GOVERNANCE AND THE EFFICIENCY OF ECONOMIC SYSTEMS
GESY

Discussion Paper No. 368

# Overconfidence and Managers´ Responsibility Hoarding

Petra Nieken *
Abdolkarim Sadrieh **
Nannan Zhou ***

* University of Bonn
** University of Magdeburg
*** University of Cologne

December 2011

Financial support from the Deutsche Forschungsgemeinschaft through SFB/TR 15 is gratefully acknowledged.

# Overconfidence and Managers' Responsibility Hoarding*

Petra Nieken† Abdolkarim Sadrieh‡ Nannan Zhou§

December 28, 2011

**Abstract**

Overconfidence is a well-established behavioral phenomenon that involves an overestimation of own capabilities. We introduce a model, in which managers and agents exert effort in a joint production, after the manager decides on the allocation of the tasks. A rational manager tends to delegate the critical task to the agent more often than given by the efficient task allocation. In contrast, an overconfident manager is more likely to hoard responsibility, i.e. to delegate the critical task less often than a rational manager. In fact, a manager with a sufficiently high ability and a moderate degree of overconfidence increases the total welfare by hoarding responsibility and exerting more effort than a rational manager. Finally, we derive the conditions under which responsibility hoarding can persist in an organization, showing that the bias survives as long as the overconfident manager can rationalize the observed output by underestimating the ability of the agent.

**Key Words:** organizational behavior, management performance, bounded rationality, behavioral bias

**JEL Classification:** C72, D03, D82, M12, M54

*Petra Nieken gratefully acknowledges financial support by the Deutsche Forschungsgemeinschaft (DFG, grant SFB/TR 15). Abdolkarim Sadrieh gratefully acknowledges financial support by the Deutsche Forschungsgemeinschaft (DFG grant SA 1350/3-1) and of the Land Sachsen-Anhalt.

†University of Bonn, Department of Economics, Adenauerallee 24-42, 53113 Bonn, Germany, petra.nieken@uni-bonn.de.

‡University of Magdeburg, Faculty of Economics and Management, Postbox 4120, 39016 Magdeburg, Germany, sadrieh@ovgu.de

§University of Cologne, Faculty of Management, Economics and Social Sciences, Albertus-Magnus-Platz, 50923 Cologne, Germany, nannan.zhou@uni-koeln.de

# 1 Introduction

Overconfidence is a well-established behavioral pattern that involves overestimating the own capabilities, especially in tasks with a partially stochastic outcome (Svenson (1981), Lichtenstein et al. (1982), Russo and Schoemaker (1992), Soll (1996)). Although the degree of overconfidence may vary with the type of task (Grieco and Hogarth (2009)), it is generally found to persist when individuals assess the probability of their own success or the relative standing of their performance compared to others (Klayman et al. (1999)).

While the behavioral pattern of overconfidence and its effects on financial decision-making have been studied extensively (see for instance De Bondt and Thaler (1996), Barber and Odean (2000), Deaves et al. (2008), or Malmendier and Tate (2005)), the effects of overconfidence on organizational performance are not fully understood yet. In particular, the question how manager overconfidence affects organizational performance by biasing the manager's delegation and task distribution choices has not been studied so far.

To investigate these effects, we introduce a model, in which a manager and an agent can exert effort into a joint production that consists of two distinct tasks with unequal impact on the output. The allocation of the tasks is at the discretion of the manager, who decides whether to perform the "critical" task (i.e. the task with the higher impact on the output) himself or to delegate it to the agent.[1]

We show that an overconfident manager tends to *hoard responsibility*, i.e. to assign the critical task more often to himself than a fully rational manager would. Responsibility hoarding takes place, even though it is individually suboptimal for the manager, who suffers from a higher cost of effort by performing the critical task instead of the other task.

We also show that, despite adding to the overconfident manager's effort cost, responsibility hoarding may actually increase the total welfare of the involved parties. As long as the overconfident manager's self-perception bias is not too large, the total welfare effect can be positive, because the amount of effort exerted by the overconfident manager is closer to the efficient level than the amount provided by a fully rational manager, who chooses a payoff maximizing effort level, generally below the welfare maximizing level. Hence, by overestimating his own productivity and exerting a correspondingly greater amount of effort, the overconfident manager typically engages in less free-riding than his rational counterpart.

Finally, we show that responsibility hoarding can persist in an organization, as long as the overconfident manager can rationalize the overestimation of the own ability by underestimating the ability of the agent. The more leeway an overconfident manager has to rationalize the observed outcome without having to adapt his positively biased assessment of the own ability level, the more likely it is to observe persistent responsibility hoarding in an organization. We determine the conditions for persistent responsibility hoarding in our model without assuming that managers are biased in the way that they update information. Adding biased information processing to the model (see e.g. Brunnermeier and Parker (2005), Mobius et al. (2011)) will intuitively increase the range of parameter values for which responsibility hoarding is a persistent

[1] We use the male pronoun for the manager and the female pronoun for the agent, because males are generally found to exhibit a higher degree of overconfidence than females (see e.g. Barber and Odean (2001), Correll (2001), Bengtsson et al. (2005)).

bias.[2]

Most of the existing literature on overconfidence in managerial settings is focused on the exaggerated investment risks taken by overconfident managers. While Barber and Odean (2000) and Deaves et al. (2008) report excessive trading by overconfident traders, Camerer and Lovallo (1999) observe excessive market entries in an experimental setting. Malmendier and Tate (2005) argue that managerial overconfidence leads to distortive investment behavior and demonstrate that data on CEO investments in the own company are in line with their overconfidence model. Hackbarth (2008) shows that overconfident managers tend to bias the capital structure of the firm towards higher debt levels. Similar results can be found in Ben-David et al. (2007) who show that companies with overconfident CFOs have a significantly different capital structure than other firms. Furthermore, Malmendier and Tate (2008) find that overconfident managers also tend to overpay in mergers and even initiate value-destroying ones. Interestingly, this bias is sometimes advantageous for the firm value. For instance, Palomino and Sadrieh (2011) show that managerial overconfidence can be advantageous concerning financial decisions. They analyze a model in which overconfident portfolio managers, who share profits, may exhibit risk attitudes that are more in line with the investors' risk attitude than fully rational risk-averse managers. Analyzing data of large publicly traded firms from 1980 to 1994, Galasso and Simcoe (2011) present evidence that overconfident CEOs have a significantly higher probability to initiate corporate innovation.

In a team production setting, Gervais and Goldstein (2007) study a firm with complementary production technology and show that the presence of an overconfident agent can increase the firm output, because it helps the agents to coordinate on a high effort level, and therefore, overcome the free-rider problem. Furthermore, Santos-Pinto (2008) shows that firms can benefit from using interdependent incentive schemes when workers exhibit have wrong beliefs about their coworkers' abilities. Regarding individual performance, Weinberg (2009) for instance shows that a moderate overestimation of the own ability can be advantageous compared to a correct assessment, because it allows the overconfident individuals to undertake more challenging tasks that raise their expected output and utility. Recent experimental findings by Sautmann (2011) support the notion that overconfident agents accept lower wage offers, while Santos-Pinto (2010) shows that firms using tournaments as incentives can achieve greater profits, if agents have a positive self-image. Similarly, Ludwig et al. (2011) find that moderate overconfidence can improve the agent's performance in a Tullock contest relative to an unbiased opponent. These results are supported by the recent experiment of Kinari et al. (2011), reporting a significant positive impact of overconfidence on productivity in tournaments. Furthermore, Englmaier (2011) argues that firms should hire overoptimistic managers to ensure the implementation of certain investment strategies in R&D tournaments.

Regarding the literature on the delegation of tasks in a principal-agent model, we are, to the best of our knowledge, the first to take overconfidence into account. Prendergast (1995) suggests a model where a manager has discretion over task assignments and may exhibit responsibility hoarding, i.e. may delegate less tasks to the agent than a rational manager would. The result is

[2] Some authors have presented models to analyze the conditions under which overconfidence may emerge (see e.g. Rabin and Schrag (1999)). In contrast, our model takes the existence of overconfidence as given, but analyzes the conditions under which it can survive in an organizational setting.

driven by the assumption that the manager can earn future rents from the on-the-job training that performing the additional tasks provides. This model of rational responsibility hoarding is especially useful when studying professions with extraordinary high rents for job experience, e.g. surgeons, pilots, or lawyers. Note, however, that – as our analysis shows - even in such settings any degree of rational responsibility hoarding may be amplified by the manager's overconfidence.

If the output of several tasks cannot be measured separately and the principal has to delegate at least one task, Itoh (1994) and Itoh (2001) find that the principal will execute some tasks himself or delegate all tasks to only one agent if the agents are risk averse. Gürtler (2008) extends this model and compares partial delegation, where the principal carries out one task and the other task is carried out by the agent, to complete delegation, where each agent specializes in one of the tasks.

## 2 Basic model

Consider a joint production setting, in which a manager and an agent can exert effort to generate a shared output. The total output $Y$ is a function of the outcomes $Y_1$ and $Y_2$ of the tasks 1 and 2, correspondingly. A crucial assumption is that the two tasks differ in their impact on the total output, where w.l.o.g. we assume that task 1 is the critical task, i.e. it tends to have higher impact on total output than the non-critical task 2. Using an additive production function, we introduce the parameter $\lambda$ that measures the relative impact of task 1 compared to task 2. Hence, the total output $Y$ is defined as:

$$Y = \lambda Y_1 + (1-\lambda) Y_2, \text{ with } \lambda \in \left(\frac{1}{2}, 1\right)$$

For simplicity, both manager and agent are risk neutral and benefit to the same extent from the total output, i.e. both individuals receive the same share of $\frac{1}{2}Y$. We assume that the effort levels as well as the output of both tasks are not observable by the firm. Only the total output $Y$ is observable for all parties. Either player can be assigned to perform either task, where the task allocation is chosen by the manager at the outset of each period. The allocation must be complete and bijective, i.e. both tasks must be allocated and each must be allocated to a different player, because no player can perform both tasks in the same period. The outcome of each task $j$ is endogenous, depending on the true ability $a_i$ and the chosen effort $e_i$ of the player $i$ performing the task:

$$Y_j = a_i \cdot e_i, \text{ with } i = M[anager], A[gent] \text{ and } j = 1, 2.$$

Furthermore, the individual effort cost $C(e_i)$ is strictly convex:

$$C(e_i) = \frac{c}{2} e_i^2, \; c \in \mathbb{R}^+.$$

To simplify the discussion, we distinguish between those task allocation choices, in which the critical task 1 is performed by the manager (*non-delegation*), and those, in which the critical task 1 is allocated to the agent (*delegation*). More formally we define:

**Definition 1** *A **delegation choice** is a task allocation in which the critical task 1 is allocated to the agent and the non-critical task 2 is allocated to the manager. In case of **non-delegation** the critical task 1 is allocated to the manager and the non-critical task 2 is carried out by the agent.*

**Definition 2** *A task allocation is **individually optimal** if it maximizes the manager's utility.*

**Definition 3** *A task allocation is **efficient** if it maximizes the total welfare of all involved parties.*

Following Gervais and Goldstein (2007) we characterize an overconfident manager as someone who systematically overestimates his own ability:

**Definition 4** *An **overconfident** manager has an overly optimistic perception of his own ability, i.e.*

$$a_M^{OC} = a_M + b$$

*where $a_M$ denotes the manager's true ability and the parameter $b > 0$ his self-perception bias or the degree of his overconfidence.*

Finally, we use the definitions above to characterize responsibility hoarding.

**Definition 5** ***Responsibility hoarding** occurs when a manager performs the critical task (task 1) himself, even though a delegation choice is individually optimal for him.*

# 3 Perfect information on agent's ability

In the first step, we assume that the agent's ability is common knowledge, i.e. both the manager and the agent have perfect information on the agent's ability. We start by investigating the efficient task allocation choice (first-best case) and then proceed to the delegation choices of a fully rational manager and of an overconfident manager. Comparing the three results, we first show that fully rational managers delegate the critical task more often than is efficient. Next, we show that overconfidence always leads to less delegation compared to an equilibrium with fully rational managers. Finally, we prove that the manager's biased self-perception may increase efficiency, because responsibility hoarding can be beneficial for the total welfare of the involved parties, as long as the increase in the overconfident manager's contribution to firm output over-compensates the loss due to his individually suboptimal delegation and effort choices.

## 3.1 Efficient task allocation (first-best case)

Assume that the critical task 1 is carried out by the manager and task 2 is assigned to the agent. The outcomes of the two tasks are then given by

$$\begin{aligned} Y_1^{nd} &= a_M e_M \\ Y_2^{nd} &= a_A e_A \end{aligned} \tag{1}$$

where the index $nd$ denotes the case of non-delegation.

In the first-best case the total welfare of the involved parties is maximized by:

$$\max_{e_M,e_A} W^{nd} = (\lambda a_M e_M + (1-\lambda)\, a_A e_A) - \frac{c}{2}e_M^2 - \frac{c}{2}e_A^2$$

which leads to the first-best effort levels given by

$$\begin{aligned} e_M^{ndFB} &= \frac{\lambda a_M}{c} \\ e_A^{ndFB} &= \frac{(1-\lambda)\, a_A}{c}. \end{aligned}$$

Hence, the total welfare in case of non-delegation is equal to

$$W^{ndFB} = \frac{\lambda^2 a_M^2}{2c} + \frac{(1-\lambda)^2\, a_A^2}{2c}. \tag{2}$$

Next, assume that the critical task 1 is assigned to the agent and task 2 is carried out by the manager. Now, the outcomes of both tasks are given by

$$\begin{aligned} Y_1^d &= a_A e_A \\ Y_2^d &= a_M e_M \end{aligned} \tag{3}$$

where the index $d$ denotes the case of delegation.

In this case, the welfare maximization problem becomes

$$\max_{e_M,e_A} W^d = ((1-\lambda)\, a_M e_M + \lambda a_A e_A) - \frac{c}{2}e_M^2 - \frac{c}{2}e_A^2$$

which leads to the first-best effort levels described by

$$\begin{aligned} e_M^{dFB} &= \frac{(1-\lambda)\, a_M}{c} \\ e_A^{dFB} &= \frac{\lambda a_A}{c}. \end{aligned}$$

Hence, the total welfare in case of delegation is equal to

$$W^{dFB} = \frac{(1-\lambda)^2\, a_M^2}{2c} + \frac{\lambda^2 a_A^2}{2c}. \tag{4}$$

Comparing (2) and (4), delegation is efficient if and only if

$$a_M \leq a_A.$$

**Proposition 1** *In the efficient task allocation the critical task should be allocated to the agent if and only if her ability is at least as high as the manager's ability, i.e.* $a_M \leq a_A$. *Otherwise, the critical task should better be assigned to the manager.*

It is straightforward that maximizing the total welfare requires that the critical task (i.e. the task with a higher impact on the total output) to be carried out by the individual with the

higher ability. Moreover, the positive welfare effect of delegation is the greater the higher the agent's ability is. However, it is not obvious that the welfare maximizing task allocation will generally be implemented when the task allocation is chosen by the manager.

### 3.2 Optimal delegation choice of a fully rational manager

Assume that the task allocation is chosen by a fully rational manager, maximizing his individual utility. Assume that the manager does not delegate, i.e. the critical task (task 1) is carried out by the manager and other task (task 2) is assigned to the agent. The outcomes of the two tasks are then given by (1).

In contrast to the first-best case, the manager's utility is now maximized with:

$$\max_{e_M} U_M^{nd} = \frac{1}{2}\left(\lambda a_M e_M + (1-\lambda)\, a_A e_A\right) - \frac{c}{2} e_M^2$$

which leads to his individually optimal effort level described by

$$e_M^{nd*} = \frac{\lambda a_M}{2c}.$$

Furthermore, the agent's optimization is given by

$$\max_{e_A} U_A^{nd} = \frac{1}{2}\left(\lambda a_M e_M + (1-\lambda)\, a_A e_A\right) - \frac{c}{2} e_A^2$$

and her individually optimal effort level is

$$e_A^{nd*} = \frac{(1-\lambda)\, a_A}{2c}.$$

Since we assume that the manager has perfect information about the agent's ability, his utility is equal to

$$U_M^{nd*} = \frac{\lambda^2 a_M^2}{8c} + \frac{(1-\lambda)^2 a_A^2}{4c}. \tag{5}$$

Next, assume that the manager delegates, i.e. the critical task 1 is assigned to the agent and task 2 is carried out by the manager. Now, the outcomes of both tasks are given by (3).

In this case, the manager's optimization problem is

$$\max_{e_M} U_M^{d} = \frac{1}{2}\left((1-\lambda)\, a_M e_M + \lambda a_A e_A\right) - \frac{c}{2} e_M^2$$

with

$$e_M^{d*} = \frac{(1-\lambda)\, a_M}{2c}$$

as his individually optimal effort level.

For the agent, the optimization is given by

$$\max_{e_A} U_A^{d} = \frac{1}{2}\left((1-\lambda)\, a_M e_M + \lambda a_A e_A\right) - \frac{c}{2} e_A^2$$

which leads to an individually optimal effort level of

$$e_A^{d*} = \frac{\lambda a_A}{2c}.$$

Hence, the manager's utility in case of delegation is equal to

$$U_M^{d*} = \frac{(1-\lambda)^2 a_M^2}{8c} + \frac{\lambda^2 a_A^2}{4c}. \tag{6}$$

Now, by comparing (5) and (6), the fully rational manager chooses delegation if and only if

$$a_M \leq \sqrt{2} a_A.$$

It is straightforward that the fully rational manager prefers to delegate the critical task as long as his own ability is smaller than the agent's ability, i.e. as long as $a_M \leq a_A$. Moreover, note that there is a range of values (i.e. $a_M \in \left(a_A; \sqrt{2}a_A\right]$) for which the manager also delegates the critical task to the agent, even though his ability is strictly higher than the agent's ability. This is due to the fact that in equilibrium the critical task 1 is performed with higher levels of effort and, thus, with a higher effort cost, than the other task. Hence, delegating the task may pay, because delegation reduces the manager's effort cost more than it reduces the expected outcome of the critical task when it is performed by the agent with the somewhat lower ability. As the fully rational manager cannot commit to the efficient task allocation this may lead to inefficient job distributions and lower total welfare in equilibrium. However, once the agent's ability falls below the threshold $\frac{\sqrt{2}}{2} a_M$, the manager prefers to perform the critical task himself, because the benefit from the own higher ability surpasses the higher effort cost.

### 3.3 Optimal delegation choice of an overconfident manager

In this section, we examine the task allocation choice of an overconfident manager, assuming that overconfidence leads to an overly optimistic perception of the own abilities. Recall that the self-perceived ability of an overconfident manager is given by

$$a_M^{OC} = a_M + b, \text{ with } b > 0.$$

Given this slight modification of the model, we derive the equilibrium choices of the overconfident manager and the agent and compare these to the case with a fully rational manager. By substituting $a_M^{OC}$ for $a_M$ and following the same procedure applied in the previous chapter, we derive the condition under which the overconfident manager chooses delegation:

$$a_M \leq \sqrt{2} a_A - b.$$

By decreasing the right-hand side of the inequality, any positive self-perception bias $b$ lowers the threshold for non-delegation, reducing the range of values for which delegation is chosen by the overconfident manager. Hence, it is obvious that an overconfident manager is more likely to hoard responsibility than a fully rational manager of the same true ability. In particular, the higher the self-perception bias $b$, the larger the range of ability values for which a fully rational

manager delegates the critical task, but an overconfident manager will not, i.e. the greater the range of ability values in which the critical task is carried out by the manager. We summarize our findings in the following proposition:

**Proposition 2** *With perfect information on the agent's ability parameter $a_A$, any positive self-perception bias $b > 0$ leads to responsibility hoarding by the overconfident manager. In particular, the range of manager types choosing delegation is strictly decreasing in the managers' degree of overconfidence $b$.*

As we have shown in the last section, fully rational managers cannot commit to the efficient task allocation as they have an incentive to lower their own effort cost by delegating the critical task to the agent as long as the agent's ability is sufficiently high. In contrast, overconfident managers overestimate their own ability, and therefore, allocate the critical task more often to themselves than fully rational managers. In particular, overconfident managers are more likely to hoard responsibility the larger their self-perception bias is. However, the task allocations chosen by overconfident managers may be closer to the efficient allocation than those of rational managers. Hence, overconfidence can be considered as a commitment device for managers to take more responsibilities and increase the efficiency of the job distribution, positively affecting the total welfare.

### 3.4 Is overconfidence beneficial or harmful?

As we have shown in the previous section, overconfidence may lead to less delegation and can, thus, improve the efficiency of the task allocations. Since an overconfident manager in general exerts more effort, the total output of the firm is often higher with an overconfident manager than with a rational manager. The higher effort level, however, also leads to a higher cost of effort provision for the overconfident manager. Hence, it is not clear whether the manager's overconfidence is generally beneficial or harmful with regard to total welfare. In this section, we show that in many cases, including some in which the task allocation is not individually optimal for the manager, overconfidence is beneficial regarding the total welfare of the involved parties. Comparing the total welfare in equilibrium with a fully rational manager to that with an overconfident manager, we establish the following proposition:

**Proposition 3** *If the manager's self-perception bias $b$ is on a moderate level relative to his true ability (i.e. $b < 2a_M$) and his true ability is sufficiently high (i.e. $a_M > \sqrt{2}a_A$) or sufficiently low (i.e. $a_M < \sqrt{2}a_A - b$), the total welfare of the involved parties in equilibrium is strictly higher with an overconfident manager than with a fully rational manager. For any ability value of the manager between those two thresholds (i.e. $\sqrt{2}a_A - b \leq a_M \leq \sqrt{2}a_A$), this result still holds if the manager's true ability is at least as high as the agent's true ability (i.e. $a_M \geq a_A$).*

**Proof.** See the appendix.

This result has several interesting implications. First, note that the manager's overconfidence is not generally harmful and can even be beneficial for the total welfare, if it is not too strong. On the one hand, the overconfident manager overestimates his own ability, and

therefore, exerts more effort than the fully rational manager, irrespective of the task allocation. On the other hand, the overconfident manager also expects a higher total outcome when carrying out the critical task himself, and thus, is more likely than his fully rational counterpart to allocate the critical task to himself. This type of responsibility hoarding behavior is efficiency enhancing, if the manager is more able than the agent. Hence, overconfidence helps to reduce free-riding. Indeed, this positive incentive effect of manager overconfidence can even over-compensate the negative effect of individually suboptimal task allocation as long as the manager is at least as productive as the agent. Note that this finding is also in line with our result from the first-best case stating that the delegation of the critical task is only efficient if the agent is more productive than the manager. In particular, the total welfare in equilibrium with an overconfident manager is closer to the efficient allocation than with a fully rational manager of the same true ability. Hence, all involved parties may in fact benefit from a moderate level of manager overconfidence.

### 3.5 Optimal degree of overconfidence

As the manager's overconfidence can be beneficial for total welfare, it is straightforward to proceed in our analysis with the determination of the optimal degree of overconfidence with respect to the total welfare. In this regard, we can show

**Proposition 4** *The total welfare is highest if the manager's self-perception bias (or degree of overconfidence) is equal to his true ability, i.e.* $b^* = a_M$.

**Proof.** See the appendix.

Note that the positive welfare effect of overconfidence is strictly increasing in the manager's true ability. Intuitively, the higher the manager's true ability is, the less harmful is his biased self-perception, the more likely responsibility hoarding may positively affect the total welfare. Moreover, it is also straightforward to see that the manager's effort choice exactly matches the efficient level, if the degree of his overconfidence is equal to his true ability. We summarize this result in the following corollary:

**Corollary 1** *If a manager's degree of overconfidence is equal to the true value of his ability, i.e.* $b = a_M$*, his effort choice in equilibrium is exactly equal to the efficient effort level, both in case of delegation and non-delegation.*

**Proof.** The results follows directly by substituting $a_M$ for $b$ into the overconfident manager's incentive conditions. ■

## 4 Persistence of the manager's overconfidence and the underestimation of the agent's ability

In the previous section, we have demonstrated that manager overconfidence can lead to less delegation, resulting in more efficient task allocations in a perfect information setting. The question that remains to be answered is whether the managers' overconfidence and responsibility

hoarding behavior can persist over time, given imperfect information and feedback. If managers quickly learn to correct their overconfident assessment of the own ability, then overconfidence and responsibility hoarding will not persist. However, if the feedback from previous outcomes cannot be used to correct overconfidence, we can establish that responsibility hoarding can be a persistent phenomenon with a sustained effect on organizational performance.

We derive the conditions under which overconfidence (and responsibility hoarding) can persist, when managers have imperfect information on the agent's ability and receive feedback only on previous performance. We restrict our analysis to the case that the manager only receives feedback on the total output of the firm (or the organizational unit). Obviously, persistence of overconfidence with more exact information, e.g. on all ability and effort parameters, would not be feasible. In the more realistic situation that we analyze, we assume that the agent's ability parameter is her private information. More specifically, we assume that the manager uses an estimate of the agent's ability parameter denoted by $\widehat{a}_A$. Since, the feedback information is restricted to the total output, the overconfident manager faces one known parameter (his own effort level), two unknown parameters (the agent's ability and effort level), and one parameter that he believes to know, but actually does not (his own ability). Under these circumstances, we show that the overconfident manager may not be able to learn that his self-assessment is biased, because he can construct a consistent model that explains the observed total outcome with an overestimated own ability parameter and an underestimated ability parameter for the agent.[3] As long as the productivity of the agent can be underestimated sufficiently, the manager's overconfidence can persist. We summarize this result in the following proposition:

**Proposition 5** *If the agent's true ability $a_A$ is not known to the manager and sufficiently high, i.e. $a_A \geq \frac{\lambda}{(1-\lambda)}\sqrt{(a_M b + b^2)}$, the manager's overconfidence persists, because the manager rationalizes the observed outcome information by underestimating the agent's ability. The higher the manager's self-perception bias $b$ is, the stronger the underestimation of the agent's ability will be.*

**Proof.** See the appendix.

A straightforward corollary to the proposition in this section is concerned with the limits of persistent overconfidence:

**Corollary 2** *Manager overconfidence is persistent at some positive level iff $a_A > 0$.*

The corollary simply points out that depending on the ability parameters there always might be some level of overconfidence that is persistent, as long as the agent's ability is not zero. Intuitively, it is clear that overconfidence can only persist, as long as the overconfident manager has the possibility to underestimate the agent's contribution to the observed total output, and thus, the ability of the agent. The range for the underestimation drops if the agent's ability is decreased, leaving less and less room for persistently overconfident managers. If the agent's ability is zero, she would not contribute at all to the total output and persistent

[3] Young (2002)shows that some games cannot be learned by rational players and demonstrates a class of learning environments in which convergence to equilibrium behavior fails to occur for any learning process, including the Bayesian updating of objectively correct priors.

overconfidence would no longer be possible. But, note that the extreme case of zero ability has no empirical relevance, because it describes a situation in which the agent cannot contribute to the output of the firm. Hence, the corollary shows that for any situation with empirical relevance, there is at least some level of persistent overconfidence, leading to some amount of persistent responsibility hoarding by overconfident managers.

# 5 Discussion

Using the results of the sections above, we discuss the range of optimal and persistent manager overconfidence and responsibility hoarding constellations in this section. The constellations are exhibited in Figure 1. It shows the four functions that determine the different outcome regions in the ability space. The manager's ability is plotted on the horizontal and the agent's (possibly estimated) ability is plotted on the vertical axis.[4]

The dashed bisecting line depicts the function $a_A = a_M$ which separates the area of efficient delegation choices above the line from the area of efficient non-delegation choices below the line. The solid line running through the origin depicts the function $a_A = \frac{\sqrt{2}}{2} a_M$ and separates the area of individually optimal delegation (i.e. rational delegation) above the line from the area of individually optimal non-delegation (i.e. rational non-delegation) below the line. The area of individually optimal delegation is larger than the area of individually optimal non-delegation, because – as we show in section 3 – as long as the agent's ability is not too low, the rational manager prefers to avoid the high cost of effort associated with performing the critical task himself.

The solid line that intersects the vertical axis above the origin depicts the function $\widehat{a}_A = \frac{\sqrt{2}}{2}(a_M + b)$ and separates the area of delegation (the dotted area above the line) from the area of non-delegation (below the line) chosen by an overconfident manager. Note that in the dotted area both the fully rational and the overconfident manager choose to delegate the critical task to the agent, while below the line running through the origin (the hatched area), both the fully rational and the overconfident manager choose to carry out the critical task themselves. The area enclosed by the functions $a_A = \frac{\sqrt{2}}{2} a_M$ and $\widehat{a}_A = \frac{\sqrt{2}}{2}(a_M + b)$ contains all ability constellations for responsibility hoarding, in which the overconfident manager still assigns the critical task to himself, but the rational manager does not.

The function $a_A = \frac{\lambda}{(1-\lambda)} \sqrt{(a_M b + b^2)}$ separates the area of persistent (above) from the area of non-persistent manager overconfidence (below). Note that this separation is only valid in the area where responsibility hoarding occurs, i.e. the area enclosed by the functions $a_A = \frac{\sqrt{2}}{2} a_M$ and $\widehat{a}_A = \frac{\sqrt{2}}{2}(a_M + b)$. Our graph shows a large area of non-persistent (the shaded area and the dark grey area) and a relatively small area of persistent manager overconfidence with responsibility hoarding by the overconfident manager (the white area and the white-dotted area). Intuitively, it seems clear that manager overconfidence has a lower chance to persist, if the manager carries out the critical task himself. The reason is that the overconfident manager always has more room to rationalize his overly optimistic self-perception by underestimating the agent's contribution when the critical task is carried out by the agent.

[4] We have fixed $\lambda = 0.55$ and $b = 4$ to make a two-dimensional plot possible.

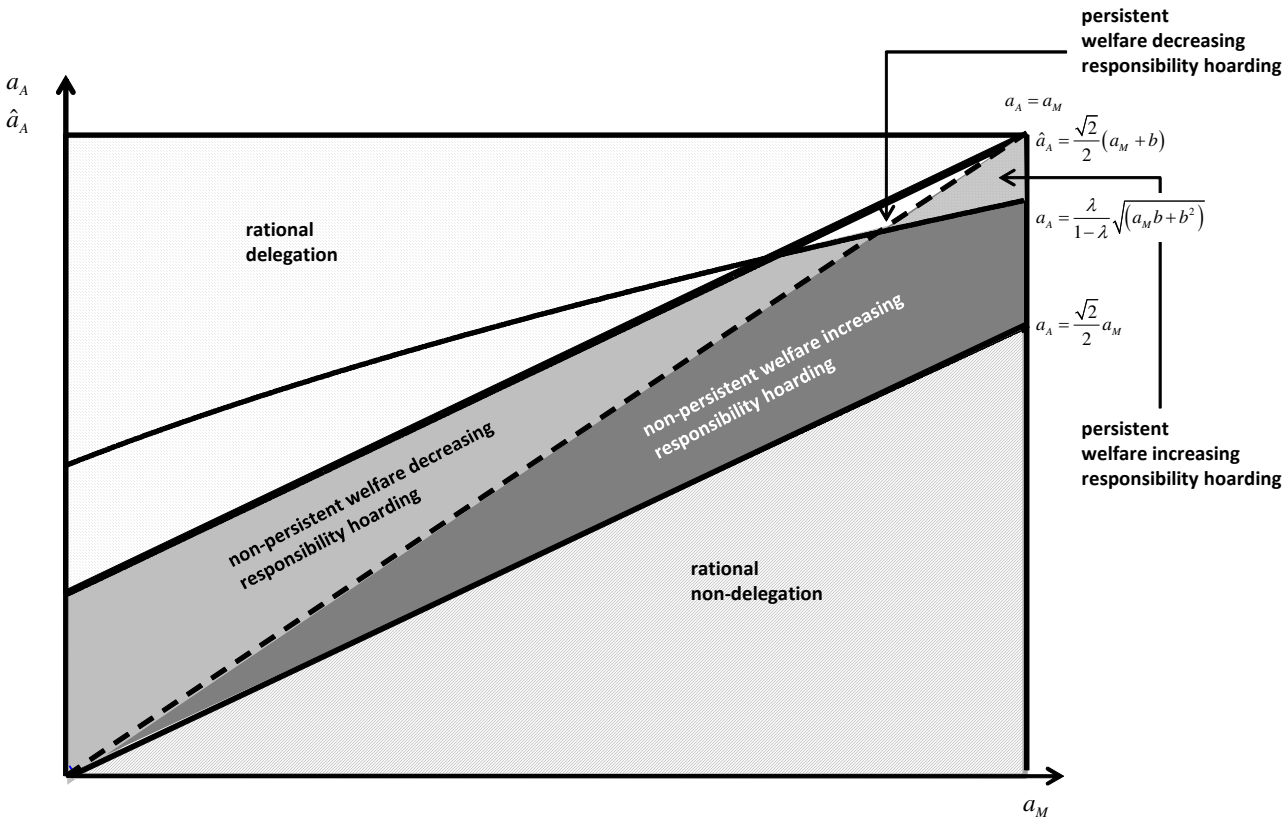


Figure 1: Range of optimal and persistent overconfidence and responsibility-hoarding constellations depending on manager's and agent's abilities

The difference between the tasks concerning their impact on the firm outcome, i.e. the value of the parameter $\lambda$, in fact, affects the location of the persistency curve (the function $a_A = \frac{\lambda}{(1-\lambda)}\sqrt{(a_M b + b^2)}$) and, thus, the size of persistent overconfidence areas in the graph (the white area and the white-dotted area). The more important the critical task is when compared to the other task (i.e. the higher $\lambda$), the smaller are the areas of persistent overconfidence and responsibility hoarding. The more asymmetric a task constellation is, the more difficult it is for the overconfident manager to find a feasible set of parameters, in which the overestimation of the own ability can be compensated by underestimating the ability of the agent.

A similar but more subtle effect exists concerning the self-perception bias $b$. As $b$ increases, the area of responsibility hoarding obviously also increases. Note, however, that an increase in the level of overconfidence $b$ also means that the persistency curve shifts upwards, reducing the area of persistent responsibility hoarding. Hence, more overconfident managers will tend to carry out the critical task more frequently, but are also more likely to receive feedback that lets them revise their self-assessment and reduce their overconfidence.

Another implication of our analysis is that both the manager's and the agent's abilities must be relatively high to allow for persistent manager overconfidence. This is because the agent's ability must be high enough to provide the relatively high degree of underestimation that persistency of overconfidence requires. Since persistent overconfidence of the manager is more likely to occur, when the ability levels of the two players are rather close to each other,

responsibility hoarding is most likely to be observed, when the overconfident manager's true ability is close to the ability level of a high ability agent.

Finally, there are constellations of ability parameters for which manager overconfidence and responsibility hoarding have a sustained effect on the total welfare of all involved parties (the white area and the white-dotted area). However, persistent manager overconfidence and responsibility hoarding are welfare increasing only if the manager is indeed more able than the agent (the white-dotted area). This finding is in line with our results of the first-best case that the critical task should always be carried out by the more able individual due to its higher impact on total outcome.

# 6 Conclusions and managerial implications

We study the consequences of manager overconfidence for organizational performance in a setting in which the manager chooses the allocation of tasks. We prove that an overconfident manager may exhibit responsibility hoarding behavior, i.e. assign the critical task more often to himself than a rational manager would. We show that while responsibility hoarding generally decreases the manager's individual utility, it tends to increase the firm output and the total welfare of the involved parties, when compared to the case of a fully rational manager. The reason for this seemingly counter-intuitive result is that overconfident managers generally exert higher levels of effort than rational managers, because they overestimate their own productivity. In this regard, overconfidence counterbalances shirking, causing managers to take up more responsibility and to reduce the inefficiency of their effort minimizing task allocation.

Hence, our results imply that firms will not generally avoid overconfident and responsibility hoarding managers, but may even prefer them to fully rational managers. In a situation where the firm cannot establish a contract to enforce the efficient allocation of tasks, moderate overconfidence of a manager can mitigate the negative effects of free-riding. The firm may prefer to hire a moderately overconfident manager to avoid the incomplete contract problem. In connection with the well-established evidence that men are generally more overconfident than women (see e.g. Barber and Odean (2001), Correll (2001), Bengtsson et al. (2005)), our result may also provide a further possible explanation why leadership positions are more often occupied by men than by women.[5]

Moreover, we have shown that an overconfident manager's biased self-perception and his responsibility hoarding behavior can persist, as long as the manager can rationalize observed outcomes, by underestimating the ability of the agent. Notably, the probability of persistent overconfidence does not only depend on the level of overconfidence, but also on the absolute level of the players' true abilities. The higher the ability levels in a workplace, the more likely it is to observe persistent overconfidence. This is due to the fact that high-ability agents can be underestimated to a greater extent than low-ability agents. The more an agent's ability can be underestimated, the easier it is for an overconfident manager to rationalize the observed output without having to adapt the overestimation of his own ability. Hence, responsibility hoarding is more likely to be widespread and persistent in workplaces with high ability workers

[5] For a similar result, based on a different type of equilibrium see also Palomino and Peyrache (2010).

and low accountability of work output than in settings with low ability workers or with a high traceability of exerted work effort.

Responsibility hoarding is also more likely to persist in situations, in which the asymmetry between tasks is relatively low. The more similar tasks are in their impact on total output, the easier it is for the overconfident manager to rationalize the observed total output by underestimating the contribution of the agent. If, in contrast, the task that the agent performs has very little impact on total output, the overconfident manager will find it difficult to rationalize observed low output levels without having to adapt the biased assessment of his own ability.

Note that if there is persistent responsibility hoarding at a workplace, the agent's work satisfaction will most probably decrease over time, due to the continued underestimation of her true ability. Hence, while the biased perception of the overconfident manager motivates him to exert more effort than a rational manager would, it may also cause lower satisfaction levels amongst the agents, leading to more tensions at the workplace and higher turnover rates. Interestingly, a high turnover rate amongst agents may even further support the persistence of the manager's overconfidence, because a constant input of new agents tends to reduce the power of the statistical evidence that would be needed for the overconfident manager to discover his self-perception bias.

Finally, our analysis also implies that allowing overconfident employees to choose their tasks may lead to less free-riding in management than predicted in a model with fully rational employees. Especially when the cost of a centrally planned task allocation is high, allowing overconfident employees to volunteer for high-effort tasks may be a cost efficient second-best solution.

## References


Barber, B. M. and T. Odean (2000). Trading is hazardous to your wealth: the common stock investment performance of individual investors. *Journal of Finance 55*, 773–806.

Barber, B. M. and T. Odean (2001). Boys will be boys: gender, overconfidence, and common stock investment. *Quarterly Journal of Economics 116*, 261–292.

Ben-David, I., J. Graham, and C. Harvey (2007). Managerial overconfidence and corporate policies. *NBER Working Paper 13711*.

Bengtsson, C., M. Persson, and P. Willenhag (2005). Gender and overconfidence. *Economics Letters 86*, 199–203.

Brunnermeier, M. K. and J. A. Parker (2005). Optimal expectations. *American Economic Review 95*(4), 1092–1118.

Camerer, C. and D. Lovallo (1999). Overconfidence and excess entry: An experimental approach. *American Economic Review 89*(1), 306–318.

Correll, S. J. (2001). Gender and the career choice process: the role of biased self-assessments. *American Journal of Sociology 106*, 1691–1730.

De Bondt, W. F. M. and R. H. Thaler (1996). Financial decision-making in markets and firms: A behavioral perspective. In *Handbook in Operations Research and Management Science, Vol. 9*. North Holland.

Deaves, R., E. Lüders, and G. Luo (2008). An experimental test of the impact of overconfidence and gender on trading activity. *Review of Finance 13*, 555–575.

Englmaier, F. (2011). Commitment in r&d tournaments via strategic delegation to overoptimistic managers. *Managerial and Decision Economics 32*, 63–69.

Galasso, A. and T. S. Simcoe (2011). CEO overconfidence and innovation. *Management Science 57*, 1469–1484.

Gervais, S. and I. Goldstein (2007). The positive effects of biased self-perceptions in firms. *Review of Finance 11*, 453–496.

Grieco, D. and R. M. Hogarth (2009). Overconfidence in absolute and relative performance: The regression hypothesis and bayesian updating. *Journal of Economic Psychology 30*, 756–771.

Gürtler, O. (2008). On delegation under relational contracts. *International Journal of the Economics of Business 15*, 85–98.

Hackbarth, D. (2008). Managerial traits and capital structure decisions. *Journal of Financial and Quantitative Analysis 43*, 843–881.

Itoh, H. (1994). Job design, delegation and cooperation: A principal-agent analysis. *European Economic Review 38*, 691–700.

Itoh, H. (2001). Job design and incentives in hierarchies with team production. *Hitotsubashi Journal of Commerce and Management 36*, 1–17.

Kinari, Y., N. Mizutani, F. Ohtake, and H. Okudaira (2011). Overconfidence increases productivity. *Working Paper*.

Klayman, J., J. B. Soll, C. González-Vallejo, and S. Barlas (1999). Overconfidence: It depends on how, what, and whom you ask. *Organizational Behavior and Human Decision Processes 79*, 216–247.

Lichtenstein, S., B. Fischhoff, and L. D. Phillips (1982). Calibration of probabilities: The state of the art to 1980. In D. Kahneman, P. Slovic, and A. Tversky (Eds.), *Judgement under uncertainty: Heuristics and biases*, pp. 306–334. New York: Cambridge University Press.

Ludwig, S., P. C. Wichardt, and H. Wickhorst (2011). Overconfidence can improve an agent's relative and absolute performance in contests. *Economics Letters 110*(3), 193–196.

Malmendier, U. and G. Tate (2005). CEO overconfidence and corporate investment. *The Journal of Finance 60*, 2661–2700.

Malmendier, U. and G. Tate (2008). Who makes acquisitions? CEO overconfidence and the market's reaction. *Journal of Financial Economics 89*(1), 20–43.

Mobius, M. M., M. Niederle, P. Niehaus, and T. S. Rosenblat (2011). Managing self-confidence: Theory and experimental evidence. *NBER Working Paper No. 17014*.

Palomino, F. and E. Peyrache (2010). Psychological bias and gender wage gap. *Journal of Economic Behavior & Organization 76*, 563–573.

Palomino, F. and A. Sadrieh (2011). Overconfidence and delegated portfolio management. *Journal of Financial Intermediation 20*(2), 159–177.

Prendergast, C. J. (1995). A theory of responsibility in organizations. *Journal of Labor Economics 13*, 387–400.

Rabin, M. and J. L. Schrag (1999). First impressions matter: A model of confirmatory bias. *Quarterly Journal of Economics 114*(1), 37–82.

Russo, J. E. and P. J. H. Schoemaker (1992). Managing overconfidence. *Sloan Management Review 33*, 7–17.

Santos-Pinto, L. (2008). Positive self-image and incentives in organisations. *The Economic Journal 118*, 1315–1332.

Santos-Pinto, L. (2010). Positive self-image in tournaments. *International Economic Review 51*(2), 475–496.

Sautmann, A. (2011). Contracts for agents with biased beliefs: Some theory and an experiment. *Working Paper*.

Soll, J. B. (1996). Determinants of Overconfidence and Miscalibration: The Roles of Random Error and Ecological Structure. *Organizational Behavior and Human Decision Processes 65*, 117–137.

Svenson, O. (1981). Are we all less risky and more skillful than our fellow drivers? *Acta Psychologica 47*, 143–148.

Weinberg, B. A. (2009). A model of overconfidence. *Pacific Economic Review 14*(4), 502–515.

Young, H. P. (2002). Bounded rationality and learning. on the limits to rational learning. *European Economic Review 46*, 791–799.

# 7 Appendix

## 7.1 Proof of Proposition 3

First, we consider the case if $a_M > \sqrt{2}a_A$. In this case, both a fully rational manager and an overconfident manager will choose non-delegation. This is also the individually optimal task allocation for the manager. The total welfare with a fully rational manager is equal to the sum of the utilities of manager and agent which is given by

$$\begin{aligned} W^{nd*} &= U_M^{nd*} + U_A^{nd*} \\ &= \left(\lambda a_M e_M^{nd*} + (1-\lambda)\, a_A e_A^{nd*}\right) - \frac{c}{2} e_M^{nd*2} - \frac{c}{2} e_A^{nd*2} \\ &= \frac{3\lambda^2 a_M^2}{8c} + \frac{3\,(1-\lambda)^2\, a_A^2}{8c}. \end{aligned} \tag{7}$$

With an overconfident manager it is equal to

$$\begin{aligned} W^{OCnd*} &= U_M^{OCnd*} + U_A^{OCnd*} \\ &= \left(\lambda a_M e_M^{OCnd*} + (1-\lambda)\, a_A e_A^{OCnd*}\right) - \frac{c}{2} e_M^{OCnd*2} - \frac{c}{2} e_A^{OCnd*2} \\ &= \frac{3\lambda^2 a_M^2}{8c} + \frac{3\,(1-\lambda)^2\, a_A^2}{8c} + \frac{\lambda^2\,(2a_M - b)\, b}{8c}. \end{aligned} \tag{8}$$

Comparing (7) and (8), it follows directly that the total welfare with an overconfident manager is strictly higher than with a fully rational manager if $b < 2a_M$.

Second, we consider the case if $a_M < \sqrt{2}a_A - b$. In this case, both a fully rational manager and an overconfident manager will choose delegation, which is also the individually optimal task allocation for the manager. The total welfare with a fully rational manager is given by

$$\begin{aligned} W^{d*} &= U_M^{d*} + U_A^{d*} \\ &= \left((1-\lambda)\, a_M e_M^{d*} + \lambda a_A e_A^{d*}\right) - \frac{c}{2} e_M^{d*2} - \frac{c}{2} e_A^{d*2} \\ &= \frac{3\,(1-\lambda)^2\, a_M^2}{8c} + \frac{3\lambda^2 a_A^2}{8c}. \end{aligned} \tag{9}$$

With an overconfident manager total welfare is

$$\begin{aligned} W^{OCd*} &= U_M^{OCd*} + U_A^{OCd*} \\ &= \left((1-\lambda)\, a_M e_M^{OCd*} + \lambda a_A e_A^{OCd*}\right) - \frac{c}{2} e_M^{OCd*2} - \frac{c}{2} e_A^{OCd*2} \\ &= \frac{3\,(1-\lambda)^2\, a_M^2}{8c} + \frac{3\lambda^2 a_A^2}{8c} + \frac{(1-\lambda)^2\,(2a_M - b)\, b}{8c}. \end{aligned} \tag{10}$$

Again by comparing (9) and (10), the total welfare with an overconfident manager is strictly higher than with a fully rational manager if $b < 2a_M$.

Finally, we consider the non-trivial case, in which $a_M \leq \sqrt{2}a_A \leq a_M + b$. In this case, a fully rational manager chooses to delegate the critical task to the agent, while an overconfident manager carries out the critical task himself.

Comparing (8) and (9), the total welfare is higher with an overconfident manager if

$$(2\lambda - 1)(a_M + a_A)(a_M - a_A) + \frac{\lambda^2}{3}(2a_M - b)\, b \geq 0$$

Note that as long as the manager's ability is at least as high as the agent's, i.e. $a_M \geq a_A$, and the manager's degree of overconfidence is on a moderate level, i.e. $b < 2a_M$, the total welfare with an overconfident manager is higher than with a fully rational manager. ■

## 7.2 Proof of Proposition 4

First, we consider the case $a_M > \sqrt{2}a_A - b$, in which an overconfident manager chooses not to delegate the critical task. In this case, the total welfare is given by

$$U^{OCnd*} = \frac{3\lambda^2 a_M^2}{8c} + \frac{3(1-\lambda)^2 a_A^2}{8c} + \frac{\lambda^2(2a_M - b)\, b}{8c}.$$

By solving the following optimization problem

$$\begin{aligned} &\max_b U^{OCnd*} \\ &s.t.\ b > \sqrt{2}a_A - a_M \end{aligned}$$

we obtain

$$\begin{aligned} & \frac{\partial U^{OCnd*}}{\partial b} \doteq 0 \\ \Leftrightarrow\ & \frac{\lambda^2}{4c}(a_M - b) = 0 \\ \Leftrightarrow\ & b^* = a_M. \end{aligned}$$

Note that the second-order condition is automatically satisfied as $U^{OCnd*}$ is strictly concave in $b$. Furthermore, the constraint $b > \sqrt{2}a_A - a_M$ is also satisfied as long as $a_M > \frac{\sqrt{2}}{2}a_A$. Hence,

$$b^* = a_M \text{ if } a_M > \frac{\sqrt{2}}{2}a_A$$

Second, we consider the case $a_M \leq \sqrt{2}a_A - b$, in which an overconfident manager chooses to delegate the critical task. In this case, the total welfare is given by

$$U^{OCd*} = \frac{3(1-\lambda)^2 a_M^2}{8c} + \frac{3\lambda^2 a_A^2}{8c} + \frac{(1-\lambda)^2(2a_M - b)\, b}{8c}.$$

Again, by solving the following optimization problem

$$\begin{aligned} &\max_b U^{OCd*} \\ &s.t.\ b \leq \sqrt{2}a_A - a_M \end{aligned}$$

we obtain

$$\begin{aligned} & \frac{\partial U^{OCd*}}{\partial b} \doteq 0 \\ \Leftrightarrow \quad & \frac{(1-\lambda)^2}{4c}(a_M - b) = 0 \\ \Leftrightarrow \quad & b^* = a_M. \end{aligned}$$

Note that the second-order condition is automatically satisfied as $U^{OCd*}$ is strictly concave in $b$. Furthermore the constraint $b \leq \sqrt{2}a_A - a_M$ is also satisfied as long as $a_M \leq \frac{\sqrt{2}}{2}a_A$. Hence,

$$b^* = a_M \text{ if } a_M \leq \frac{\sqrt{2}}{2}a_A$$

and the optimal degree of overconfidence is given $b^* = a_M$. ■

### 7.3 Proof of Proposition 6

We prove the results by first analyzing the case of non-delegation and then the case of delegation. We derive the two conditions for sustained overconfidence. We then show that as long as the critical task contributes more to the total output than the other task, i.e. as long as $\frac{1}{2} < \lambda < 1$, the condition stated in the proposition is binding for both delegation and non-delegation situations. Finally, we show that the higher the manager's self-perception bias, the more the agent's ability is underestimated.

*1. Persistence of overconfidence in the case of non-delegation*

Our essential assumption is that the manager will not revise his assessment of the own ability as long as he observes outcomes that can be rationalized by varying the two unknown parameters, i.e. the agent's ability and effort level. As long as any observed outcome can be rationalized by the manager, overconfidence is persistent. In the following, we derive the sufficient condition for the persistence of overconfidence in case of non-delegation.

Recall that the total output in case of non-delegation is given by

$$Y^{OCnd*} = \lambda a_M e_M^{OCnd*} + (1-\lambda)\, a_A e_A^{OCnd*}.$$

If an overconfident manager observes this total output, he overestimates his own contribution and underestimates the agent's contribution as follows:

$$Y_P^{OCnd} = \lambda\,(a_M + b)\, e_M^{OCnd*} + (1-\lambda)\,\widehat{a}_A \widehat{e}_A^{OCnd*}.$$

This biased model (i.e. the overconfident rationalization of the observed output) is only feasible as long as the following condition holds:

$$\begin{aligned} & Y^{OCnd*} \geq \lambda\,(a_M + b)\, e_M^{OCnd*} \\ \Leftrightarrow \quad & \lambda a_M e_M^{OCnd*} + (1-\lambda)\, a_A e_A^{OCnd*} \geq \lambda\,(a_M + b)\, e_M^{OCnd*} \\ \Leftrightarrow \quad & a_A \geq \frac{\lambda}{(1-\lambda)}\sqrt{(a_M b + b^2)}. \end{aligned}$$

Next, we check for the degree of underestimation of the agent's contribution in case of non-delegation. Let $\tau \equiv a_A - \widehat{a}_A$ denote the underestimation of the agent's ability. Since $Y^{OCnd*} = Y_P^{OCnd}$ we can determine the level of underestimation by solving the following equation:

$$\begin{aligned}
& Y^{OCnd*} = Y_P^{OCnd} \\
\Leftrightarrow \quad & \lambda a_M e_M^{OCnd*} + (1-\lambda)\, a_A e_A^{OCnd*} = \lambda\,(a_M + b)\, e_M^{OCnd*} + (1-\lambda)\,\widehat{a}_A \widehat{e}_A^{OCnd*} \\
\Leftrightarrow \quad & a_A - \widehat{a}_A = \frac{\lambda^2}{(1-\lambda)^2} \frac{a_M b + b^2}{(a_A + \widehat{a}_A)} \\
\Leftrightarrow \quad & \tau = \frac{\lambda^2}{(1-\lambda)^2} \frac{a_M b + b^2}{(a_A + \widehat{a}_A)} > 0.
\end{aligned}$$

Since $\tau$ is higher than zero, we have established a positive underestimation of the agent's ability that increases in the manager's self-perception bias $b$.

*2. Persistence of overconfidence in the case of delegation*

Now, we derive the sufficient condition for the persistence of overconfidence in case the manager delegates the critical task to the agent. Recall that the total output in the case of delegation is

$$Y^{OCd*} = (1-\lambda)\, a_M e_M^{OCd*} + \lambda a_A e_A^{OCd*}.$$

The overconfident manager rationalizes the observation of this output as follows

$$Y_P^{OCd} = (1-\lambda)\,(a_M + b)\, e_M^{OCd*} + \lambda \widehat{a}_A \widehat{e}_A^{OCd*}.$$

The overestimation of the own contribution (underestimation of the agent's ability) is only feasible as long as the following condition holds:

$$\begin{aligned}
& Y^{OCd*} \geq (1-\lambda)\,(a_M + b)\, e_M^{OCd*} \\
\Leftrightarrow \quad & (1-\lambda)\, a_M e_M^{OCd*} + \lambda a_A e_A^{OCd*} \geq (1-\lambda)\,(a_M + b)\, e_M^{OCd*} \\
\Leftrightarrow \quad & a_A \geq \frac{(1-\lambda)}{\lambda} \sqrt{(a_M b + b^2)}.
\end{aligned}$$

Analogous to the non-delegation case, we check for the degree of underestimation of the agent's contribution in case of delegation. Let $\tau \equiv a_A - \widehat{a}_A$ denote the underestimation of the agent's ability. Since $Y^{d*} = Y_P^d$ we can determine the level of underestimation by solving the following equation:

$$\begin{aligned}
& Y^{OCd*} = Y_P^{OCd} \\
\Leftrightarrow \quad & (1-\lambda)\, a_M e_M^{OCd*} + \lambda a_A e_A^{OCd*} = (1-\lambda)\,(a_M + b)\, e_M^{OCd*} + \lambda \widehat{a}_A \widehat{e}_A^{OCd*} \\
\Leftrightarrow \quad & \tau = \frac{(1-\lambda)^2}{\lambda^2} \frac{a_M b + b^2}{(a_A + \widehat{a}_A)} > 0.
\end{aligned}$$

Again, we find that $\tau$ is higher than zero, i.e. the overconfident manager underestimates the agent's ability and the underestimation increases in the manager's self-perception bias $b$.

*3. General conditions for both cases*

Taking the results of the two parts together, we can show that for all cases in which the critical task contributes more to the total output than the other task, i.e. as long as $\frac{1}{2} < \lambda < 1$, the condition for persistence in the second case (delegation) is generally more restrictive than in the first case:

$$\frac{\lambda}{1-\lambda} > \frac{1-\lambda}{\lambda}, \text{ for any } \lambda \in \left(\frac{1}{2}, 1\right).$$

Hence, if $a_A \geq \frac{\lambda}{(1-\lambda)}\sqrt{(a_M b + b^2)}$ is true, then the condition $a_A \geq \frac{(1-\lambda)}{\lambda}\sqrt{(a_M b + b^2)}$ also holds, allowing us to use the former as a general condition in the proposition. ■