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A Note on “Modeling the Birth and Death of Cartels with An Application to Evaluating Competition Policy” by Harrington and Chang (2009)

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A Note on "Modeling the Birth and Death of Cartels with An Application to Evaluating Competition Policy" by Harrington and Chang (2009)[1]

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In the December 2009 issue of the Journal of European Economic Association, Harrington and Chang presented a model of dynamic cartel formation and dissolution where an industry of firms interact repeatedly over an infinite time horizon. Absent antitrust intervention, there is a "marginal industry" in which firms are indifferent between collusion and competing because the short-run gain of cheating for each firm equals its long-run benefit from colluding. An efficacious antitrust innovation works its effect by increasing a firm's short-run benefit from cheating to a level that exceeds its long-run gains from colluding. In this way, the policy-innovation moves the "marginal type" from a population of sustainable, longer-lived cartels to a population of unstable, shorter-lived ones. The model generates intuitive predictions that can be used to assess the efficacy of antitrust innovations (such as the leniency program): The impact of an efficacious policy on the duration of discovered cartels is *time-dependent*. In particular, following an antitrust innovation that increases probability of detection, the marginal cartels immediately break up and the ensuing cartel discovery comes from a population of longer-lasting cartels. Because of such a sample selection effect, the average duration of discovered cartels increases in the short-run. That is, the short-run distribution of cartel duration dominates the steady-state pre-innovation distribution in the sense of first order stochastic dominance (FOSD) (Theorem 7 of Harrington and Chang); in the long run, the duration decreases due to the enhanced overall deterrence. That is, the post-innovation steady-state distribution of cartel duration dominates the short-run one in the sense of FOSD (Theorem 8 of Harrington and Chang).

These theoretical predictions can be tested empirically but not direct ways. This is because the estimation of the cartel duration from discovered cartels must consider the censoring of

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duration for cartels ending due to antitrust interventions (Levenstein and Suslow (forthcoming)). For such cartels, we can only infer that collusion would have exceeded the observed cartel duration at the time of the cartel's dissolution. In this note, I provide two stronger theorems than Theorems 7 and 8 in Harrington and Chang. My results can be directly corroborated in a empirical model of *survival analysis*— a by now standard approach to the analysis of cartel durations.[2] They relate to the probability that a cartel survives for t periods conditional on the event that the cartel survives for at least t periods, i.e., the *dissolution hazard of discovered cartels*.

In particular, I show within Harrington and Chang's framework that (1) in the short-run an after an antitrust innovation that raises the probability of detection, the distribution of cartel duration shifts and dominates, in the sense of hazard rate dominance (HRD), the pre-innovation distribution; and that (2) in the long run after the innovation, the distribution readjusts and dominates the short-run distribution in the sense of HRD.

2. The Model

2.1. *Industrial Behavior*

Consider the following dynamic model of cartel formation and dissolution that is adapted from Harrington and Chang (2009). There is a population of oligopolistic industries. Time is discrete and N identical firms play an infinitely repeated Prisoner's Dilemma in each industry. In each period, there is a stochastic realization of a market's profitability that is summarized by π. Each firm earns π if they collude; if not, they compete and each firm earns $\alpha\pi$; without loss of generality, I normalize α to 0. A cartel participant earns $\theta\pi$ (with $\theta > 1$) by unilaterally deviating from a collusive arrangement, where θ represents the *value of deviation*. θ is drawn from a distribution F with support $[\underline{\theta}, \bar{\theta}]$ and positive continuous density f. At the beginning of each period, π is observed by the firms prior to deciding how to behave. π is given by a distribution G with support $[\underline{\pi}, \bar{\pi}]$ and positive continuous density g. The firms discount time at the same rate; their discount factor is δ where $0 < \delta < 1$.

At the beginning of each period, industries are either cartelized or not. Industries that were cartelized at the end of the previous period are currently cartelized; Industries that were not cartelized at the end of the previous period have an opportunity to do so with probability p (with $0 < p < 1$). If a cartel collapses at the end of a period – either due to self-defect or an

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antitrust intervention – then with probability p the industry has an opportunity to re-cartelize in the next period. Let y^0 (resp. w^0) denote the present value of a firm's payoff when an industry is cartelized (resp. not cartelized). If the industry is not currently cartelized, then with probability p it can cartelize with each firm earning y^0; with probability $1-p$ the firms continue to compete and each firm earns 0 in the present period and w^0 in the following periods. It follows that $w^0 = py^0 + (1-p)\delta w^0$.

An antitrust policy is a pair of parameters $\langle \sigma, \gamma \rangle$, where $\sigma \in (0,1)$ is the probability that the antitrust authority detects and penalizes a cartel at the end of each period. The (present value of) total amount of fines that a cartel participant pays is γy^0, where $\gamma > 0$ is the *fine multiplier*.

Suppose that a cartel has formed and that π is realized. If a firm sticks to the collusive agreement, then it earns π in the present period and with probability $1-\sigma$ it escapes detection and continues to earn y^0 in the following periods; and with probability σ it is caught and pays a fine of γy^0 at the end of the present period and earns w^0 in the following periods. If, however, the firm unilaterally deviates from the agreement, then it earns $\theta\pi$ in the present period and w^0 in the following periods; Moreover, at the end of the present period it will be fined for an amount of γy^0 with probability σ.[3] Write $y \equiv (1-\delta)y^0$ as the re-scaled payoff. A cartel is sustainable if a firm's payoff from collusion exceeds that from cheating, i.e., if $\pi + \delta[(1-\sigma)y^0 + \sigma(w^0 - \gamma y^0)] \geq \theta\pi + \delta(w^0 - \sigma\gamma y^0)$. Substituting and rearranging the inequality, we have that a cartel is sustainable if $\pi \leq \frac{\delta(1-\sigma)(1-p)y}{[1-\delta(1-p)](\theta-1)}$. Denote $\frac{\delta(1-\sigma)(1-p)y}{[1-\delta(1-p)](\theta-1)}$ by $\varphi(y)$. It follows that the present value of collusion is:

$$y^0 = \int_{\underline{\pi}}^{\varphi(y)} \left\{ \pi + \frac{\delta}{1-\delta} \left[(1-\sigma)y^0 + \sigma(w^0 - \gamma y^0)\right] \right\} g(\pi)d\pi + \int_{\varphi(y)}^{\overline{\pi}} \frac{\delta}{1-\delta}(w^0 - \sigma\gamma y^0)g(\pi)d\pi, \quad (1)$$

where the first (resp. second) term of the right-hand side of equation (1) is a firm's expected payoff when a cartel can (resp. cannot) be sustained. Substituting and rearranging (1) we have, due to Harrington and Chang (2009), that y is given by:

$$y = \int_{\underline{\pi}}^{\varphi(y)} \left\{ (1-\delta)\pi + y\left[\delta - \frac{\delta\sigma(1-p)(1-\delta)}{1-\delta(1-p)}\right] \right\} g(\pi)d\pi + \int_{\varphi(y)}^{\overline{\pi}} \frac{\delta p y}{1-\delta(1-p)} g(\pi)d\pi$$

[3] The assumption that cheating cartel participants do not escape prosecution is consistent with the actual practice of the European Commission and the European Court of Justice. See, e,g., Judgment of The Court of Justice in Case C-260/09 P on 10 February 2011, para 19. See also Commission Decision in Case COMP/38.695 on 11 June 2008, para 531.

$$- \delta\sigma\gamma y. \tag{2}$$

Denote the right-hand side of equation (2) by $\psi(y)$ and let $y^* = \max\{0 \le y \le \mu \mid \psi(y) = y\}$ be the maximal fixed point of $\psi(\cdot)$. The steady-state probability that a cartel survives in any period– the joint probability that it survives both market fluctuations and detection – is given by $q(\sigma, \theta) = G(\varphi(y^*))\,(1-\sigma)$.

Let $s(t; \sigma, \theta)$ denote the the steady-state share of cartels with a duration of t periods in a type-θ industry under policy σ, where $t \in \{0, 1, 2, ...\}$; $1 - \sum_{\hat{t}=0}^{t-1} s(\hat{t}, \sigma, \theta)$ is then the share that survive for at least t periods. $t = 0$ means that industry θ is not cartelized. These steady-state shares follow from Harrington and Chang (2009):

$$s(0; \sigma, \theta) = \frac{1 - q(\sigma, \theta)}{1 - (1-p)q(\sigma, \theta)}; \tag{3}$$

$$s(t; \sigma, \theta) = \frac{p\,(1 - q(\sigma, \theta))\,q(\sigma, \theta)^t}{1 - (1-p)q(\sigma, \theta)} \text{ for all } t \in \{1, 2, ...\}; \tag{4}$$

$$1 - \sum_{\hat{t}=0}^{t-1} s(\hat{t}; \sigma, \theta) = \frac{p\,q(\sigma, \theta)^t}{1 - (1-p)q(\sigma, \theta)}. \tag{5}$$

2.2. *The Hazard Rate of Cartel Dissolution*

From the results above, I derive theoretical predictions that can be tested empirically. They relate to the probability that a cartel survives for t periods conditional on the event that the cartel survives for at least t periods, i.e., the *dissolution hazard of discovered cartels*. An antitrust innovation, such as a leniency program, affects the hazard over time. I model an antitrust innovation as an exogenous change in the detection rate from σ_1 to σ_2 (with $\sigma_2 > \sigma_1$). Equations (3), (4) and (5) give the *steady-state dissolution hazard of discovered cartels in industry* θ prior to the innovation:

$$h(t; \sigma_1, \theta) = \frac{s(t; \sigma_1, \theta)}{1 - \sum_{\hat{t}=0}^{t-1} s(\hat{t}, \sigma_1, \theta)} = 1 - q(\sigma_1, \theta), \quad t \in \{1, 2, ...\}; \tag{6}$$

and the *average steady-state dissolution hazard of discovered cartels* prior to the innovation:

$$\tilde{h}(t; \sigma_1) = \frac{\int_{\Theta_1} s(t; \sigma_1, \theta) f(\theta) d\theta}{1 - \int_{\Theta_1} \sum_{\hat{t}=0}^{t-1} s(\hat{t}, \sigma_1, \theta) f(\theta) d\theta}, \quad t \in \{1, 2, ...\}, \tag{7}$$

where Θ_1 is the set of industries in which collusion can be sustained prior to the innovation. Rearranging (7), we have that

$$\tilde{h}(t; \sigma_1) = \int_{\Theta_1} \left[\frac{s(t; \sigma_1, \theta)}{1 - \sum_{\hat{t}=0}^{t-1} s(\hat{t}, \sigma_1, \theta)} \times \frac{\left(1 - \sum_{\hat{t}=0}^{t-1} s(\hat{t}, \sigma_1, \theta)\right) f(\theta)}{\int_{\Theta_1} \left(1 - \sum_{\hat{t}=0}^{t-1} s(\hat{t}, \sigma_1, \theta)\right) f(\theta) d\theta} \right] d\theta$$

$$= \int_{\Theta_1} \left[h(t;\sigma_1,\theta) \times \frac{\left(1 - \sum_{\hat{t}=0}^{t-1} s(\hat{t},\sigma_1,\theta)\right) f(\theta)}{\int_{\Theta_1} \left(1 - \sum_{\hat{t}=0}^{t-1} s(\hat{t},\sigma_1,\theta)\right) f(\theta) d\theta} \right] d\theta. \tag{8}$$

$\tilde{h}(t;\sigma_1)$ is then the weighted average of $h(t;\sigma_1,\theta)$, where the associated weight is the probability that a cartel with a duration of at least t periods is of type θ.

After the innovation, a subset of industries in Θ_1 become no longer capable of sustaining collusion and the distribution of cartels shifts, immediately, from

$$\frac{\left(1 - \sum_{\hat{t}=0}^{t-1} s(\hat{t},\sigma_1,\theta)\right) f(\theta)}{\int_{\Theta_1} \left(1 - \sum_{\hat{t}=0}^{t-1} s(\hat{t},\sigma_1,\theta)\right) f(\theta) d\theta} \text{ to } \frac{\left(1 - \sum_{\hat{t}=0}^{t-1} s(\hat{t},\sigma_1,\theta)\right) f(\theta)}{\int_{\Theta_2} \left(1 - \sum_{\hat{t}=0}^{t-1} s(\hat{t},\sigma_1,\theta)\right) f(\theta) d\theta},$$

where Θ_2 denotes the set of industries capable of sustaining collusion after the innovation. But in the short run durations stay unadjusted for the remaining cartels, i.e., their dissolution hazard is unchanged. The average dissolution hazard shifts, in the short run, to:

$$\tilde{h}(t;\sigma_1,\sigma_2) = \int_{\Theta_2} \left[h(t;\sigma_1,\theta) \times \frac{\left(1 - \sum_{\hat{t}=0}^{t-1} s(\hat{t},\sigma_1,\theta)\right) f(\theta)}{\int_{\Theta_2} \left(1 - \sum_{\hat{t}=0}^{t-1} s(\hat{t},\sigma_1,\theta)\right) f(\theta) d\theta} \right] d\theta. \tag{9}$$

The transition from the short run to the new steady state involves the duration of the surviving cartels adjusting in each industry: The industry-level hazard shifts from $h(t;\sigma_1,\theta)$ to $h(t;\sigma_2,\theta) = \frac{s(t;\sigma_2,\theta)}{1-\sum_{\hat{t}=0}^{t-1} s(\hat{t},\sigma_2,\theta)}$. As a result, the average hazard readjusts, in the long run, to

$$\tilde{h}(t;\sigma_2) = \int_{\Theta_2} \left[h(t;\sigma_2,\theta) \times \frac{\left(1 - \sum_{\hat{t}=0}^{t-1} s(\hat{t},\sigma_2,\theta)\right) f(\theta)}{\int_{\Theta_2} \left(1 - \sum_{\hat{t}=0}^{t-1} s(\hat{t},\sigma_2,\theta)\right) f(\theta) d\theta} \right] d\theta.$$

I start by providing two lemmas that will be used in the proof of the main results of this note.

Lemma 1. (Hazard Rate Dominance): *The steady-state dissolution hazard in an industry is increasing in the profitability of deviation and the detection rate. That is,*

$\frac{\partial}{\partial\theta} h(t;\sigma,\theta) \geq 0$ *and* $\frac{\partial}{\partial\sigma} h(t;\sigma,\theta) \geq 0$ *for all* $\theta \in [\underline{\theta}, \tilde{\theta}(\sigma)]$, *for all* $\sigma \in (0,1)$, *and for all* $t \in \{1,2,...\}$.

Proof. Note that y^* is a function of θ and σ. Taking the derivative of $q(\sigma,\theta)$ with respect to θ, we have that $\frac{\partial}{\partial\theta} q(\sigma,\theta) = (1-\sigma) \times \frac{\partial}{\partial\varphi(y^*)} G(\varphi(y^*)) \times \frac{\partial}{\partial\theta}\varphi(y^*)$. By construction, $1-\sigma > 0$ and $\frac{\partial}{\partial\varphi(y^*)} G(\varphi(y^*)) \geq 0$. It follows from Harrington and Change (2009) that $\frac{\partial}{\partial\theta}\varphi(y^*) \leq 0$. Therefore, $\frac{\partial}{\partial\theta} q(\sigma,\theta) \leq 0$. That is, making deviation more profitable reduces the probability that a cartel will survive in any period. Taking the derivative of (6) with respective to θ, we obtain that $\frac{\partial}{\partial\theta} h(t;\sigma,\theta) = -\frac{\partial}{\partial\theta} q(\sigma,\theta) \geq 0$ for all $\theta \in [\underline{\theta}, \tilde{\theta}(\sigma)]$.

By performing similar steps, we obtain that $\frac{\partial}{\partial\sigma} h(t;\sigma,\theta) = -\frac{\partial}{\partial\sigma} q(\sigma,\theta) \geq 0$ for all $\theta \in [\underline{\theta}, \tilde{\theta}(\sigma)]$.

Q.E.D.

Lemma 2. *If σ is sufficiently small then there exists $\tilde{\theta}(\sigma) \in [\underline{\theta}, \bar{\theta}]$ such that $y^* \gtreqless 0$ for all $\theta \lesseqgtr \tilde{\theta}(\sigma)$. Moreover, $\tilde{\theta}(\sigma)$ is decreasing in σ. That is, there exists a cut-off value of deviation below which cartels can be sustained in steady state when the probability of detection is sufficiently low. Moreover, raising the detection rate reduces the cut-off value.*

Proof. The proof is given in Harrington and Chang (2009). I omit it.

We now arrive at the main result of the theoretical model:

Theorem 1. *If $\sigma_1 < \sigma_2$, then $\tilde{h}(t;\sigma_1) \geq \tilde{h}(t;\sigma_1,\sigma_2)$ for all $t \in \{1, 2, ...\}$. That is, an increase in the detection rate leads to an immediate fall in the average dissolution hazard of discovered cartels after an innovation.*

Proof. Due to Lemma 2, we can write Θ_1 and Θ_2 – the sets of industries capable of sustaining collusion under policies σ_1 and σ_2– as $[\theta, \theta]$ and $[\theta, \theta]$, respectively. For the sake of brevity, write

$$\rho(\sigma_1,\theta) = \frac{\left(1-\sum_{\hat{t}=0}^{t-1} s(\hat{t},\sigma_1,\theta)\right) f(\theta)}{\int_{\underline{\theta}}^{\tilde{\theta}(\sigma_1)} \left(1-\sum_{\hat{t}=0}^{t-1} s(\hat{t},\sigma_1,\theta)\right) f(\theta) d\theta} \text{ and } \rho(\sigma_1,\sigma_2,\theta) = \frac{\left(1-\sum_{\hat{t}=0}^{t-1} s(\hat{t},\sigma_1,\theta)\right) f(\theta)}{\int_{\underline{\theta}}^{\tilde{\theta}(\sigma_2)} \left(1-\sum_{\hat{t}=0}^{t-1} s(\hat{t},\sigma_1,\theta)\right) f(\theta) d\theta} \tag{10}$$

as the distribution of cartels under policy σ_1 in steady state and in distribution in the short run, respectively. Suppose that an antitrust innovation raises the probability of detection from σ_1 to σ_2 and the economy is in its steady state prior to the innovation. The stationary dissolution hazard under policy σ_1 can be rewritten as follows:

$$\begin{aligned}
\tilde{h}(t;\sigma_1) = \int_{\underline{\theta}}^{\tilde{\theta}(\sigma_2)} \rho(\sigma_1,\theta) h(t;\sigma_1,\theta) d\theta + \int_{\tilde{\theta}(\sigma_2)}^{\tilde{\theta}(\sigma_1)} \rho(\sigma_1,\theta) h(t;\sigma_1,\theta) d\theta = \frac{\rho(\sigma_1,\theta)}{\rho(\sigma_1,\sigma_2,\theta)} \\
\times \int_{\underline{\theta}}^{\tilde{\theta}(\sigma_2)} \rho(\sigma_1,\sigma_2,\theta) h(t;\sigma_1,\theta) d\theta + \frac{\rho(\sigma_1,\sigma_2,\theta)-\rho(\sigma_1,\theta)}{\rho(\sigma_1,\sigma_2,\theta)} \\
\times \int_{\tilde{\theta}(\sigma_2)}^{\tilde{\theta}(\sigma_1)} \frac{\rho(\sigma_1,\theta)\rho(\sigma_1,\sigma_2,\theta)}{\rho(\sigma_1,\sigma_2,\theta)-\rho(\sigma_1,\theta)} h(t;\sigma_1,\theta) d\theta.
\end{aligned}$$

It follows that

$$\tilde{h}(t;\sigma_1) - \tilde{h}(t;\sigma_1,\sigma_2) = \left(1 - \frac{\rho(\sigma_1,\theta)}{\rho(\sigma_1,\sigma_2,\theta)}\right) \times \left(\int_{\tilde{\theta}(\sigma_2)}^{\tilde{\theta}(\sigma_1)} \frac{\rho(\sigma_1,\theta)\rho(\sigma_1,\sigma_2,\theta)}{\rho(\sigma_1,\sigma_2,\theta)-\rho(\sigma_1,\theta)} h(t;\sigma_1,\theta) d\theta\right.$$

$$- \int_{\underline{\theta}}^{\tilde{\theta}(\sigma_2)} \rho(\sigma_1, \sigma_2, \theta) h(t; \sigma_1, \theta) d\theta \Bigg). \tag{11}$$

If $\sigma_2 > \sigma_1$, then $\tilde{\theta}(\sigma_1) \geq \tilde{\theta}(\sigma_2)$ due to Lemma 2. $\tilde{\theta}(\sigma_1) \geq \tilde{\theta}(\sigma_2)$ implies that

$$\int_{\underline{\theta}}^{\tilde{\theta}(\sigma_1)} \left(1 - \sum_{\hat{t}=0}^{t-1} s(\hat{t}, \sigma_1, \theta)\right) f(\theta) d\theta \geq \int_{\underline{\theta}}^{\tilde{\theta}(\sigma_2)} \left(1 - \sum_{\hat{t}=0}^{t-1} s(\hat{t}, \sigma_1, \theta)\right) f(\theta) d\theta.$$

Therefore,

$$\rho(\sigma_1, \sigma_2, \theta) = \frac{\left(1 - \sum_{\hat{t}=0}^{t-1} s(\hat{t}, \sigma_1, \theta)\right) f(\theta)}{\int_{\underline{\theta}}^{\tilde{\theta}(\sigma_2)} \left(1 - \sum_{\hat{t}=0}^{t-1} s(\hat{t}, \sigma_1, \theta)\right) f(\theta) d\theta} \geq \frac{\left(1 - \sum_{\hat{t}=0}^{t-1} s(\hat{t}, \sigma_1, \theta)\right) f(\theta)}{\int_{\underline{\theta}}^{\tilde{\theta}(\sigma_1)} \left(1 - \sum_{\hat{t}=0}^{t-1} s(\hat{t}, \sigma_1, \theta)\right) f(\theta) d\theta} = \rho(\sigma_1, \theta).$$

We therefore obtain that $1 - \frac{\rho(\sigma_1,\theta)}{\rho(\sigma_1,\sigma_2,\theta)} \geq 0$. Substituting (10) into the second term on the right-hand side of equation (11) and rearranging, we have that

$$\tilde{h}(t; \sigma_1) - \tilde{h}(t; \sigma_1, \sigma_2) = \left(1 - \frac{\rho(\sigma_1, \theta)}{\rho(\sigma_1, \sigma_2, \theta)}\right) \times \left\{ \int_{\tilde{\theta}(\sigma_2)}^{\tilde{\theta}(\sigma_1)} \left[\frac{\left(1 - \sum_{\hat{t}=0}^{t-1} s(\hat{t}, \sigma_1, \theta)\right) f(\theta)}{\int_{\tilde{\theta}(\sigma_2)}^{\tilde{\theta}(\sigma_1)} \left(1 - \sum_{\hat{t}=0}^{t-1} s(\hat{t}, \sigma_1, \theta)\right) f(\theta) d\theta} \right] h(t; \sigma_1, \theta) d\theta \right.$$

$$\left. - \int_{\underline{\theta}}^{\tilde{\theta}(\sigma_2)} \left[\frac{\left(1 - \sum_{\hat{t}=0}^{t-1} s(\hat{t}, \sigma_1, \theta)\right) f(\theta)}{\int_{\underline{\theta}}^{\tilde{\theta}(\sigma_2)} \left(1 - \sum_{\hat{t}=0}^{t-1} s(\hat{t}, \sigma_1, \theta)\right) f(\theta) d\theta} \right] h(t; \sigma_1, \theta) d\theta \right\}.$$

Due to Lemma 1, we have that $h(t; \sigma, \theta_a) \geq h(t; \sigma, \theta_b)$ for all $\theta_a \in (\tilde{\theta}(\sigma_2), \tilde{\theta}(\sigma_1)]$ and for all $\theta_b \in [\underline{\theta}, \tilde{\theta}(\sigma_2)]$. It follows that

$$\int_{\tilde{\theta}(\sigma_2)}^{\tilde{\theta}(\sigma_1)} \left[\frac{\left(1 - \sum_{\hat{t}=0}^{t-1} s(\hat{t}, \sigma_1, \theta)\right) f(\theta)}{\int_{\tilde{\theta}(\sigma_2)}^{\tilde{\theta}(\sigma_1)} \left(1 - \sum_{\hat{t}=0}^{t-1} s(\hat{t}, \sigma_1, \theta)\right) f(\theta) d\theta} \right] h(t; \sigma_1, \theta) d\theta \geq$$

$$\int_{\underline{\theta}}^{\tilde{\theta}(\sigma_2)} \left[\frac{\left(1 - \sum_{\hat{t}=0}^{t-1} s(\hat{t}, \sigma_1, \theta)\right) f(\theta)}{\int_{\underline{\theta}}^{\tilde{\theta}(\sigma_2)} \left(1 - \sum_{\hat{t}=0}^{t-1} s(\hat{t}, \sigma_1, \theta)\right) f(\theta) d\theta} \right] h(t; \sigma_1, \theta) d\theta.$$

Therefore, if $\sigma_2 > \sigma_1$, then $\tilde{h}(t; \sigma_1) \geq \tilde{h}(t; \sigma_1, \sigma_2)$ for all $t \in \{1, 2, ...\}$.

Q.E.D.

Theorem 2. *If $\sigma_1 < \sigma_2$, then $\tilde{h}(t; \sigma_1, \sigma_2) \leq \tilde{h}(t; \sigma_2)$ for all $t \in \{1, 2, ...\}$. That is, after the immediate fall in the average hazard of discovered cartels following an increase in the detection rate, the hazard readjusts above the short-run levels.*

Proof. Integration by parts yield that

$$\tilde{h}(t; \sigma_2) = \int_{\underline{\theta}}^{\tilde{\theta}(\sigma_2)} \left(h(t; \sigma_2, \theta) \times \left\{ \int_{\underline{\theta}}^{\tilde{\theta}(\sigma_2)} \left[\frac{\left(1 - \sum_{\hat{t}=0}^{t-1} s(\hat{t}, \sigma_2, \theta)\right) f(\theta)}{\int_{\underline{\theta}}^{\tilde{\theta}(\sigma_2)} \left(1 - \sum_{\hat{t}=0}^{t-1} s(\hat{t}, \sigma_2, \theta)\right) f(\theta) d\theta} \right] d\theta \right\} \right) d\theta$$

$$- \int_{\underline{\theta}}^{\tilde{\theta}(\sigma_2)} \left(\frac{\partial}{\partial \theta} h(t;\sigma_2,\theta) \times \left\{ \int_{\underline{\theta}}^{\tilde{\theta}(\sigma_2)} \left[\frac{\left(1 - \sum_{\hat{t}=0}^{t-1} s(\hat{t},\sigma_2,\theta)\right) f(\theta)}{\int_{\underline{\theta}}^{\tilde{\theta}(\sigma_2)} \left(1 - \sum_{\hat{t}=0}^{t-1} s(\hat{t},\sigma_2,\theta)\right) f(\theta) d\theta} \right] d\theta \right\} \right) d\theta$$

$$= \int_{\underline{\theta}}^{\tilde{\theta}(\sigma_2)} h(t;\sigma_2,\theta) d\theta - \left(h(t;\sigma_2,\tilde{\theta}(\sigma_2)) - h(t;\sigma_2,\underline{\theta}) \right).$$

Performing similar steps, we obtain that $\tilde{h}(t;\sigma_1,\sigma_2) = \int_{\underline{\theta}}^{\tilde{\theta}(\sigma_2)} h(t;\sigma_1,\theta) d\theta - \big(h(t;\sigma_1,\tilde{\theta}(\sigma_2)) -h(t;\sigma_1,\underline{\theta}) \big)$. Due to Lemma 1, $\frac{\partial}{\partial \theta} h(t;\sigma,\theta) \geq 0$ for all $\theta \in [\underline{\theta},\tilde{\theta}(\sigma)]$. Therefore, $\sigma_1 < \sigma_2$ implies $h(t;\sigma_1,\theta) \leq h(t;\sigma_2,\theta)$ for all $\theta \in [\underline{\theta},\tilde{\theta}(\sigma_1)]$. Because, $(\underline{\theta},\tilde{\theta}(\sigma_2)) \subseteq [\underline{\theta},\tilde{\theta}(\sigma_1)]$, it follows that that $h(t;\sigma_1,\theta) \leq h(t;\sigma_2,\theta)$ for all $\theta \in (\underline{\theta},\tilde{\theta}(\sigma_2))$. It follows that $\tilde{h}(t;\sigma_2) \geq \tilde{h}(t;\sigma_1,\sigma_2)$.

Q.E.D.

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