\rho_1$ is low and, conversely, in lower precision when $\rho_1$ is high: the desire to build a reputation for accuracy leads a certifier perceived as insufficiently accurate to increase effort so as to detect bad quality products more often; however, a certifier perceived as "too accurate" needs to decrease precision in order to achieve a more balanced reputation and attract more future bad sellers. The overall effect of reputation-building on welfare is therefore ambiguous. Reputation has a disciplining effect, as a certifier needs to build a certain level of credibility to attract good sellers, which, in turn, makes certification attractive to bad sellers. However, there can also be "bad reputation" effects, whereby the certifier lowers the precision of his signal relative to a no-reputation benchmark, in an effort to cater to future bad sellers. Note that the certifier's incentives to manipulate the precision of his signal are purely driven by reputation, that is, he has no short-term incentive to distort information production. This contrasts with existing models of reputation of experts, such as Benabou and Laroque (1992) or Mathis, McAndrews, and Rochet (2009), in which the intermediary trades off long-term incentives to be truthful against short-term incentives to distort information in order to reap immediate profits. In these models, the intermediary always prefers being perceived as more accurate, but is at some point willing to milk his reputation to enjoy higher current benefits. Hence, while reputation might not be enough to perfectly discipline the certifier, there is more information transmission when the intermediary cares about his reputation than when he does not. On the contrary, in our model, for $\rho_1 > \overline{\rho}$, there is less information provided when the intermediary cares about his reputation.

# 4 Multiple certifiers: single- and multihoming.

In this section, we study how the entry of a second certifier affects reputational incentives. Specifically, we assume that while certifier $A$ is a monopoly in the first period, a second certifier, $B$, enters the market in period 2 with a random reputation $\rho_2^B$ drawn form the uniform distribution on $[0, 1]$.[22] As in the monopoly case we assume that a seller who apply for certification bears a cost $(1 - z)\phi$ and the each certifier charges a fee $z\phi$ for its services. We contrast two market structures: in the first one ("singlehoming"), the seller is constrained to solicit certification from one certifier only; in the second ("multihoming"), a seller may solicit both certifiers. We show that allowing for multihoming shifts reputational incentives and reverses the conclusions of the singlehoming case.

## 4.1 Singlehoming

We start with the case where sellers are constrained to singlehome. We slightly modify the timing to allow for the entry of a competitor in period 2. Specifically,

In period 1,

1a. The seller observes the quality $\theta$ of his product and decides to solicit certification from $A$ or not,

1b. If solicited, certifier A makes effort $e_1$ and publishes the signal $\sigma \in \{\varnothing, b\}$ ,

1c. Buyers observe the signal and independently submit bids for the product.

In period 2,

2a. $\rho_2^B$ is realized and observed by all parties.

2b. The seller observes the quality $\theta$ and of his product and decides to solicit certification from $A$ or from $B$,

[22] Our results would hold for any distribution. Randomness allows to smooth profit functions, it is introduced only for technical reasons.

2c. The certifier who have been sollicited chooses effort $e_j$ and publishes the signal $\sigma^j \in \{\varnothing, b\}$,

2d. Buyers observe signals and independently submit bids for the product.

The key feature of the competition taking place in period 2 is that the certifier with the highest reputation captures all the market. Letting $\rho_t^j$ denote the reputation of certifier $j \in \{A, B\}$ in period $t$, we formalize this result in the following lemma.

**Lemma 2.** *In period 2, the incumbent certifier A is active if and only if $\rho_2^A > \rho_2^B$. In this case, his profit is identical to the monopoly profit.*

*Proof.* In the Appendix. □

The intuition for this result is as follows. Good sellers exert a positive externality on bad sellers: an increase in the participation of good sellers $\overline{\lambda}$ improves average quality in the pool of applicants and hence incentives for bad sellers to participate. In fact, the certifier cannot attract bad sellers unless he is able to attract good sellers. Because good sellers have a preference for precision, they pick the certifier with the highest reputation, which leaves the certifier with the lowest reputation inactive. This dynamic is reminiscent of the literature on two-sided markets in which platforms have a similar incentive to "cater" to the side of the market which exerts the strongest positive externality on the other side.[23] Note that the seller's choice of a certifier at $t = 2$ may in itself convey information to buyers. As a result, it is in principle possible to sustain an equilibrium in which only the certifier with the lowest reputation is active. However, this equilibrium relies on out-of-equilibrium beliefs which are ruled out by a simple refinement criterion.

Lemma 2 implies that the incumbent looses the market whenever $\rho_2^B > \rho_2^A$, which happens with prior probability $1 - \rho_2^A$. Therefore, the expected profit of the incumbent reads $\pi^{sh}(\rho_2^A) = \rho_2^A \pi(\rho_2^A)$. Turning to the period-1 choice of effort, we derive the following result.

[23] See for instance Caillaud and Jullien (2003).

**Proposition 3.** *In the singlehoming case, competition mitigates bad reputation effects:*

$$e_1^*(\rho_1) \leq 0 \Rightarrow e_1^*(\rho_1) \leq e_1^{sh}(\rho_1)$$

*Proof.* In the Appendix □

When sellers singlehome, competition lowers incentives for the certifier to pander to bad types. Intuitively, the inefficiency in the monopoly case comes from the excessive weight that the certifier put on bad sellers relative to good ones when trying to optimize his reputation. Competition corrects in part this bias by increasing the value of attracting good sellers. One can show that the function $\pi_2^{sh}(\rho) = \rho\pi_2(\rho)$ is either nondecreasing or hump-shaped with a maximum reached at point strictly larger than $\rho_2^*$, the certifier's bliss reputation under monopoly. As a result, the region in which certifier $A$'s reputation is beyond his preferred value shrinks, or even disappear, and within this region the net benefit of decreasing his perceived accuracy decreases. Note however that the overall impact of competition is unclear for lower values of $\rho_1^A$ as the threat of being displaced has another effect on incentives to provide effort: it scales down expected profits in period 2, which lowers the expected benefit from providing effort.

## 4.2 Multihoming

In this subsection, we relax the constraint that sellers have to choose one certifier and allow them to "multihome." That is, every seller has the possibility to solicit certification from both certifiers $A$ and $B$. The timing of the game is identical to the one in subsection 4.1, except that in period 2, the seller can now apply for certification from certifier A, B or both (step 2.b). Note that multihoming is common practice in the market for corporate credit ratings, where the overwhelming majority of large bond issues are rated by both Moody's and Standard & Poors (see, for instance, Bongaerts, Cremers, and Goetzmann, 2012). We assume that the signals produced by certifiers $A$ and $B$ are independently distributed conditional on product quality. This assumption is inessential -we only need that $A$'s signal is not a sufficient statistics for $B$'s signal and vice versa- but simplifies the analysis. We also impose that the seller simultaneously applies for ratings if he applies to more than one. Our results would hold if we allowed for sequential applications, as long as applications are public. However, we abstract from the issue of shopping, whereby can secretly ask for a rating and disclose it only if it is good enough. (Rating shopping is studied in Skreta and Veldkamp, 2009, and Bolton, Freixas, and Shapiro, 2012.). Finally, to ensure interior solutions, we make the following assumption, which is the counterpart of Assumption 2 in the monopoly case.

**Assumption 4.** $\phi(1+z) < (1-\alpha_H)^2$ *and* $\beta < \frac{\phi(1+z)(1-\alpha_L)^2}{\phi(1+z)(1-\alpha_L)^2+[(1-\alpha_L)^2-\phi(1+z)]F[1-\phi(1+z)]}$

As in the previous cases, we start with the equilibrium of the certification market in period 2, in which the following result obtains.

**Lemma 3.** *The equilibrium is such that sellers either multihome, i.e. solicit certification from both certifiers, or do not solicit certification at all.*

Intuitively, an equilibrium in which only one certifier is active can only be sustained by the out-of-equilibrium belief that a seller who deviates and apply for a second rating has a sufficiently high probability of being a bad type. This belief is ruled out by a simple refinement which attributes this type of deviation to the type who is more likely to benefit from it, that is, a good-quality seller. This implies that the net increase in price for a good-quality seller following a second rating is high enough to cover the cost of this second rating, which makes the deviation profitable.[24] Note that this result is consistent with the empirical observation mentioned above that multi-rated issuances are pervasive for corporate bonds (Bongaerts, Cremers, and Goetzmann, 2012). The key consequence of Lemma 3 is that rather than generating competition, a market structure in which multihoming is possible results in both certifiers sharing the same pool of clients. In fact, the equilibrium is such that both certifiers enjoy the same level of profit, regardless their respective reputations.

[24]Under assumption 4 the cost of an extra rating $\phi z$ is bounded above.

**Lemma 4.** *For a given a realization $\rho_2^B$, both certifiers make the same period-2 profit:*

$$\tilde{\pi}^{mh}(\rho_2^A, \rho_B^2) = \beta \frac{z}{1+z}[1-(1+z)\frac{1-q_2^{mh}(\rho_2^A,\rho_2^B)}{q_2^{mh}(\rho_2^A,\rho_2^B)}\phi]F[(1+z)\frac{1-q_2^{mh}(\rho_2^A,\rho_2^B)}{q_2^{mh}(\rho_2^A,\rho_2^B)}\phi]$$

Viewed *ex ante*, certifier A's continuation profit in period 2 reads

$$\pi^{mh}(\rho_2^A) = \int_0^1 \tilde{\pi}^{mh}(\rho_2^A, \rho_2^B)\,\mathrm{d}\rho_2^B$$

Let $\rho_2^{*mh}(\rho_B)$ the bliss reputation of certifier $A$ given $\rho_2^B$.

**Proposition 4.** *If $\pi_2$ is hump-shaped, then $\tilde{\pi_2}^{mh}(\rho_2^A, \rho_2^B)$ is either nonincreasing or hump-shaped in $\rho_2^A$. Furthermore, $\rho_2^{*mh}(\rho_2^B) < \rho_2^*$ and $\frac{\partial \rho_2^{*mh}(\rho_2^B)}{\partial \rho_2^B} \leq 0$.*

*Proof.* In the Appendix. □

Notice that $\frac{1-q_{"}^{mh}}{q_2^{mh}}$ is the ratio between the probability for a bad seller of being unable to trade because of rejection and the probability that he trades. Because he faces a tradeoff between the credibility of certification and the need to attract bad sellers who fear they might not be able to trade, the certifier obtains a profit which is maximized for some interior value of this ratio, which corresponds to the profit-maximizing "informativeness" of the overall certification process. When a second certifier is active in the market, the other certifier can only compensate the additional information which the latter generates by having himself a lower reputation. And the more reputable the other certifier, the more so. The extra cost $z\phi$ has the additional effect of lowering the bliss reputation of the certifier even further. Since $z\phi$ plays a screening role, it would make high reputations less desirable even if no other certifier was providing additional information. Note however, that even if that extra cost was vanishingly small, the incumbent preferred reputation would still be lower than in the monopoly case.

As lower reputations become relatively more desirable to a certifier under multihoming than under a monopoly, reputational concerns could, in turn, adversely impact information production in period 1. We assume for simplicity that certifiers submit their ratings simultaneously, meaning that a certifier cannot learn about the type of the other by observing his action before disclosing his own rating. We also make the extra assumption that $(1-x)F(x)$ is concave.[25]

**Proposition 5.** *Multihoming exacerbates bad reputation effects,*

$$e_1^*(\rho_1) \leq 0 \Rightarrow e_1^{mh}(\rho_1) \leq e_1^*(\rho_1).$$

*Proof.* In the Appendix. □

When sellers can multihome, the entry of a second certifier has the opposite effect as exclusive competition (singlehoming). Because certifiers do not compete to attract sellers but instead share the same customer base, a reputation for accuracy becomes less valuable than in the monopoly case. Certifiers exert an externality on each other: the more reputable the competitor, the less valuable a good reputation is. This exacerbates bad reputation effects: when reputation for accuracy is high enough to generate a negative effort for a monopolist, then the prospect of entry threat provides further incentives to decrease effort. Note that we have assumed, for simplicity, that competition only takes place in period 2, which makes the analysis of period 1 more tractable. If there were two competitors in both periods, the welfare impact of multihoming in period 1 would then be ambiguous: (a) on the one hand, multihoming lowers incentives to exert effort, which adversely impacts welfare; (b) on the other hand, more information is conveyed because the entrant produces an extra signal.

# 5 Conclusion

Recent years have witnessed the emergence of a compelling need for efficient certification: technologies become more complex, market participants are more sophisticated, which increases the scope for information asymmetries; there has been an increasing demand for green or fair trade products. All these evolutions tie in with a more influential role for certifiers. Furthermore, externalities create a need for regulation, as

[25]This is true for instance if the density $f$ is nonincreasing.

in financial markets for instance. A few certification intermediaries may accordingly end up exerting a considerable influence on the allocation of resources in the whole economy. In the same time, recent years have witnessed one of the most dramatic failures in the certification industry: rating agencies have massively failed to predict the subprime crisis and have instead played an important role by overrating structured securities. The question of the ability and incentives of certifiers to generate and transmit accurate information is therefore absolutely critical. In this paper, we investigate the role of reputation as an incentive mechanism for information production. We argue that reputation in certification markets is essentially two-sided, in that the reputation that certifiers ideally would like to achieve is not always a reputation for providing high-quality information. Therefore, reputation may provide wrong incentives, if the certifier cares about developing a reputation for being imprecise, which we show happens when he is perceived as "too accurate". Furthermore, the presence of multiple certifiers sometimes exacerbates these inefficiencies due to wrong incentives.

Our paper offers several possibilities of extension in different directions. First, we abstract from the issue of optimal pricing by certifiers in order to insulate the impact of reputation on the certifier's profit. While assuming some rigidity in prices (i.e., prices cannot instantly adjust to changes in reputation) seems a reasonable assumption in the short run, it would be interesting to study how pricing interacts with reputation, in a context where a certifier tries to attract different types of applicants. In particular, the interplay between reputation and competition is particularly promising. Second, the idea that reputation is multi-sided, in that it reflects the desire to attract different pools of customers, could generate new interesting insights on other markets. In particular, two-sided markets where a platform connects two types of agents (e.g., media, operating systems) would constitute a natural application. Finally, the idea that reputation is multi-sided could be related to the literature on multi-sided communication. In the spirit of this literature, a very interesting question is whether a sender willing to build a two-sided reputation should talk privately or publicly to each of his audiences.

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# 6 Appendix

## 6.1 Monopoly

**Proof of Proposition 1** Let us define $g(x) \equiv \beta z(1-x)F(x)$ and $k(\rho) \equiv \frac{\rho\alpha_H+(1-\rho)\alpha_L}{1-(\rho\alpha_H+(1-\rho)\alpha_L)}\phi$.

Both functions $F(x)$ and $1-x$ log-concave, so $g(x)$ is also log-concave in $x$. Since $g$ is nonnegative on $[0,1]$, it is also quasi-concave on $[0,1]$. $k(\rho_2) \in [0,1]$ when $\rho_2 \in [0,1]$ and $k'(\rho_2) \geq 0$. Therefore, $\pi_2(\rho_2) = g[k(\rho_2)]$ is quasi-concave in $\rho_2$ on $[0,1]$.

$\pi_2'(0)$ has the same sign as $g'(k(0)) = \beta z[(1-\frac{\alpha_L}{1-\alpha_L}\phi)f(\frac{\alpha_L}{1-\alpha_L}\phi) - F(\frac{\alpha_L}{1-\alpha_L}\phi)] \geq 0$.

$\pi_2'(1)$ has the sign of $g'(k(1)) = \beta z[(1-\frac{\alpha_H}{1-\alpha_H}\phi)f(\frac{\alpha_H}{1-\alpha_H}\phi) - F(\frac{\alpha_H}{1-\alpha_H}\phi)] \leq 0$.
Because $\pi_2$ is quasi-concave, it cannot change monotonicities more than once. Therefore, $\pi_2$ is a unimodal function: there is a unique $\rho_2^* \in (0,1)$ such $\pi_2'(\rho_2^*) = 0$. □

**Proof of Proposition 2** First of all, let us make the extra assumption that $\gamma_1(e_1) \in (0,1)$ for all $e_1 \in [-\epsilon, \epsilon]$. Sufficient conditions for this are:

**Assumption 5.** *$\phi < 1 - \alpha_H - \epsilon$ and $\beta < \beta_1' \equiv \frac{1}{1+\max\limits_{q_1\in[1-\alpha_H-\epsilon,1-\alpha_L+\epsilon]}\frac{1}{\phi}(1-\frac{\phi}{q_1})F(\frac{1-q_1}{q_1}\phi)}$*

$\gamma_1$ is interior, hence it equals $\frac{\beta}{1-\beta}(1-\frac{\phi}{q_1})\frac{1}{\phi}F(\frac{1-q_1}{q_1}\phi)$ (see the Proof of Lemma 1). Therefore, we have $\frac{(1-\beta)\gamma_1}{\beta F(\bar{\lambda}_1)+(1-\beta)\gamma_1} = \frac{q_1-\phi}{q_1-\phi+\phi q_1}$.

Let us define the following function (where the dependence of $q_1, \rho^+$ and $\rho^-$ on both $\rho_1$ and $e_1$ is made explicit):

$$L(\rho_1, e_1) \equiv \frac{q_1(\rho_1,e_1)-\phi}{q_1(\rho_1,e_1)-\phi+\phi q_1(\rho_1,e_1)}[\pi_2(\rho^+(\rho_1,e_1)) - \pi_2(\rho^-(\rho_1,e_1))] - ce_1$$

A solution to (5) is either $e_1^* = -\epsilon$ if $L(\rho_1,-\epsilon) < 0$, $e_1^* = \epsilon$ if $L(\rho_1,\epsilon) > 0$, or $e_1^*$ such that $L(\rho_1, e_1^*) = 0$.

Let us assume that $c$ is large enough such that $\sup_{\rho_1,e_1} L_2(\rho_1, e_1) < 0$.[26] This ensures that there is a unique solution $e_1^*$ to (5) for all $\rho_1$.[27]

Furthermore, $L$ is continuously differentiable in each argument, so $e_1^*(\rho_1)$ is continuous in $\rho_1$.

Consider $\rho_1 \in \{0, 1\}$ . $\forall e_1, \rho^+(\rho_1, e_1) = \rho^-(\rho_1, e_1) = \rho_1$, so $L(\rho_1, e_1) = -ce_1$.

Therefore, we have $e_1^*(0) = e_1^*(1) = 0$.

By the implicit function theorem, when the solution to (5) is interior,

$$\frac{\partial e_1^*}{\partial \rho_1}(\rho_1) = -\frac{L_1(\rho_1, e_1^*(\rho_1))}{L_2(\rho_1, e_1^*(\rho_1))}$$

$$= \frac{\frac{(\alpha_H-\alpha_L)\phi^2}{[q_1(\rho_1,e_1^*)-\phi+\phi q_1(\rho_1,e_1^*)]^2}[\pi_2(\rho^+(\rho_1, e_1^*)) - \pi_2(\rho^-(\rho_1, e_1^*))]}{L_2(\rho_1, e_1^*)}$$

$$-\frac{\frac{q_1(\rho_1,e_1^*)-\phi}{q_1(\rho_1,e_1^*)-\phi+\phi q_1(\rho_1,e_1^*)}\left\{\pi_2'[\rho^+(\rho_1, e^*(\rho_1))]\frac{\partial \rho^+}{\partial \rho_1}(\rho_1, e^*(\rho_1)) - \pi_2'[\rho^-(\rho_1, e^*(\rho_1))]\frac{\partial \rho^-}{\partial \rho_1}(\rho_1, e^*(\rho_1))\right\}}{L_2(\rho_1, e_1^*)}$$

We have:

$$\frac{\partial \rho^+}{\partial \rho_1}(\rho_1, e_1) = \frac{(\alpha_H + e_1)(\alpha_L + e_1)}{[\rho_1\alpha_H + (1-\rho_1)\alpha_L + e_1]^2} \geq 0$$

and

$$\frac{\partial \rho^-}{\partial \rho_1}(\rho_1, e_1) = \frac{(1-\alpha_H - e_1)(1-\alpha_L - e_1)}{[\rho_1(1-\alpha_H) + (1-\rho_1)(1-\alpha_L) - e_1]^2} \geq 0.$$

This implies, recalling that $e_1^*(0) = e_1^*(1) = 0$ and that $L_2(\rho_1, e_1^*(\rho_1))) < 0$ :

- $\frac{\partial e_1^*}{\partial \rho_1}(0)$ has the sign of $\pi_2'(0)[\frac{\alpha_H}{\alpha_L} - \frac{1-\alpha_H}{1-\alpha_L}]$, i.e. is nonnegative.
- $\frac{\partial e_1^*}{\partial \rho_1}(1)$ has the sign of $\pi_2'(1)[\frac{\alpha_L}{\alpha_H} - \frac{1-\alpha_L}{1-\alpha_H}]$, i.e. is nonnegative.

By continuity of $e_1^*(\rho_1)$ and from $e_1^*(0) = e_1^*(1) = 0$, there exists at least a $\overline{\rho}$ such that $e_1^*(\overline{\rho}) = 0$.

[26] $L_i(.,.)$ refers to the partial derivative of $L$ with respect to the $i$-th variable.

[27] This assumption is sufficient to get uniqueness, but not necessary. It is indeed enough that $L(\rho_1, e_1) = 0 \Rightarrow L_2(\rho_1, e_1) < 0$ but since a solution to $L(\rho_1, e_1) = 0$ can only be defined implicitly, this is much more cumbersome to write.

$\overline{\rho}$ is such that $L(\overline{\rho},0)=0$, which is equivalent to

$$\pi_2[\rho^+(\overline{\rho},0)] = \pi_2[\rho^-(\overline{\rho},0)]$$

$\rho^+(\overline{\rho},0) \neq \rho^-(\overline{\rho},0)$ because $\overline{\rho} \notin \{0,1\}$ , so the single-peakedness of $\pi_2$ implies that

$$\pi_2'[\rho^+(\overline{\rho},0)] < 0 < \pi_2'[\rho^-(\overline{\rho},0)]$$

We derive that

$$\frac{\partial e_1^*}{\partial \rho_1}(\overline{\rho}) = -\frac{\frac{q_1(\overline{\rho},0)-\phi}{q_1(\overline{\rho},0)-\phi+\phi q_1(\overline{\rho},0)}\left\{\pi_2'[\rho^+(\overline{\rho},0)]\frac{\partial \rho^+}{\partial \rho_1}(\overline{\rho},0) - \pi_2'[\rho^-(\overline{\rho},0)]\frac{\partial \rho^-}{\partial \rho_1}(\overline{\rho},0)\right\}}{L_2(\overline{\rho},0)} < 0.$$

Therefore, by continuity of $e_1^*(\rho_1)$, $\overline{\rho}$ must be unique. From uniqueness of $\overline{\rho}$ and from $\frac{\partial e^*}{\partial \rho_1}(\overline{\rho}) < 0$, we derive that $e_1^*(\rho_1) > 0$ for $\rho_1 \in (0;\overline{\rho})$ and $e_1^*(\rho_1) < 0$ for $\rho_1 \in (\overline{\rho};1)$.

□

## 6.2 Singlehoming

**Proof of Lemma 2** Let denote by $P_j (j = A, B)$ the price that a seller obtains following certification from certifier $j$. Furthermore, let us specify as a refinement that, whenever Bayes' rule cannot apply, the posterior belief following a deviation is such that all the weight is put on the type most likely to deviate. More precisely, suppose $P_j$ cannot be derived from Bayes' rule. A good seller deviates from $i$ to $j$ if $P_j > P_i$, while a bad seller deviates if $(1-\rho_2^j\alpha_H - (1-\rho_2^j)\alpha_L)P_j > (1-\rho_2^i\alpha_H - (1-\rho_2^i)\alpha_L)P_i$. Therefore, the set of $P_j$ for which a good seller deviates is a subset (resp. superset) of the set of $P_j$ for which a bad seller deviates if $\rho_2^i > \rho_2^j$ (resp. $\rho_2^i < \rho_2^j$). We impose that a deviation from the more reputable to the less reputable certifier comes from a bad seller, while a deviation in the opposite direction comes from a good seller. This adaptation of the $D1$ criterion allows to rule unnatural equilibria where good sellers have to solicit the less reputable certifier.[28]

[28]Notice that the D1 refinement does not apply strictly speaking, because we have a continuum of types.

If $\rho_2^A = \rho_2^B$, let us assume that all seller types go to A. This assumption is innocuous, as $\rho_2^A = \rho_2^B$ is a zero probability event. If $\rho_2^A \neq \rho_2^B$, suppose w.l.o.g. that $\rho_2^A > \rho_2^B$. Let us consider different cases:

- $P_A < P_B$ : this implies that no good seller ever goes to $A$. Therefore, $P_A$ cannot be pinned down by Bayes' rule. Since $\rho_2^A > \rho_2^B$, our refinement imposes that $P_A = 1$, which violates $P_A < P_B$.
- $P_A = P_B = 1$ : this is impossible, as a low-quality seller would then have, from Assumption 2, an incentive to solicit, say, $A$, which is inconsistent with $P_A = 1$.
- $P_A = P_B < 1$ : then good sellers are indifferent between $A$ and $B$. But, since $\rho_2^A > \rho_2^B$, bad sellers must prefer strictly $B$ to $A$, so $P_A$ should be equal to 1.

Therefore, we must have $P_A > P_B$. This implies that no good seller ever goes to $B$, hence no bad seller either. $P_B$ cannot be pinned down by Bayes' rule, and our refinement imposes $P_B = 0$. All the sellers who solicit certification then go to A and the price $P_A$ is then determined as in the monopoly case.

**Proof of Proposition 3** Let us define

$$L^{sh}(\rho_1, e_1) \equiv \frac{q_1(\rho_1, e_1) - \phi}{q_1(\rho_1, e_1) - \phi + \phi q_1(\rho_1, e_1)}[\rho^+(\rho_1, e_1)\pi_2(\rho^+(\rho_1, e_1)) - \rho^-(\rho_1, e_1)\pi_2(\rho^-(\rho_1, e_1))] - ce_1.$$

A solution to the incumbent's problem is either $e_1^{sh} = -\epsilon$ if $L^{sh}(\rho_1, -\epsilon) < 0$, $e_1^{sh} = \epsilon$ if $L^{sh}(\rho_1, \epsilon) > 0$, or $e_1^{sh}$ such that $L^{sh}(\rho_1, e_1^{sh}) = 0$. As in the monopoly case, we impose that $c$ is large enough, so that $\frac{\partial L^{sh}}{\partial e_1}(\rho_1, e_1) < 0$. This ensures the uniqueness of $e_1^{sh}$.

If $e_1^* = -\epsilon$, the result is obvious. If $-\epsilon < e_1^* \leq 0$, $e_1^*$ is defined by

$$L(\rho_1, e_1^*) = \frac{q_1(\rho_1, e_1^*) - \phi}{q_1(\rho_1, e_1^*) - \phi + \phi q_1(\rho_1, e_1^*)}[\pi_2(\rho^+(\rho_1, e_1^*)) - \pi_2(\rho^-(\rho_1, e_1^*))] - ce_1^* = 0$$

$$\begin{aligned} L^{sh}(\rho_1, e_1^*) &= \tfrac{q_1(\rho_1, e_1^*) - \phi}{q_1(\rho_1, e_1^*) - \phi + \phi q_1(\rho_1, e_1^*)}[\rho^+(\rho_1, e_1^*)\pi_2(\rho^+(\rho_1, e_1^*)) - \rho^-(\rho_1, e_1^*)\pi_2(\rho^-(\rho_1, e_1^*))] - ce_1^* \\ &= \tfrac{q_1(\rho_1, e_1^*) - \phi}{q_1(\rho_1, e_1^*) - \phi + \phi q_1(\rho_1, e_1^*)}[(1 - \rho^-(\rho_1, e_1^*))\pi_2(\rho^-(\rho_1, e_1^*)) - (1 - \rho^+(\rho_1, e_1^*))\pi_2(\rho^+(\rho_1, e_1^*))] \end{aligned}$$

Furthermore, $e_1^* \leq 0 \Leftrightarrow \pi_2(\rho^+(\rho_1, e_1^*)) \leq \pi_2(\rho^-(\rho_1, e_1^*))$.

Since $\frac{q_1(\rho_1,e_1^*)-\phi}{q_1(\rho_1,e_1^*)-\phi+\phi q_1(\rho_1,e_1^*)} > 0$, we have:

$$L^{sh}(\rho_1, e_1^*) \geq [\rho^+(\rho_1, e_1^*) - \rho^-(\rho_1, e_1^*)]\pi_2(\rho^+(\rho_1, e_1^*)) \geq 0.$$

Finally, from $\frac{\partial L^{sh}}{\partial e_1}(\rho_1, e_1) < 0$, we derive $e_1^* \leq e_1^{sh}$. □

## 6.3 Multihoming

**Proof of Lemma 3** Let $P_{AB}, P_A$ and $P_B$ the prices that buyers are willing to pay following certification by both A and B, A only, and B only. Suppose furthermore that $\rho_A \geq \rho_B$.

- Consider an equilibrium in which no seller multihomes. For simplicity, assume that the seller never goes to $B$ (the proof is unchanged if no one goes to $A$ or if the seller randomizes, in the case $\rho_2^A = \rho_2^B$). From Assumption 2, we know that there is neither zero nor full participation of bad sellers. Therefore, $P_A$ is given by the indifference condition of bad sellers: $P_A = \frac{\phi}{1-\rho_A\alpha_H-(1-\rho_A)\alpha_L}$. $P_{AB}$ and $P_B$ cannot be derived from Bayes' rule. Our refinement imposes that $P_{AB}$ be set to 1, as good sellers always have stronger incentives to deviate to multihoming than bad sellers.

  In order for such an equilibrium to exist, we must therefore have $1 < P_A + z\phi$.

  Since $P_A + z\phi \leq \frac{\phi}{1-\alpha_H} + z\phi$, and $\phi \leq \frac{(1-\alpha_H)^2}{1+z}$ (from Assumption 6), we conclude: $P_A + z\phi \leq 1 - \alpha_H - \frac{z\alpha_H(1-\alpha_H)}{1+z} < 1$.

  This establishes that a good seller who solicits $A$ only should deviate and solicit an extra rating.

  If $P_A$ cannot be pinned down by Bayes' rule but $P_B$ can, the result is a fortiori true because $\rho_A \geq \rho_B$, so incentives to deviate are even bigger.

  This proves that there is no equilibrium with no multihoming.

- Suppose now that there is both singlehoming and multihoming in equilibrium. A good seller must then be indifferent between multihoming and singlehoming, say with A only. Then, the bad seller strictly prefers to singlehome, so we must have $P_{AB} = 1$. The indifference condition for the good seller thus reads $P_A = 1 - z\phi$, which is impossible under Assumption 6.

  Therefore, the good seller multihomes with probability 1, and singlehoming does not occur in equilibrium. □

**Proof of Lemma 4** As in the monopoly case, an equilibrium features a triple $(\overline{\lambda}_2^{mh}, \gamma_2^{mh}, P_2^{mh})$. Consider some $q_2^{mh} \in [(1-\alpha_H)^2, (1-\alpha_L)^2]$. If $P_2^{mh} > (1+z)\phi$, we must have:[29]

$$P_2^{mh} - \overline{\lambda}_2^{mh} = (1+z)\phi \tag{6}$$

$$\gamma_2^{mh} \in \underset{\tilde{\gamma}\in[0,1]}{\operatorname{argmax}} \tilde{\gamma}[q_2^{mh} \frac{\beta F(\overline{\lambda}_2^{mh})}{\beta F(\overline{\lambda}_2^{mh}) + (1-\beta)\gamma_2^{mh} q_2^{mh}} - (1+z)\phi] \tag{7}$$

$$P_2^{mh} = Pr(\theta = \theta_g | \sigma^A = \sigma^B = \varnothing) = \frac{\beta F(\overline{\lambda}_2^{mh})}{\beta F(\overline{\lambda}_2^{mh}) + (1-\beta)\gamma_2^{mh} q_2^{mh}}. \tag{8}$$

The solution of this system must be unique, as $P_2^{mh}$ is a decreasing function of $\gamma_2^{mh}$. Furthermore, at any solution involving certification with positive probability, we have $P_2^{mh} > (1+z)\phi$, so $\overline{\lambda}_2^{mh} > 0$. Again, we restrict attention to the case where $\gamma_2^{mh} \in (0,1)$.

For all $q_2^{mh} \in [(1-\alpha_H)^2, (1-\alpha_L)^2]$, the interior solution is:

$$\begin{cases} \overline{\lambda}_2^{mh} &= \frac{1-q_2^{mh}}{q_2^{mh}}(1+z)\phi \\ \gamma_2^{mh} &= \frac{\beta}{1-\beta}(\frac{1}{1+z} - \frac{\phi}{q_2^{mh}})\frac{1}{\phi}F(\frac{1-q_2^{mh}}{q_2^{mh}}(1+z)\phi) \\ P_2^{mh} &= \frac{\phi}{q_2^{mh}} \end{cases}$$

Necessary conditions for the solution to be interior are:

**Assumption 6.** $\phi(1+z) < (1-\alpha_H)^2$

**Assumption 7.** $\beta < \frac{1}{1+\max_{q^{mh}\in[(1-\alpha_H)^2,(1-\alpha_L)^2]} \frac{1}{\phi}(\frac{1}{1+z} - \frac{\phi}{q^{mh}})F(\frac{1-q^{mh}}{q^{mh}}(1+z)\phi)}$

[29]Notice that we rule out again the equilibrium in which $\overline{\lambda}_2^{mh} = \gamma_2^{mh} = 0$, so that we can always apply Bayes' rule to pin down $P_2^{mh}$.

Assumptions 6 and 7 are the counterpart of Assumption 2 for multihoming. The former ensures that the minimal joint probability of being certified by two different certifiers is high enough, so that the demand of low quality sellers is always positive. The latter allows to make sure that there is never full participation of low-quality sellers.

We conclude by computing the profit as a function of $q_2^{mh}$ :

$$\pi_2^{mh}(q_2^{mh}) = [\beta F(\overline{\lambda}_2^{mh}) + (1-\beta)\gamma_2^{mh}]z\phi = \beta\frac{z}{1+z}[1 - \frac{1-q_2^{mh}}{q_2^{mh}}(1+z)\phi]F[\frac{1-q_2^{mh}}{q_2^{mh}}(1+z)\phi].$$

**Proof of Proposition 4**

**Some notation and preliminary computations** In order to compare the profits under monopoly and multihoming, let us first define $t(x, \rho_2^B) \equiv (1+z)\frac{x+(\rho_2^B\alpha_H+(1-\rho_2^B)\alpha_L)\phi}{1-(\rho_2^B\alpha_H+(1-\rho_2^B)\alpha_L)}$. Recalling that

$$g(x) = \beta z(1-x)F(x),$$

$$k(\rho) = \frac{\rho\alpha_H + (1-\rho)\alpha_L}{1-(\rho\alpha_H + (1-\rho)\alpha_L)}\phi,$$

$$q_2^{mh}(\rho_2^A, \rho_2^B) = [1 - (\rho_2^A\alpha_H + (1-\rho_2^A)\alpha_L)][1 - (\rho_2^B\alpha_H + (1-\rho_2 B)\alpha_L)],$$

one rewrites

$$\pi_2(\rho_2) = g[k(\rho_2)]$$

$$\tilde{\pi}_2^{mh}(\rho_2^A, \rho_2^B) = \frac{1}{1+z}g[t(k(\rho_2^A), \rho_2^B)].$$

*Proof.* We have established that $g(.)$ is quasi-concave on $[0,1]$. Furthermore, $t(k(\rho_2^A), \rho_2^B) = \frac{1-q_2^{mh}}{q_2^{mh}}(1+z)\phi \in [0,1]$ from Assumption 6. Finally, from $k'(\rho) \geq 0$ and $t_1(.,.) \geq 0$, we derive that $\tilde{\pi}_2^{mh}(\rho_2^A, \rho_2^B)$ is quasi-concave in $\rho_2^A$.[30]

By definition of $\rho_2^*$, we have $g'(k(\rho_2^*)) = 0$.

$$\frac{\partial \pi_2^{mh}}{\partial \rho_2^A}(\rho_2^*, \rho_2^B) = \frac{1}{1+z}g'[t(k(\rho_2^*), \rho_2^B)]t_1(k(\rho_2^*), \rho_2^B)k'(\rho_2^*).$$

[30] $t_i(x, \rho_2^B)$ refers to the partial derivative of $t$ with respect to the $i$-th variable.

This expression is negative since $t(x, \rho_2^B) > x$ for all $(x, \rho_2^B)$ and $g'(x) < 0$ for $x \geq k(\rho_2^*)$.

This implies that $\rho_2^{*mh}(\rho_2^B) < \rho_2^*$ for all $\rho_2^B$.

If $\rho_2^{*mh} > 0$, we have

$$\frac{\partial \pi_2^{mh}}{\partial \rho_2^A}(\rho_2^{*mh}, \rho_2^B) = \frac{1}{1+z} g'[t(k(\rho_2^{*mh}), \rho_2^B)] t_1(k(\rho_2^{*mh}), \rho_2^B) k'(\rho_2^{*mh}) = 0.$$

Since $t_1$ and $k'$ are positive and $g$ is unimodal, we derive that $t(k(\rho_2^{*mh}), \rho_2^B)$ is equal to some constant (actually $k(\rho_2^*)$). It follows that

$$\frac{\partial \rho_2^{*mh}(\rho_2^B)}{\partial \rho_2^B} = -\frac{t_2(k(\rho_2^{*mh}), \rho_2^B)}{t_1(k(\rho_2^{*mh}), \rho_2^B) k'(\rho_2^{*mh})}.$$

Finally, $t_2(.,.) > 0$ implies that $\frac{\partial \rho_2^{*mh}(\rho_2^B)}{\partial \rho_2^B} \leq 0$. $\rho_2^{*mh} = 0$ is possible only if $\rho_2^B$ is large enough. □

**Proof of Proposition 5** Let us first prove the following lemma:

**Lemma 5.** *The following implication holds:*

$$\pi_2(\rho^+(\rho_1, e_1)) \leq \pi_2(\rho^-(\rho_1, e_1)) \Rightarrow \pi_2^{mh}(\rho^+(\rho_1, e_1)) - \pi_2^{mh}(\rho^-(\rho_1, e_1)) \leq \pi_2(\rho^+(\rho_1, e_1)) - \pi_2(\rho^-(\rho_1, e_1)).$$

*Proof.* Using the notation introduced earlier, we have

$$\pi_2^{mh}(\rho) = \frac{1}{1+z} \int_0^1 g[t(k(\rho), \rho_2^B)] \, d\rho_2^B \text{ and } \pi_2(\rho) = g[k(\rho)].$$

Dropping arguments of $\rho^+$ and $\rho^-$, one writes

$$\pi_2^{mh}(\rho^+(\rho_1, e_1)) - \pi_2^{mh}(\rho^-(\rho_1, e_1)) - [\pi_2(\rho^+(\rho_1, e_1)) - \pi_2(\rho^-(\rho_1, e_1))]$$

$$= \frac{1}{1+z}[\int_0^1 g[t(k(\rho^+), \rho_2^B)] - g[t(k(\rho^-), \rho_2^B)] \, d\rho_2^B - [g[k(\rho^+)] - g[k(\rho^-)]].$$

One notices first that

$$\frac{1}{1+z}[\int_0^1 g[t(k(\rho^+),\rho_2^B)] - g[t(k(\rho^-),\rho_2^B)]\,\mathrm{d}\rho_2^B$$

$$= \frac{1}{1+z}\int_0^1 \int_{k(\rho^-)}^{k(\rho^+)} g'(t(s,\rho_2^B))t_1(s,\rho_2^B)\,\mathrm{d}s\,\mathrm{d}\rho_2^B$$

$$= \int_0^1 \frac{1}{1-(\rho_2^B\alpha_H+(1-\rho_2^B)\alpha_L)}\int_{k(\rho^-)}^{k(\rho^+)} g'(t(s,\rho_2^B))\,\mathrm{d}s\,\mathrm{d}\rho_2^B.$$

Finally, we derive

$$\pi_2^{mh}(\rho^+) - \pi_2^{mh}(\rho^-) - [\pi_2(\rho^+) - \pi_2(\rho^-)]$$

$$= \int_0^1 \frac{1}{1-(\rho_2^B\alpha_H+(1-\rho_2^B)\alpha_L)}\left\{\int_{k(\rho^-)}^{k(\rho^+)}[g'(t(s,\rho_2^B)) - g'(s)]\,\mathrm{d}s + (\rho_2^B\alpha_H+(1-\rho_2^B)\alpha_L)[g(k(\rho^+) - g(k(\rho^-))]\right\}\mathrm{d}\rho_2^B$$

Given that $\rho^+ \geq \rho^-$, we have $k(\rho^+) \geq k(\rho^-)$.

Furthermore, $t(s,\rho_2^B) = (1+z)\frac{s+(\rho_2^B\alpha_H+(1-\rho_2^B)\alpha_L)\phi}{1-(\rho_2^B\alpha_H+(1-\rho_2^B)\alpha_L)} > s$ for all $\rho_2^B$.

Since $g$ is concave, we derive that $\int_{k(\rho^-)}^{k(\rho^+)}[g'(t(s,\rho_2^B)) - g'(s)]\,\mathrm{d}s \leq 0$.

It follows that

$$\pi_2(\rho^+) - \pi_2(\rho^-) \leq 0 \Rightarrow \pi_2^{mh}(\rho^+) - \pi_2^{mh}(\rho^-) - [\pi_2(\rho^+) - \pi_2(\rho^-)] \leq 0.$$

□

We now turn to the Proof of Proposition 5. Let us define

$$L^{mh}(\rho_1,e_1) \equiv \frac{q_1(\rho_1,e_1)-\phi}{q_1(\rho_1,e_1)-\phi+\phi q_1(\rho_1,e_1)}[\pi_2^{mh}(\rho^+(\rho_1,e_1)) - \pi_2^{mh}(\rho^-(\rho_1,e_1))] - ce_1$$

A solution to the incumbent's problem is either $e_1^{mh} = -\epsilon$ if $L^{mh}(\rho_1,-\epsilon) < 0$, $e_1^{mh} = \epsilon$ if $L^{mh}(\rho_1,\epsilon) > 0$, or $e_1^{mh}$ such that $L^{mh}(\rho_1,e_1^{mh}) = 0$. As usual, we impose that $c$ is large enough, so that $\frac{\partial L^{mh}}{\partial e_1}(\rho_1,e_1) < 0$. This ensures the uniqueness of $e_1^{mh}$.

If $-\epsilon < e_1^* \leq 0$, $e_1^*$ is defined by

$$L(\rho_1, e_1^*) = \frac{q_1(\rho_1, e_1^*) - \phi}{q_1(\rho_1, e_1^*) - \phi + \phi q_1(\rho_1, e_1^*)}[\pi_2(\rho^+(\rho_1, e_1^*)) - \pi_2(\rho^-(\rho_1, e_1^*))] - ce_1^* = 0$$

In order to compare $e_1^*$ and $e_1^{mh}$, let us derive $L^{mh}(\rho_1, e_1^*)$ :

$$L^{mh}(\rho_1, e_1^*) = \frac{q_1(\rho_1, e_1^*) - \phi}{q_1(\rho_1, e_1^*) - \phi + \phi q_1(\rho_1, e_1^*)}[\pi_2^{mh}(\rho^+(\rho_1, e_1^*)) - \pi_2^{mh}(\rho^-(\rho_1, e_1^*))] - ce_1^*$$

$$= \frac{q_1(\rho_1, e_1^*) - \phi}{q_1(\rho_1, e_1^*) - \phi + \phi q_1(\rho_1, e_1^*)}[\pi_2^{mh}(\rho^+(\rho_1, e_1^*)) - \pi_2^{mh}(\rho^-(\rho_1, e_1^*)) - (\pi_2(\rho^+(\rho_1, e_1^*)) - \pi_2(\rho^-(\rho_1, e_1^*)))].$$

From Lemma 5, we have $e_1^* \leq 0, L^{mh}(\rho_1, e_1^*) \leq 0$. We conclude from $\frac{\partial L^{mh}}{\partial e_1}(\rho_1, e_1) < 0$ that

$$-\epsilon < e_1^* \leq 0 \Rightarrow e_1^{mh} \leq e_1^*.$$

Suppose that $e_1^* = -\epsilon$. We must therefore have

$$\frac{q_1(\rho_1, -\epsilon) - \phi}{q_1(\rho_1, -\epsilon) - \phi + \phi q_1(\rho_1, -\epsilon)}[\pi_2(\rho^+(\rho_1, -\epsilon)) - \pi_2(\rho^-(\rho_1, -\epsilon))] + c\epsilon < 0.$$

From Lemma 5, we have

$$\pi_2^{mh}(\rho^+(\rho_1, -\epsilon)) - \pi_2^{mh}(\rho^-(\rho_1, -\epsilon)) \leq \pi_2(\rho^+(\rho_1, -\epsilon)) - \pi_2(\rho^-(\rho_1, -\epsilon))$$

We conclude that

$$\frac{q_1(\rho_1, -\epsilon) - \phi}{q_1(\rho_1, -\epsilon) - \phi + \phi q_1(\rho_1, -\epsilon)}[\pi_2^{mh}(\rho^+(\rho_1, -\epsilon)) - \pi_2^{mh}(\rho^-(\rho_1, -\epsilon))] < -c\epsilon,$$

which implies that $e_1^{mh} = -\epsilon$.

Finally,

$$e_1^* \leq 0 \Rightarrow e_1^{mh} \leq e_1^*.$$

□