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Authority and Incentives in Organizations

Matthias Kräkel *

* University of Bonn

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Speaker: Prof. Dr. Klaus M. Schmidt · Department of Economics · University of Munich · D-80539 Munich,
Phone: +49(89)2180 2250 · Fax: +49(89)2180 3510

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Matthias Kräkel, *University of Bonn***

Abstract

The paper analyzes the choice of organizational structure as solution to the trade-off between controlling behavior based on authority rights and minimizing costs for implementing high efforts. The analysis includes the owner of a firm, a top manager and two division heads. If it is more expensive to incentivize the division heads, the owner will prefer full delegation of authority to them to replace their high incentive pay by incentives based on private benefits of control. In that situation, decentralization is optimal given that selfish behavior is more important than cooperation for maximizing returns, but concentrated delegation of full authority to a single division head is optimal for cooperation being crucial. If, however, incentivizing the division heads is clearly less expensive than creating incentives for the top manager, the owner will choose centralization given that cooperation is the dominating issue, but partial delegation if selfish behavior is crucial.

Key words: authority, centralization, contracts, decentralization, moral hazard.

JEL classification: D21, D23, D86, L22.

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** Matthias Kräkel, Department of Economics, University of Bonn, Adenauerallee 24–42, D-53113 Bonn, Germany, phone: +49-228-739211, fax: +49-228-739210, e-mail: m.kraekel@uni-bonn.de.

1 Introduction

In many environments, hierarchies have evolved as optimal organizational form to deal with complex tasks (e.g., Chandler 1977, Williamson 1981). Complexity of tasks arises from the division of labor within the corporation and from complementarities between the organizational units. Concerning economic activities, we can observe large corporations that are controlled by a central decision maker at the top (e.g., the CEO). This top manager determines the business strategy of the corporation and exerts effort that influences the performance of all organizational units at lower hierarchy levels.

Given such raw form of a hierarchy, the fundamental question then is how should authority rights be allocated between the top manager and the organizational units to create the best organizational structure? In this paper, I will give an answer from incentive perspective. I consider a stylized hierarchy model with four players – the owner of the corporation, who chooses optimal incentive contracts for the three other players and decides on the allocation of authority rights, two division heads, who choose efforts to increase the success probability of their organizational units or divisions, and the top manager whose effort choice influences the performance of both divisions.

The model allows for externalities between the two divisions. If one division is successful and behaves cooperatively, this success will also contribute to the returns of the other division. For example, suppose that one division produces certain goods and the other division sells these goods. On the one hand, the production department's decision which technology to use (e.g., applying a standard or an innovative production technique) influences the timing, quality and degree of diversity of produced output, which influences the success of the sales division. On the other hand, the sales division's choice of distributive channels influences both own sales and the internal demand for output from the production department. Similar externalities can arise between two divisions if one division produces an intermediate good or service that is used by the other division for producing a consumer good and selling it to customers.

This paper combines the organizational-design setting of Choe and Ishiguro (2012) with the moral-hazard limited-liability approach to analyze two kinds of incentives. First, following Choe and Ishiguro (2012), I assume that players receive private benefits of control from having decision authority over a division, which incentivizes the players as their private benefits increase in the performance of the division. Second, inspired by Che and Yoo (2001), Laffont and Martimort (2002), Hermalin (2005), and Schmitz (2005a, 2013), among others, I use a binary-effort moral-hazard model with limited liability to solve for the optimal incentive contracts for the top manager and the two division heads. This setting has the big advantage that it allows to derive the optimal incentive scheme without restricting the class of feasible contracts.

As in the paper by Choe and Ishiguro, there are six possible allocations of authority rights leading to six different organizational structures. (1) If the top manager receives the decision rights for both divisions, we will obtain a centralized organization. (2) Alternatively, all decision rights can be given to one of the division heads, leading to concentrated delegation. (3) Hierarchical delegation arises if the top manager has decision authority over one division whose head possesses the decision rights over the other division. (4) We can speak of partial delegation, if one division head has authority over his own division but the top manager decides on the other division. (5) Decentralization will exist, if each division head decides on his own division. (6) If each division head has decision rights over the other division, respectively, the organizational structure can be called cross-authority delegation.

I will show that the owner chooses the optimal organizational structure against the background of two issues, which may be conflicting: On the one hand, the allocation of authority influences the players' decisions towards more selfish or more cooperative behavior, respectively. For example, under decentralization each division head solely cares for his own division, which fosters selfish behavior and works against cooperation. Thus, the owner has to take into account whether cooperative or selfish behavior is more important to maximize overall returns of the firm. On the other hand, allocating decision authority to the top manager and/or the division heads provides them with incentives, which do not directly lead to labor costs for the owner. Consequently, the owner uses these incentives to replace incentives based on pay for performance, which would imply positive labor costs.

The analysis of the optimal compensation shows that, under any organizational structure, the owner cannot do better than paying the top manager and the division heads on the basis of overall firm performance. For the top manager the optimal contract is unique since his effort influences the performance of both divisions, but for the division heads multiple contracts exist that yield optimal incentives.

If it is more difficult to motivate the division heads than the manager (i.e., the division heads' costs from exerting high effort are larger than those of the manager), explicit pay for performance for incentivizing the division heads would be quite high. In this situation, the owner prefers delegation of authority to the division heads to replace their monetary incentives by incentives based on private benefits of control. Decentralization will be optimal if selfish behavior of the two divisions is more important than cooperation for maximizing firm returns. If, however, cooperative behavior is crucial for maximizing returns, it will be optimal to give one player full decision authority so that he pays attention to the whole firm. Since, in the given situation, saving of explicit incentive pay for the division heads is of main interest for the owner, full authority should be given to one of the division heads, leading to concentrated delegation as optimal structure.

If it is more costly to incentivize the manager, either centralization or partial delegation will be optimal, because these two structures allow to replace monetary incentives for the manager by incentives based on private benefits of control. Centralization will be optimal if cooperative behavior is important to maximize returns, since under centralization the manager has full authority and, thus, cares about the whole firm. If, however, selfish behavior is crucial for high returns, the owner will choose partial delegation as optimal organizational structure.

Cross-authority delegation and hierarchical delegation are never optimal from an incentive perspective. Under these two structures, players get authority over a division whose performance cannot be influenced by their effort choices. Thus, players receive an additional utility from private benefits of control, but this utility solely increases their already positive rents without benefiting the owner.

The following table roughly summarizes the main findings on the optimal organizational structure:

	selfish behavior important	cooperation important
more costly to incentivize division heads	decentralization	concentrated delegation
more costly to incentivize manager	partial delegation	centralization

The paper is related to two strands of the literature. First, there are parallels to the literature on organizational design and the allocation of authority rights in organizations. The seminal paper by Aghion and Tirole (1997) introduces delegation of authority to subordinate managers as an incentive device. Aghion et al. (2002) investigate in a partial-contracting setting how delegation of authority is optimally used for inducing cooperative behavior. Bester and Krähmer (2008) discuss a moral-hazard model with limited liability and investigate the interplay of authority and incentives. However, they consider only one agent so that the choice between different organizational structures is not an issue. Jost and Rohlfing-Bastian (2013) analyze a moral-hazard limited-liability framework in which a firm owner can either delegate the right to coordinate tasks to one of two agents (decentralization) or keep the decision authority (centralization). They show under which conditions the two organizational structures are optimal for incentive reasons. Thiele (2013) analyzes how decentralization and centralization can solve the trade-off between more accurate information from subjective performance evaluation and possible collusion between the agents at lower tiers of the hierarchy. Here, centralization refers to a situation in which the principal evaluates the agents, whereas decentralization leads to the delegation of the evaluation task to a supervisor.

As indicated above, my paper is most closely related to Choe and Ishiguro (2012). They address the same six organizational structures and investigate which organization is optimal from the owner's point of view. However, in the model by Choe and Ishiguro, the manager's and division heads' incentives are exclusively exogenous. On the one hand, players have incentives based on private benefits of control from received decision authority, as in my paper. On the other hand, the manager is motivated by intrinsic concerns for firm success, whereas the division heads have intrinsic concerns for their respective divisions. In my paper, I show that replacing intrinsic motivation by optimal incentive contracts yields a new effect – namely, the allocation of authority rights as a means of substituting pay for performance by incentives from having authority. This new effect leads to completely different results compared to Choe and Ishiguro (2012). In particular, cross-authority delegation and hierarchical delegation can be optimal in the setting of Choe and Ishiguro (2012) but are never optimal in my model, whereas concentrated delegation can be optimal in my model, which can never be the case in Choe and Ishiguro (2012). Moreover, in the model by Choe and Ishiguro, centralization (decentralization) will be optimal if it is less (more) costly to motivate the top manager than the two division heads. These findings are just reversed in my setting due to the substitution effect explained before. Since both alternatives – intrinsic motivation and optimal endogenous incentives based on division performance – seem realistic, my analysis complements the one by Choe and Ishiguro by pointing to an important new effect under optimal contracts.

The second strand of related literature belongs to the principal-agent theory and analyzes optimal contracts under moral hazard and limited liability. In that case, the principal typically has to leave a positive rent to the agent when creating incentives. If incentives based on private benefits of control have to be supplemented by monetary incentives, the same contractual friction can also be observed in my model. Limited liability has been introduced as contractual friction by the seminal paper of Sappington (1983) and later used by many others to address incentive problems under moral hazard (e.g., Innes 1990, Demougin and Fluet 1998, Schmitz 2005a, 2005b, Ohlendorf and Schmitz 2012, Kräkel and Schöttner 2012). Within this class of models, my paper is closest to Schmitz (2013), who also uses a binary effort approach and looks for optimal incentives to implement high effort. Contrary to my paper, Schmitz (2013) does not consider the allocation of authority within organizations and the corresponding optimal contract. He analyzes two sequential production stages which are conflicting and for which the principal can hire either one agent or two agents.

The paper is organized as follows. The next section introduces the model. In Section 3, I derive the optimal contracts for the manager and the two division heads. Section 4 describes the different organizational structures. Section 5 compares the expected profits and presents the optimal organizational structure for the different parameter constellations. Section 6 concludes.

2 The Model

The following model combines the organizational design set-up of Choe and Ishiguro (2012) with the binary-effort approach that is often used to discuss moral-hazard problems in principal-agent models under limited liability (e.g., Schmitz 2013). I consider a firm that consists of four risk neutral parties – an owner O, a manager M, and two division heads A and B. Owner O decides on the organizational structure of the firm and chooses incentive contracts for the three other parties. Manager M exerts effort $e_M \in \{0,1\}$ which leads to effort costs $k \cdot e_M$ with $k > 0$. Division head i $(i = A, B)$ also chooses effort, denoted by $e_i \in \{0,1\}$ leading to costs $c \cdot e_i$ with $c > 0$. The three effort choices influence the performance of the two divisions and, hence, overall firm performance. As the usual tie-breaking rule I assume that, if an individual is indifferent between low and high effort, he will choose high effort.

Division i $(i = A, B)$ is successful with probability $P_i \equiv P_i(e_i + e_M) \in (0,1)$ and fails with probability $1 - P_i(e_i + e_M)$. Hence, manager M's effort choice influences both divisions. For example, if M spends effort to improve overall firm reputation this will help both divisions in selling their products. To compute explicit solutions, I assume that each positive effort level $e_i = 1$ or $e_M = 1$ adds the probability mass $\rho > 0$ to the success probability of division i whereas zero effort adds zero probability to P_i:

$$P_i\left(e_i, e_M\right) = \begin{cases} 2\rho & \text{if} \quad e_i + e_M = 2 \\ \rho & \text{if} \quad e_i + e_M = 1 \\ 0 & \text{if} \quad e_i + e_M = 0 \end{cases}$$

with $2\rho < 1$. We have a moral-hazard problem since the owner can observe the success of each division, which is also contractible, but does not observe the effort choices of the three other parties.

As Choe and Ishiguro (2012), I assume that the divisions are interconnected so that the success of one division also contributes to the returns of the other division. In particular, if division A succeeds, this will yield returns $h(a)$ for division A and $q(a)$ for division B. Similarly, if division B is successful, this outcome will increase the returns of division B by $h(b)$ and the returns of division A by $q(b)$. If a division fails, this will contribute zero returns to either division. As indicated by the notation, the specific returns depend on the endogenous decisions (a, b) with $a \in \{\hat{S}, \hat{C}\}$ and $b \in \{\hat{S}, \hat{C}\}$. As will become clear from the following, "$\hat{S}$" stands for selfish behavior and "$\hat{C}$" for cooperative behavior. In addition to the specific returns introduced before, $h(\cdot)$ and $q(\cdot)$, each successful division yields basic returns $R > 0$ that directly accrue to owner O.

The decision rights on a and b are allocated by the owner O to the three other parties. For example, division head A may obtain authority on a and division head B on b so that we have decentralization as organizational structure, or all decision rights may be allocated to manager M leading to a centralized organization. Let the allocation of decision authority be denoted by $\Delta := \{\chi_{Mj}, \chi_{Aj}, \chi_{Bj}\}_{j=A,B}$. The indicator variable χ_{iA} (χ_{iB}) takes the value 1 if player i has decision authority over division A (division B) and, hence, chooses a (b). However, χ_{iA} (χ_{iB}) takes the value 0 if player i is not allowed to choose a (b).

To simplify matters, I follow Choe and Ishiguro (2012, p. 493) by assuming that $q(\hat{C}) := q > q(\hat{S}) = 0$ and $h(\hat{S}) := h > h(\hat{C}) = 0$. Hence, if a division is successful, selfish behavior by the authorized decision maker will add positive returns h to this division, but zero returns to the other division. However, cooperative behavior increases the returns of the other division by q but adds zero returns to the division for which the decision maker is responsible. Altogether, given efforts $\mathbf{e} = (e_M, e_A, e_B)$, the expected specific returns of division A sum up to

$$E[\pi_A|\mathbf{e}] = P_A \cdot h(a) + P_B \cdot q(b)$$

and those of division B to

$$E[\pi_B|\mathbf{e}] = P_B \cdot h(b) + P_A \cdot q(a).$$

Following Choe and Ishiguro (2012), I assume that a party receives private benefits of control from having decision authority over a division, which is parameterized by $\lambda \in (0, 1)$. Hence, i's ($i = A, B, M$) expected payoff from private benefits of control are given by $\lambda \cdot \sum_{j=A,B} \chi_{ij} E[\pi_j|\mathbf{e}]$. Note that, according to Choe and Ishiguro (2012), the parameter λ is used to express a party's utility from having decision authority. It is not a sharing parameter which would imply that only the remaining part of the expected specific returns goes to the owner. Contrary to Choe and Ishiguro (2012) but in line with Schmitz (2013), I assume that the owner can choose incentive contracts for the three other parties based on the contractible success of the two divisions. Let $\mathbf{w}^i = (w^i_{11}, w^i_{10}, w^i_{01}, w^i_{00})$ denote the wage schedule that owner O offers to player i ($i = A, B, M$) where w^i_{11} (w^i_{00}) represents the payment to i if both divisions succeed (fail), w^i_{10} the payment if division A succeeds and division B fails, and w^i_{01} the payment if division A fails and division B succeeds. Finally, I assume that player i ($i = A, B, M$) is protected by limited liability in terms of $w^i_{11}, w^i_{10}, w^i_{01}, w^i_{00} \geq 0$, and that his reservation value is standardized to zero.

To summarize, manager M maximizes expected utility

$$\begin{aligned} EU_M\left(e_M|e_A,e_B\right) &= P_A P_B w_{11}^M + P_A\left(1-P_B\right)w_{10}^M + \left(1-P_A\right)P_B w_{01}^M \\ &\quad + \left(1-P_A\right)\left(1-P_B\right)w_{00}^M + \lambda\cdot\sum_{j=A,B}\chi_{Mj}E[\pi_j|\mathbf{e}] - k\cdot e_M, \end{aligned}$$

and division head A

$$\begin{aligned} EU_A\left(e_A|e_M,e_B\right) &= P_A P_B w_{11}^A + P_A\left(1-P_B\right)w_{10}^A + \left(1-P_A\right)P_B w_{01}^A \\ &\quad + \left(1-P_A\right)\left(1-P_B\right)w_{00}^A + \lambda\cdot\sum_{j=A,B}\chi_{Aj}E[\pi_j|\mathbf{e}] - c\cdot e_A. \end{aligned}$$

The objective function of B, denoted by $EU_B\left(e_B|e_M,e_A\right)$, is derived analogously to $EU_A\left(e_A|e_M,e_B\right)$. Owner O maximizes expected profits

$$\begin{aligned} \pi = &\sum_{j=A,B}E[\pi_j|\mathbf{e}] + P_A P_B\left(2R-\sum\nolimits_{i=A,B,M}w_{11}^i\right) \\ &+ P_A\left(1-P_B\right)\left(R-\sum\nolimits_{i=A,B,M}w_{10}^i\right) + \left(1-P_A\right)P_B\left(R-\sum\nolimits_{i=A,B,M}w_{01}^i\right) \\ &- \left(1-P_A\right)\left(1-P_B\right)\sum\nolimits_{i=A,B,M}w_{00}^i. \end{aligned} \tag{1}$$

I follow Laffont and Martimort (2002, p. 155) and Schmitz (2005a, p. 322; 2013, p. 110), among many others, and assume that the basic return R is sufficiently large so that O always wants to implement high efforts $e_A = e_B = e_M = 1$. By this simplifying assumption I can skip the analysis of all the remaining effort combinations, which would lead to many additional computations without leading to really new insights.

The timing of events is the following: First, owner O chooses an allocation of decision rights, Δ, and offers contracts $\mathbf{w}^i$ $(i = A, B, M)$ to the three other parties. Thereafter, A, B and M decide whether to accept or reject the respective contract. If they accept, they will simultaneously choose efforts e_i $(i = A, B, M)$ and decisions (a, b) to maximize their objective functions.

Finally, nature decides on the success of the two divisions and payoffs are realized.

3 Optimal Contracts

At any stage of the game, all players know that, for given Δ, the authorized decision makers will choose (a, b) to maximize their respective objective functions. These decisions are anticipated by owner O at the beginning of the game. Since he always wants to implement high efforts, we can directly solve for the optimal contracts $\mathbf{w}^{i*}$ $(i = A, B, M)$ that implement $e_i = 1$ at lowest expected labor costs for any given allocation of authority. Note that due to the limited-liability constraints, which guarantee non-negative wages, we can ignore the participation constraints of players A, B and M: since each player has a zero reservation value and zero cost from choosing zero effort each feasible contract that satisfies the limited-liability constraints will be accepted.

In the following, we have to look for those contracts under which $e_A = e_B = e_M = 1$ is a Nash equilibrium. Manager M's expected utility for $e_A = e_B = e_M = 1$ is given by $EU_M\,(1|1,1)$. If M deviates to $e_M = 0$, his expected utility will be $EU_M\,(0|1,1)$. Hence, M will not deviate from high effort if $EU_M\,(1|1,1) \geq EU_M\,(0|1,1)$. Similarly, we must have $EU_A\,(1|1,1) \geq EU_A\,(0|1,1)$ and $EU_B\,(1|1,1) \geq EU_B\,(0|1,1)$ so that players A and B do not deviate from high effort either. Altogether, owner O minimizes expected labor costs for implementing high efforts $e_i = 1$ subject to the three Nash equilibrium conditions $EU_i\,(1|1,1) \geq EU_i\,(0|1,1)$ $(i = A, B, M)$. The corresponding wages describe the optimal contracts $\mathbf{w}^{i*} = (w_{11}^{i*}, w_{10}^{i*}, w_{01}^{i*}, w_{00}^{i*})$

($i = A, B, M$) that are chosen by O at the first stage of the game:

Proposition 1 *Let* $\Lambda_M := \lambda[\chi_{MA}\,(h\,(a) + q\,(b)) + \chi_{MB}\,(h\,(b) + q\,(a))]$*,* $\Lambda_A := \lambda[\chi_{AA}h\,(a) + \chi_{AB}q\,(a)]$*, and* $\Lambda_B := \lambda[\chi_{BA}q\,(b) + \chi_{BB}h\,(b)]$*.*

(a) If $\Lambda_M \geq k/\rho$*, then contract* $\mathbf{w}^{M*} = (0, 0, 0, 0)$ *will be optimal; otherwise, O optimally chooses* $\mathbf{w}^{M*} = (w_{11}^{M*}, 0, 0, 0)$ *with*

$$w_{11}^{M*} = \frac{k}{3\rho^2} - \frac{\Lambda_M}{3\rho}. \tag{2}$$

(b) If $\Lambda_A \geq c/\rho$*, then contract* $\mathbf{w}^{A*} = (0, 0, 0, 0)$ *will be optimal; otherwise O optimally chooses* $\mathbf{w}^{A*} = (w_{11}^{A*}, w_{10}^{A*}, 0, 0)$ *with*

$$2\rho w_{11}^{A*} + (1 - 2\rho)\,w_{10}^{A*} = \frac{c}{\rho} - \Lambda_A. \tag{3}$$

(c) If $\Lambda_B \geq c/\rho$*, then contract* $\mathbf{w}^{B*} = (0, 0, 0, 0)$ *will be optimal; otherwise O optimally chooses* $\mathbf{w}^{B*} = (w_{11}^{B*}, 0, w_{01}^{B*}, 0)$ *with*

$$2\rho w_{11}^{B*} + (1 - 2\rho)\,w_{01}^{B*} = \frac{c}{\rho} - \Lambda_B. \tag{4}$$

Proof. See the Appendix. ■

Whether owner O induces incentives for A, B and M by offering positive wages in case of success, crucially depends on the magnitude of the already existing incentives based on private benefits of control, described by $\rho\Lambda_i$ ($i = A, B, M$). Hence, if $\rho\Lambda_i$ exceeds player i's additional effort costs for choosing high instead of low effort, then i is sufficiently motivated without any additional wage premium so that O optimally saves labor costs by offering zero wages for any event. However, if incentives based on private benefits of control are not large enough – in particular, if a player has not received any authority – then owner O must counterbalance missing motivation by offering sufficiently large wage premiums. As is shown by the right-hand sides of (2), (3) and (4), for each of the three players A, B and M it holds that the larger the already existing incentives based on private benefits of control the lower will be optimal expected wages because both kinds of incentives are substitutes in the Nash equilibrium conditions.

If the owner has to offer supplementary monetary incentives, the optimal contract for manager M will be unique: since M positively contributes to the performance of both divisions in the same way, he will only obtain a positive wage if both divisions are successful. If division heads A and B have to be incentivized via wages, the respectively optimal contract will not be unique. The effort choice of division head i ($i = A, B$) only influences the success of his own division, but this success contributes to overall firm success. Thus, the performance of i's division or overall firm performance or a combination of both can be used as alternative instruments to create incentives for division head i.

The results of Proposition 1 imply that if owner O has to offer positive wages, he can restrict his choice of optimal contracts to those with $w_{11}^{i*} > 0$ and $w_{10}^{i*} = w_{01}^{i*} = w_{00}^{i*} = 0$ ($i = A, B, M$):

Corollary 1 *If the owner has to induce incentives via wages, contracts based on overall firm performance with*

$$w_{11}^{M*} = \frac{k}{3\rho^2} - \frac{\Lambda_M}{3\rho} \quad \text{and} \quad w_{11}^{j*} = \frac{c}{2\rho^2} - \frac{\Lambda_j}{2\rho} \qquad (j = A, B)$$

and $w_{10}^{i*} = w_{01}^{i*} = w_{00}^{i*} = 0$ $(i = A, B, M)$ *at least weakly dominate all other contracts.*

The result of Corollary 1 highlights an important difference to Choe and Ishiguro (2012). In their paper, Choe and Ishiguro consider two types of incentives. First, players have exogenous incentives based on private benefits of control. This assumption is identical to the one used in my paper. The second kind of incentives in Choe and Ishiguro (2012) stems from intrinsic motivation of the players, which is also exogenously given. Intrinsic motivation of division heads A and B depends on the success of their respective division, but the intrinsic motivation of manager M, who contributes to the success of both divisions, depends on overall firm success (see Choe and Ishiguro 2012, p. 492). In my paper, this second kind of incentives – intrinsic motivation – is replaced by optimal endogenous incentives based on contracts. Since I assume limited liability, players receive positive rents in this paper as well as in Choe and Ishiguro (2012). However, the crucial difference between both settings is that the allocation of decision rights can be used in Choe and Ishiguro (2012) to align the interests of at most one division head with the owner's interests (and that of manager M), whereas in my paper optimal contracts lead to aligned interests of all four parties O, M, A and B. In the following, I will use the optimal contracts to solve for the equilibrium allocation of authority and the corresponding organizational structures.

4 Allocation of Decision Authority

At the first stage of the game, O has to decide on $\Delta = \{\chi_{Mj}, \chi_{Aj}, \chi_{Bj}\}_{j=A,B}$, which allocates decision authority over a and b among the players A, B and M. In principle, there are $3^2 = 9$ possible allocations. However, since the two divisions as well as their division heads A and B are identical we can skip three allocations without restricting the scope of the analysis. The remaining allocations and their corresponding organizational structures, as suggested by Choe and Ishiguro (2012), are summarized in the following table:[1]

authority over a	authority over b	organizational structure
M	M	centralization (C)
A	A	concentrated delegation (CD)
M	A	hierarchical delegation (HD)
A	M	partial delegation (PD)
A	B	decentralization (D)
B	A	cross-authority delegation (CA)

The first (second) column contains the player that receives authority over a (b) and the third column shows the corresponding organizational structure with the respective abbreviation in parentheses. A centralized organizational structure (C) arises if the hierarchically highest decision maker, manager M, receives both decision rights. An organizational structure is defined as concentrated delegation (CD) if both decision rights are allocated to a single division head. We have a three-tier hierarchy, called hierarchical delegation (HD), if manager M has decision authority over division A, and division head A has decision authority over division B. Partial delegation (PD) is given if division head A has authority over his own division but manager M decides on division B. There is decentralization (D) if each division head decides on his own division. Finally, it is possible that each division head

[1]The combination "B, B" is skipped since it is similar to "A, A". In addition, I skip "M, B" and "B, M" because they are similar to "M, A" and "A, M".

has decision authority over the other division, respectively, which is called cross-authority delegation (CA).

For these six organizational structures, optimal expected firm profits – π^C, π^{CD}, π^{HD}, $\pi^{PD}_{k>c}$, $\pi^{PD}_{k<c}$, π^D, and π^{CA} – can be derived (see the Appendix). The results show that cooperation will only be chosen if one player has full decision authority over a and b, and cooperation is more important than selfish behavior for maximizing returns. Hence, under centralization, manager M will choose cooperative behavior instead of selfish one if $q > h$, and in case of concentrated delegation, division head A will prefer cooperative behavior to selfish one if $q > h$. In all other cases, the players prefer to behave selfishly. In the next step, we can use pairwise comparisons of expected profits to analyze which organizational structure is optimal under which conditions. The following section will present the findings.

5 Optimal Organizational Structure

An immediate observation leads to the first result:

Proposition 2 *Hierarchical delegation and cross-authority delegation are never chosen by the owner.*

Proof. Comparing π^{HD} with $\pi^{PD}_{k>c}$ and $\pi^{PD}_{k<c}$ immediately shows that partial delegation always dominates hierarchical delegation, irrespective of whether $k > c$ or $k < c$. Expected profits π^{CA} are lower than the expected profits under any other organizational structure. ■

Cross-authority delegation is clearly dominated by all other organizational structures. Hierarchical delegation is also not optimal in economizing on implementation costs. The comparison with partial delegation shows that, under both HD and PD, owner O can save implementation costs for the manager M if private benefits of control are sufficiently large, but partial delegation additionally saves costs from one of the division heads. If private benefits of control are only moderate or rather small so that O either saves implementation costs only for M or no costs at all under both organizational structures, then partial delegation anyhow yields higher expected returns at identical implementation costs compared to hierarchical delegation.

The intuition for the inferiority of hierarchical delegation and cross-authority delegation is based on the fact that implementation costs will only be saved if a player gets authority over a division that can be influenced by his own effort choice. Otherwise, the player receives additional utility from authority, but this utility solely increases his already positive rent without influencing behavior. For this reason, cross-authority delegation is strictly inferior since no player i ($i = A, B, M$) gets authority over an organizational unit whose performance can be improved by i's effort choice. Similarly, under hierarchical delegation, incentives are wasted for division head A, who has authority over division B but cannot influence division B's success probability.

The findings of Proposition 2 are in stark contrast to those in Choe and Ishiguro (2012), where both hierarchical delegation and cross-authority delegation can be optimal. Consider, for example, hierarchical delegation.[2] Choe and Ishiguro show that, if q is large, both decisions, a and b, will be cooperative and lead to higher profits the larger q. Consequently, hierarchical delegation can be optimal in situations where the specific returns from cooperative behavior (i.e., q) considerably exceed specific returns from selfish

[2] The following argumentation analogously holds for cross-authority delegation.

behavior (i.e., h). Choe and Ishiguro conclude that "hierarchical delegation can emerge as an optimal organizational form when both coordination and motivation are important" (Choe and Ishiguro 2012, p. 491).

In my setting with endogenous incentives and optimal contracts, however, decisions are always selfish under hierarchical delegation. The objective functions (5) and (8) in the Appendix show that, under $\chi_{MA} = \chi_{AB} = 1$ both decision makers M and A prefer selfish behavior to maximize their private benefits of control. Proposition 1 then points to a fundamental incentive problem that arises under hierarchical delegation. As we know from above, owner O profits from implicit incentives based on private benefits of control if they replace explicit wage premiums. Under hierarchical delegation, A's implicit incentives from decision authority are given by

$$\rho\Lambda_A = \rho\lambda[\chi_{AA}h(a) + \chi_{AB}q(a)].$$

Since $\chi_{MA} = \chi_{AB} = 1$ whereas all other indicator variables are zero and since under purely selfish behavior we obtain $h(a) = h > q(a) = 0$, implicit incentives of division head A are $\rho\Lambda_A = 0$ – despite delegation. This finding stems from the important fact that division head A is not responsible for his own division so that no implicit incentives are generated.[3]

Note that the result on the inferiority of hierarchical delegation and cross-authority delegation is also related to the incentive intensity principle of Milgrom and Roberts (1992). According to this principle, the "intensity of

[3]Technically, A receives the extra utility $2\rho\lambda[\chi_{AB}(h(b) + q(a))] = 2\rho\lambda h(b)$ from delegation (see (8) in the Appendix), but A's incentive constraint is independent of $2\rho\lambda h(b)$ because he cannot influence the success of division B. Thus, A may obtain the private benefits of control, $\lambda h(b)$, with positive probability, but this probability is $P_B(e_B, e_M) = 2\rho$ irrespective of whether A chooses $e_A = 1$ or $e_A = 0$.

incentives should increase with the marginal productivity of effort and with the agent's ability to respond to incentives" (Milgrom and Roberts 1992, p. 599). Hierarchical delegation and cross-authority delegation do not work well from an incentive perspective since division heads cannot respond with their effort choices to increased authority.

The remaining four alternatives – centralization (C), concentrated delegation (CD), partial delegation (PD), and decentralization (D) – can be optimal organizational structures for certain parameter constellations. If it is more costly to incentivize the division heads than the manager, we will obtain the following results:

Proposition 3 *Let $c > k$.*

(a) If the specific returns to selfish behavior exceed the specific returns to cooperative behavior (i.e., $h > q$), then decentralization is the optimal organizational structure.

(b) If cooperative behavior is more important than selfish behavior (i.e., $h < q$), the optimal organizational structure crucially depends on the magnitude of $q - h$:

(i) Suppose $\frac{2}{3}k \leq \lambda q \rho$. There exists a cut-off value $\tilde{q}$ so that concentrated delegation will be optimal iff $q > \tilde{q}$; otherwise decentralization is optimal.

(ii) Suppose $\frac{2}{3}k > \lambda q \rho$. There exists a cut-off value $\tilde{h}$ so that centralization will be optimal iff $h < \tilde{h}$; otherwise decentralization is optimal.

Proof. See the Appendix. ■

Result (a) of Proposition 3 can be explained as follows. Recall that the compensation of A, B and M for exerting high effort consists of private benefits of control – which are for free for owner O – and of explicit wage payments – which directly increase O's labor costs. The higher a player's private benefits of control the lower are O's labor costs for this player and vice versa. If it is more costly to incentivize a division head than the manager (i.e., $c > k$), it will be profitable for O to avoid explicit wage payments to the division heads by giving decision authority and, hence, private benefits of control to them. Decentralization is the only organization where both decision heads get implicit incentives via delegated decision rights. As a consequence, O can reduce his labor costs considerably under this organizational structure since in many situations he only has to pay the rather moderate wage w_{11}^{M*} to manager M without paying anything else to A and B. If, in addition, selfish behavior is more important than cooperative behavior ($h > q$), decentralization will be always optimal since division heads prefer selfish behavior under decentralization.

If cooperative behavior is more important than selfish behavior for maximizing returns ($h < q$), either centralization or concentrated delegation may become optimal (result (b) of Proposition 3) because under these two organizational forms the player that gets full decision authority chooses cooperation in the given situation. However, in case of centralization neither division head gets implicit incentives and under concentrated delegation only one division head is incentivized via delegated authority. As $c > k$ still favors the delegation of decision rights to division heads for saving implementation costs, now two opposing effects exist for the owner – realizing high specific returns q versus saving high implementation costs for two (instead of one) division heads. As a consequence, the optimal organizational structure now depends on an additional parameter condition.

Suppose the owner's costs of creating monetary incentives for the manager are quite large relative to specific returns from cooperation (i.e., $\frac{2}{3}k > \lambda q\rho$). Then, ensuring returns from cooperation and, at the same time, saving implementation costs for the manager will be the best solution to the trade-off described in the previous paragraph. This intuition explains result (b)(ii) of Proposition 3. If, as in result (b)(i), savings from replacing M's monetary incentives by implicit ones via delegated authority are small relative to the returns from cooperation (i.e., $\frac{2}{3}k \leq \lambda q\rho$), the owner will prefer concentrated delegation. In that case, he ensures specific returns from cooperation and saves implementation costs for one division head. However, if returns from cooperation are not sufficiently large and/or returns from selfish behavior are not sufficiently small, the owner will still choose decentralization as optimal organization in spite of $h < q$.

The next result deals with situations in which it is quite costly for the owner to incentivize the manager and in which the specific returns to selfish behavior exceed the specific returns to cooperative behavior:

Proposition 4 *Let $c < k$ and $h > q$. If $c > \min\{\frac{2}{3}k, \frac{2}{3}\lambda h\rho\}$, then decentralization will be optimal. If $\frac{2}{3}k - \frac{2}{3}\lambda h\rho < c < \min\{\frac{2}{3}k, \frac{2}{3}\lambda h\rho\}$, then partial delegation will be optimal. For the remaining parameter values centralization is optimal.*

Proof. See the Appendix. ■

The proposition shows that if the costs of inducing high effort to the manager and the division heads do not differ very much (i.e., $c > \min\{\frac{2}{3}k, \frac{2}{3}\lambda h\rho\}$), the owner's organizational choice will be the same as in Proposition 3(a) with $c > k$: he prefers decentralization to save implementation costs of the division heads and to ensure selfish behavior. If, however, the division heads' effort costs c take lower values, it will be optimal for the owner to save costs for incentivizing the manager. The owner, therefore, prefers either partial delegation or centralization. Under either organizational form the manager gets implicit incentives via delegated authority. If c is of moderate size (i.e., $\frac{2}{3}k - \frac{2}{3}\lambda h\rho < c < \min\{\frac{2}{3}k, \frac{2}{3}\lambda h\rho\}$), it is optimal to save implementation costs for the manager and one division head by using partial delegation. If c is small – and k is large –, it will be optimal to give the manager full decision authority to reduce implementation costs for his effort choice considerably. The owner chooses centralization in that situation which gives the manager authority over both divisions and, hence, doubles the respective cost reduction.

The results of Proposition 3(a) and Proposition 4 describe the optimal organizational form if selfish behavior is more important than cooperation for maximizing overall returns. Figure 1 summarizes these findings. We can see that only three of the four organizational alternatives can be optimal for $h > q$. If it is sufficiently expensive for the owner to motivate the division heads, decentralization will be optimal to give each division head authority rights and thus implicit incentives. If, however, it is more costly to incentivize the manager, the owner will prefer either partial delegation or centralization.

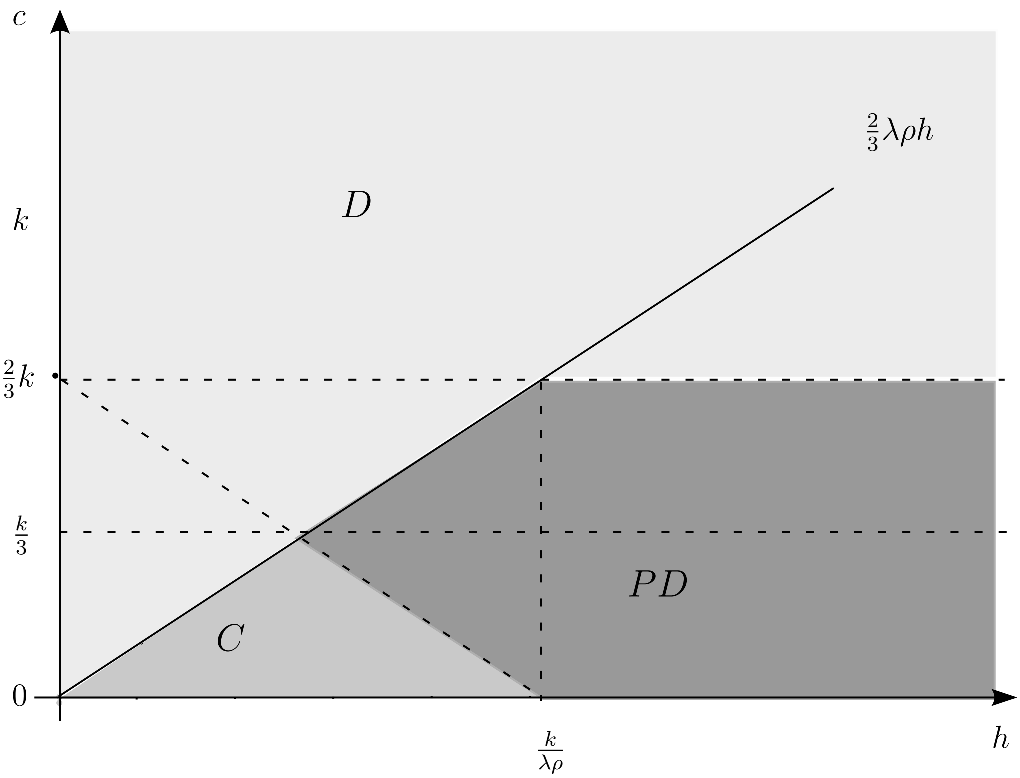

Figure 1: Optimal organizational structure for $h > q$

Now, I consider the remaining parameter constellations with $h < q$ and $c < k$. The comparison of expected profits leads to the results of Proposition 5:

Proposition 5 *Let $c < k$ and $h < q$.*

(a) Suppose $c < \frac{2}{3}k$. If $q - h$ is sufficiently large, centralization will be optimal.

(b) Suppose $c \geq \frac{2}{3}k$. If $q - h$ is sufficiently large, concentrated delegation will be optimal.

Proof. See the Appendix. ■

Proposition 5 shows that if the specific returns to cooperative behavior sufficiently exceed those to selfish behavior, only centralization or concentrated delegation can be optimal. Both organizational forms ensure cooperative behavior of the players, which is of major interest for the owner in the given situation. Saving implementation costs again is the owner's second aim. If the costs for motivating the manager are large (i.e., $c < \frac{2}{3}k$), the owner will give full decision authority to the manager by using centralization. If, however, motivating the division heads is more costly (i.e., $c \geq \frac{2}{3}k$), the owner will delegate full decision authority to one of the decision heads via concentrated delegation.

Figure 2 captures the complete results for $h < q$. Different to the case of $h > q$, now the exact look of the figure crucially depends on the concrete values of the parameters although the results of Proposition 3(b) and Proposition 5 already describe some general findings. The reason for the difference to Figure 1 stems from the fact that players always chooses selfish behavior under $h > q$ so that the value of q is irrelevant for Figure 1. For the situation with $h < q$, however, the values of both h and q are important for the choice of the optimal organization since players choose selfish behavior in case of decentralization and partial delegation, but cooperative behavior under centralization and concentrated delegation. For the construction of Figure 2, I use $\lambda = \rho = 1/2$ and $\lambda h \rho = \frac{3}{4}k$, implying $h = 3k$.[4]

Figure 2 confirms the findings of Proposition 5 that, for $q - h$ being sufficiently large, centralization (concentrated delegation) will be optimal if

[4]See the additional material for the referees on a detailed description of how Figure 2 can be obtained.

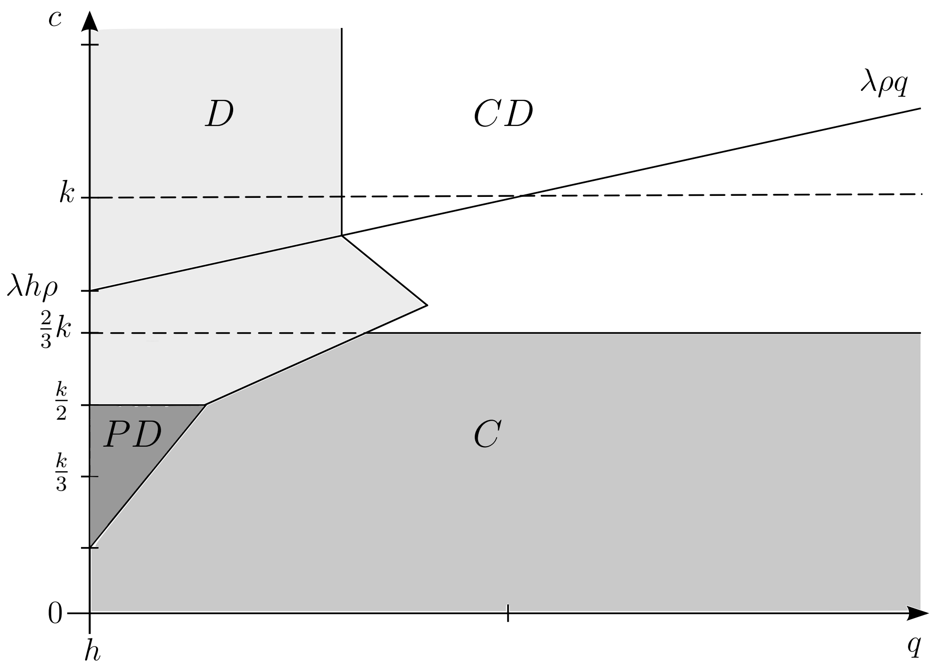

Figure 2: Optimal organizational structure for $h < q$

$c < \frac{2}{3}k$ ($c \geq \frac{2}{3}k$). If $q-h$ takes lower values, partial delegation or decentralization can be optimal although cooperative behavior maximizes overall returns but players choose selfish behavior under the two organizational forms. This observation can be explained by the fact that $h = 3k$ in Figure 2. Thus, even selfish behavior leads to some specific returns, which makes partial delegation and decentralization attractive for the owner. Suppose, on the contrary, that $h \to 0$. In that extreme case, the organizational forms "partial delegation" and "decentralization" would completely disappear in Figure 2, because specific returns and saved implementation costs would both be zero.

6 Conclusion

In this paper, I analyze the optimal organizational structure of a corporation that consists of an owner, a top manager, and two divisions with corresponding division heads. There exist possible externalities between the divisions: if a division behaves cooperatively, the division's success will increase the performance of the other division. The owner of the corporation can control the behavior of the top manager and the division heads by two instruments. On the one hand, the owner can use optimal incentive contracts based on the performance of the two divisions. This instrument determines the effort choices of the top manager and the division heads. On the other hand, the owner allocates decision authority over the two divisions to the three players. The behavioral implications of this allocation are twofold: (1) the authorized player is directed towards more selfish or more cooperative behavior, (2) the allocation influences the players' private benefits of control and, thus, their incentives for choosing high efforts. The paper shows that the interplay of these two effects determines the optimal organizational structure.

Four different structures can be optimal – decentralization, concentrated delegation, centralization and partial delegation. Decentralization will be optimal if selfish behavior is more important than cooperative behavior to maximize firm profits and if it is more costly to incentivize division heads than the top manager. If, however, cooperation is more important than selfish behavior and motivating division heads is still more costly, then concentrated delegation turns out to be optimal. If it is more costly to motivate the top manager than the division heads, the two remaining organizational structures can be optimal. Centralization is beneficial since the top manager, who has full decision authority, flexibly either chooses selfish or cooperative behavior, depending on which one is more effective. However, centralization has the drawback that private benefits of control can only reduce the owner's labor costs for the top manager. Partial delegation always leads to selfish behavior and, hence, is disadvantageous if cooperative behavior is considerably more important than selfish behavior. However, compared to centralization, partial delegation can reduce the labor costs for the top manager and one division head, which is beneficial from the owner's perspective.

Appendix

Proof of Proposition 1:

We have

$$\begin{aligned} EU_M\left(1|1,1\right) &= 4\rho^2 w_{11}^M + 2\rho\left(1-2\rho\right)\left(w_{10}^M + w_{01}^M\right) + \left(1-2\rho\right)^2 w_{00}^M \\ &\quad + 2\rho\lambda\left[\chi_{MA}\left(h\left(a\right)+q\left(b\right)\right) + \chi_{MB}\left(h\left(b\right)+q\left(a\right)\right)\right] - k, \end{aligned} \tag{5}$$

and, in case of deviation to low effort $e_M = 0$,

$$\begin{aligned} EU_M\left(0|1,1\right) &= \rho^2 w_{11}^M + \rho\left(1-\rho\right)\left(w_{10}^M + w_{01}^M\right) + \left(1-\rho\right)^2 w_{00}^M \\ &\quad + \rho\lambda\left[\chi_{MA}\left(h\left(a\right)+q\left(b\right)\right) + \chi_{MB}\left(h\left(b\right)+q\left(a\right)\right)\right]. \end{aligned}$$

Condition $EU_M\left(1|1,1\right) \geq EU_M\left(0|1,1\right)$ can therefore be written as

$$\begin{aligned} & 3\rho^2 w_{11}^M - \rho\left(3\rho-1\right)\left(w_{10}^M + w_{01}^M\right) - \rho\left(2-3\rho\right) w_{00}^M \\ & +\rho\lambda\left[\chi_{MA}\left(h\left(a\right)+q\left(b\right)\right) + \chi_{MB}\left(h\left(b\right)+q\left(a\right)\right)\right] \geq k. \end{aligned} \tag{6}$$

Owner O's expected labor costs from inducing high effort to M are

$$4\rho^2 w_{11}^M + 2\rho\left(1-2\rho\right)\left(w_{10}^M + w_{01}^M\right) + \left(1-2\rho\right)^2 w_{00}^M. \tag{7}$$

There are two possibilities: if M has got decision rights for a and/or b and his private benefits of control are sufficiently large so that $\rho\lambda[\chi_{MA}(h(a)+q(b))+\chi_{MB}(h(b)+q(a))] \geq k$, then M's motivation is already large enough so that O optimally chooses $w_{11}^M = w_{10}^M = w_{01}^M = w_{00}^M = 0$ to save labor costs; otherwise, O will minimize (7) subject to (6). Obviously, $w_{00}^M = 0$ is optimal. In addition, without loss of generality, we can set $w_{01}^M = 0$. Thus, the problem reduces to

$$\min_{w_{11}^M, w_{10}^M} 4\rho^2 w_{11}^M + 2\rho(1-2\rho) w_{10}^M \quad \text{subject to}$$
$$3\rho w_{11}^M - (3\rho - 1) w_{10}^M + \Lambda_M \geq \frac{k}{\rho}$$

with $\Lambda_M := \lambda[\chi_{MA}(h(a)+q(b)) + \chi_{MB}(h(b)+q(a))]$. If $\rho \geq 1/3$, then $w_{10}^M = 0$ and $w_{11}^M = \frac{k}{3\rho^2} - \frac{\Lambda_M}{3\rho}$ are optimal; otherwise – that is, $\rho < 1/3$ – owner O optimally chooses from the iso-cost curves with costs C_M that are described by

$$w_{10}^M = \frac{C_M}{2\rho(1-2\rho)} - \frac{2\rho}{1-2\rho} w_{11}^M$$

the one that corresponds to the lowest possible costs C_M and, at the same time, satisfies M's Nash equilibrium condition

$$w_{10}^M \geq \frac{\frac{k}{\rho} - \Lambda_M}{1-3\rho} - \frac{3\rho}{1-3\rho} w_{11}^M.$$

Since the absolute value of the slope of the iso-cost curves is smaller than the absolute value of the slope of the Nash equilibrium condition – i.e., $\frac{2\rho}{1-2\rho} < \frac{3\rho}{1-3\rho}$ – it is again optimal for O to choose $w_{10}^M = 0$ and $w_{11}^M = \frac{k}{3\rho^2} - \frac{\Lambda_M}{3\rho}$.

Given $e_A = e_B = e_M = 1$, player A's expected utility amounts to

$$\begin{aligned} EU_A(1|1,1) = {} & 4\rho^2 w_{11}^A + 2\rho(1-2\rho)\left(w_{10}^A + w_{01}^A\right) + (1-2\rho)^2 w_{0.0}^A \\ & + 2\rho\lambda[\chi_{AA}(h(a)+q(b)) + \chi_{AB}(h(b)+q(a))] - c \end{aligned} \tag{8}$$

Deviating to low effort leads to

$$\begin{aligned}EU_A(0|1,1) &= 2\rho^2 w_{11}^A + \rho(1-2\rho)\,w_{10}^A + 2\rho(1-\rho)\,w_{01}^A + (1-\rho)(1-2\rho)\,w_{00}^A \\ &\quad + \lambda\left[\chi_{AA}(\rho h(a) + 2\rho q(b)) + \chi_{AB}(2\rho h(b) + \rho q(a))\right].\end{aligned}$$

Therefore, A will not deviate to low effort if

$$2\rho^2\left(w_{11}^A - w_{01}^A\right) + \rho(1-2\rho)\left(w_{10}^A - w_{00}^A\right) + \rho\lambda\left[\chi_{AA}h(a) + \chi_{AB}q(a)\right] \geq c.$$

Since O wants to minimize expected labor costs

$$4\rho^2 w_{11}^A + 2\rho(1-2\rho)\left(w_{10}^A + w_{01}^A\right) + (1-2\rho)^2 w_{00}^A,$$

he optimally chooses $w_{00}^A = w_{01}^A = 0$. The cost minimization problem thus boils down to

$$\min_{w_{11}^A, w_{10}^A} 4\rho^2 w_{11}^A + 2\rho(1-2\rho)\,w_{10}^A \quad \text{subject to}$$

$$2\rho^2 w_{11}^A + \rho(1-2\rho)\,w_{10}^A + \rho\lambda\left[\chi_{AA}h(a) + \chi_{AB}q(a)\right] \geq c. \tag{9}$$

Again, if private benefits of control are sufficiently large –, i.e., $\rho\lambda[\chi_{AA}h(a) + \chi_{AB}q(a)] \geq c$ – then the choice of $w_{11}^A = w_{10}^A = 0$ is optimal. Otherwise, O minimizes costs by minimizing $2\rho w_{11}^A + (1-2\rho)\,w_{10}^A$ subject to (9). Thus, the best O can do is to choose w_{11}^A and w_{10}^A so that (9) becomes binding. Optimal wages w_{11}^A and w_{10}^A are therefore described by

$$2\rho w_{11}^A + (1-2\rho)\,w_{10}^A = \frac{c}{\rho} - \Lambda_A$$

with $\Lambda_A := \lambda\left[\chi_{AA}h(a) + \chi_{AB}q(a)\right]$.

Analogous results can be found for player B: optimal is always $w_{00}^B = w_{10}^B = 0$. If $\lambda\rho[\chi_{BA}q(b) + \chi_{BB}h(b)] \geq c$, then $w_{11}^B = w_{01}^B = 0$ is optimal; otherwise optimal incentives for B are described by

$$2\rho w_{11}^B + (1-2\rho)\,w_{01}^B = \frac{c}{\rho} - \Lambda_B$$

with $\Lambda_B = \lambda[\chi_{BA} q(b) + \chi_{BB} h(b)]$.

Optimal compensation and expected firm profits:

Under *centralization*, we have $\chi_{MA} = \chi_{MB} = 1$ whereas the other indicator variables are zero. According to his objective function (5), for given wages, manager M will choose $a, b = \hat{S}$ if $h > q$, and $a, b = \hat{C}$ if $h < q$. From Proposition 1, we know that his compensation will be

$$w_{11}^{M*} = \begin{cases} 0 & \text{if} \quad h > q \quad \text{and} \quad 2\lambda h \geq \frac{k}{\rho} \\ 0 & \text{if} \quad h < q \quad \text{and} \quad 2\lambda q \geq \frac{k}{\rho} \\ \frac{k}{3\rho^2} - \frac{2\lambda h}{3\rho} & \text{if} \quad h > q \quad \text{and} \quad 2\lambda h < \frac{k}{\rho} \\ \frac{k}{3\rho^2} - \frac{2\lambda q}{3\rho} & \text{if} \quad h < q \quad \text{and} \quad 2\lambda q < \frac{k}{\rho} \end{cases}$$

Since the two division heads have zero authority, they must be fully compensated via explicit incentive pay. From Corollary 1 we obtain $w_{11}^{A*} = \frac{c}{2\rho^2}$ and $w_{11}^{B*} = \frac{c}{2\rho^2}$. According to (1), owner O's expected profits with a centralized organization are

$$\pi^C = \begin{cases} 4(R\rho + h\rho - c) & \text{if} \quad h > q \quad \text{and} \quad 2\lambda h \geq \frac{k}{\rho} \\ 4(R\rho + q\rho - c) & \text{if} \quad h < q \quad \text{and} \quad 2\lambda q \geq \frac{k}{\rho} \\ 4R\rho + \left(4 + \frac{8}{3}\lambda\right) h\rho - 4c - \frac{4}{3}k & \text{if} \quad h > q \quad \text{and} \quad 2\lambda h < \frac{k}{\rho} \\ 4R\rho + \left(4 + \frac{8}{3}\lambda\right) q\rho - 4c - \frac{4}{3}k & \text{if} \quad h < q \quad \text{and} \quad 2\lambda q < \frac{k}{\rho}. \end{cases}$$

If the owner has chosen *concentrated delegation*, division head A has full decision authority so that $\chi_{AA} = \chi_{AB} = 1$. From (8) it follows that, for given wages, he will choose $a, b = \hat{S}$ if $h > q$, and $a, b = \hat{C}$ if $h < q$. His compensation is therefore

$$w_{11}^{A*} = \begin{cases} 0 & \text{if} \quad h > q \quad \text{and} \quad \lambda h \geq \frac{c}{\rho} \\ 0 & \text{if} \quad h < q \quad \text{and} \quad \lambda q \geq \frac{c}{\rho} \\ \frac{c}{2\rho^2} - \frac{\lambda h}{2\rho} & \text{if} \quad h > q \quad \text{and} \quad \lambda h < \frac{c}{\rho} \\ \frac{c}{2\rho^2} - \frac{\lambda q}{2\rho} & \text{if} \quad h < q \quad \text{and} \quad \lambda q < \frac{c}{\rho}. \end{cases}$$

Players B and M do not have any authority and must be fully compensated via $w_{11}^{B*} > 0$ and $w_{11}^{M*} > 0$. Corollary 1 yields $w_{11}^{B*} = \frac{c}{2\rho^2}$ and $w_{11}^{M*} = \frac{k}{3\rho^2}$. Owner O's expected profits are

$$\pi^{CD} = \begin{cases} 4R\rho + 4h\rho - 2c - \frac{4}{3}k & \text{if} \quad h > q \quad \text{and} \quad \lambda h \geq \frac{c}{\rho} \\ 4R\rho + 4q\rho - 2c - \frac{4}{3}k & \text{if} \quad h < q \quad \text{and} \quad \lambda q \geq \frac{c}{\rho} \\ 4R\rho + (4 + 2\lambda)\, h\rho - 4c - \frac{4}{3}k & \text{if} \quad h > q \quad \text{and} \quad \lambda h < \frac{c}{\rho} \\ 4R\rho + (4 + 2\lambda)\, q\rho - 4c - \frac{4}{3}k & \text{if} \quad h < q \quad \text{and} \quad \lambda q < \frac{c}{\rho}. \end{cases}$$

In case of *hierarchical delegation*, M has authority over division A (i.e., $\chi_{MA} = 1$) and chooses $a = \hat{S}$ to maximize (5), implying $h(a) = h$ and $q(a) = 0$. Player A has authority over division B (that is, $\chi_{AB} = 1$). According to (8), for given wages, he chooses $b = \hat{S}$, implying $h(b) = h$ and $q(b) = 0$. Hence, M's compensation is given by

$$w_{11}^{M*} = \begin{cases} 0 & \text{if} \quad \lambda h \geq \frac{k}{\rho} \\ \frac{k}{3\rho^2} - \frac{\lambda h}{3\rho} & \text{if} \quad \lambda h < \frac{k}{\rho}, \end{cases}$$

whereas the two division heads receive wages $w_{11}^{A*} = w_{11}^{B*} = \frac{c}{2\rho^2}$.[5] Owner O's expected profits can be written as

$$\pi^{HD} = \begin{cases} 4\left(R\rho + h\rho - c\right) & \text{if} \quad \lambda h \geq \frac{k}{\rho} \\ 4R\rho + \left(4 + \frac{4}{3}\lambda\right) h\rho - 4c - \frac{4}{3}k & \text{if} \quad \lambda h < \frac{k}{\rho}. \end{cases}$$

[5] Note that A has authority over division B (i.e., $\chi_{AB} = 1$), but $q(a) = 0$ so that player A does not have incentives from delegated authority.

Partial delegation is characterized by $\chi_{AA} = \chi_{MB} = 1$. (5) and (8) show that M optimally chooses $b = \hat{S}$, which implies $h(b) = h$ and $q(b) = 0$, and A chooses $a = \hat{S}$, implying $h(a) = h$ and $q(a) = 0$. The corresponding wages are therefore

$$w_{11}^{M*} = \begin{cases} 0 & \text{if} \quad \lambda h \geq \frac{k}{\rho} \\ \frac{k}{3\rho^2} - \frac{\lambda h}{3\rho} & \text{if} \quad \lambda h < \frac{k}{\rho} \end{cases} \quad \text{and} \quad w_{11}^{A*} = \begin{cases} 0 & \text{if} \quad \lambda h \geq \frac{c}{\rho} \\ \frac{c}{2\rho^2} - \frac{\lambda h}{2\rho} & \text{if} \quad \lambda h < \frac{c}{\rho}, \end{cases}$$

whereas B is offered wage $w_{11}^{B*} = \frac{c}{2\rho^2}$. Owner O's expected profits crucially depend on the relation of M's and A's effort costs. If $k > c$, then

$$\pi_{k>c}^{PD} = \begin{cases} 4R\rho + \left(4 + \frac{10}{3}\lambda\right) h\rho - 4c - \frac{4}{3}k & \text{if} \quad \lambda h < \frac{c}{\rho} \\ 4R\rho + \left(4 + \frac{4}{3}\lambda\right) h\rho - 2c - \frac{4}{3}k & \text{if} \quad \frac{c}{\rho} \leq \lambda h < \frac{k}{\rho} \\ 4R\rho + 4h\rho - 2c & \text{if} \quad \frac{k}{\rho} \leq \lambda h, \end{cases}$$

but if $k < c$, then

$$\pi_{k<c}^{PD} = \begin{cases} 4R\rho + \left(4 + \frac{10}{3}\lambda\right) h\rho - 4c - \frac{4}{3}k & \text{if} \quad \lambda h < \frac{k}{\rho} \\ 4R\rho + (4 + 2\lambda) h\rho - 4c & \text{if} \quad \frac{k}{\rho} \leq \lambda h < \frac{c}{\rho} \\ 4R\rho + 4h\rho - 2c & \text{if} \quad \frac{c}{\rho} \leq \lambda h. \end{cases}$$

Not surprisingly, in case of *decentralization* ($\chi_{AA} = \chi_{BB} = 1$), both division heads behave selfishly: $a = b = \hat{S}$, which implies $h(a) = h(b) = h$ and $q(a) = q(b) = 0$. The division heads' wages are thus

$$w_{11}^{A*} = w_{11}^{B*} = \begin{cases} 0 & \text{if} \quad \lambda h \geq \frac{c}{\rho} \\ \frac{c}{2\rho^2} - \frac{\lambda h}{2\rho} & \text{if} \quad \lambda h < \frac{c}{\rho}, \end{cases}$$

and manager M obtains $w_{11}^{M*} = \frac{k}{3\rho^2}$, leading to expected profits

$$\pi^{D} = \begin{cases} 4R\rho + 4h\rho - \frac{4}{3}k & \text{if} \quad \lambda h \geq \frac{c}{\rho} \\ 4R\rho + (4 + 4\lambda) h\rho - 4c - \frac{4}{3}k & \text{if} \quad \lambda h < \frac{c}{\rho}, \end{cases}$$

for owner O.

Under *cross-authority delegation* ($\chi_{BA} = \chi_{AB} = 1$), both division heads prefer selfish behavior $a = b = \hat{S}$, implying $h(a) = h(b) = h$ and $q(a) = q(b) = 0$. From Proposition 1 and Corollary 1, we obtain $w_{11}^{M*} = \frac{k}{3\rho^2}$ and $w_{11}^{A*} = w_{11}^{B*} = \frac{c}{2\rho^2}$, leading to profits $\pi^{CA} = 4R\rho + 4h\rho - 4c - \frac{4}{3}k$.

Proof of Proposition 3:

To prove the proposition, I start with the following useful observation:

Lemma 1 *If $h > q$, then $\pi^D > \pi^{CD}$.*

Proof. If $\lambda h \geq \frac{c}{\rho}$, then $\pi^D = \pi^{CD} + 2c$. If $\lambda h < \frac{c}{\rho}$, then $\pi^D = \pi^{CD} + 2\lambda h\rho$.

■

Next, the following observation can be proved:

Lemma 2 *If $c > k$, then $\pi^D > \pi^{PD}_{k<c}$.*

Proof. If $\lambda h < \frac{k}{\rho}$, then $\pi^D > \pi^{PD}_{k<c} \Leftrightarrow 4\lambda h\rho > \frac{10}{3}\lambda h\rho$, which is true. If $\frac{k}{\rho} \leq \lambda h < \frac{c}{\rho}$, then $\pi^D > \pi^{PD}_{k<c} \Leftrightarrow \lambda h > \frac{2}{3}\frac{k}{\rho}$, which is true. If $\frac{c}{\rho} \leq \lambda h$, then $\pi^D > \pi^{PD}_{k<c} \Leftrightarrow c > \frac{2}{3}k$, which is true. ■

For $h > q$ and $c > k$, straightforward calculations show that $\pi^D > \pi^C$, which completes the proof of result (a).

Now, consider result (b) with $h < q$ and $c > k$. PD cannot be optimal due to Lemma 2. The comparison of CD and C can be summarized as follows:

Lemma 3 *Let $h < q$ and $c > k$. There will be $\pi^{CD} > \pi^C$, iff $\lambda q\rho \geq \frac{2}{3}k$.*

Proof. If $\lambda q < \frac{k}{2\rho}$, then $\pi^C > \pi^{CD} \Leftrightarrow \frac{8}{3}\lambda q\rho > 2\lambda q\rho$ is true. If $\frac{k}{2\rho} \leq \lambda q < \frac{c}{\rho}$, then $\pi^C > \pi^{CD} \Leftrightarrow \frac{2}{3}k > \lambda q\rho$. If $\lambda q \geq \frac{c}{\rho}$, then $\pi^C > \pi^{CD} \Leftrightarrow \frac{2}{3}k > c$ is false, which completes the proof. ■

According to Lemma 3, only CD or D can be optimal under $\lambda q\rho \geq \frac{2}{3}k$. Comparing profits leads to three possible cases: If $\lambda h < c/\rho$ and $\lambda q < c/\rho$, then $\pi^{CD} > \pi^{D} \Leftrightarrow q > [(2+2\lambda)\,h]/\,(2+\lambda)$. If $\lambda h < c/\rho$ and $\lambda q \geq c/\rho$, then $\pi^{CD} > \pi^{D} \Leftrightarrow q > [(2+2\lambda)\,h\rho - c]/\,(2\rho)$. If $\lambda h \geq c/\rho$ and $\lambda q \geq c/\rho$, then $\pi^{CD} > \pi^{D} \Leftrightarrow q > [2h\rho + c]/\,(2\rho)$. Altogether, if q is sufficiently large, CD will be optimal; otherwise D is optimal.

In case of $\lambda q\rho < \frac{2}{3}k$, only C or D can be optimal (see Lemma 3). The comparison of profits yields two different cases: If $\lambda h\rho < \lambda q\rho < k/2$, then $\pi^{C} > \pi^{D} \Leftrightarrow (1+\frac{2}{3}\lambda)q > (1+\lambda)h$. If $\lambda h\rho < \lambda q\rho \in [k/2, \frac{2}{3}k)$, then $\pi^{C} > \pi^{D} \Leftrightarrow q\rho > (1+\lambda)h\rho - \frac{1}{3}k$. Thus, if h is sufficiently small, C will be optimal; otherwise D is optimal.

Proof of Proposition 4:

Note that CD cannot be optimal due to Lemma 1. Comparing the profits for D and C yields the following result:

Lemma 4 *Let $h > q$ and $c < k$. If $c > \frac{1}{3}k$, then $\pi^{D} > \pi^{C}$. If $c \leq \frac{1}{3}k$, then $\pi^{D} > \pi^{C}$ iff $c > \frac{2}{3}\lambda h\rho$.*

Proof. Suppose $c \geq k/2$. Then the comparison of π^{D} and π^{C} shows that $\pi^{D} > \pi^{C}$ holds for all values of $\lambda h\rho$. Now, consider $c < k/2$. The comparison of profits leads to three different cases. (1) If $\lambda h\rho < c$, then $\pi^{D} > \pi^{C}$ is always satisfied. (2) If $c \leq \lambda h\rho < k/2$, then $\pi^{D} > \pi^{C} \Leftrightarrow \lambda h\rho < \frac{3}{2}c$. The last inequality is satisfied under $c \leq \lambda h\rho < k/2$ if $\frac{3}{2}c > \frac{k}{2} \Leftrightarrow c > \frac{1}{3}k$. (3) If $\lambda h\rho \geq k/2$, then $\pi^{D} > \pi^{C} \Leftrightarrow c > \frac{1}{3}k$. Thus, as long as $c > \frac{1}{3}k$, decentralization leads to higher profits than centralization, but the same will only be true for $c \leq \frac{1}{3}k$ if additionally $\lambda h\rho < \frac{3}{2}c \Leftrightarrow c > \frac{2}{3}\lambda h\rho$ holds. ■

Next, we can compare the profits for D and PD:

Lemma 5 *Let $h > q$ and $c < k$. If $c > \frac{2}{3}k$, then $\pi^D > \pi^{PD}_{k>c}$. If $c \leq \frac{2}{3}k$, then $\pi^D > \pi^{PD}_{k>c}$ iff $c > \frac{2}{3}\lambda h\rho$.*

Proof. When comparing π^D and $\pi^{PD}_{k>c}$, we have to differentiate between three cases: (1) If $\lambda h\rho < c$, then $\pi^D > \pi^{PD}_{k>c}$ always holds. (2) If $c \leq \lambda h\rho < k$, then $\pi^D > \pi^{PD}_{k>c} \Leftrightarrow \lambda h\rho < \frac{3}{2}c$. The last inequality is satisfied under $c \leq \lambda h\rho < k$ if $\frac{3}{2}c > k \Leftrightarrow c > \frac{2}{3}k$. (3) If $\lambda h\rho \geq k$, then $\pi^D > \pi^{PD}_{k>c} \Leftrightarrow c > \frac{2}{3}k$. To sum up, if $c > \frac{2}{3}k$ then decentralization dominates partial delegation, but the same is only true for $c \leq \frac{2}{3}k$ if $\lambda h\rho$ satisfies $\lambda h\rho < \frac{3}{2}c \Leftrightarrow c > \frac{2}{3}\lambda h\rho$. ■

Finally, we have to compare the profits for C and PD:

Lemma 6 *Let $h > q$ and $c < k$. If $c > \frac{1}{3}k$, then $\pi^{PD}_{k>c} > \pi^C$. If $c \leq \frac{1}{3}k$, then $\pi^{PD}_{k>c} > \pi^C$ iff $c > \frac{2}{3}\lambda h\rho$ or $c > \frac{2}{3}k - \frac{2}{3}\lambda h\rho$.*

Proof. For $c > \frac{1}{2}k$, inequality $\pi^{PD}_{k>c} > \pi^C$ holds for all values of $\lambda h\rho$. Now, suppose $c \leq \frac{1}{2}k$. When comparing profits, we have do differentiate between four constellations: (1) If $\lambda h\rho < c$, then $\pi^{PD}_{k>c} > \pi^C$ always holds. (2) If $c \leq \lambda h\rho < k/2$, then $\pi^{PD}_{k>c} > \pi^C \Leftrightarrow \lambda h\rho < \frac{3}{2}c$. The last inequality is always satisfied under $c \leq \lambda h\rho < k/2$ if $\frac{3}{2}c > \frac{k}{2} \Leftrightarrow c > \frac{1}{3}k$. (3) If $k/2 \leq \lambda h\rho < k$, then $\pi^{PD}_{k>c} > \pi^C \Leftrightarrow \lambda h\rho > k - \frac{3}{2}c$. The last inequality is satisfied under $k/2 \leq \lambda h\rho < k$ if $k - \frac{3}{2}c < k/2 \Leftrightarrow c > \frac{1}{3}k$. (4) If $\lambda h\rho \geq k$, then $\pi^{PD}_{k>c} > \pi^C$ always holds. Hence, if $c > \frac{1}{3}k$ then partial delegation will yield higher expected profits than centralization. For $c \leq \frac{1}{3}k$, however, the same will only be true if $\lambda h\rho < \frac{3}{2}c \Leftrightarrow c > \frac{2}{3}\lambda h\rho$ or if $\lambda h\rho > k - \frac{3}{2}c \Leftrightarrow c > \frac{2}{3}k - \frac{2}{3}\lambda h\rho$. ■

Lemmas 4–6 together prove the results of Proposition 4, which in connection with the result of Proposition 3(a) yield Figure 1.

Proof of Proposition 5:

The comparison of profits for C and CD leads to the following observation:

Lemma 7 *Let $h < q$ and $c < k$. If $c < \frac{2}{3}k$, then $\pi^C > \pi^{CD}$. If $c \geq \frac{2}{3}k$, then $\pi^C > \pi^{CD}$ iff $\lambda q \rho < \frac{2}{3}k$.*

Proof. Given $c < \frac{1}{2}k$, the comparison of π^C and π^{CD} shows that $\pi^C > \pi^{CD}$ holds for all values of $\lambda q \rho$. For the comparison of profits under $c \geq \frac{1}{2}k$, we have to differentiate between three cases: (1) If $\lambda q \rho < k/2$, then $\pi^C > \pi^{CD}$ always holds. (2) If $k/2 \leq \lambda q \rho < c$, then $\pi^C > \pi^{CD} \Leftrightarrow \lambda q \rho < \frac{2}{3}k$. The last inequality is satisfied under $k/2 \leq \lambda q \rho < c$ if $\frac{2}{3}k > c$. (3) If $\lambda q \rho \geq c$, then $\pi^C > \pi^{CD} \Leftrightarrow c < \frac{2}{3}k$. Hence, if $c < \frac{2}{3}k$ then centralization will yield higher expected profits than concentrated delegation. For $c \geq \frac{2}{3}k$, however, the same will only be true if $\lambda q \rho < \frac{2}{3}k$. $\blacksquare$

Comparing π^D and $\pi^{PD}_{k>c}$ yields:

Lemma 8 *Let $h < q$ and $c < k$. If $c > \frac{2}{3}k$, then $\pi^D > \pi^{PD}_{k>c}$. If $c \leq \frac{2}{3}k$, then $\pi^D > \pi^{PD}_{k>c}$ iff $\lambda h \rho < \frac{3}{2}c$.*

Proof. We have to differentiate between three cases: (1) If $\lambda h \rho < c$, then $\pi^D > \pi^{PD}_{k>c}$ always holds. (2) If $c \leq \lambda h \rho < k$, then $\pi^D > \pi^{PD}_{k>c} \Leftrightarrow \lambda h \rho < \frac{3}{2}c$. The last inequality is satisfied under $c \leq \lambda h \rho < k$ if $\frac{3}{2}c > k \Leftrightarrow c > \frac{2}{3}k$. (3) If $\lambda h \rho \geq k$, then $\pi^D > \pi^{PD}_{k>c} \Leftrightarrow c > \frac{2}{3}k$. Thus, if $c > \frac{2}{3}k$ then decentralization will dominate partial delegation. If $c \leq \frac{2}{3}k$, the same will only hold if $\lambda h \rho < \frac{3}{2}c$. $\blacksquare$

Lemmas 7 and 8 show that CD (PD) is dominated by another organizational form if $c < \frac{2}{3}k$ ($c > \frac{2}{3}k$). Thus, we do not have to compare CD and PD with each other. Comparing the profits for C with those for PD and

D, and comparing the profits for CD with those for D leads to the following results:[6]

Lemma 9 *Let $h < q$ and $c < k$. If $q-h$ is sufficiently large, then $\pi^C > \pi^{PD}_{k>c}$ and $\pi^C > \pi^D$ and $\pi^{CD} > \pi^D$.*

First, consider result (a) of Proposition 5 with $c < \frac{2}{3}k$. In this situation, CD cannot be optimal (Lemma 7). If $q - h$ is large, $\pi^C > \max\{\pi^{PD}_{k>c}, \pi^D\}$ (Lemma 9) so that C will be optimal. Result (b) refers to $c \geq \frac{2}{3}k$. According to Lemma 8, PD cannot be optimal. If $q - h$ is large and therefore q is large, then $\pi^{CD} > \pi^C$ (Lemma 7). Since $\pi^{CD} > \pi^D$ for large values of $q - h$ (Lemma 9), CD is optimal.

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[6]The proof of Lemma 9 is relegated to the additional material for the referees.

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Additional Material for the Referees

Proof of Lemma 9:

First, **centralization** is compared to **partial delegation**.

Suppose $k/2 < c$:

(1) If $\lambda h\rho < c$ and $\lambda q\rho < k/2$:

$$\pi^C > \pi^{PD}_{k>c} \Leftrightarrow \left(1 + \frac{2}{3}\lambda\right) q > \left(1 + \frac{5}{6}\lambda\right) h.$$

(2) If $\lambda h\rho < c$ and $\lambda q\rho > k/2$:

$$\pi^C > \pi^{PD}_{k>c} \Leftrightarrow q\rho > \left(1 + \frac{5}{6}\lambda\right) h\rho - \frac{1}{3}k.$$

(3) If $c < \lambda h\rho < k$ and $\lambda q\rho > k/2$:

$$\pi^C > \pi^{PD}_{k>c} \Leftrightarrow q\rho > \left(1 + \frac{1}{3}\lambda\right) h\rho + \frac{1}{2}c - \frac{1}{3}k. \tag{10}$$

(4) $\lambda h\rho > k$ and $\lambda q\rho > k/2$:

$$\pi^C > \pi^{PD}_{k>c} \Leftrightarrow q\rho > h\rho + \frac{1}{2}c.$$

Now, suppose $k/2 > c$: We find the same cases and conditions (1)–(4) as under $k/2 < c$. However, there is the additional case that $c < \lambda h\rho < k$ and $\lambda q\rho < k/2$:

$$\pi^C > \pi^{PD}_{k>c} \Leftrightarrow \left(1 + \frac{2}{3}\lambda\right) q\rho > \left(1 + \frac{1}{3}\lambda\right) h\rho + \frac{1}{2}c.$$

Altogether, if $q - h$ is sufficiently large, $\pi^C > \pi^{PD}_{k>c}$ will be satisfied.

Next, **centralization** is compared to **decentralization**.

Suppose $k/2 < c$:

(1) If $\lambda h\rho < c$ and $\lambda q\rho < k/2$:

$$\pi^C > \pi^D \Leftrightarrow \left(1 + \frac{2}{3}\lambda\right) q > (1+\lambda)\, h.$$

(2) If $\lambda h\rho < c$ and $\lambda q\rho > k/2$:

$$\pi^C > \pi^D \Leftrightarrow q\rho > (1+\lambda)\, h\rho - \frac{1}{3}k.$$

(3) If $\lambda h\rho > c$ and $\lambda q\rho > k/2$:

$$\pi^C > \pi^D \Leftrightarrow q\rho - c > h\rho - \frac{1}{3}k. \tag{11}$$

Now, suppose $k/2 > c$: We find the same cases and conditions (1)–(3) as under $k/2 < c$. However, there is the additional case that $\lambda h\rho > c$ and $\lambda q\rho < k/2$:

$$\pi^C > \pi^D \Leftrightarrow \left(1 + \frac{2}{3}\lambda\right) q\rho - c > h\rho.$$

Thus, if $q - h$ is sufficiently large, $\pi^C > \pi^D$ will be satisfied.

Finally, **concentrated delegation** is compared to **decentralization**:

(1) If $\lambda h\rho < c$ and $\lambda q\rho < c$:

$$\pi^{CD} > \pi^D \Leftrightarrow (2+\lambda)\, q > (2+2\lambda)\, h. \tag{12}$$

(2) If $\lambda h\rho < c$ and $\lambda q\rho > c$:

$$\pi^{CD} > \pi^D \Leftrightarrow 2q\rho + c > (2+2\lambda)\, h\rho. \tag{13}$$

(3) If $\lambda h\rho > c$ and $\lambda q\rho > c$:

$$\pi^{CD} > \pi^D \Leftrightarrow 2q\rho - c > 2h\rho. \tag{14}$$

If $q - h$ is sufficiently large, $\pi^{CD} > \pi^D$ will be satisfied in each of the three cases.

Construction of Figure 2:

The figure uses $\lambda = \rho = 1/2$ and $\lambda h\rho = \frac{3}{4}k \Rightarrow h = 3k$. Note that the horizontal axis starts at $q = h = 3k$, since only values with $q > h$ are feasible.

First, consider $c > k$. Since PD is dominated by D (Lemma 2) and C is dominated by CD (Lemma 3), only D and CD are candidate solutions in this parameter region. Moreover, we have $\lambda h\rho < c$ in the relevant range so that (12) (i.e., $q > (2 + 2\lambda)\, h/\, (2 + \lambda)] = 3.6k$) and (13) (i.e., $c > 4.5k - q$) describe the solution.

For $\frac{2}{3}k < c < k$, again D and CD are the only candidate solutions since C is dominated by CD (Lemma 7), and PD is dominated by D (Lemma 8). The solution for the interval $\lambda h\rho < c < k$ is described by (13) (i.e., $c > 4.5k - q$), and the solution for $\frac{2}{3}k < c < \lambda h\rho$ by (14) (i.e., $c < q - 3k$).

For $\frac{1}{2}k < c < \frac{2}{3}k$, the organizational form C dominates CD (Lemma 7) and D dominates PD (Lemma 8; note that $\lambda h\rho < \frac{3}{2}c \Leftrightarrow c > k/2$ is satisfied here). Hence, either C or D is optimal. The respective solution is given by condition (11) (i.e., $c < \frac{q}{2} - \frac{7}{6}k$).

Finally, for $c < \frac{1}{2}k$, the form C dominates CD (Lemma 7), and PD dominates D (Lemma 8; now $\lambda h\rho > \frac{3}{2}c \Leftrightarrow c < k/2$ holds). The comparison between C and PD is described by (10) (i.e., $c < q - \frac{17}{6}k$).