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Competition with Exclusive Contracts in Vertically Related Markets: An Equilibrium Non-Existence Result

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Abstract

I develop a model in the spirit of Ordover, Saloner, and Salop (1990), in which two upstream firms compete to supply a homogeneous input to two downstream firms, who compete in prices with differentiated products in a downstream market. Upstream firms are allowed to offer exclusive two-part tariff contracts to the downstream firms. I show that, under very general conditions, this game does not have a subgame-perfect equilibrium in pure strategies. The intuition is that variable parts in such an equilibrium would have to be pairwise-proof. But when variable parts are pairwise-proof, downstream competitive externalities are not internalized, and there exists a profitable deviation. I contrast this non-existence result with earlier papers that found equilibria in similar models.

1 Introduction

The anticompetitive effects of exclusive dealing agreements have long been a hotly debated issue among economists and antitrust practitioners. Such contracts were seen with suspicion by competition authorities in the first half of the twentieth century. The

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main theory of harm was the so-called vertical foreclosure theory, according to which exclusive dealing contracts allow a manufacturer to exclude its upstream rivals from the input market. Authors associated with the Chicago School challenged this view on the ground that a rational downstream buyer would never accept to sign an exclusive dealing contract for anticompetitive reasons (Posner (1976); Bork (1978)).

From the 1990s onward, a more recent strategic approach has been revisiting these issues using modern game-theoretical tools. Paper in this literature can be divided in two groups. A first strand of literature analyzes triangular market structures in which, by assumption, the upstream or the downstream market is supplied by a monopoly (Hart and Tirole (1990); O'Brien and Shaffer (1997); Bernheim and Whinston (1998); Marx and Shaffer (2007); Miklós-Thal, Rey, and Vergé (2011)). A second strand of literature develops naked exclusion models, in which an upstream incumbent signs exclusive contracts with downstream buyers before a potential upstream entrant makes its entry decision (Rasmusen, Ramseyer, and Wiley (1991); Segal and Whinston (2000); Fumagalli and Motta (2006); Simpson and Wickelgren (2007); Abito and Wright (2008)). In naked exclusion models, the entrant cannot offer exclusive dealing contracts before entering, and there is therefore no competition for exclusives.[1]

Yet, in most exclusive dealing cases, entrants are already present in the market at the time the incumbent makes exclusive offers, and there are several firms at both layers of the production chain.[2] In the words of Whinston (2006), "developing models that reflect this reality is a high priority". In this paper, I show that developing such models involves non-trivial theoretical complications.

I start with Ordover, Saloner, and Salop (1990)'s well-known framework, in which two identical upstream firms, U_1 and U_2, compete to supply a homogeneous input to two differentiated downstream firms, D_1 and D_2. In the first stage of the game, upstream firms announce their input supply contracts simultaneously. Contrary to Ordover, Saloner, and Salop (1990), I allow upstream firms to offer exclusive two-part tariff contracts to the downstream firms. In the second stage, downstream firms elect their upstream suppliers, and, in the last stage, set their downstream prices simultaneously.

[1]Exceptions to this classification include Besanko and Perry (1994), in which there is an oligopoly in both markets but upstream firms are restricted to use linear wholesale prices, and a recent contribution by Spector (2011), in which both the incumbent and the entrant can offer exclusives, but downstream buyers do not compete against each other.

[2]Spector (2011) discusses this point in his introduction.

All offers and acceptance decisions are publicly observable.

The following outcome would be a natural equilibrium candidate: U_1 supplies D_1 with a two-part tariff (w_1, T_1), and U_2 supplies D_2 with a two-part tariff (w_2, T_2); due to upstream competition, the fixed parts of the tariffs, T_1 and T_2, are set so as to redistribute upstream profits to the downstream firms; variable parts w_1 and w_2 should be pairwise-stable as in Bonanno and Vickers (1988) and Shaffer (1991), i.e., w_i should maximize the joint profits of U_i and D_i, taking w_j as given, $i \neq j$ in $\{1, 2\}$. It is well-known from the strategic delegation literature that such w_i's are strictly larger than the upstream marginal costs, because high variable parts tend to soften downstream competition when prices are strategic complements.

The problem is that, in this outcome, the industry profit is not maximized, because competition externalities between downstream firms are not internalized. In particular, upstream variable part and downstream prices are too low from the point of view of industry profit maximization. This opens the door to the following deviation: U_1 first becomes the upstream supplier of D_2 by slightly undercutting T_2; next, it slightly increases w_1 and decreases T_1 to make sure that D_1 does not switch to U_2. Since the channel profit of structure $U_1 - D_1$ was maximized at the initial w_1, U_1 starts making losses on D_1, but these losses are second-order. On the other hand, since D_1 now has a higher marginal cost, it increases its downstream price in the continuation subgame, which tends to raise D_2's downstream demand, and therefore D_2's input demand. This implies a first-order increase in the profits that U_1 earns from D_2, and therefore makes the deviation profitable. I formalize this argument and show that, under general conditions, the two-part tariff competition game with exclusive contracts does not have a subgame-perfect equilibrium in pure strategies.

This non-existence problem looks surprising in light of Shaffer (1991)'s and Chen and Riordan (2007)'s results. Shaffer (1991) solves a model similar to mine, except that he has a large number of identical upstream firms competing in the input market. He argues that this game has an equilibrium, and that in any equilibrium, upstream firms make zero profit and variable parts are pairwise-stable. However, he did not check for the deviation I developed in the previous paragraph. In Section 4.3, I explain in greater detail how this deviation (and other potential issues) affects equilibrium characterization in Shaffer (1991)'s model. The bottom line is that the set of equilibria may be either empty, or much larger than what Shaffer (1991) claimed.

Chen and Riordan (2007)'s model is also very close to mine, but they assume that downstream consumers are uniformly distributed on the Hotelling segment, and that downstream firms can perfectly price discriminate. All-out competition for every consumer drives (personalized) downstream prices down to the most efficient firm's marginal cost (net of transport cost). This mechanism destroys the strategic delegation effect, and ensures that the only pairwise-stable variable parts are equal to upstream marginal costs. This also neutralizes the deviation explained above, and ensures that Chen and Riordan (2007)'s equilibrium is indeed an equilibrium. In my model, under a very general class of demand functions and as long as downstream firms cannot price discriminate, pairwise-stable variable parts are always strictly larger than cost and an equilibrium therefore always fails to exist. This issue makes it difficult to assess the robustness of Chen and Riordan (2007)'s results to more common downstream demand systems.

Lemma 1, proven in the appendix, may be of independent interest. In this paper, I allow demand functions to be kinked at points where a firm's demand becomes just equal to zero. The model therefore includes linear demands as a special case, contrary to most of the industrial organization literature, which usually works with demand functions which are continuously differentiable everywhere. The problem is that, in this framework, the contraction mapping theorem cannot be applied to prove uniqueness of the Nash equilibrium in the downstream competition subgame, because the best-response map is not necessarily a contraction. Lemma 1 says that equilibrium existence and uniqueness still obtains provided that the usual duopoly stability condition holds at every point such that both firms' demands are positive.

The rest of the paper is organized as follows. I present the model in Section 2, solve its second and third stages in Section 3, and prove equilibrium non-existence in Section 4. Section 5 concludes. The appendix contains the proof of Lemma 1.

2 The Model

There are four firms in the industry: two upstream firms, U_1 and U_2, and two downstream firms, D_1 and D_2. Upstream firms produce an intermediate input at constant unit cost $m \geq 0$. Downstream firms purchase this input and transform it into the final product on a one-to-one basis. Downstream firms incur no additional costs.

In line with the vertical integration literature (Ordover, Saloner, and Salop (1990); Chen (2001)), the intermediate input supplied by upstream firms is homogeneous and final products are differentiated. In the downstream market, firms D_1 and D_2 compete by simultaneously setting prices p_1 and p_2. The demand addressed to D_i, $i = 1, 2$, can be written as $q_i(p)$, where $p = (p_1, p_2)$ denotes the downstream price vector. Downstream firms are symmetric, which implies that the demand addressed to firm D_i can be written as $q_i(p) = q(p_i, p_j)$, where function $q(., .)$ does not depend on i.

Demands are downward-sloping, final products are substitutes, and the total demand is (weakly) decreasing in prices. Formally, if $x > x'$, then: $q(x', y) \geq q(x, y)$ (with a strict inequality if $q(x', y) > 0$), $q(y, x) \geq q(y, x')$ (with a strict inequality if $q(x', y) > 0$ and $q(y, x') > 0$), and $q(x', y) + q(y, x') \geq q(x, y) + q(y, x)$. $q(.)$ is continuous, and it is also twice continuously differentiable at every point (p_i, p_j) such that $q(p_i, p_j) > 0$ and $q(p_j, p_i) > 0$.[3]

In the upstream market, $U1$ and $U2$ offer two-part tariffs. I assume that contracts and acceptance decisions are publicly observed. Upstream firms are allowed to discriminate between downstream buyers and to offer negative fixed fees (slotting fees). Upstream contracts are exclusive, i.e. if D_k signs an exclusive dealing contract with U_i, then it cannot sign another contract with U_j. Formally, a contract between firms U_i and D_k is a pair (w_k^i, T_k^i), where w_k^i (resp. T_k^i) is the variable (resp. fixed) part of the two-part tariff. Once U_i and D_k have signed a contract, U_i commits to supply any quantity of input q_k that firm D_k may demand against payment $w_k^i q_k + T_k^i$. I restrict the action set of upstream firms to contracts with variable parts no smaller than m. I discuss this assumption in Section 4.2. By contrast, I do not impose any restrictions on the sign of T_k^i, i.e. slotting fees are allowed.

The game unfolds as follows:

1. U_1 and U_2 offer their contracts (w_k^i, T_k^i), $1 \leq i, k \leq 2$, simultaneously.

2. D_1 and D_2 observe all upstream contracts, simultaneously decide which contract to accept, and pay the corresponding fixed fees.

3. Downstream firms' acceptance decisions become common knowledge. The down-

[3] I do not assume that demand functions are twice continuously differentiable everywhere, because this assumption is not satisfied with standard Shubik and Levitan (1980) linear demands (linear demands are kinked at points where a firm's demand is exactly equal to zero).

stream firms which accepted at least one upstream contract in stage 2 set their downstream prices simultaneously.[4]

4. Downstream demands are realized, downstream firms order the relevant quantities of input from their upstream suppliers, pay the corresponding variable parts, transform the input into final products, and ship the goods to their downstream consumers.

A key assumption here is that downstream firms produce to order, i.e., they start production after the final consumers have formulated their demand. This assumption is common in the vertical relations literature (see, among others, Ordover, Saloner, and Salop (1990), Chen (2001) and Chen and Riordan (2007)). As discussed in Rey and Tirole (2007), it makes sense in industries in which final consumers are patient enough and the production cycle is fast enough.

I look for subgame-perfect equilibria in pure strategies on the equilibrium path. I will, however, allow downstream firms to mix in stage 2 off the equilibrium path. I explain and motivate this choice of solution concept in Section 3.2.

3 Solution: Stages 2 and 3

3.1 Downstream Competition

Consider stage 3 of the game, and assume that D_k has signed a contract with a variable part equal to w_k, $k = 1, 2$. I adopt the convention that, if D_k has signed no contract, then its variable part is $w_k = \infty$. D_k's profit in stage 3 (gross of the fixed fee) can be written as:

$$\pi_k^D(p_1, p_2, w_k) = (p_k - w_k)q(p_k, p_l) \equiv \pi^D(p_k, p_l, w_k)$$

I make the following assumptions on this profit function: for all p_l, w_k, $\pi^D(., p_l, w_k)$ is strictly quasi-concave on the set of prices p_k such that $q(p_k, p_l) > 0$; for all w_k, $\sup_{p_l \geq 0} \inf \left(\arg\max_{p_k \geq w_k} \pi^D(p_k, p_l, w_k) \right)$ is finite;[5] prices are strategic complements and the duopoly stability condition holds: for all w_k, for all (p_k, p_l) such that $q(p_k, p_l) > 0$

[4] If a downstream firm has not signed any supply contract, then it exits the industry and the price of its downstream product is set to infinity.

[5] With this assumption, I can work with compact action sets and use a fixed point theorem to prove equilibrium existence (see Appendix A).

and $q(p_l, p_k) > 0$, $\partial^2_{12}\pi^D(p_k, p_l, w_k) > 0$ and $\partial^2_{11}\pi^D(p_k, p_l, w_k) + \partial^2_{12}\pi^D(p_k, p_l, w_k) < 0$.[6] I prove the following lemma:[7]

Lemma 1. *For all $w_1, w_2 \geq m$, the downstream competition subgame has a unique Nash equilibrium.*
Equilibrium downstream prices are continuously differentiable and strictly increasing at every point (w_1, w_2) such that the equilibrium is interior.

Proof. See Appendix A. □

Existence follows directly from Theorem 1.2 in Fudenberg and Tirole (1991). I have to write my own proof for uniqueness, because authors usually assume that payoff functions are globally strictly quasi-concave and differentiable everywhere, and that the stability condition holds globally. The problem is that, because demands may be kinked, the best response map is not necessarily a contraction. Take for instance the commonly used demand system of Shubik and Levitan (1980), let $p_1 \geq 0$, and denote by $BR_2(p_1)$ the best-response function of D_2. If p_1 is such that both firms' demands are strictly positive at price vector $(p_1, BR_2(p_1))$, then the slope of BR_2 is locally given by $|\partial^2_{12}\pi^D / \partial^2_{11}\pi^D|$, which, under linear demands, is a constant strictly smaller than 1. By contrast, if p_1 is such that $q(p_1, BR_2(p_1)) = 0$ and $q(p_1, BR_2(p_1) + \varepsilon) > 0$ for all $\varepsilon > 0$ (i.e., D_2 best replies by just cornering the market), then the slope of BR_2 is locally given by $|\partial_1 q / \partial_2 q|$,[8] which, under linear demand, is a constant strictly larger than 1. It follows that, when demands are allowed to be kinked at points where a firm just corners the market, the contraction mapping theorem cannot be applied. Using a different line of proof, I show in Appendix A that uniqueness of the Nash equilibrium still obtains in such a framework.

Denote by $\hat{p}_k(w_1, w_2)$ the equilibrium downstream price set by D_k. By symmetry and uniqueness, this function can be rewritten as $\hat{p}_k(w_1, w_2) = \hat{p}(w_k, w_l)$. I define downstream firms' equilibrium demands in stage 3:

$$\hat{q}_k(w_1, w_2) \equiv \hat{q}(w_k, w_l) \equiv q(\hat{p}(w_k, w_l), \hat{p}(w_l, w_k)),$$

[6] I denote by $\partial_k f$ the partial derivative of f with respect to its k-th argument, and by $\partial^2_{ij} f$ the second partial derivative of f with respect to its i-th and j-th arguments.

[7] For $k = 1, 2$, I restrict D_k's strategy space to $[w_k, \infty]$. This refines away equilibria in which one downstream firm sets a price below its own marginal cost, and the other downstream firm best replies by setting a price such that the first downstream firm gets 0 demand.

[8] Here, $\partial_1 q$ and $\partial_2 q$ are one-sided partial derivatives.

and downstream firms' equilibrium profits:

$$\hat{\pi}_k^D(w_1, w_2) \equiv \hat{\pi}^D(w_k, w_l) \equiv \pi^D(\hat{p}(w_k, w_l), \hat{p}(w_l, w_k), w_k).$$

I also denote the equilibrium upstream profits derived from selling the input to firm D_k by

$$\hat{\pi}_k^U(w_1, w_2) \equiv \hat{\pi}^U(w_k, w_l) \equiv (w_k - m)\hat{q}(w_k, w_l).$$

I make the following assumption

Assumption 1. $\partial_2 \hat{q}(w_k, w_l) > 0$ *whenever* $\hat{q}(w_k, w_l) > 0$ *and* $\hat{q}(w_l, w_k) > 0$.

An increase in D_l's cost has a direct positive impact on D_k's equilibrium demand (D_l increases its price), and an indirect one (D_k changes its price as well). Assumption 1 means that direct effects dominate indirect ones.

I close this section with the following remark:

Example 1. *Consider the Shubik and Levitan (1980) demand system:*

$$q(p_k, p_l) = \begin{cases} \frac{1}{2}\left(1 - p_k - \gamma\left(p_k - \frac{p_k + p_l}{2}\right)\right) & \text{if } \frac{(2+\gamma)p_l - 2}{\gamma} \leq p_k \leq \frac{\gamma p_l + 2}{\gamma + 2}, \\ \frac{1+\gamma}{2+\gamma}(1 - p_k) & \text{if } p_k \leq \min\left(\frac{(2+\gamma)p_l - 2}{\gamma}, 1\right), \\ 0 & \text{otherwise.} \end{cases}$$

Then, all the assumptions made so far are satisfied.

Proof. The proof is standard and available from the author upon request. □

3.2 Supplier Choice Stage

I stated at the end of Section 2 that I would look for subgame-perfect equilibria in pure strategies, but that I would also need to allow downstream firms to mix in stage 2 off the equilibrium path. In this section, I move back to stage 2, and motivate this choice of solution concept. I first prove the following result:

Lemma 2. *When demands are linear and γ is high enough, there exist profiles of upstream contracts such that the supplier choice game in stage 2 does not have an equilibrium in pure strategies.*

	$(0, a(1-a)-\varepsilon)$	(a, η)	Exit
$(0, a(1-a)-\varepsilon)$	$(\varepsilon - a(1-a), \varepsilon - a(1-a))$	$(\varepsilon, -\eta)$	$(\frac{1}{4} + \varepsilon - a(1-a), 0)$
(a, η)	$(-\eta, \varepsilon)$	$(-\eta, -\eta)$	$(\frac{(1-a)^2}{4} - \eta, 0)$
Exit	$(0, \frac{1}{4} + \varepsilon - a(1-a))$	$(0, \frac{(1-a)^2}{4} - \eta)$	$(0, 0)$

Figure 1: Payoff Matrix

Proof. To begin with, set $\gamma = \infty$, so that downstream products are homogeneous, and consider the following profile of upstream offers:

- $w_1^1 = w_2^2 = 0$ and $w_1^2 = w_2^1 = a < 1/2$.
- $T_1^1 = T_2^2 = a(1-a) - \varepsilon$ and $T_1^2 = T_2^1 = \eta$, where $\varepsilon, \eta > 0$.

If D_i chooses contract $(0, a(1-a)-\varepsilon)$ and D_j does not accept any offer, then D_i's profit is:

$$\max_p p(1-p) + \varepsilon - a(1-a) = \frac{1}{4} + \varepsilon - a(1-a).$$

If D_i chooses contract (a, η) and D_j does not accept any offer, then D_i earns:

$$\max_p (p-a)(1-p) - \eta = \frac{(1-a)^2}{4} - \eta.$$

Finally, if D_i chooses contract $(0, a(1-a)-\varepsilon)$ and D_j chooses contract (a, η), then D_i gets ε, and D_j gets $-\eta$. Figure 1 represents the game in matrix form. The only pure-strategy Nash equilibrium candidates have one firm choosing contract $(0, a(1-a)-\varepsilon)$ and the other firm exiting the industry. However, when ε and η are small enough, there exists a continuum of values of $a < 1/2$ such that

$$\left(\frac{(1-a)^2}{4} - \eta\right) - \left(\frac{1}{4} + \varepsilon - a(1-a)\right) = \frac{1}{4}a(2-3a) - \varepsilon - \eta$$

is strictly positive. When a belongs to this continuum, there is no pure-strategy Nash equilibrium. It is then straightforward to extend this result to high but finite values of γ. $\square$

When demands are linear and γ is high enough, there exist subgames starting in stage 2 which do not have any subgame-perfect equilibria in pure strategies. It follows that, for these values of γ, the whole game does not have any subgame-perfect equilibria in pure strategies. To avoid this issue, I allow downstream firms to mix over their supplier choices in stage 2. Since the supplier choice game is finite, it always has a mixed strategy equilibrium. Therefore, there may exist subgame-perfect equilibria in which firms mix in stage 2, but not in stages 1 and 3.

In the following, I focus on subgame-perfect equilibria in which mixing does not take place on the equilibrium path. I am guessing, but I have never seen this stated explicitly, that this is the solution concept that the existing literature studying competition in two-part tariffs contracts has been working with.

4 Solution: Stage 1

4.1 Equilibrium non-existence

In this section, I move back to stage 1 and, prove that the whole game does not have an equilibrium:

Proposition 1. *The two-part tariff competition game with exclusive contracts has no equilibria.*

4.2 Proof of Proposition 1

The proof proceeds in several steps. To begin with, I rule out equilibrium candidates in which one or two downstream firms are inactive (Lemma 3).[9] Next, I turn my attention to equilibrium candidates in which both downstream firms are active. I show that upstream firms must make zero profit on the equilibrium path and that, for a downstream firm, accepting the contract it is meant to choose on the equilibrium path strictly dominates exiting the industry (Lemma 4). Next, I prove that the variable parts at which downstream firms end up purchasing on path should be pairwise-stable in the sense of Bonanno and Vickers (1988) and Shaffer (1991) (Lemmas 5 and 6). I conclude the proof with Lemma 7, which shows that, even if variable parts are pairwise-stable, there still exist profitable deviations for the upstream firms.

Lemma 3. *There is no equilibrium in which one downstream firm is inactive.*

Proof.

[9] A firm is active if it accepts a contract and its equilibrium quantity is positive.

No equilibrium in which both downstream firms are inactive on path. If both downstream firms are inactive on path, then all firms make zero profit. Let us first show that, for all $(k, i) \in \{1, 2\}^2$:

$$\text{For all } w \geq m, \ \hat{\pi}^D(w_k^i, w) - T_k^i \leq 0. \tag{1}$$

If no firm accepts a contract on the equilibrium candidate path, then, for all k, i, $\pi^D(w_k^i, \infty) - T_k^i \leq 0$. This implies condition (1) for all (k, i).

Next, assume only one firm accepts a contract on path: to fix ideas, suppose D_1 accepts U_1's contract. Then, $\hat{\pi}(w_1^2, \infty) - T_2^1 \leq \hat{\pi}(w_1^1, \infty) - T_1^1 = 0$, and condition (1) holds for $k = 1$ and $i = 1, 2$. Besides, $\hat{q}(w_1^1, \infty) = 0$ (D_1 is inactive), and $T_1^1 = 0$ (no firm makes positive profits). Therefore, $q(w_1^1, \infty) = 0$. It follows that, for all $w \geq w_1^1$ and $w' \geq 0$, $q(w, w') = 0$ (since q is non-increasing in its first argument and non-decreasing in its second argument) and $q(w', w) = q(w', \infty)$ (since the total demand is non-increasing in prices, q is non-decreasing in its second argument and $q(w, w') = 0$). Since only D_1 accepts a contract on path, $\hat{\pi}^D(w_2^i, w_1^1) - T_2^i \leq 0$, $i = 1, 2$. But since $q(w', w) = q(w', \infty)$ for all $w \geq w_1^1$ and $w' \geq 0$, it also follows that $\hat{\pi}^D(w_2^i, \infty) - T_2^i \leq 0$ for all i. This implies condition (1) for $k = 2$.

Last, assume both downstream firms accept a contract on the equilibrium path: to fix ideas, suppose they both sign a contract with U_1. Then, $\hat{q}(w_1^1, w_2^1) = \hat{q}(w_2^1, w_1^1) = 0$. Therefore, $q(w_1^1, w_2^1) = q(w_2^1, w_1^1) = 0$. I can then proceed as in the previous paragraph to show that $q(w, w') = 0$ for all $w \geq w_1^i$, for all $w' \geq 0$, $i = 1, 2$, and that condition (1) holds for all k, i.

Now, consider the following deviation: U_1 offers (m, ε) to D_1 and (∞, ∞) to D_2, where $\varepsilon > 0$. Since downstream products are differentiated, and since D_2's marginal cost cannot be lower than m, $\hat{\pi}^D(m, w) - \varepsilon > 0$ for all $w \geq m$, provided that ε is small enough. By condition (1) for $k = 1$ and $i = 2$, it follows that it is a strictly dominant strategy for D_1 to accept U_1's contract. Therefore, in any equilibrium of stage 2, D_1 accepts the deviation, U_1 makes a profit of ε, and the deviation is profitable.

No equilibrium in which only one downstream firm is inactive on path. Assume that only D_1 is active and that U_1 is its upstream supplier. Assume first that D_2 does not accept any offer on path. Then, T_2^1 and T_2^2 are non-negative, and

$\hat{\pi}^D(w_2^1, w_1^1) - T_2^1 \leq 0$. Besides, since products are differentiated and $w_1^1 \geq m$, we also have that $T_2^1 > 0$ or $w_2^1 > m$ (otherwise $\hat{\pi}^D(w_2^1, w_1^1) - T_2^1 \leq 0$ would not hold). I claim that U_2 can profitably deviate by offering (∞, ∞) to D_1 and (m, ε) to D_2, where $\varepsilon > 0$. Consider the acceptance choice subgame following this deviation. D_1 can either accept U_1's contract or exit the industry. If D_1 accepts U_1's contract, then D_2 strictly prefers accepting U_2's contract (when ε is small enough), since $\hat{\pi}^D(m, w_1^1) - \varepsilon > 0 \geq \hat{\pi}^D(w_2^1, w_1^1) - T_2^1$, where the first inequality follows from the fact that products are differentiated and $w_1^1 \geq m$. If D_1 exits, then D_2 still strictly prefers U_2's contract, since, using the fact that $w_2^1 > m$ or $T_2^1 > 0$, $\hat{\pi}^D(m, \infty) - \varepsilon > \hat{\pi}^D(w_2^1, \infty) - T_2^1$ for ε small enough. Therefore, in any equilibrium of the supplier choice subgame, D_2 accepts U_2's contract, and U_2 makes a profit of ε.

Next, assume D_2 accepts U_2's offer on the equilibrium path (but stays inactive in the downstream market). Then, D_2 makes zero profit on path (otherwise U_2 would be making losses and would have incentives to withdraw its offers), and $\hat{\pi}^D(w_2^1, w_1^1) - T_2^1 \leq 0$. As before, we also have that $T_2^1 > 0$ or $w_2^1 > m$, and U_2 can deviate by offering (∞, ∞) to D_1 and (m, ε) to D_2, where $\varepsilon > 0$.

Last, assume D_2 accepts U_1's offer on the equilibrium path. Then, $T_2^1 \leq 0$. Assume by contradiction that $T_2^1 < 0$. Then, $\hat{\pi}^U(w_1^1, w_2^1) + T_1^1 > 0$, otherwise U_1 would be making strictly negative profits. U_2 can deviate by offering $(w_1^1, T_1^1 - \varepsilon)$ to D_1 and (∞, ∞) to D_2. In the subgame following this deviation, it is a dominant strategy for D_2 to accept U_1's offer. We know that D_1 makes non-negative profits when accepting U_1's offer. Since $\varepsilon > 0$, it makes strictly higher profits when accepting U_2's offer. Therefore, at the unique Nash equilibrium of the acceptance stage, D_1 buys from U_2 and D_2 buys from U_1. U_2 earns $\hat{\pi}^U(w_1^1, w_2^1) + T_1^1 - \varepsilon$, which is strictly positive for ε small enough. Therefore, $T_2^1 = 0$, and $w_2^1 > m$. Then, as in the previous paragraph, U_2 can deviate by offering (∞, ∞) to D_1 and (m, ε) to D_2. □

The lengthy proof of Lemma 3 reveals an important issue that I will have to deal with many times in this section. Starting from a given equilibrium candidate, and following a deviation by an upstream firm in stage 1, there may be multiple equilibria in the continuation subgame starting in stage 2. To destroy the equilibrium candidate, I need to ensure that in any equilibrium of the continuation subgame, the profit of the upstream deviator increases. The assumption that the upstream firms cannot set variable parts below m proves very useful here, as it ensures that a downstream firm would always strictly prefer accepting a contract (m, ε) rather than exiting the industry altogether.

Let us now move on to equilibrium candidates in which both downstream firms are active on path. Then, both upstream firms make zero profit, and exiting the industry is a strictly dominated strategy for the downstream firms:

Lemma 4. *Assume there exists an equilibrium in which both downstream firms are active. Denote by (w_k^A, T_k^A) (resp. (w_k^R, T_k^R)) the contract that is accepted (resp. rejected) by D_k on the equilibrium path, $k = 1, 2$. Then, upstream contracts satisfy the following properties:*

1. $\hat{\pi}^D(w_k^A, w_l^A) - T_k^A \geq \hat{\pi}^D(w_k^R, w_l^A) - T_k^R$.
2. $T_k^A = -\hat{\pi}^U(w_k^A, w_l^A)$, $k \neq l$ *in* $\{1, 2\}$.
3. $\hat{\pi}^D(w_k^A, w) - T_k^A > 0$ *for all* $k \in \{1, 2\}$, *for all* $w \geq m$.

Proof. There exists an equilibrium in stage 2 in which downstream firms accept contracts $\left\{(w_k^A, T_k^A)\right\}_{k=1,2}$ if and only if:

$$\hat{\pi}^D(w_k^A, w_l^A) - T_k^A \geq \max\left(\hat{\pi}^D(w_k^R, w_l^A) - T_k^R, 0\right), \ k \neq l \text{ in } \{1, 2\}.$$

This implies the first bullet point of the lemma.

Now, let us focus on the second bullet point. Assume by contradiction that U_1 supplies both downstream firms on the equilibrium candidate path, and that $\hat{\pi}^U(w_1^A, w_2^A) + T_1^A > 0$. There are two cases to consider. Assume first that $\hat{\pi}^D(w_2^A, w_1^A) - T_2^A = 0$, i.e. D_2 makes 0 profit on the equilibrium path. Then, the profit that U_1 earns from selling the input to D_2 is equal to:

$$\hat{\pi}^U(w_2^A, w_1^A) + T_2^A = \underbrace{\hat{\pi}^U(w_2^A, w_1^A)}_{\geq 0 \text{ since } w_2^A \geq m} + \underbrace{\hat{\pi}^D(w_2^A, w_1^A)}_{>0 \text{ since } D_2 \text{ is active}} > 0.$$

In this case, I claim that U_2 can profitably deviate by offering contract $(w_1^A, T_1^A - \varepsilon)$ to D_1 and contract $(w_2^A, T_2^A - \varepsilon)$ to D_2 ($\varepsilon > 0$). Accepting U_2's contract obviously strictly dominates accepting U_1's contract. Besides, for $k \neq l$ in $\{1,2\}$:

$$\hat{\pi}^D(w_k^A, \infty) - T_k^A + \varepsilon > \hat{\pi}^D(w_k^A, w_l^A) - T_k^A + \varepsilon > 0,$$

i.e. in this subgame, accepting U_2's contract also strictly dominates accepting no contract at all. It follows that the only Nash equilibrium in this subgame has both downstream firms accepting U_2's deviation. The deviation is profitable for U_2 provided that ε is small enough.

Now, assume $\hat{\pi}^D(w_2^A, w_1^A) - T_2^A > 0$. U_2 can deviate by offering contract $(w_1^A, T_1^A - \varepsilon)$ to D_1 ($\varepsilon > 0$) and contract (∞, ∞) to D_2. Then, it is a dominant strategy for D_1 to accept U_2's contract and for D_2 to stick to U_1's contract, and the deviation is profitable when ε is small enough.

Now, assume that U_1 supplies D_1 and U_2 supplies D_2 on the equilibrium candidate path. Assume by contradiction that $\hat{\pi}^U(w_1^A, w_2^A) + T_1^A > 0$. Clearly, $\hat{\pi}^U(w_2^A, w_2^A) + T_2^A \geq 0$, otherwise U_2 can profitably deviate by withdrawing its offers.

Assume first that $w_1^A > m$. Then, U_2 can profitably deviate by offering $\big(w_1^A, \min(-\varepsilon, T_1^A - \varepsilon)\big)$ to D_1 and $(w_2^A, T_2^A - \varepsilon)$ to D_2. It is a strictly dominant strategy for D_1 to accept U_2's contract. Besides, conditional on D_1 accepting U_2's contract, D_2 is strictly better off accepting U_2's contract rather than exiting or accepting U_1's contract. Therefore, the only equilibrium of the supplier choice subgame has both downstream firms accepting U_2's deviation. U_2's profit is either

$$\begin{array}{lccc} & \underbrace{\left(\hat{\pi}^U(w_1^A, w_2^A) + T_1^A\right)}_{>0} & + & \underbrace{\left(\hat{\pi}^U(w_2^A, w_1^A) + T_2^A\right)}_{\geq 0} - 2\varepsilon, \\ \text{or} & \underbrace{\left(\hat{\pi}^U(w_1^A, w_2^A)\right)}_{>0} & + & \underbrace{\left(\hat{\pi}^U(w_2^A, w_1^A) + T_2^A\right)}_{\geq 0} - 2\varepsilon. \end{array}$$

Both expressions are strictly positive and strictly larger than $\hat{\pi}^U(w_2^A, w_1^A) + T_2^A$ when ε is small enough: the deviation is profitable.

Now, assume $w_1^A = m$. Then, $T_1^A > 0$, otherwise U_1 would not be making positive profits. Then, U_2 can profitably deviate by offering (m, ε) to D_1 ($\varepsilon > 0$) and $(w_2^A, T_2^A - \frac{\varepsilon}{2})$ to D_2. If ε is small enough, then, from D_1's point of view, accepting U_2's contract strictly dominates accepting U_1's contract (obvious) and exiting (because products are differentiated and D_2's marginal cost cannot be lower than m). Besides, conditional on D_1 accepting U_2's contract, D_2 is strictly better off accepting U_2's contract rather than exiting or accepting U_1's contract. Therefore, the only equilibrium of the supplier choice subgame has both downstream firms accepting U_2's deviation. U_2's profit is:

$$\underbrace{\left(\hat{\pi}^U(w_2^A, w_1^A) + T_2^A\right)}_{\geq 0} + \frac{\varepsilon}{2} > 0,$$

and the deviation is therefore profitable.

Finally, I prove the third bullet point of the lemma. We know that $T_k^A = -\hat{\pi}^U(w_k^A, w_l^A)$, $k \neq l$ in $\{1, 2\}$. If $w_1^A > m$, then $T_1^A < 0$. Therefore, when accepting contract (w_1^A, T_1^A), D_1 obtains a profit of at least $-T_1^A$, which is strictly larger than zero. Next, if $w_1^A = m$, then $T_1^A = 0$. No matter which contract D_2 accepts, its marginal cost will always be larger than or equal to m. Therefore, if D_1 accepts (w_1^A, T_1^A), then it will always make strictly positive profits, since downstream products are differentiated. $\square$

Intuitively, since upstream firms are competing in prices with homogeneous products, we cannot expect them to make positive profits in equilibrium. While this result seems obvious, the proof turns out to be tedious, because of the potential equilibrium multiplicity in stage 2 that I mentioned before. The third bullet point of the lemma says that, for D_k, accepting (w_k^A, T_k^A) strictly dominates exiting, irrespective of the variable part at which D_l is purchasing. This result is useful, as it will allow me to ignore downstream firms' exit option when looking for the equilibria of the supplier choice game, thereby turning this game into a two-by-two game.

The following concept will be useful to look for equilibria in which both downstream firms are active:

Definition 1. *A pair of linear upstream prices $(w_1^\star, w_2^\star)$ satisfies the Bonanno-Vickers-Shaffer (BVS) conditions if $\hat{q}(w_1^\star, w_2^\star) > 0$, $\hat{q}(w_2^\star, w_1^\star) > 0$, and for $k \neq l$ in $\{1, 2\}$,*

$$w_k^\star \in \arg\max_{w \geq m} \left(\hat{\pi}^D(w, w_l^\star) + \hat{\pi}^U(w, w_l^\star)\right). \tag{2}$$

In words, the BVS conditions are satisfied when both downstream firms can be active and upstream prices are pairwise-stable. As is well-known in the literature

(Bonanno and Vickers (1988); Shaffer (1991)), such upstream prices are strictly larger than marginal cost:

Lemma 5. *If* $(w_1^\star, w_2^\star)$ *satisfies the BVS conditions, then* $w_1^\star > m$ *and* $w_2^\star > m$.

Proof. Assume by contradiction that $w_k^\star = m$. Differentiating $\hat{\pi}^D(w, w_l^\star) + \hat{\pi}^U(w, w_l^\star)$ with respect to w at point $w = m$ and using the envelope theorem, we get:

$$\partial_1 \left(\hat{\pi}^D(m, w_l^\star) + \hat{\pi}^U(m, w_l^\star)\right) = \left[\hat{p}(m, w_l^\star) - m\right] \partial_2 q \left[\hat{p}(m, w_l^\star), \hat{p}(m, w_k^\star)\right] \partial_2 \hat{p}(w_l^\star, m) > 0,$$

contradiction! □

The intuition is that high upstream prices commit downstream firms to set high downstream prices. Such a commitment is desirable when prices are strategic complements. In a model in which upstream firms do not compete, Bonanno and Vickers (1988) show that downstream firms purchase at upstream prices which are consistent with the BVS conditions. Shaffer (1991) argues that the same equilibrium outcome emerges in a model with a large number of identical upstream firms. The following lemma confirms that, if there is an equilibrium in which both downstream firms are active, then the upstream variable parts at which downstream firms end up purchasing have to be consistent with the BVS conditions:

Lemma 6. *Assume there exists an equilibrium in which both downstream firms are active. Denote by* (w_k^A, T_k^A) *(resp.* (w_k^R, T_k^R)*) the contract that is accepted (resp. rejected) by* D_k *on the equilibrium path,* $k = 1, 2$*. Then,* (w_1^A, w_2^A) *satisfies the BVS conditions.*

Proof. Consider an equilibrium candidate in which both downstream firms are active. Then, it follows from Lemma 4 that $T_k^A = -\hat{\pi}^U(w_k^A, w_l^A)$, $k \neq l$ in $\{1, 2\}$, and that, from D_k's point of view, accepting contract (w_k^A, T_k^A) strictly dominates accepting no contract at all.

Assume by contradiction that (w_1^A, w_2^A) does not satisfy the BVS conditions. Since both firms are assumed to be active when accepting their equilibrium candidate contracts, this means that condition (2) is not satisfied for some firm, say, D_1. There exists $\hat{w} \geq m$ such that

$$\hat{\pi}^D(\hat{w}, w_2^A) + \hat{\pi}^U(\hat{w}, w_2^A) > \hat{\pi}^D(w_1^A, w_2^A) + \hat{\pi}^U(w_1^A, w_2^A). \tag{3}$$

Assume first that U_1 supplies both D_1 and D_2. Then, U_2 can profitably deviate by offering contract (∞, ∞) to D_2, and a contract with a variable part equal to $\hat{w}$ and a fixed part equal to

$$\hat{T} = \hat{\pi}^D(\hat{w}, w_2^A) - \big(\hat{\pi}^D(w_1^A, w_2^A) + \hat{\pi}^U(w_1^A, w_2^A)\big) - \varepsilon \tag{4}$$

to D_1. It is a strictly dominant strategy for D_2 to stick to U_1's contract. Besides, given that D_2 accepts U_1's contract, D_1 strictly prefers accepting U_2's contract, since:

$$\hat{\pi}^D(\hat{w}, w_2^A) - \hat{T} = \hat{\pi}^D(w_1^A, w_2^A) + \hat{\pi}^U(w_1^A, w_2^A) + \varepsilon.$$

Therefore, at the only equilibrium of the subgame starting in stage 2, D_1 accepts U_2's contract, and D_2 accepts U_1's contract. The profit of U_2 is equal to:

$$\hat{\pi}^D(\hat{w}, w_2^A) + \hat{\pi}^U(\hat{w}, w_2^A) - \big(\hat{\pi}^D(w_1^A, w_2^A) + \hat{\pi}^U(w_1^A, w_2^A)\big) - \varepsilon, \tag{5}$$

which is strictly positive when ε is small enough: the deviation is profitable.

Next, assume U_1 supplies D_1 and U_2 supplies D_2. Then, U_1 can profitably deviate by offering (∞, ∞) to D_2 and $(\hat{w}, \hat{T})$ to D_1, where $\hat{w}$ and $\hat{T}$ are defined in equations (3) and (4), respectively. It is a strictly dominant strategy for D_2 to keep accepting U_2's contract. Besides, since $\varepsilon > 0$, conditional on D_2 sticking to U_2's contract, D_1 strictly prefers accepting U_1's deviation rather than exiting or accepting (w_1^R, T_1^R). U_1's profit is equal to expression (5), which is strictly positive when ε is small enough: the deviation is profitable. $\square$

However, even when downstream firms accept tariffs of which the variable parts satisfy the BVS conditions, there exist profitable deviations:

Lemma 7. *There is no equilibrium in which downstream firms accept tariffs with variable parts satisfying the BVS conditions.*

Proof. Assume by contradiction that such an equilibrium exists. Denote by (w_k^A, T_k^A) (resp. (w_k^R, T_k^R)) the contract that is accepted (resp. rejected) by D_k on the equilibrium path, $k = 1, 2$. Then, we know from Lemma 4 that $T_k^A = -\hat{\pi}^U(w_k^A, w_l^A)$, $k \neq l$ in $\{1, 2\}$.

Assume first that U_1 supplies both downstream firms on the equilibrium candidate path. Suppose U_2 deviates and offers $(w_1^A + \varepsilon, \hat{T}_1)$ to D_1 and $(w_2^A, T_2^A - \eta)$ to D_2, where

ε, η are small positive numbers. Using Lemma 4 and the fact that $\eta > 0$, it follows that accepting U_2's contract is a strictly dominant strategy for D_2. The equilibrium of the supplier choices subgame is unique and such that D_1 accepts U_2's contract as well if and only if:

$$\hat{\pi}^D(w_1^A + \varepsilon, w_2^A) - \hat{T}_1 > \hat{\pi}^D(w_1^A, w_2^A) + \hat{\pi}^U(w_1^A, w_2^A), \tag{6}$$

where I have used the fact that $T_1^A = -\hat{\pi}^U(w_1^A, w_2^A)$. Adding $\hat{\pi}^U(w_1^A + \varepsilon, w_2^A)$ to both sides of inequality (6) and rearranging terms, I get:

$$\begin{aligned} &\hat{\pi}^U(w_1^A + \varepsilon, w_2^A) + \hat{T}_1 < \\ &\qquad \left(\hat{\pi}^D(w_1^A + \varepsilon, w_2^A) + \hat{\pi}^U(w_1^A + \varepsilon, w_2^A)\right) - \left(\hat{\pi}^D(w_1^A, w_2^A) + \hat{\pi}^U(w_1^A, w_2^A)\right). \end{aligned} \tag{7}$$

Since the right-hand side is continuously differentiable, I can use Taylor's theorem. There exists a function $h_1(.)$ such that $\lim_{\varepsilon \to 0} h_1(\varepsilon) = 0$ and

$$\begin{aligned} &\left(\hat{\pi}^D(w_1^A + \varepsilon, w_2^A) + \hat{\pi}^U(w_1^A + \varepsilon, w_2^A)\right) - \left(\hat{\pi}^D(w_1^A, w_2^A) + \hat{\pi}^U(w_1^A, w_2^A)\right) \\ &\qquad = \partial_1 \left(\hat{\pi}^D(w_1^A, w_2^A) + \hat{\pi}^U(w_1^A, w_2^A)\right) \varepsilon + h_1(\varepsilon)\varepsilon, \\ &\qquad = h_1(\varepsilon)\varepsilon, \end{aligned}$$

where the last line follows from the fact that (w_1^A, w_2^A) satisfies the BVS conditions. I set $\hat{T}_1$ so that $\hat{\pi}^U(w_1^A + \varepsilon, w_2^A) + \hat{T}_1 = h_1(\varepsilon)\varepsilon - \delta$, where $\delta > 0$ is a small number. Notice that inequality (7) holds as long as $\delta > 0$. Now, notice that the profit that U_2 makes from selling the input to D_2 is equal to

$$\begin{aligned} \hat{\pi}^U(w_2^A, w_1^A + \varepsilon) + T_2^A - \eta &= \hat{\pi}^U(w_2^A, w_1^A) + T_2^A - \eta + \varepsilon\partial_2\hat{\pi}^U(w_2^A, w_1^A) + \varepsilon h_2(\varepsilon) \\ &= \varepsilon\partial_2\hat{\pi}^U(w_2^A, w_1^A) + \varepsilon h_2(\varepsilon) - \eta, \end{aligned}$$

where, again, $\lim_{\varepsilon \to 0} h_2(\varepsilon) = 0$, and I have used the fact that $\hat{\pi}^U(w_2^A, w_1^A) + T_2^A = 0$. It follows that the total profit that U_2 earns when it deviates is equal to:

$$\Pi = \varepsilon(\partial_2\hat{\pi}^U(w_2^A, w_1^A) + h_1(\varepsilon) + h_2(\varepsilon)) - \eta - \delta. \tag{8}$$

Since $h_1(\varepsilon) \to_{\varepsilon \to 0} 0$, $h_2(\varepsilon) \to_{\varepsilon \to 0} 0$ and $\partial_2\hat{\pi}^U(w_2^A, w_1^A) > 0$ (Assumption 1), there exists

$\overline{\varepsilon} > 0$ such that $\partial_2 \hat{\pi}^U(w_2^A, w_1^A) + h_1(\overline{\varepsilon}) + h_2(\overline{\varepsilon}) > 0$. I take $\varepsilon = \overline{\varepsilon}$, and I make η and δ small enough, so that the right-hand side of equation (8) is strictly positive. With these values of ε, η and δ, the deviation is indeed strictly profitable. Therefore, there is no equilibrium in which an upstream firm supplies both downstream firms with variable parts consistent with the BVS conditions.

Next, assume that U_1 supplies D_1 and U_2 supplies D_2 on the equilibrium candidate path. Then, I claim that

$$\hat{\pi}^D(w_k^A, w_l^A) - T_k^A = \hat{\pi}^D(w_k^R, w_l^A) - T_k^R, \; k \neq l \text{ in } \{1, 2\}. \tag{9}$$

Assume by contradiction that

$$\hat{\pi}^D(w_1^A, w_2^A) - T_1^A > \hat{\pi}^D(w_1^R, w_2^A) - T_1^R.$$

Then, U_1 can profitably deviate by offering (∞, ∞) to D_2 and $(w_1^A, T_1^A + \varepsilon)$ ($\varepsilon > 0$) to D_1. It is straightforward to check that, when ε is small enough, there exists a unique equilibrium of stage 2 in which D_1 accepts U_1's contract and D_2 accepts U_2's contract. Therefore, U_1's profit increases from 0 to $\varepsilon > 0$, contradiction! Therefore, condition (9) holds.

Now, consider the following deviation: U_1 offers $(w_1^A + \varepsilon, \hat{T}_1)$ to D_1 and $(w_2^A, T_2^A - \eta)$ to D_2, where ε, $\eta > 0$. As before, it is a strictly dominant strategy for D_2 to accept U_1's contract. The equilibrium of the suppliers choice subgame is unique and such that D_1 accepts U_1's offer as well if

$$\hat{\pi}^D(w_1^A + \varepsilon, w_2^A) - \hat{T}_1 > \hat{\pi}^D(w_1^R, w_2^A) - T_1^R = \hat{\pi}^D(w_1^A, w_2^A) - T_1^A, \tag{10}$$

where the equality follows from condition (9). Now, notice that inequality (10) is the same as inequality (6). It follows from the first part of this proof that we can find ε and $\hat{T}_1$ such that both downstream firms accept U_1's contracts at the unique equilibrium of the supplier choice subgame, and that U_1 makes strictly positive profits (if we make η as small as needed). This concludes the proof. $\square$

4.3 Comparison with the existing literature

In a model with a large number of upstream firms, Shaffer (1991) claims that the two-part tariff competition game with exclusive contracts has an equilibrium, and that, in any equilibrium, downstream firms purchase at variable parts consistent with the BVS conditions, and upstream firms make zero profit on path. The problem is that Shaffer does not tell us anything about the tariffs offered by the upstream firms whose contracts are not accepted on path.

To see why this matters, assume first that all upstream firms offer the same contracts (using my notations: $(w_k^i, T_k^i) = (w_k^A, T_k^A)$ for all k, i). Then, just as in the proof of Lemma 7, an upstream firm, call it U, can deviate by offering a slightly lower fixed part to D_2, and a contract with a higher variable part and a lower fixed part to D_1. U would make a second-order loss on D_1 and a first-order gain on D_2, which would make the deviation profitable.

This deviation might no longer be effective if some upstream firms offer contracts different from the contracts that downstream firms are meant to accept on the equilibrium path. In this case, downstream firms might coordinate on another Nash equilibrium of the supplier choice subgame following U's deviation, and variable parts consistent with the BVS conditions might be sustainable in equilibrium. The problem is that this argument also applies to Lemma 6: variable parts which are not consistent with BVS might also be sustainable in equilibrium, because downstream firms might react to a deviation from an upstream firm by coordinating on another Nash equilibrium. By the same token, outcomes in which upstream firms make positive profits might also be sustainable, i.e., Lemma 4 might not extend to Shaffer (1991)'s framework. To sum up, it is not clear whether there exists an equilibrium with variable parts consistent with the BVS conditions in Shaffer's paper, and it is not clear what the set of equilibria looks like either.

Chen and Riordan (2007) solve a model in which final consumers are uniformly distributed on the Hotelling segment, and downstream firms can perfectly price discriminate between consumers. In equilibrium, every consumer ends up being supplied by the most efficient downstream firm (i.e., the firm with the lowest marginal cost net of transport costs) at a price equal to the marginal cost (net of transport costs) of the least efficient firm. From the point of view of downstream firm D_1, a variable part

w_1 above marginal cost implies that (a) some downstream consumers will be lost to D_2, and that (b) some consumers will receive inefficiently high prices. A high w_1 only leads D_2 to increase its prices for the consumers it will eventually supply. It follows that, under downstream price discrimination, the optimal variable part is always equal to marginal cost, and that the only pair of upstream prices consistent with the BVS conditions is (m, m). The proof of Lemma 7 cannot be extended to a setting with $w_1^A = w_2^A = m$, because if U_1 increases the variable part of D_1, it does not capture any of the additional profit D_2 makes, since the variable part it offers to D_2 is m. As pointed out in the introduction, it is unclear how Chen and Riordan (2007)'s results on the joint impact of exclusive contracts and vertical integration extend to settings without downstream price discrimination.

5 Conclusion

I have extended Ordover, Saloner, and Salop (1990)'s model by allowing upstream firms to offer exclusive two-part tariff contracts, shown that this model does not have an equilibrium, and compared this non-existence result to the existing literature. This non-existence result is bad news, because exclusive-dealing contracts and non-linear tariffs are prevalent in many vertically related industries, and competition authorities are often concerned about their anticompetitive effects. I very much hope that it will stimulate a new literature aiming to better understand the impact of these contracts.

A Proof of Lemma 1

Proof.

Existence For $k = 1, 2$, let

$$BR_k(p_l, w_k) \equiv \inf \left(\arg \max_{p_k \geq w_k} \pi^D(p_k, p_l, w_k) \right)$$

the best-response function of D_k to D_l's price. Define also $\overline{p}_k \equiv \sup_{p_l \geq 0} BR_k(p_l, w_k)$, and remember that $\overline{p}_k$ is finite by assumption. Consider an auxiliary game in which the action set of D_k, $k = 1, 2$, is restricted to $[w_k, \overline{p}_k]$. Theorem 1.2 in Fudenberg and Tirole

(1991) ensures that this auxiliary game has a pure strategy Nash equilibrium, and it follows immediately that this equilibrium is also an equilibrium of the unrestricted game.

Uniqueness This part proceeds in several steps.

Step 1: If $q(\hat{p}, BR_k(\hat{p}, w_k)) = 0$, then $q(\tilde{p}, BR_k(\tilde{p}, w_k)) = 0$ for all $\tilde{p} \geq \hat{p}$.

Let $\tilde{p} > \hat{p}$ and assume $q(\hat{p}, BR_k(\hat{p}, w_k)) = 0$. If $q(\tilde{p}, x) = 0$ for all x, or if $q(BR_k(\hat{p}, w_k), \hat{p}) = 0$, then the conclusion follows trivially.

Conversely, assume that $q(BR_k(\hat{p}, w_k), \hat{p}) > 0$ and that there exists x such that $q(\tilde{p}, x) > 0$. For $p_l \in [\hat{p}, \tilde{p}]$, let $\rho_0(p_l)$ the highest p_k such that $q(p_l, p_k) = 0$ and $\bar{\rho}(p_l)$ the smallest p_k such that $q(p_k, p_l) = 0$. Define the following function:

$$f^{p_l} : p_k \in (\rho_0(p_l), \bar{\rho}(p_l)) \mapsto \partial_1 \pi^D (p_k, p_l, w_k).$$

It follows from the stability condition that $f^{p_l\prime}(p_k) = \partial^2_{11} \pi^D(p_k, p_l, w_k) < 0$, i.e., $f^{p_l}(.)$ is strictly decreasing on interval $(\rho_0(p_l), \bar{\rho}(p_l))$. Therefore, $f^{p_l}(.)$ has a limit as p_k approaches $\rho_0(p_l)$ from the right, and this limit is either finite or equal to $+\infty$. From now on, I let

$$\phi_0(p_l) \equiv \lim_{p_k \to \rho_0(p_l)^+} f^{p_l}(p_k).$$

Notice that $\phi_0(p_l) > f^{p_l}(p_k)$ for all $p_k > \rho_0(p_l)$. Besides, since $\pi^D(., p_l, w_k)$ is strictly quasi-concave on the set of prices such that $q(., p_l) > 0$, it is straightforward to show that $q(p_l, BR_k(p_l, w_k)) = 0$ if and only if $\phi_0(p_l) \leq 0$. Therefore, $\phi_0(\hat{p}) \leq 0$, and all I need to do is show that $\phi_0(.)$ is non-increasing.

For all $\varepsilon > 0$, let $\rho_\varepsilon(p_l)(> \rho_0(p_l))$ the unique solution (in p_k) of equation $q(p_l, p_k) = \varepsilon$, and $\phi_\varepsilon(p_l) \equiv f^{p_l}(\rho_\varepsilon(p_l))$. Then, for all p_l,

$$\phi_0(p_l) = \lim_{\varepsilon \to 0^+} \phi_\varepsilon(p_l).$$

Differentiating ϕ_ε with respect to p_l for $\varepsilon > 0$, I get:

$$\begin{aligned}
\phi'_\varepsilon(p_l) &= \rho'_\varepsilon(p_l)\partial^2_{11}\pi^D(\rho_\varepsilon(p_l), p_l, w_k) + \partial^2_{12}\pi^D(\rho_\varepsilon(p_l), p_l, w_k), \\
&= \frac{-\partial_1 q(p_l, \rho_\varepsilon(p_l))}{\partial_2 q(p_l, \rho_\varepsilon(p_l))}\partial^2_{11}\pi^D(\rho_\varepsilon(p_l), p_l, w_k) + \partial^2_{12}\pi^D(\rho_\varepsilon(p_l), p_l, w_k), \\
&\leq \partial^2_{11}\pi^D(\rho_\varepsilon(p_l), p_l, w_k) + \partial^2_{12}\pi^D(\rho_\varepsilon(p_l), p_l, w_k), \\
&< 0.
\end{aligned}$$

where the second line follows from the implicit function theorem, the third line follows from the local concavity of π^D and the fact that the total demand is non-increasing in prices, and the last line follows from the stability condition. This implies that $\phi_\varepsilon(.)$ is strictly decreasing for all $\varepsilon > 0$. At the limit, $\phi_0(.)$ is therefore non-increasing. This concludes the proof of this step.

Step 2: There is at most one interior equilibrium.

An equilibrium is interior if both firms supply a strictly positive quantity. In an interior equilibrium, both downstream markups are strictly positive: if a firm has a strictly negative markup, then it can profitably deviate by setting its markup to zero instead; if its markup is equal to zero, then it can slightly increase its price and still get a positive demand, since products are differentiated.

Assume that both $(\hat{p}_1, \hat{p}_2)$ and $(\tilde{p}_1, \tilde{p}_2)$ are interior Nash equilibria, and that $(\hat{p}_1, \hat{p}_2) \neq (\tilde{p}_1, \tilde{p}_2)$. Assume without loss of generality that $\hat{p}_1 < \tilde{p}_1$. Since equilibrium $(\tilde{p}_1, \tilde{p}_2)$ is interior, $q(\tilde{p}_1, BR_2(\tilde{p}_1, w_2)) = q(\tilde{p}_1, \tilde{p}_2) > 0$. It follows from Step 1 that $q(p, BR_2(p, w_2)) > 0$ for all $p \in [\hat{p}_1, \tilde{p}_1]$. Besides, since equilibrium $(\hat{p}_1, \hat{p}_2)$ is interior, it follows immediately that $q(BR_2(p, w_2), p) > 0$ for all $p \in [\hat{p}_1, \tilde{p}_1]$. Therefore, firm 2's best response is interior for all $p \in [\hat{p}_1, \tilde{p}_1]$. It follows from the implicit function theorem and from the stability condition that $BR_2(p_1, w_2)$ is continuously differentiable in p_1 on interval $[\hat{p}_1, \tilde{p}_1]$, and that $\partial_1 BR_2(p_1, w_2) \in (0, 1)$. This implies that $\hat{p}_2 < \tilde{p}_2$ and, using the mean value inequality, that

$$\begin{aligned}
|\tilde{p}_2 - \hat{p}_2| = |BR_2(\tilde{p}_1, w_2) - BR_2(\hat{p}_1, w_2)| &\leq \sup_{p_1 \in [\hat{p}_1, \tilde{p}_1]} |\partial_1 BR_2(p_1, w_2)||\tilde{p}_1 - \hat{p}_1| \\
&< |\tilde{p}_1 - \hat{p}_1|,
\end{aligned}$$

where the strict inequality follows from the fact that I am taking the supremum of a continuous function on a compact set. But since $\hat{p}_2 < \tilde{p}_2$, I can use the exact same argument to show that $|\tilde{p}_1 - \hat{p}_1| < |\tilde{p}_2 - \hat{p}_2|$, contradiction! This establishes Step 2.

Step 3: There is at most one corner equilibrium.

Assume there exist two distinct corner equilibrium outcomes:[10] $(\hat{p}_1, \hat{p}_2)$ and $(\tilde{p}_1, \tilde{p}_2)$. Assume by contradiction that $q(\hat{p}_1, \hat{p}_2) = q(\tilde{p}_2, \tilde{p}_1) = 0$. Then, we also have that $q(w_1, \hat{p}_2) = q(w_2, \tilde{p}_1) = 0$, and that $q(w_1, w_2) = q(w_2, w_1) = 0$. It follows that both firms are getting 0 demand in both equilibria, which means that these equilibrium outcomes are the same, a contradiction.

Now, assume $q(\hat{p}_1, \hat{p}_2) = q(\tilde{p}_1, \tilde{p}_2) = 0$. Let $p_2^m (= BR_2(\infty, w_2))$ firm D_2's monopoly price. If $q(w_1, p_2^m) = 0$, then $\hat{p}_2 = \tilde{p}_2 = p_2^m$, $q(\tilde{p}_1, \tilde{p}_2) = q(\hat{p}_1, \hat{p}_2) = 0$, and $q(\tilde{p}_2, \tilde{p}_1) = q(\hat{p}_2, \hat{p}_1)$. Therefore, both equilibria lead to the same outcome: contradiction! Conversely, assume $q(w_1, p_2^m) > 0$. If $\hat{p}_1 > w_1$, then $\hat{p}_2$ is either equal to p_2^m or to the highest p_2 such that $q(\hat{p}_1, p_2) = 0$. In both cases, D_1 can profitably deviate by setting $p_1 = w_1 + \varepsilon$. It follows that $\hat{p}_1 = \tilde{p}_1 = w_1$. By strict quasi-concavity, we also have that $\hat{p}_2 = \tilde{p}_2$, which is a contradiction.

Step 4: Corner and interior equilibria cannot coexist.

Assume by contradiction that there exists one interior equilibrium $((\hat{p}_1, \hat{p}_2))$ and one corner equilibrium $((\tilde{p}_1, \tilde{p}_2))$. Assume that, in the corner equilibrium, $q(\tilde{p}_2, \tilde{p}_1) > 0$ and $q(\tilde{p}_1, \tilde{p}_2) = 0$. As in the previous step, if $q(w_1, p_2^m) > 0$, then $\tilde{p}_1 = w_1$ and $\tilde{p}_2$ is the highest p_2 such that $q(w_1, p_2) = 0$. Therefore, $q(w_1, BR_2(w_1, w_2)) = 0$. Since $q(\hat{p}_1, BR_2(\hat{p}_1, w_2)) > 0$, it follows from Step 1 that $\hat{p}_1 < w_1$, which is a contradiction. Conversely, if $q(w_1, p_2^m) = 0$, then we also have that $q(w_1, BR_2(w_1, w_2)) = 0$, and we obtain the same contradiction.

Combining steps 2, 3 and 4, I conclude that the equilibrium is unique.

Differentiability Assume there exists an interior equilibrium when upstream prices are $(\hat{w}_1, \hat{w}_2)$. Equilibrium downstream prices solve $\partial_1 \pi(p_1, p_2, \hat{w}_1) = 0$ and $\partial_1 \pi(p_2, p_1, \hat{w}_2) = 0$. Since $\partial_1 \pi$ is (locally) continuously differentiable, I can apply the implicit function theorem to conclude that there is a neighborhood of $(\hat{w}_1, \hat{w}_2)$ such that, for all (w_1, w_2) in this neighborhood, the equilibrium is interior, and equilibrium downstream prices

[10] Two equilibrium outcomes are distinct if at least one firm's equilibrium demand changes across equilibria.

are continuously differentiable. The fact that downstream prices are increasing in upstream prices follows readily from a monotone comparative statics argument (see Vives (1999), p.35), and from the fact that the downstream equilibrium is unique. □

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