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Externalities in Recruiting*

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Abstract

External recruiting at least weakly improves the quality of the pool of applicants, but the incentive implications are less clear. Using a contest model, this paper investigates the pure incentive effects of external recruiting. Our results show that if workers are heterogeneous, the opening of a firm's career system may lead to a homogenization of the pool of contestants and, thus, encourage the firm's high ability workers to exert more effort. If this positive effect outweighs the discouragement of low ability workers, the firm will benefit from external recruiting. If, however, the discouragement effect dominates the homogenization effect, the firm should disregard external recruiting. In addition, product market competition makes opening of the career system less attractive for a firm since it increases the incentives of its competitors' workers and hence strengthens the competitors.

Key Words: contest; externalities; recruiting; wage policy.

JEL Classification: C72; J2; J3.

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1 Introduction

External recruitment of workers is frequently applied by firms (e.g., for lack of appropriate internal candidates). At first sight, this observation seems puzzling since, contrary to outsiders, internal candidates have accumulated firm-specific human capital. In addition, by recruiting externally, the firm harms its reputation of honoring good performance of its workers via job-promotion to higher hierarchy levels. As a consequence, career incentives of internal workers may be destroyed so that the workers optimally react by reducing their efforts or even deciding to quit. Practitioners like the human resource expert John Sullivan, former Chief Talent Officer at Agilent and responsible for over 40,000 employees, question this view. He speculates that external recruitment may have positive incentive effects: "It keeps our employees on the edge because they know they must compete against outsiders for jobs" (Sullivan 1999). Moreover, expanding the pool of applicants by external job candidates at least weakly improves the pool's average quality and, therefore, leads to a better staffing than without external applicants.

Whereas the last argument – improving the pool of applicants – is quite obvious, the incentive implications of external recruiting are not straightforward. In our paper, we use a contest model to investigate these incentive effects. In a first step, the firm decides on whether allowing external workers to apply or not and then chooses optimal contest prizes. Thereafter, the relevant pool of workers – either only internal workers or internal as well as external workers – competes for a vacant position in a recruitment contest. To focus on the pure incentive view, we assume that external candidates do not have superior talents. Thus, if a firm admits external candidates, the well-known benefit of improving the pool of applicants cannot play any role.

The results of the model show under which conditions a firm profits from opening its career system to outsiders and when not. If the firm's current workforce is heterogeneous, a purely internal competition for vacant positions can be rather weak. For example, if workers have very different talents and if they know each other for a long time (e.g., as members of the same entrance cohort) internal career competition will be rather low since everybody knows the presumable winner. We show that allowing external workers to apply in such a situation can make the competition stronger, even if the outsiders' talents are observable by the internal workers. Expanding the pool of applicants leads to a discouragement of a firm's workforce but possibly also to a more homogeneous field of applicants, which increases incentives. If this advantage dominates discouragement, the firm will optimally decide in favor of external recruiting.

We consider two firms employing heterogeneous workers. Workers have either a high or a low ability. If a firm has to fill a vacancy and thinks about external recruiting, it must keep the following externalities in mind:[1] Since the number of workers competing for the vacant position increases, external recruiting discourages own high-ability and low-ability workers. If the ability difference between the two types of workers is sufficiently large and the number of high-ability workers exceeds a critical value, then the low-ability workers will be completely discouraged and remain passive. Only the high-ability workers will hence be active in the competition. These workers' incentives are boosted by the homogenization of the effective set of players. If this advantage outweighs the lost incentives of the low-ability workers, the hiring firm will admit external applicants from a pure incentive perspective. Otherwise, disregarding external candidates will be optimal for the firm.

This paper completely focuses on incentives. Including the quality of the recruiting decision (i.e., the ability of the worker that is assigned to the vacant position) would further strengthen the argument for external recruiting, even if external candidates do not have superior talents: Without external candi-

[1] See Konrad (2009), chapter 5, on other externalities in contests.

dates, both internal low-ability and internal high-ability workers may have a positive probability of being promoted. If, as in the situation described above, allowing external workers to apply discourages low-ability workers, the vacant position is filled with a high-ability worker for sure.

Our results offer some testable implications with regard to inside versus outside recruiting: Given our findings, we would expect that firms with a more homogeneous workforce are less prone to recruit higher-level positions from outside. The reason is that in these firms internal competition for promotion is already strong. A more homogeneous workforce could for instance be the result of extensive screening when recruiting junior employees. Such scrutiny in the selection of juniors can be found in industries like top management consulting and large law firms. We would thus expect to find less recruitment from outside in these industries, a prediction that is supported by empirical evidence (see, e.g., Wilkins and Gulati (1998) on promotion-to-partnership tournaments in large law firms). Clearly, this prediction can be diluted if outsiders offer additional benefits such as bringing with them an important client base.

In addition, our model predicts that an outsider who enters the firm should have a higher ability than the average inside worker. The reason is that opening a firm's career system for outsiders only attracts high ability external workers to compete with insiders. Thus, any external candidate who wins the competition will be of high ability whereas inside the firm there are both, low and high ability workers. There exists anecdotal evidence that, indeed, external recruits are more productive than internal hires on average (see, e.g., Baker et al. 1994).

In the final part of the paper, we address those externalities in recruiting that arise if firms serve the same product market. If the two firms A and B compete for the same customers but only firm A has a vacant position, this firm A is less likely to allow for external applications compared to the basic model with separate product markets. Under product market competition, opening of A's career system for external workers generates a positive externality for the other firm B. The workforce of firm B gets incentives for free, which makes B a stronger competitor to A in the product market. Consequently, external recruiting becomes less attractive for firm A.

Our theoretical result predicts that hiring from outside will be less frequent if product market competition is more intense. This theoretical finding is supported by the empirical study of Bayo-Moriones and Ortín-Ángel (2006), who analyze the recruitment and promotion decisions of 653 Spanish firms. They find that the degree of competition has a positive and statistically highly significant impact on the use of internal promotions. The authors conclude: "Further theoretical research is needed to understand why product market competition so strongly enhances the use of internal promotions" (p. 466). Our model offers a possible explanation – firms focus on internal promotions under intense product market competition to avoid positive externalities on rival firms.

The remainder of the paper is organized as follows: We start with a brief overview of the related literature. In Sections 3 and 4, the basic model is described and solved. Section 5 considers product market competition. Section 6 concludes.

2 Related Literature

Our paper is related to the previous work on contests,[2] in particular to those contest papers that also address competition between heterogeneous contestants. For such setting, Lazear and Rosen (1981), Nalebuff and Stiglitz (1983) and O'Keeffe et al. (1984) have argued that handicapping the more able contestants can increase overall incentives. However, this kind of hand-

[2] For an overview see Konrad (2009).

icap is only possible when the ability of each worker is known to the firm. In our setting, the firm cannot observe workers' individual abilities, which renders the use of handicaps impossible. We show that the firm has another possibility to create a more balanced contest when only the distribution of types in- and outside the firm is known: By allowing external candidates to apply, internal low ability workers will be discouraged and incentives for the remaining high ability workers are increased.

Cornes and Hartley (2005) analyze asymmetric contests, applying a general form of the Tullock contest-success function. They show that, depending on the degree of heterogeneity among the players, only the strongest contestants are active in equilibrium. As Baye et al. (1996) and Siegel (2009, 2010) point out, a similar finding also holds for the all-pay auction with complete information. In equilibrium, only the strongest contestants choose positive efforts with a positive probability.

This intuitive finding that a more homogeneous pool of contestants leads to stronger competition and higher efforts in equilibrium has also been confirmed empirically. For instance, the importance of a "competitive balance" in sports leagues has been widely acknowledged. This can be seen, e.g., from the prevalence of policies which aim at achieving that balance. Examples include the "rookie draft system" in sports leagues such as the NFL which gives weaker teams an advantage in hiring new talent and in the elaborate revenue sharing rules for broadcasting revenues found in many sports. See, Szymanski (2003) and Szymanski and Késenne (2004) for details on these and other examples.

The contest literature has studied many ways to homogenize the pool of contestants, such as head starts, bid-caps, handicaps or excluding (strong) contestants.[3] We contribute to this literature by giving a converse to the

[3] See, e.g., Baye et al. (1993), Che and Gale (1998), Kirkegaard (2008, 2012) and the references therein.

exclusion results of, e.g., Baye et al. (1993): We show that including additional contestants whose efforts do not count towards overall efforts increase competition to the designer's advantage. Yet unlike excluding particular contestants it does not require knowledge of contestants' identities from the designer. We consider a contest which is not perfectly discriminating, concretely, a Tullock-type contest. Our results can however be expected to be robust with respect to the choice of contest model. Consider for instance an all-pay auction as in Baye et al. (1993) where contestants have unit effort costs and where the two strongest contestants have valuations v_H and $v_L < v_H$ for winning. In this case, overall efforts are given by

$$\frac{v_L}{2}\left(1+\frac{v_L}{v_H}\right) \in \left[\frac{v_L}{2}, v_L\right].$$

Then, by an effect parallel to the one in our model, including another contestant with valuation v_H whose efforts do not benefit the designer leads to total efforts of $v_H/2$ from the previous contestants which is an improvement if $v_H - v_L$ is sufficiently large.

3 The Basic Model

We consider two adjacent hierarchy levels in two firms A and B. There is a set N of n employees working at the lower hierarchy level in either of the firms. Workers are either of high-ability type H or of low-ability type L. For $T = L, H$ and $F = A, B$ we denote by N_{FT} the set of all type T workers employed in the lower hierarchy level of firm F and denote its cardinality by n_{FT}. The numbers n_{FT} are common knowledge of all players, but only the individual worker knows his own type. In other words, we have asymmetric information in the sense that workers exactly know their respective types but the corresponding firm only knows the relative frequency of types. Moreover, we denote by N_F the set of all workers employed in the lower hierarchy level of firm F, and by N_T the set of all type T workers employed at one of the firm's lower hierarchy level. The respective cardinalities are denoted by n_F and n_T and we assume that $n_F \geq 2$ for $F = A, B$. The two firms A and B and all n workers are assumed to be risk neutral. Workers are protected by limited liability so that their wages must be non-negative. Furthermore, each worker has a zero reservation value.

Nature chooses one of the two firms randomly to have a vacant position at the higher hierarchy level that must be filled. The respective firm F can either promote one of its n_F internal candidates or fill the vacancy with an external hire. In other words, firms A and B have comparable technologies in the sense that working on the lower level of either firm qualifies a worker to fill a vacancy at the higher level of both firms.

The n workers choose non-negative efforts e_i at personal cost e_i/t_i with $t_i \in \{t_L, t_H\}$, $t_H > t_L > 0$, reflecting worker i's talent or ability ($i \in N$). Hence, firm F has n_{FL} (n_{FH}) workers of talent t_L (t_H). Workers' efforts e_i ($i \in N_F$) lead to the value $v\left(\sum_{i \in N_F} e_i\right)$ for employer F with $v(\cdot) > 0$, $v'(\cdot) > 0$, $\lim_{x \to \infty} v'(x) = 0$ and $v''(\cdot) < 0$. In words, the value function is monotonically increasing, strictly concave with vanishing increments as well as strictly positive for all feasible arguments. Neither efforts e_i nor the value $v\left(\sum_{i \in N_F} e_i\right)$ are directly observable by the employer. For example, the firm's value of workers' efforts will be realized in the future or it corresponds to a rather complex good or service whose quality cannot be directly determined.[4]

However, an employer can use a coarse signal on relative performance for filling the vacant position. With probability $p_i(e_1, \ldots, e_i, \ldots, e_m)$, this signal tells firm F that worker i has performed best, so that worker i gets the contract offer for the vacant position. Here, m denotes the number of workers that are included in the employer's chosen career system (i.e., either

[4]See MacLeod (2003), p. 219, on this point.

$m = n$ or $m = n_F$). Let M denote the set of these workers. In any case, the firm does not have information on who has performed second-best and so on. This kind of coarse signal particularly holds for those situations where the m workers compete against each other in the same market with only the winner becoming visible. For example, we can think of competition between salesmen for a certain key customer where the only public information is the identity of the salesman who is accepted by the customer. As a second example, we can imagine a situation with different industrial researchers competing in the same innovation race.[5] Competition immediately stops when one of them has made the innovation. In that situation, it is difficult to know who would have succeeded next. Given these examples, the value function $v\left(\sum_{i \in N_F} e_i\right)$ indicates that, from the firm's point of view, finishing the observable task (e.g., acquiring a key customer or making an innovation) is only one valuable aspect of workers' effort choices.

To simplify matters, we adopt the signal structure that is frequently used in the literature on innovation races (e.g., Loury 1979, Dasgupta and Stiglitz 1980, Denicolo 2000, Baye and Hoppe 2003):[6]

$$p_i\left(e_1, \ldots, e_i, \ldots, e_m\right) = \begin{cases} \frac{e_i}{\sum_{j \in M} e_j} & \text{if } \sum_{j \in M} e_j > 0 \\ \frac{1}{m} & \text{otherwise.} \end{cases}$$

In order to focus on different firms that compete with their career systems

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[5] If an industrial researcher is hired from outside, such employee poaching can be interpreted as a form of knowledge spillover, which is very successful in high-technology industries; see, e.g., Levin (1988). However, in our setting employee poaching is used as a pure incentive device.

[6] Let $G(\tau_i | e_i) = 1 - \exp(-h \cdot e_i \cdot \tau_i)$ denote the probability that i succeeds (i.e., acquires a certain key customer or solves a certain problem by making an innovation) before time τ_i and $g(\tau_i | e_i)$ the corresponding density. Let the workers' success times be stochastically independent. i's conditional probability of succeeding first is $P(i \text{ wins} | \tau_i) = \prod_{j \in M \setminus \{i\}} P(\tau_j > \tau_i) = \prod_{j \in M \setminus \{i\}} [1 - G(\tau_i | e_j)] = \exp(-h \cdot \tau_i \sum_{j \in M \setminus \{i\}} e_j)$. Thus, i's unconditional winning probability is $\int_0^\infty \exp(-h\tau_i \sum_{j \in M \setminus \{i\}} e_j) g(\tau_i | e_i) \, d\tau_i = \int_0^\infty h e_i \exp(-h\tau_i \sum_{j \in M} e_j) d\tau_i = e_i / (\sum_{j \in M} e_j)$, which is the well-known contest-success function suggested by Tullock (1980).

in the same labor market we assume that each firm can credibly commit to assign the best performer to the higher hierarchy level in case of a vacancy.[7] Moreover, we neglect other possible incentive schemes. The only possibility of a firm to generate incentives is to design a recruiting contest for the vacant position at the higher level. Here, firm F can either restrict competition to internal candidates or widen worker competition by accepting external candidates as well. To install a recruiting contest, the firm announces a wage $w \geq 0$ that is attached to the vacant job.[8] The best performing worker gets this job. All other workers get zero wages as optimal contest loser prizes since workers are protected by limited liability and have zero reservation values.[9] We concentrate on incentive issues and, at the end of Section 4, shortly comment on the consequences of job assignment on firm profits.

We can summarize the time schedule of the basic model as follows:

1	2	3	4	5
nature chooses vacancy in A or B	firm decides on external recruiting	firm chooses wage w	workers choose efforts e_i	payments are made

At the first stage of the game, nature randomly selects one of the firms A and B to have a vacancy on the higher hierarchy level. At stage 2, this firm

[7] E.g., the signal on the best performer is verifiable.

[8] Note that the wage does not depend on whether an insider or an outsider fills the vacancy. First, large corporations often use wages being attached to jobs to avoid a huge number of individual negotiations with their workers. Second, in the given setting workers do not differ from the viewpoint of the two firms and a third party so that equal opportunity laws would prohibit unequal treatment of internal and external workers; see Schotter and Weigelt (1992) on contests and equal opportunity laws.

[9] In other words, since the firm does not have more information on workers' ranking, any positive loser prize would only increase the firm's labor costs and decrease workers' incentives.

F has to make the policy decision whether to accept external candidates or not. For the chosen career system – with or without external recruiting – the firm solves

$$\max_{w \geq 0} v \left(\sum\nolimits_{i \in N_F} e_i \right) - w \tag{1}$$

at stage 3. The optimal wage attached to the vacant job also describes the contract offered to each of the internal workers at the lower hierarchy level. Any worker will accept a feasible contract with $w \geq 0$ since workers have zero reservation values but a non-negative payoff when participating in the career game and choosing zero effort. Thus, we do not have to care for the workers' participation constraints when solving the game. In stage 4, all n workers observe the firm's recruiting policy (including w) and simultaneously choose efforts to compete for the vacant position. Finally, the best performing worker that is assigned to the vacant higher-level job gets w, whereas the other workers get zero. The firm F that has filled its vacancy earns profit (1) and the other firm $\hat{F} \in \{A, B\} \backslash \{F\}$ receives $v \left(\sum_{i \in N_{\hat{F}}} e_i \right)$. After having solved the game of the basic model we will turn to the case of both firms competing in the same product market.

4 Solution to the Basic Model

We solve the game by backwards induction starting with stage 4, where the m workers simultaneously choose their efforts. Of course, if workers of firm $\hat{F}$ cannot apply for the vacant position since firm F has excluded candidates from outside they will optimally choose zero efforts in order to save effort costs. However, workers of firm F are always included in the recruiting contest. Let m_H denote the number of H-type workers and m_L the number of L-type workers that are allowed to apply for the vacant job with wage $w > 0$. We obtain the following result:

Proposition 1 *There exists a unique and symmetric equilibrium in which workers of the same type choose identical effort levels. If* $t_H(m_H - 1) \geq m_H t_L$*, then* L*-type workers choose* $e_L^* = 0$ *in equilibrium and* H*-type workers* $e_H^* = \frac{m_H - 1}{m_H^2} t_H w$*, otherwise*

$$e_L^* = w\frac{t_H t_L (m-1)(m_H t_L - (m_H - 1) t_H)}{(m_H t_L + m_L t_H)^2} \quad \textit{and} \tag{2}$$

$$e_H^* = w\frac{t_H t_L (m-1)(m_L t_H - (m_L - 1) t_L)}{(m_H t_L + m_L t_H)^2}. \tag{3}$$

Proof. See Appendix. ■

Proposition 1 shows that we have two possible outcomes at the contest stage. Either outcome is symmetric in the sense that H-type workers choose identical efforts and L-type workers choose identical efforts. If the H-type workers are sufficiently more able than the L-type workers, the latter ones will be completely discouraged and drop out of the competition by choosing zero effort.[10] The larger the number of H-type workers the more likely will be this outcome. In particular, for $m_H \to \infty$ the L-type workers will even drop out if the H-type workers have only a marginally higher ability since condition $t_H \geq \frac{m_H}{m_H - 1} t_L$ becomes $t_H \geq t_L$. The number of H-type workers also discourages the high-ability workers. They will not drop out, but their equilibrium effort level monotonically decreases in m_H. Recall that either $m_H = n_{AH} + n_{BH}$ or $m_H = n_{FH}$. Hence, if L-type workers drop out under pure internal competition they will drop out as well if firm F opens its career system for external hires, whereas the opposite result does not necessarily hold. Altogether, opening the career system to outsiders can generate strong externalities by discouraging the weak internal workers.

[10]Note that this result is not specific to the Tullock contest-success function. It is due to the fact that marginal effort costs are positive at zero. If marginal effort costs were zero at zero effort, workers would not drop out but the discouragement effect would be qualitatively the same.

If $t_H(m_H - 1) < m_H t_L$, the recruiting contest will have an equilibrium with both types of workers exerting positive efforts. From (2) and (3) we can see that equilibrium efforts increase in the wage w and that $e_H^* > e_L^*$ since $m_L t_H - (m_L - 1) t_L > m_H t_L - (m_H - 1) t_H$. Moreover, the level of a worker's equilibrium effort crucially depends on two factors – the number of contestants and the degree of heterogeneity between the workers. These two factors can be highlighted by considering them separately. In order to point out the impact of the number of contestants, let $m_H = m_L = \bar{m}$. In that case, we obtain

$$e_L^* + e_H^* = \frac{w t_H t_L (2\bar{m} - 1)}{\bar{m}^2 (t_L + t_H)},$$

which is clearly decreasing in $\bar{m}$. Thus, analogously to the case of a corner solution considered in the paragraph before, each worker is discouraged if the number of opponents increases.

To emphasize the role of heterogeneity let, for illustrating purposes, $m_H = m_L = 1$.[11] The sum of equilibrium efforts boils down to

$$e_L^* + e_H^* = w \frac{t_H t_L}{t_L + t_H}.$$

Hence, for a given amount of collective talent, $t_L + t_H$, workers' efforts are maximized if heterogeneity diminishes (i.e., $t_L = t_H$). This finding is quite intuitive and also in line with results in other contest models: The closer the race between the contestants the more effort each player will choose in equilibrium. Both effects – discouragement by a larger number of contestants and encouragement by a small degree of heterogeneity among the workers – are crucial for firm F's decision whether to allow external recruiting or not.

Anticipating the workers' behavior in the recruiting contest, at stages 2 and 3 firm F solves the design problem for filling the vacancy at its higher

[11] Since by assumption of the basic model, $n_{FL} + n_{FH} \geq 2$, we are in a situation where external workers are excluded.

hierarchy level. Let V denote the inverse of the marginal value function $v'(\cdot)$. Then we get the following results:

Proposition 2 *Let firm F strictly prefer a positive wage.*[12] *F allows external workers to apply for the vacancy iff*

$$t_H \frac{n_{FH}-1}{n_{FH}} < t_L \leq t_H \frac{n_H - 1}{n_H} \quad \textit{and} \tag{4}$$

$$\frac{(n_F-1)\, n_H^2}{n_{FH}\,(n_H-1)\, n_{FL}} - \frac{n_{FH}}{n_{FL}} < \frac{t_H}{t_L}. \tag{5}$$

In that case, F optimally chooses

$$w^* = \Phi_1 \cdot V(\Phi_1) \quad \textit{with} \quad \Phi_1 = \frac{n_H^2}{n_{FH}\,(n_H-1)\, t_H}. \tag{6}$$

In all other cases, F does not admit external applications and chooses

$$w^* = \Phi_2 \cdot V(\Phi_2) \quad \textit{with} \quad \Phi_2 = \begin{cases} \frac{n_{FH}}{(n_{FH}-1)t_H} & \textit{if } t_L \leq t_H \frac{n_{FH}-1}{n_{FH}} \\ \frac{n_{FH}t_L + n_{FL}t_H}{t_H t_L (n_F-1)} & \textit{otherwise.} \end{cases} \tag{7}$$

Proof. See Appendix. ■

Remark *There exist feasible parameter constellations that satisfy (4) and (5) at the same time. Consider, for example, $n_{\hat{F}H} = n_{FL} = \eta > 0$ and $n_{FH} = 1$ with $\hat{F}$ denoting the other firm. For this parameter constellation, conditions (4) and (5) boil down to*

$$0 < t_L \leq t_H \frac{\eta}{1+\eta} \quad \textit{and} \quad t_L < t_H \frac{\eta}{\left((\eta+1)^2 - 1\right)}.$$

There are feasible values of t_L and t_H that satisfy both inequalities for any positive integer η.

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[12] Hence, we must have that $v'(0) \cdot \min\left\{\frac{n_{FH}-1}{n_{FH}}, \frac{t_L(n_{FH}+n_{FL}-1)}{n_{FH}t_L + n_{FL}t_H}, \frac{n_{FH}(n_H-1)}{n_H^2}\right\} t_H > 1$.

From Proposition 1 we know that L-type workers will drop out and choose zero effort, if the number of H-type workers is sufficiently large. Hence, from the perspective of firm F we can differentiate between three cases – (1) the number of internal H-type workers is so large that L-type workers even drop out without external competition, (2) L-type workers only drop out if F opens the career system for external candidates but not under pure internal competition, (3) L-type workers never drop out. Proposition 2 shows that only in case (2) firm F may be interested in allowing external applications. In that case, F strictly benefits from the strong externalities induced by the outsiders. F will prefer an open career system if the increased effort levels of its H-type workers exceed the lost efforts of its L-type workers who become completely discouraged and drop out. In particular, three effects are at work that crucially influence firm F's decision to allow external recruiting: (i) Since the L-type workers drop out, there is pure homogeneous competition among H-type workers. As equilibrium efforts are highest the more homogeneous the players, F strictly profits from an active homogeneous workforce. (ii) Firm F loses the valuable efforts of his L-type workers, who exert zero efforts. (iii) Allowing external candidates changes the number of active contestants. In general, a single worker will be discouraged and, hence, supply less effort the larger the number of his opponents. Whereas F strictly benefits from (i) and suffers from (ii) the direction of this third effect is not clear. On the one hand, the number of active players decreases as L-type workers drop out, which encourages each remaining H-type worker. On the other hand, additional H-type workers from the other firm enter the competition, which increases the number of active players.

We can identify these three effects when looking at condition (5).[13] This inequality is more likely to be satisfied if t_H is rather large and t_L rather small. The larger t_H the more F will profit from enhanced competition between his

[13]Condition (4) only states that we are in case (2).

H-type workers. The smaller t_L the smaller will be F's losses from his L-type workers, who become completely passive. A similar interpretation can be obtained for n_{FL}: Condition (5) is equivalent to

$$\frac{t_L\left(n_{FH}+n_{FL}-1\right)}{\left(n_{FH}t_L+n_{FL}t_H\right)}<n_{FH}\frac{n_H-1}{n_H^2}.$$

Differentiating the left-hand side with respect to n_{FL} gives

$$\frac{\partial}{\partial n_{FL}}\left(\frac{t_L\left(n_{FH}+n_{FL}-1\right)}{\left(n_{FH}t_L+n_{FL}t_H\right)}\right)=\frac{n_{FH}t_L\left(t_L-\dfrac{n_{FH}-1}{n_{FH}}t_H\right)}{\left(t_Ln_{FH}+n_{FL}t_H\right)^2},$$

which is strictly positive according to (4). Hence, the smaller n_{FL} the smaller will be F's losses from completely discouraging all of his L-type workers and the more F will tend to open its career system for external workers. Finally, the left-hand side of (5) is non-decreasing (and for $n_H>2$ strictly increasing) in n_H. This finding is quite intuitive, following effect (iii) above. Recall that n_H also contains the number of H-type workers of the other firm, $n_{\hat{F}H}$. The larger this number, the larger will be the number of active contestants when allowing external candidates to apply. Since the equilibrium effort level of a single H-type worker decreases in the number of opponents when the field of players is completely homogeneous (see Proposition 1), a larger value of $n_{\hat{F}H}$ makes opening the career system for firm F less attractive.

The argument given at the end of the last paragraph exactly explains why firm F does not open its career system in case (1) described above. The only effect of such opening would be a discouragement of the internal H-type workers since m_H increases from $m_H=n_{FH}$ to $m_H=n_{FH}+n_{\hat{F}H}$. The remaining case (3) deals with the scenario where L-type workers never give up by choosing zero efforts. At first sight, it is not clear whether opening of the career system may be profitable for F. Of course, allowing external applications unambiguously increases the number of contestants, which discourages each internal worker. However, maybe the additional contestants lead to a better mixture of workers so that the field becomes more homogeneous. Proposition 2 shows that this possible advantage is not strong enough to justify opening of the career system.

In this paper, we do not address the firm's consequences of assigning a worker with certain talent t to the vacant position at the higher hierarchy level. However, since the vacant position is typically accompanied by higher responsibility and influence on firm profits, the firm should prefer $t = t_H$ to $t = t_L$ for the new job holder. Note that given such preference the firm additionally profits in case (2) from ensuring the assignment of an H-type worker to the higher position. Since all L-type workers drop out of the competition and, thus, have a zero probability of winning the contest, opening the career system guarantees optimal selection of workers as a by-product.

Finally, equations (6) and (7) show that the workers' abilities and the numbers of different types of workers play an ambiguous role for firm F's choice of the optimal wage, w^*. This can be exemplarily seen from (6). Note that $V(\cdot)$ is monotonically decreasing since the value function v is increasing and concave. On the one hand, a high talent t_H corresponds to high equilibrium efforts and makes investing in incentives rather attractive for F. On the other hand, marginal returns from effort supply are decreasing due to the concavity of the value function, which makes incentivizing workers less attractive to F.

5 Product Market Competition

We now turn to the case where both firms compete in the same product market. Again, firm F has to fill a vacancy and has to decide whether or not to open its career system for the workers of its competitor $\hat{F}$.

The basic structure of the model remains the same as in Section 3. However, under product market competition, the profit of firm F does not only depend on its own workers' efforts but also on the efforts of its competitor $\hat{F}$'s workers. The higher the total effort of the rival firm's workforce, the lower should be F's profit. This effect seems to be natural if firms directly compete against each other. To model this effect, firm F is assumed to maximize profit

$$\psi\left(\sum\nolimits_{i\in N_F} e_i - \sum\nolimits_{j\in N_{\hat{F}}} e_j\right) - w \tag{8}$$

where the function ψ has the following properties: ψ is a monotonically increasing, strictly positive, continuously differentiable and bounded function on $\mathbb{R}$ which is strictly concave on $\mathbb{R}^+$ and for which $\psi(x)+\psi(-x)$ is constant in x. The last assumption captures the idea that the two firms are competing for a market of fixed size.

Since the contest game between the workers remains the same, equilibrium efforts for a given wage w are still described by Proposition 1. As can be seen from (8), the introduction of product market competition renders external recruiting less attractive. The reason is that the recruiting contest gives incentives to all participating workers, which includes the workforce of the competing firm in case of external recruiting. Therefore, we can only expect new insights for the case where the firm would open its career system to external workers in the absence of competition, described by conditions (4) and (5) of Proposition 2. Let Ψ denote the inverse of function ψ'. Then we obtain the following result:

Proposition 3 *Suppose that conditions (4) and (5) hold. Furthermore, let firm F strictly prefer a positive wage. F still allows external workers to apply despite product market competition iff $n_{FH} > n_{\hat{F}H}$ and*

$$\frac{(n_F-1)\,n_H^2}{(n_{FH}-n_{\hat{F}H})\,(n_H-1)\,n_{FL}} - \frac{n_{FH}}{n_{FL}} < \frac{t_H}{t_L}. \tag{9}$$

In that case, F optimally chooses

$$w^* = \Phi_3 \cdot \Psi(\Phi_3) \quad \textit{with} \quad \Phi_3 = \frac{n_H^2}{(n_{FH} - n_{\hat{F}H})(n_H - 1)\, t_H}. \tag{10}$$

Otherwise, F does not admit external applications and chooses a wage w^* corresponding to the second case of (7) with V being replaced by Ψ.

Proof. See Appendix. ■

Proposition 3 shows that with product market competition two additional conditions – $n_{FH} > n_{\hat{F}H}$ and inequality (9) – need to hold for F to open up its career system. Firm F now has to consider the negative externalities in form of the career incentives for the workers in firm $\hat{F}$. These externalities only arise for H-type workers since the L-type workers in both firms will be completely discouraged and drop out of the job-competition. Firm F thus has to consider the number of H-type workers $n_{\hat{F}H}$ at the competing firm, which yields the two additional conditions. If $n_{FH} < n_{\hat{F}H}$ firm $\hat{F}$ will gain more from career incentives than firm F since $\hat{F}$ employs more H-type workers. In that case, firm F would unambiguously harm itself by opening its career system for external hires. Thus, $n_{FH} > n_{\hat{F}H}$ describes a necessary condition for F to admit external candidates.

In addition, opening the career system requires condition (9) to hold. Again, the number of H-type workers of the other firm $\hat{F}$ turns out to be crucial. The larger $n_{\hat{F}H}$ the more the H-type workers in both firms will be discouraged since the equilibrium effort level of the H-type workers,

$$e_H^* = \frac{(n_{FH} + n_{\hat{F}H}) - 1}{(n_{FH} + n_{\hat{F}H})^2} t_H w,$$

decreases in $n_{\hat{F}H}$. This effect should harm firm F more than firm $\hat{F}$ because of $n_{FH} > n_{\hat{F}H}$. Thus, the larger $n_{\hat{F}H}$ the less condition (9) should be satisfied. The comparison of conditions (5) and (9) shows that this conjecture is correct. The only difference between (5) and (9) is the replacement of n_{FH} by $n_{FH} - n_{\hat{F}H}$ in the denominator of the first expression at the left-hand side. Hence, condition (9) is stricter than condition (5) so that under product market competition firm F will open its recruiting system less often to external applicants than without competition. Since the left-hand side of (9) is monotonically increasing in $n_{\hat{F}H}$, (9) is less likely to be satisfied for large values of $n_{\hat{F}H}$.

6 Conclusion

We have addressed two kinds of externalities that arise if a firm chooses external recruiting. First, opening the career system can lead to both negative and positive externalities for worker competition. Negative externalities always arise since, for a given vacancy, the enlarged pool of applicants leads to worker discouragement. Positive externalities are generated if external recruiting induces a homogenization of active players which boosts the incentives of a firm's high-ability workers. The firm prefers external recruiting, if the positive externalities from homogenization dominate the negative ones from worker discouragement. Second, there are externalities in case of product market competition. Suppose there are two firms competing in the same market. If one firm opens its career system for the workers of the other firm, the latter one will profit from the incentives its workers receive without paying for them. Thus, the second firm becomes a stronger competitor, which harms the first firm. Consequently, strong product market competition makes opening of the career system less attractive for a firm.

Appendix

Proof of Proposition 1:

If $e_{L1},\ldots,e_{Lm_L}$ denote the efforts of the L-type workers and $e_{H1},\ldots,e_{Hm_H}$ those of the H-type workers, L-type worker α will maximize

$$EU_{L\alpha}\left(e_{L\alpha}\right)=\frac{e_{L\alpha}}{e_{L\alpha}+\sum_{i\in\{1,\ldots,m_L\}\setminus\{\alpha\}}e_{Li}+\sum_{j\in\{1,\ldots,m_H\}}e_{Hj}}w-\frac{e_{L\alpha}}{t_L},$$

whereas H-type worker β chooses effort $e_{H\beta}$ to maximize

$$EU_{H\beta}\left(e_{H\beta}\right)=\frac{e_{H\beta}}{e_{H\beta}+\sum_{i\in\{1,\ldots,m_L\}}e_{Li}+\sum_{j\in\{1,\ldots,m_H\}\setminus\{\beta\}}e_{Hj}}w-\frac{e_{H\beta}}{t_H}.$$

If $w>0$, there cannot be an equilibrium with each worker exerting zero effort because then one of the workers can switch to a marginal amount of positive effort and wins w for sure. Since each worker has a strictly concave objective function, worker α either optimally chooses $e^*_{L\alpha}=0$ if $EU'_{L\alpha}(0)\leq 0$, or $e^*_{L\alpha}>0$ with $EU'_{L\alpha}\left(e^*_{L\alpha}\right)=0$ if $EU'_{L\alpha}(0)>0$. In analogy, we obtain

$$e^*_{H\beta}\begin{cases}=0 & \text{if}\quad EU'_{H\beta}(0)\leq 0\\ >0 \text{ with } EU'_{H\beta}\left(e^*_{H\beta}\right)=0 & \text{if}\quad EU'_{H\beta}(0)>0.\end{cases}$$

Hence, a corner solution $e^*_{L\alpha}=0$ satisfies

$$\frac{\sum_{i\in\{1,\ldots,m_L\}\setminus\{\alpha\}}e_{Li}+\sum_{j\in\{1,\ldots,m_H\}}e_{Hj}}{\left(e^*_{L\alpha}+\sum_{i\in\{1,\ldots,m_L\}\setminus\{\alpha\}}e_{Li}+\sum_{j\in\{1,\ldots,m_H\}}e_{Hj}\right)^2}w\leq\frac{1}{t_L}\Leftrightarrow$$
$$\frac{1}{\sum_{i\in\{1,\ldots,m_L\}\setminus\{\alpha\}}e_{Li}+\sum_{j\in\{1,\ldots,m_H\}}e_{Hj}}w\leq\frac{1}{t_L},$$

and an interior solution $e^*_{L\alpha}>0$

$$\frac{1}{\sum_{i\in\{1,\ldots,m_L\}\setminus\{\alpha\}}e_{Li}+\sum_{j\in\{1,\ldots,m_H\}}e_{Hj}}w>\frac{1}{t_L}$$

with $e^*_{L\alpha}$ being described by the first-order condition

$$\frac{\sum_{i\in\{1,\dots,m_L\}\backslash\{\alpha\}} e_{Li} + \sum_{j\in\{1,\dots,m_H\}} e_{Hj}}{\left(e^*_{L\alpha} + \sum_{i\in\{1,\dots,m_L\}\backslash\{\alpha\}} e_{Li} + \sum_{j\in\{1,\dots,m_H\}} e_{Hj}\right)^2} w = \frac{1}{t_L}. \tag{11}$$

Next, we show that there is a unique equilibrium with all workers of the same type choosing identical effort levels. To show uniqueness of the Nash equilibrium we follow an approach put forward by Cornes and Hartley (2005). Let $E \equiv \sum_{i\in\{1,\dots,m_L\}} e_{Li} + \sum_{j\in\{1,\dots,m_H\}} e_{Hj}$. From (11) we know that for $e^*_{L\alpha} > 0$ we must have $\frac{E-e^*_{L\alpha}}{E^2} w = \frac{1}{t_L}$ or

$$e^*_{L\alpha} = E\left(1 - \frac{E}{wt_L}\right).$$

Let $e^*_{L\alpha}(E) \equiv \max\left\{E\left(1 - \frac{E}{wt_L}\right), 0\right\}$, which is the unique possible equilibrium value of $e_{L\alpha}$ given that the sum of all effort levels is equal to E. Similarly, define $e^*_{H\beta}(E) \equiv \max\left\{E\left(1 - \frac{E}{wt_H}\right), 0\right\}$. Then, a necessary condition for $(e_{L1}, \dots, e_{Lm_L}, e_{H1}, \dots, e_{m_H})$ being an equilibrium is that the sum of these effort levels E is equal to the sum of the equilibrium effort levels from $e^*_{L\alpha}(E)$ and $e^*_{H\beta}(E)$. Formally, we must have:

$$\begin{aligned} E &= \sum_{i\in\{1,\dots,m_L\}} e^*_{Li}(E) + \sum_{j\in\{1,\dots,m_H\}} e^*_{Hj}(E) \Leftrightarrow \\ 1 &= \sum_{i\in\{1,\dots,m_L\}} \max\left\{1 - \frac{E}{wt_L}, 0\right\} + \sum_{j\in\{1,\dots,m_H\}} \max\left\{1 - \frac{E}{wt_H}, 0\right\}. \end{aligned} \tag{12}$$

The RHS of (12) is decreasing in E, has value $m > 1$ for $E = 0$, and tends to 0 for $E \to \infty$. Hence, a unique value E^* exists satisfying (12). Since $e^*_{L\alpha}(E)$ and $e^*_{H\beta}(E)$ constitute the unique equilibrium candidate for a given value E, the unique equilibrium is given by $e^*_{L\alpha}(E^*)$ and $e^*_{H\beta}(E^*)$. Thus there exists a unique equilibrium and it has the property that all workers of the same type choose identical effort levels.

Therefore, we have symmetric solutions in the sense of $e^*_{L\alpha} = e^*_L$ ($\alpha = 1, \ldots, m_L$) and $e^*_{H\beta} = e^*_H$ ($\beta = 1, \ldots, m_H$). The condition for the corner solution $e^*_{L\alpha} = e^*_L = 0$ boils down to

$$\frac{1}{m_H e^*_H} w \leq \frac{1}{t_L}, \tag{13}$$

and the conditions for an interior solution $e^*_{L\alpha} = e^*_L > 0$ can be simplified to

$$\frac{1}{m_H e^*_H} w > \frac{1}{t_L} \quad \text{and} \tag{14}$$

$$\frac{(m_L - 1) e^*_L + m_H e^*_H}{(m_L e^*_L + m_H e^*_H)^2} w = \frac{1}{t_L}. \tag{15}$$

Analogously, we obtain

$$\frac{1}{m_L e^*_L} w \leq \frac{1}{t_H} \tag{16}$$

for $e^*_{H\beta} = e^*_H = 0$, and

$$\frac{1}{m_L e^*_L} w > \frac{1}{t_H} \quad \text{and} \tag{17}$$

$$\frac{m_L e^*_L + (m_H - 1) e^*_H}{(m_L e^*_L + m_H e^*_H)^2} w = \frac{1}{t_H} \tag{18}$$

for $e^*_{H\beta} = e^*_H > 0$.

First, we can show by contradiction that a solution $e^*_L > 0$ and $e^*_H = 0$ is not possible. For this solution (15) and (16) must hold at the same time. Inserting $e^*_H = 0$ into (15) yields $e^*_L = [t_L (m_L - 1) w] / m_L^2$. Plugging into (16) and rewriting gives $t_H m_L \leq t_L (m_L - 1)$, a contradiction.

However, a corner solution with $e^*_L = 0$ and $e^*_H > 0$ is possible. Combining (13) with (18) and $e^*_L = 0$ leads to

$$e^*_H = \frac{(m_H - 1) t_H}{m_H^2} w \quad \text{and} \quad t_H \geq \frac{m_H}{m_H - 1} t_L \quad (m_H > 1),$$

where the last inequality is clearly satisfied for $m_H \to \infty$.

Finally, an interior solution with $e_L^* > 0$ and $e_H^* > 0$ is described by the two first-order conditions (15) and (18). Straightforward computations yield (2) and (3).

Proof of Proposition 2:

If $n_L = 0$ or $n_H = 0$, competing workers are homogeneous irrespective of whether firm F allows external applicants or not. In this situation, F strictly benefits from excluding external hires since a worker's individual equilibrium effort decreases in the number of contestants.

The other possible situations can be divided into three cases. Case (1) deals with $t_L \leq t_H \frac{n_{FH}-1}{n_{FH}}$. Then L-type workers drop out with and without external recruiting (see Proposition 1). F solves

$$\max_w v\left(n_{FH}\frac{n_{FH}-1}{n_{FH}^2}t_H w\right) - w$$

when excluding external workers, and

$$\max_w v\left(n_{FH}\frac{n_H-1}{n_H^2}t_H w\right) - w$$

if it allows external workers to apply. Note that we have an immediate result without solving for the optimal wages: Since $(n_{FH}-1)/n_{FH}^2 \geq (n_H-1)/n_H^2$, the first objective function always lies above the second one so that firm F prefers to exclude external candidates. Because the firm's objective function is strictly concave, the optimal wage is described by the first-order condition

$$v'\left(\frac{n_{FH}-1}{n_{FH}}t_H w^*\right)\frac{n_{FH}-1}{n_{FH}}t_H = 1,$$

given that $v'(0)\frac{n_{FH}-1}{n_{FH}}t_H > 1$ guarantees an interior solution. The first-order condition can be rewritten to the expression given in the first line of (7).

Case (2) is characterized by $t_H \frac{n_{FH}-1}{n_{FH}} < t_L \leq t_H \frac{n_H-1}{n_H}$. Now, L-type workers drop out with external recruiting but do not drop out without external hires. Using (2) and (3), under pure internal career competition firm F maximizes

$$v\left(n_{FH} \cdot e_H^* + n_{FL} \cdot e_L^*\right) - w = v\left(\frac{t_H t_L \left(n_F - 1\right)}{\left(n_{FH} t_L + n_{FL} t_H\right)} w\right) - w. \tag{19}$$

If F additionally includes external candidates, his L-type workers will drop out and F maximizes

$$v\left(n_{FH} \frac{n_H - 1}{n_H^2} t_H w\right) - w. \tag{20}$$

Firm F will prefer external recruiting, if

$$n_{FH} \frac{n_H - 1}{n_H^2} > \frac{t_L \left(n_F - 1\right)}{\left(n_{FH} t_L + n_{FL} t_H\right)},$$

which can be rewritten to (5). If F prefers to allow external job candidates it will maximize (20), leading to (6). Otherwise, F maximizes (19), yielding the expression in the second line of (7).

Case (3) deals with $t_H \frac{n_H-1}{n_H} < t_L$. Now, L-type workers will not drop out irrespective of whether firm F allows external applicants or not. Thus, the only effect of opening the career system is an increase in the number of L-type and H-type contestants without influencing the number of effort spending internal workers. We can show that such opening does not pay for the firm since the negative incentive effect of an increased number of contestants always dominates a possibly positive incentive effect by a less heterogeneous field of contestants (see the additional pages for the referees).

Proof of Proposition 3:

Let conditions (4) and (5) be fulfilled. As before, L-type workers drop out with external recruiting but do not drop out without external hires. Using (2) and (3), under pure internal career competition firm F maximizes in analogy to (19):

$$\psi\left(n_{FH}\cdot e_H^*+n_{FL}\cdot e_L^*\right)-w=\psi\left(\frac{t_H t_L\left(n_F-1\right)}{\left(n_{FH}t_L+n_{FL}t_H\right)}w\right)-w.$$

If F additionally invites external job applicants, all L-type workers will drop out and F maximizes

$$\psi\left(n_{FH}\cdot e_H^*-n_{\hat{F}H}\cdot e_H^*\right)-w=\psi\left(\left(n_{FH}-n_{\hat{F}H}\right)\frac{n_H-1}{n_H^2}t_H w\right)-w. \quad (21)$$

Thus, for any positive wage w firm F will prefer external recruiting iff

$$\left(n_{FH}-n_{\hat{F}H}\right)\frac{n_H-1}{n_H^2}>\frac{t_L\left(n_F-1\right)}{\left(n_{FH}t_L+n_{FL}t_H\right)}.$$

This condition can only be satisfied for $n_{FH}>n_{\hat{F}H}$. In that case, it can be rewritten to (9), and F maximizes (21) leading to (10). Otherwise, we are in the analogous situation as without product market competition where F maximizes (19), yielding the expression in the second line of (7) with function V being replaced by Ψ.

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Additional pages for the referees on Proposition 2, case (3):

Let $e_L^*(m_L, m_H)$ and $e_H^*(m_L, m_H)$ denote the equilibrium efforts being described by (2) and (3). We can first show that increased heterogeneous competition via opening the career system leads to a decrease of internal workers' efforts for almost all feasible parameter constellations. For $e_L^*(n_{FL}, n_{FH})$ we obtain[14]

$$\frac{\partial e_L^*(n_{FL}, n_{FH})}{\partial n_{FH}} = \Omega_1 \cdot [n_{FH} t_L - (2n_{FH} + n_{FL} - 2) t_H]$$
$$\text{with } \Omega_1 = \Theta \cdot (n_{FL} t_H - (n_{FL} - 1) t_L) > 0 \text{ and } \Theta = \frac{w t_H t_L}{(n_{FH} t_L + n_{FL} t_H)^3}$$
$$\frac{\partial e_L^*(n_{FL}, n_{FH})}{\partial n_{FL}} = \Omega_2 \cdot [n_{FH} t_L - (2n_{FH} + n_{FL} - 2) t_H]$$
$$\text{with } \Omega_2 = \Theta \cdot (n_{FH} t_L - (n_{FH} - 1) t_H) > 0.$$

Only the term in square brackets of each derivative can be negative. For the derivatives to be positive we must have that $n_{FH} > 2n_{FH} + n_{FL} - 2 \Leftrightarrow n_{FH} + n_{FL} < 2$, which is impossible because each firm has at least two workers at the lower tier of the hierarchy. For $e_H^*(n_{FL}, n_{FH})$ the comparative statics read as

$$\frac{\partial e_H^*(n_{FL}, n_{FH})}{\partial n_{FH}} = \Omega_1 \cdot [n_{FL} t_H - (2n_{FL} + n_{FH} - 2) t_L]$$
$$\frac{\partial e_H^*(n_{FL}, n_{FH})}{\partial n_{FL}} = \Omega_2 \cdot [n_{FL} t_H - (2n_{FL} + n_{FH} - 2) t_L].$$

Similar to the derivatives before, only the term in square brackets can be negative. It is positive iff

$$n_{FL} t_H > (2n_{FL} + n_{FH} - 2) t_L.$$

[14] Of course, n_{FL} and n_{FH} are integers. However, for n_{FL} and n_{FH} being not too small $e_L^*(n_{FL}, n_{FH})$ and $e_H^*(n_{FL}, n_{FH})$ are monotonically decreasing in the number of workers of both types so that the results on marginal changes of these numbers carry over to discrete changes.

Since the talent of H-type workers is restricted to $t_H \frac{n_{FH}-1}{n_{FH}} < t_L$, to be true the inequality must at least be satisfied for $t_H = \frac{n_{FH}}{n_{FH}-1} t_L$. Inserting into the inequality yields

$$(2 - n_{FH})(n_F - 1) > 0,$$

which only holds for $n_{FH} = 1$ and $n_{FL} \geq 1$, or for $n_{FH} = 0$ and $n_{FL} \geq 2$.

Altogether, the comparative-static results point out that for $n_{FH} \geq 2$ it does not pay off for F to enlarge worker competition by allowing external applications: Internal workers become discouraged, irrespective of the mixture of the two firms' workers at the lower hierarchy level. However, we still have to check out whether increasing $e_H^*(n_{FL}, n_{FH})$ by external recruiting under $n_{FH} = 1$ or $n_{FH} = 0$ outweighs lower values of $e_L^*(n_{FL}, n_{FH})$.

We start with the case of $n_{FH} = 1$. Under pure internal recruiting, firm F maximizes

$$v\left(\frac{t_H t_L n_{FL}}{t_L + n_{FL} t_H} w\right) - w.$$

Allowing external applicants would lead to objective function

$$v\left(\frac{t_H t_L (n_H + n_L - 1)(t_L + t_H n_{FL} + (t_H - t_L)(n_L - n_H n_{FL}))}{(n_H t_L + n_L t_H)^2} w\right) - w.$$

Thus, F will open its career system for external workers if and only if

$$\frac{n_{FL}}{t_L + n_{FL} t_H} < \frac{(n_H + n_L - 1)(t_L + t_H n_{FL} + (t_H - t_L)(n_L - n_H n_{FL}))}{(n_H t_L + n_L t_H)^2}. \tag{22}$$

Note that this inequality does not hold for $n_H = 1$. Hence, we must have $n_H \geq 2$. Differentiating RHS(22) with respect to n_H yields

$$\frac{(2 - n_H) t_L + n_L (t_H - 2t_L)}{(n_H t_L + n_L t_H)^3} (t_L + t_H n_{FL} + (t_H - t_L)(n_L - n_H n_{FL}))$$
$$- (t_H - t_L) n_{FL} \frac{(n_H + n_L - 1)}{(n_H t_L + n_L t_H)^2},$$

which is negative because $n_H \geq 2$ and $t_H < 2t_L$ (since $t_H < \frac{n_H}{n_H-1}t_L, \forall n_H \geq 2$). Therefore, if (22) can be satisfied, it must at least hold for the lower limit $n_H = 2$. Inserting into (22) and substituting for $n_L = n_{FL} + n_{\hat{F}L}$ (again, $\hat{F}$ indicates the other firm) gives

$$\frac{n_{FL}}{t_L + n_{FL}t_H} < \frac{(n_{FL} + n_{\hat{F}L} + 1)(t_L + t_H n_{FL} + (t_H - t_L)(n_{\hat{F}L} - n_{FL}))}{(2t_L + (n_{FL} + n_{\hat{F}L})t_H)^2}. \tag{23}$$

Differentiating RHS(23) with respect to $n_{\hat{F}L}$ leads to

$$\frac{(t_H - 2t_L)\left[(n_{FL} + 1)t_H n_{FL} + n_{\hat{F}L}(2t_L + t_H(n_{FL} - 1))\right]}{(2t_L + (n_{FL} + n_{\hat{F}L})t_H)^3},$$

which is negative due to $t_H < 2t_L$. Thus, if (23) holds, it must at least be true for $n_{\hat{F}L} = 1$. Inserting into (23) and rearranging gives

$$\left[(t_H - t_L)n_{FL}^3 + (3n_{FL} - 2)t_L\right]t_H + (2t_H - t_L)t_L n_{FL}^2 + \left[(n_{FL} - 1)t_H^2 + 2t_L^2\right]n_{FL} < 0,$$

which cannot be true. To sum up, F will prefer to exclude external workers from competing with internal ones if $n_{FH} = 1$.

Finally, we have to consider the case of $n_{FH} = 0$. If firm F excludes applicants from the other firm, it will maximize

$$v\left(\frac{t_L(n_{FL} - 1)}{n_{FL}}w\right) - w.$$

Under the external-recruiting policy, F maximizes

$$v\left(n_{FL}\frac{t_H t_L(n_H + n_L - 1)(n_H t_L - (n_H - 1)t_H)}{(n_H t_L + n_L t_H)^2}w\right) - w.$$

F will prefer the latter policy if and only if

$$\frac{n_{FL} - 1}{n_{FL}} < n_{FL}\frac{t_H(n_H + n_L - 1)(n_H t_L - (n_H - 1)t_H)}{(n_H t_L + n_L t_H)^2}. \tag{24}$$

Since

$$\frac{\partial RHS\,(24)}{\partial t_H} = -\frac{n_H t_L\,(n_H + n_L - 1)\,(n_H\,(2t_H - t_L) + (n_L - 2)\,t_H)}{(n_H t_L + n_L t_H)^3}$$

is negative,[15] for inequality (24) to be true it must at least hold for $t_H = t_L$. Inserting $t_H = t_L$ into (24) yields

$$\frac{n_{FL} - 1}{n_{FL}} < \frac{n_{FL}\,(n_H + n_L - 1)}{(n_H + n_L)^2}.$$

Note that the RHS is decreasing in n_H: $\frac{\partial}{\partial n_H}\left(\frac{n_H + n_L - 1}{(n_H + n_L)^2}\right) = -\frac{(n_H + n_L - 2)}{(n_H + n_L)^3} < 0$ as $n_H + n_L \geq 4$. Inserting the best possible case[16] $n_H = 1$ into the last inequality gives

$$\frac{n_{FL} - 1}{n_{FL}} < \frac{n_{FL}\,(n_{FL} + n_{\hat{F}L})}{(1 + n_{FL} + n_{\hat{F}L})^2}.$$

Further, note that

$$\frac{\partial}{\partial n_{\hat{F}L}}\left(\frac{n_{FL} + n_{\hat{F}L}}{(1 + n_{FL} + n_{\hat{F}L})^2}\right) = -\frac{n_{\hat{F}L} + n_{FL} - 1}{(n_{\hat{F}L} + n_{FL} + 1)^3} < 0.$$

Therefore, plugging $n_{\hat{F}L} = 1$ into the last inequality leads to

$$\frac{n_{FL} - 1}{n_{FL}} < \frac{n_{FL}\,(n_{FL} + 1)}{(2 + n_{FL})^2} \Leftrightarrow n_{FL}^2 < 2,$$

which contradicts $n_{FL} \geq 2$. Thus, F will not prefer to open its career system for external hires if $n_{FH} = 0$.

[15] Note that we must have $n_L \geq 2$ since each firm consists of at least two workers at the lower hierarchy level and since $n_{FH} = 0$, which implies $n_{FL} \geq 2$ and, hence, $n_L \geq 2$.

[16] Recall from the beginning of the proof that we can exclude $n_H = 0$.