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Abstract

This paper studies the structure of optimal finance contracts in an agency model of outside finance, when investors possess private information. We show that, depending on the intensity of the entrepreneur's moral hazard problem, optimal contracts induce full, partial, or no revelation of the investor's private information. A partial or non–revelation of information is optimal, when it mitigates an undersupply of effort by the entrepreneur due to moral hazard.

Keywords: informed investors, optimal finance contracts, partial information revelation
JEL Classification No.: G24, D82

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1 Introduction

When dealing with entrepreneurs, the main task of investors is to provide the required initial capital. Recent literature however emphasizes that investors have also an important secondary role to play (e.g. Garmaise (2001), Inderst and Müller (2003), Kaplan and Strömberg (2005), Manove et. al. (2001)). It argues that due to their experience, investors are in a better position to judge certain aspects of a project than the entrepreneur. For instance, a bank that finances restaurants seems in a better position to estimate the potential demand for a new restaurant than a young, ambitious cook. Similarly, a venture capitalist that is involved in a number of high–tech startups is better at assessing the marketability of some additional new gadget. Indeed, exactly because investors finance multiple, similar projects, they inevitably acquire superior information about general economic conditions (Garmaise (2001)). Kaplan and Strömberg (2005) argue therefore that outside investors "may even be better informed" about risks that are *external* to the firm such as customer adaptation, competition, and the market in general. Similarly, one may say that investors are generally in a better position to judge the *economic* viability of a project, whereas the entrepreneur is indispensable for judging and overseeing the *technical* aspects of it.

This paper studies optimal financial arrangements, when investors possess relevant, private information. It thereby focuses on the question how much information an investor should reveal optimally. We study this problem in a standard Jensen&Meckling type agency model of outside finance: After the investor provides the initial investment, the entrepreneur takes some unobservable action that influences the outcome of the project. We extend this classical setup by private information on part of the investor. More specifically, we assume that the investor has superior knowledge about the state of demand that affects the project's outcome. In order to elicit the investor's information the entrepreneur offers the investor a mechanism.

This paper demonstrates that, despite the relevance of the investor's information, it may be in the entrepreneur's best interest that the investor does not reveal all his information. Indeed, the entrepreneur may actually benefit from not knowing the investor's information at all. That is, even though the investor's information is valuable in principle, the information has a negative value in equilibrium.[1] The intuition behind this result is best explained by recalling the well–known effect that the entrepreneur's moral hazard problem leads to an undersupply of effort (Jensen and Meckling (1976)). Yet, the entrepreneur's choice of effort will generally also depend on his beliefs about unknown information. For instance, an entrepreneur who expects a high demand chooses a higher effort level, because the higher demand makes his effort more worthwhile. In this case, the investor can prevent a potential undersupply of effort by not revealing bad news about demand conditions. That is, upholding the entrepreneur's belief about demand gives him enough incentives to choose an adequate effort level. However, insulating the entrepreneur from bad news implies, in a rational Bayesian world, that the investor can also not reveal too much good news, because otherwise a rational entrepreneur would deduce the bad news indirectly. Hence, not revealing information has the drawback that effort will be too low in the good state. Consequently, a non–trivial trade–off obtains. The paper derives explicit conditions under which the beneficial effect of a partial revelation outweighs its negative effect. It thereby reveals that the trade–off is delicate. The magnitude of the moral hazard problem determines whether optimal contracts involve non, some, or a full revelation of information.

The paper is related to Inderst and Müller (2003) and Manove et. al. (2001). These authors examine the use of the investor's private information as a screening device to distinguish between good and bad projects. The current paper adheres to this view and takes it one natural step further. Once the investor has completed her screening and concludes that the project is worthwhile to finance, there still remains uncertainty about how good the project actually is. That is, the fact that the investor is willing to invest reveals some, but not all her information. Hence, when the entrepreneur gets financed, he will know that his project is "good", but not how good it actually is. This paper focuses on this remaining degree of asymmetric

[1] The fact that an economic agent may gain when he remains uninformed is not unfamiliar (e.g. Kessler 1998).

information. It asks the simple question, whether it is optimal that the investor reveals her remaining private information, given that it is relevant to the entrepreneur. As in Inderst and Müller (2003) it uses an optimal contracting approach to study this question.

This paper complements the current literature on inside investors (e.g. Admati and Pfleiderer (1994), Bergemann and Hege (1997), Repullo and Suarez (1999), and Schmidt (2003)). Also this literature assumes a more active role for the investor, but her role relates to some activity. For example, Admati and Pfleiderer (1994) and Bergemann and Hege (1997) emphasize the dynamic structure of entrepreneurial finance, where the investor has to take multiple, finance related decisions. Whenever the investor's decision is not contractible, it yields a double moral hazard problem (e.g. Repullo and Suarez (1999), and Schmidt (2003)). Rather than focusing on additional activities of an inside investor, I extend the finance problem with private information of the investor and thereby focus on a different aspect.

From a technical perspective this paper provides the innovation that we analyze an adverse selection framework in which we cannot employ the revelation principle. The problem which arises is that the entrepreneur, as contract designer, chooses an unobservable action that depends on his belief. Since a revelation of information affect these beliefs, one cannot apply the classical revelation principle.[2] Effectively, the paper considers a contracting setting with limited commitment by the contract designer. We therefore use a modified revelation principle developed in Bester and Strausz (2001) to derive an optimal mechanism.

The rest of the paper is organized as follows. The next section introduces the principal–agent setup. Section 3 derives the first best solution. The Section 4 analyze the classical setup when there is only moral hazard. Section 5 studies the problem when there is moral hazard as well as adverse selection. Finally, Section 6 concludes. All formal proves are relegated to the appendix.

[2]Note that in any setup in which the revelation principle holds full information revelation cannot be suboptimal. In this sense, the failure of the revelation principle is a prerequisite for partial information revelation to be strictly optimal.

2 The Setup

Consider an entrepreneur who has a non–scalable project that requires an initial investment of $I > 0$. If the project is successful, it yields a value of $x \equiv 1$. An unsuccessful project yields zero. The probability of success is $p(e, \theta) \equiv e\theta$. That is, it increases with the entrepreneur's effort, e, and the state of demand, θ. Moreover, the higher the demand θ, the larger the marginal effect of effort.[3] The cost of effort is $c(e) = e^2/2$.

Without knowing the actual state of demand, the entrepreneur is aware that demand is high, θ_h, with probability ν and low, θ_l, with probability $1-\nu$, where $\theta_h > \theta_l$. The entrepreneur therefore rationally expects a demand of $\bar{\theta} \equiv \nu\theta_h + (1-\nu)\theta_l$. An outside investor may provide the entrepreneur with the required investment I. Since the investor has an experience with financing similar projects, she has learned whether the state of demand is θ_h or θ_l.[4] As the entrepreneur has no wealth, his liability is limited to zero so that he can payback the investor only if his project succeeds. We assume that the outcome of the project is verifiable so that a general contract can specify a repayment, $R > 0$, contingent on the project being successful.

The entrepreneur and the investor are risk neutral. That is, when the entrepreneur exerts an effort of e under the contract R then, given a state of demand θ, the payoff of the entrepreneur and the investor are

$$V(e, R|\theta) = \theta e(1 - R) - c(e);$$

and

$$U(e, R|\theta) = \theta e R - I;$$

respectively. Interest rates are normalized to zero.

[3] The multiplicative specification is the most straightforward setup that embodies these characteristics. It yields a tractable framework in which we may derive the optimal contract analytically.

[4] I abstract from any feedback effects of the entrepreneur's project on the other loans of the investor.

We assume that the investor is privately informed about the state of demand and that the entrepreneur's effort is unobservable. That is, we study a model with both adverse selection and moral hazard, where the two types of asymmetric information are attributed to different parties: The entrepreneur's choice of effort underlies a moral hazard problem, while the investor's revelation of information underlies an adverse selection problem. We further assume that the entrepreneur has all bargaining power. That is, he makes a take–or–leave–it offer R to the investor.

Summarizing, we will study the following sequence of events:

t=0: Nature chooses the state of demand $\theta \in \{\theta_h, \theta_l\}$ and informs the investor.

t=1: The entrepreneur offers a repayment schedule $R(.)$ to the investor.

t=2: The investor accepts or rejects. If she rejects, the game ends.

t=3: The entrepreneur chooses his effort $e \geq 0$.

t=4: Nature determines whether the project succeeds or fails.

Note that the investor's private information is relevant for the entrepreneur. Hence, at $t = 1$ the entrepreneur may try to elicit the investor's private information by means of some mechanism. A first question that arises is therefore what type of mechanisms one needs to consider. Since the entrepreneur at stage $t = 3$ chooses some unobservable effort, we cannot use the revelation principle to justify a restriction to mechanisms that induce truthful revelation. To see that the revelation principle is not valid, observe that the entrepreneur's choice of effort, e, will typically depend on his information about the state of demand θ. Hence, the standard argument that we may replace any mechanism by a truthful one is invalid. The replacement would change the entrepreneur's beliefs and, thereby, his choice of effort. Effectively, the revelation principle does not hold, because the entrepreneur has limited commitment. For such frameworks Bester and Strausz (2001) develop a modified revelation principle. There it is shown that if the mechanism designer, i.e., the entrepreneur, requires the agent, i.e., the investor, to send a message, one may restrict attention to direct mechanisms that simply ask the investor about her type. However, in contrast to the classical revelation principle, the optimal mechanism may require the investor to “lie” about her type with positive probability. Such lying represents a partial revelation of information.

3 Full Information Benchmark

In order to develop some intuition about the model, we start with the full information case in which both effort and the state of demand are publicly observable.

Suppose the entrepreneur can finance the project himself and, moreover, knows θ perfectly. In this case, the investor does not play a role and the entrepreneur must only decide whether to invest and which effort level to take. His payoff from investing in the project is $V(e|\theta_i) = e\theta_i - c(e) - I$. First order conditions yield the optimal, first best effort level

$$e_i^* \equiv \theta_i.$$

This effort level yields the entrepreneur a payoff of $\theta_i^2/2 - I$. Hence, in the demand state θ_i the entrepreneur realizes his project if and only if $I \leq \bar{I}_i^* \equiv \theta_i^2/2$.

Now suppose there is still full information, but the entrepreneur must, due to a lack of private funds, raise the required investment I from the investor. Since effort is observable, a general finance contract is a pair (e, R), dictating an effort level e and a repayment $R > 0$ conditional on the project being successful. The investor accepts a contract (e, R) whenever

$$\theta_i eR - I \geq 0. \tag{1}$$

It follows that the optimal contract solves the following maximization problem

$$\max_{e,R} \quad \theta e(1 - R) - e^2/2 \quad \text{s.t. (1).} \tag{2}$$

The solution to this problem is $(e, R) = (\theta_i, I/\theta_i^2)$ and yields the entrepreneur a payoff $V(\theta_i, I/\theta_i^2) = \theta_i^2/2 - I$. As before, the entrepreneur starts his project whenever $I \leq \bar{I}_i^*$. Despite the need for outside finance, the first best solution is still attainable, because all information is shared symmetrically.

4 Moral Hazard Only

In this section we analyze the finance problem as a standard agency problem. That is, we assume that the state is observable, whereas the entrepreneur's effort is not. Hence, the contract can condition repayments directly on the state of demand, while it cannot condition on the entrepreneur's choice of effort. The effort choice underlies a moral hazard problem. It follows that the contract has the form (R_l, R_h) and dictates a repayment R_i contingent on the actual state being θ_i.

In the state θ_i the entrepreneur's utility from an effort level e is

$$V(e_i, R_i|\theta_i) = \theta_i e(1 - R_i) - e_i^2/2.$$

Hence, his optimal choice of effort is

$$\hat{e}_i \equiv \theta_i(1 - R_i). \tag{3}$$

As is standard, the effort level $\hat{e}_i$ is smaller than the respective first best level e_i^*, because the entrepreneur receives only a share $1 - R_i$ of the project's return.

Anticipating the effort level $\hat{e}_i$, an investor θ_i accepts a repayment R_i whenever

$$R_i \theta_i^2 (1 - R_i) \geq I. \tag{4}$$

Hence, under moral hazard the entrepreneur's optimal contract, R_i^m, is a solution to the problem

$$\begin{array}{ll} \max\limits_{R_i} & \theta_i e_i(1 - R_i) - e_i^2/2 \\ \text{s.t.} & (3) \text{ and } (4). \end{array}$$

The problem admits a solution only when the required investment I is small enough. For $I \leq \theta_i^2/4$ the solution is

$$R_i^m \equiv \left(1 - \sqrt{1 - 4I/\theta_i^2}\right)/2.$$

Proposition 1 *Assume effort is unobservable, while the state of demand is public information. For $I \leq \theta_l^2/4$ the optimal contract is (R_l^m, R_h^m) and the project is executed in both states. For $I \in (\theta_l^2/4, \theta_h^2/4]$ the project is executed only in state θ_h under the contract R_h^m . For $I > \theta^2/4$ the project is not executed in either state.*

The proposition shows that moral hazard causes two types of inefficiencies. First, it leads to an *undersupply* of effort, because the entrepreneur receives only a share of the return from his effort level, while incurring their entire costs. Second, since under the first best the projects are realized for any $I \leq \theta_i^2/2$, whereas with moral hazard the project is only executed for $I \leq \theta_i^2/4$, *underinvestment* occurs for $I \in [\theta_i^2/4, \theta_i^2/2]$. The underinvestment effect is due to an undersupply of effort. Since moral hazard leads to a suboptimally low effort level, the project's net value is lower and, hence, it is profitable for a smaller range of investements I.

5 Informed Investors

This section analyzes optimal contracting when both the entrepreneur's effort and the investor's type are private information. Effectively, this implies that the only contractible component is the investor's message. Since the entrepreneur chooses his effort after observing the message, it is unclear whether it is optimal to induce the investor to reveal her private information truthfully. Indeed, the entrepreneur as contract designer has limited commitment and, as argued before, the revelation principle does not hold. However, with respect to Perfect Bayesian equilibria, Bester and Strausz (2001) demonstrate that in settings with limited commitment direct mechanisms are still optimal but may require partial truthtelling.

In the present framework this implies that there is no loss of generality by focusing on a menu (R_l, R_h) which gives the investor an incentive to report truthfully. However, in contrast to settings with full commitment, the optimal mechanism may require partial information revelation. That is, the optimal direct mechanism may require the investor to misreport her type with positive probability, despite her (weak) incentive to report truthfully.

Restricting our attention to menus (R_l, R_h) the Perfect Bayesian Equilibrium can be described by a combination $\Gamma = (R_l, R_h, \alpha_l, \alpha_h, \nu_l, \nu_h, e_l, e_h)$, where α_i describes the probability that the investor θ_i reports her type truthfully. The variable ν_i represents the entrepreneur's updated belief that the investor is of type θ_h given that she claimed type θ_i. Finally, e_i describes the entrepreneur's choice of effort when the investor made the claim θ_i. In order to constitute a Perfect Bayesian Equilibrium the combination Γ has to satisfy the following restrictions:

First, the investor must have a weak incentive to report her type truthfully. Hence, given the effort levels (e_l, e_h), it must hold for type θ_h that

$$\theta_h e_h R_h - I \geq \theta_h e_l R_l - I,$$

whereas for type θ_l it must hold

$$\theta_l e_l R_l - I \geq \theta_l e_h R_h - I.$$

Taken together these inequalities are equivalent to the condition

$$e_l R_l = e_h R_h. \tag{5}$$

That is, the requirement that the investor must have a weak incentive to report her type truthfully implies that she is indifferent between repayment schedule R_h and R_l. Consequently, the condition guarantees that any reporting strategy $\alpha_i < 1$, that involves some positive probability of lying, is also optimal. Since constraint (5) originates from the investor's private information, we refer to it as the *adverse selection constraint.*

Second, the entrepreneur's beliefs (ν_l, ν_h) must be Bayes' consistent with the investor's reporting strategy (α_l, α_h). That is, the beliefs ν_i satisfy Bayes

Law. This implies that

$$\nu_l = \nu_l(\alpha) \equiv \frac{\nu(1-\alpha_h)}{\nu(1-\alpha_h) + (1-\nu)\alpha_l}; \ \nu_h = \nu_h(\alpha) \equiv \frac{\nu\alpha_h}{\nu\alpha_h + (1-\nu)(1-\alpha_l)}. \quad (6)$$

Third, given the entrepreneur's beliefs (ν_l, ν_h) his effort choice (e_l, e_h) must be optimal. His expected utility from an effort level e when he faces a repayment R and has a belief $\tilde{\nu}$ is

$$V(e|R, \tilde{\nu}) = (1-\tilde{\nu})\theta_l e(1-R) + \tilde{\nu}\theta_h e(1-R) - e^2/2.$$

Consequently, his optimal effort level is

$$e(\tilde{\nu}, R) \equiv [\tilde{\nu}(\theta_h - \theta_l) + \theta_l](1-R).$$

It follows that the effort choice (e_l, e_h) must satisfy

$$e_l = e(\nu_l, R_l); \quad e_h = e(\nu_h, R_h). \quad (7)$$

The equations in (7) represent the *moral hazard constraints*. They describe the entrepreneur's unobservable behavior in response to the repayment scheme R and his beliefs $\tilde{\nu}$ about the investor's private information. Quite intuitively, the entrepreneur's effort is increasing in his belief $\tilde{\nu}$ and decreasing in the repayment R.

Finally, the combination Γ must guarantee the investor her reservation utility, since otherwise she would reject to participate. This condition translates to the individual rationality constraints

$$\theta_l e_l R_l \geq I; \quad \theta_h e_h R_h \geq I. \quad (8)$$

Summarizing, the combination Γ constitutes a Perfect Bayesian Equilibrium if and only if it satisfies (5)–(8). Our task is to derive a Perfect Bayesian Equilibrium that yields the entrepreneur the largest payoff. Given an equilibrium Γ, this payoff is

$$\begin{aligned} V(\Gamma) \equiv\ & (1-\nu)[\alpha_l(\theta_l e_l(1-R_l) - e_l^2/2) + (1-\alpha_l)(\theta_l e_h(1-R_h) - e_h^2/2)] \\ & +\nu[\alpha_h(\theta_h e_h(1-R_h) - e_h^2/2) + (1-\alpha_h)(\theta_h e_l(1-R_l) - e_l^2/2)]. \end{aligned}$$

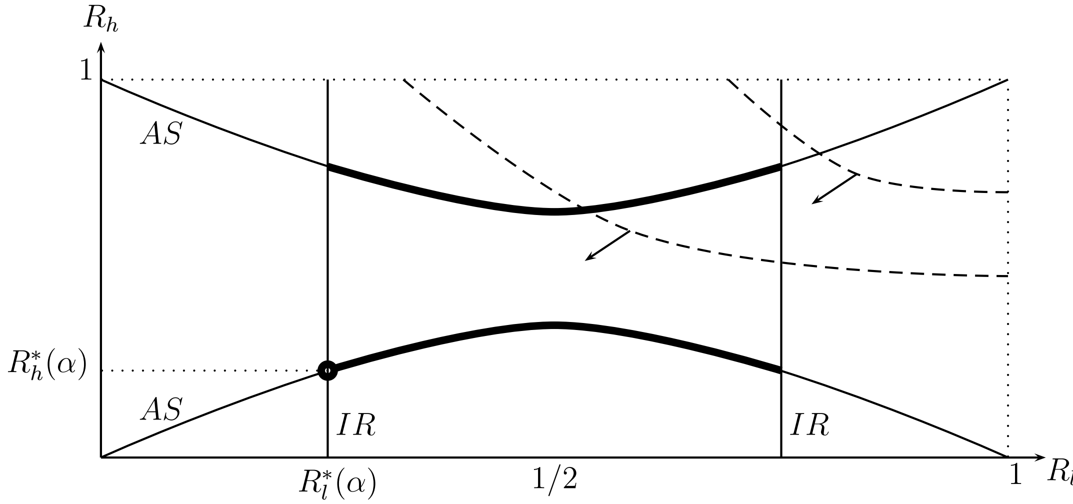

Figure 1: Adverse Selection and Individual Rationality Constraints.

Consequently, we solve the maximization problem:

$$\max_{\Gamma} \quad V(\Gamma) \text{ s.t. } (5) - (8).$$

Substitution of the moral hazard constraints (7) into the adverse selection constraints (5) yields

$$(\theta_l + \nu_l(\theta_h - \theta_l))(1 - R_l)R_l = (\theta_l + \nu_h(\theta_h - \theta_l))(1 - R_h)R_h. \tag{9}$$

Since $\theta_l < \theta_h$, the adverse selection constraints (5) imply $\theta_h e_h R_h = \theta_h e_l R_l > \theta_l e_l R_l$. Consequently, the relevant individual rationality constraint in (8) is $\theta_l e_l R_l \geq I$. Substitution of the respective moral hazard constraint in (7) transforms this individual rationality constraint into

$$\theta_l(\theta_l + \nu_l(\theta_h - \theta_l))(1 - R_l)R_l \geq I. \tag{10}$$

The constraints (9) and (10) play a crucial role in the analysis. Figure 1 displays, for a given reporting behavior α, the constraints graphically. The two parabola represent the adverse selection constraints (9). The vertical lines describe the individual rationality constraint (10). The dashed curves illustrate two iso–utility levels of the entrepreneur. As may be expected, the arrows indicate that his utility levels increase towards the origin. The figure reveals the main idea behind the subsequent analysis. The thickened parts of the parabola describe all the combinations (R_l, R_h) that satisfy the adverse selection (9) and the individual rationality constraints (10). Since the entrepreneur's utility increases towards the origin, the optimal repayment schedule is located at $(R_l^*(\alpha), R_h^*(\alpha))$. However, the figure is somewhat misleading, because it does not reveal that a specific reporting behavior α is only implementable if the required investment I is small enough. The following proposition addresses this issue and derives the optimal repayment schedule $(R_l^*(\alpha), R_h^*(\alpha))$ analytically.

Proposition 2 *A reporting strategy α is implementable if and only if*

$$I \leq \theta_l(\theta_l + \nu_l(\alpha)(\theta_h - \theta_l))/4. \tag{11}$$

The optimal repayment schedule $(R_l^*(\alpha), R_h^*(\alpha))$ that induces an implementable α is

$$R_l^*(\alpha) \equiv \frac{1}{2} - \frac{\sqrt{\nu_l(\alpha)\theta_h + (1-\nu_l(\alpha))\theta_l - 4I/\theta_l}}{2\sqrt{\nu_l(\alpha)\theta_h + (1-\nu_l(\alpha))\theta_l}}$$

and

$$R_h^*(\alpha) \equiv \frac{1}{2} - \frac{\sqrt{\nu_h(\alpha)\theta_h + (1-\nu_h(\alpha))\theta_l - 4I/\theta_l}}{2\sqrt{\theta_h\nu_h(\alpha) + (1-\nu_h(\alpha))\theta_l}}.$$

Proposition 2 shows that a reporting strategy α is implementable if and only if the required investment I is small relative to the equilibrium belief $\nu_l(\alpha)$. In particular, since $\nu_l(1,0) = 0$, full information revelation is implementable if and only if $I \leq \theta_l^2/4$. That is, whenever $I > \theta_l^2/4$ the entrepreneur cannot induce the investor to reveal all her information. However, in this case some reporting behavior α that leads to $\nu_l(\alpha) > 0$ may be implementable. Effectively, such reporting constitutes a partial revelation of information. This reasoning already indicates that partial information revelation may be optimal, simply because full revelation is impossible. In order to determine the optimal reporting strategy among all implementable reporting strategies, we define

$$\bar{I}_1 \equiv \frac{2\theta_h\theta_l^2(2\theta_h - \theta_l)}{(4\theta_h - \theta_l)^2} \text{ and } \bar{I}_2 \equiv \frac{2\theta_h\theta_l\bar{\theta}(2\theta_h - \bar{\theta})}{(4\theta_h - \bar{\theta})^2}.$$

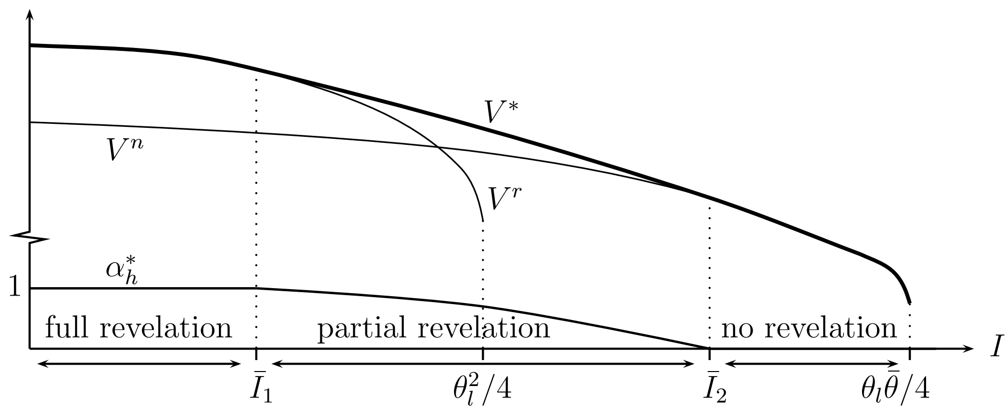

Figure 2: Optimal reporting α_h^* and the entrepreneur's payoffs.

Proposition 3 *Suppose it is optimal to ensure the investor's participation in both states θ_h and θ_l. Then for $I \leq \bar{I}_1$ the optimal contract is full revealing with $\alpha_h = \alpha_l = 1$. For $I \in [\bar{I}_2, \theta_l\bar{\theta}/4]$ the optimal contract is non–revealing with $\alpha_h = 0$ and $\alpha_l = 1$. For $I \in (\bar{I}_1, \bar{I}_2)$ the optimal contract is partially revealing with*

$$\alpha_l = 1 \textit{ and } \alpha_h = \frac{(1-\hat{R}_l)\hat{R}_l\theta_l\bar{\theta} - I}{\nu((1-\hat{R}_l)\hat{R}_l\theta_h\theta_l)} \in (0,1).$$

where

$$\hat{R}_l = \frac{1}{4}\left(1 + \sqrt{1 - 4I/(\theta_h\theta_l)}\right).$$

The lower part of Figure 2 illustrates the proposition graphically. It shows how the optimal revelation of information, α_h, falls with the required investment I. The upper part of the figure contrasts the entrepreneur's optimal payoff, V^*, to his payoffs associated with non–revelation, V^n, and full revelation, V^r. Since full revelation is only implementable for $I \leq \theta_l^2/4$, the curve V^r ends at $I = \theta_l^2/4$. Likewise, the implementation of a non–revealing contract $\alpha_h = 0$ is possible only for $I \leq \theta_l\bar{\theta}/4$. As stated by the proposition, full revelation is optimal whenever $I \leq \bar{I}_1$. In this case, $\alpha_h = 1$ is optimal and the entrepreneur's payoff V^* coincides with the full revelation payoff V^r. For $I \geq \bar{I}_2$ the optimal contract induces no revelation of information. Hence, $\alpha_h = 0$ and the optimal payoff V^* coincides with the non–revelation payoff

V^n. Finally, for the range $I \in (\bar{I}_1, \bar{I}_2)$ the optimal contract induces a partial revelation of information, $\alpha_h < 1$, that declines with I. In this range the entrepreneur's payoff V^* is strictly greater than the payoffs associated with full revelation, V^r, and non–revelation V^n. Since $\bar{I}_1 < \theta_l^2/4$ partial revelation is optimal for the range $I \in (\bar{I}_1, \theta_l^2/4)$, despite the fact that full revelation is implementable.

We now turn to the intuition behind the result that for $I \geq \bar{I}_1$ the optimal contract induces the investor not to reveal all her information. We start with the observation that in order for the investor to recoup her initial investment of I, the entrepreneur must provide an adequate level of effort. As I increases, this requirement becomes more difficult to fulfill. Moreover, as shown in Section 4 the moral hazard problem leads to an undersupply of effort, which makes it even harder to satisfy the requirement. The problem exacerbates further, when the entrepreneur learns that the state of demand is θ_l, because he then responds with an even lower effort level. Hence, to reduce the tension of the payback requirement an option is to forgo on the revelation of information. Indeed, Figure 2 illustrates that as of some level I the entrepreneur's payoff V^n of a non–revealing contract exceeds his payoff V^r from a full revealing contract. However, for values $I \in (\bar{I}_1, \bar{I}_2)$, the entrepreneur can do even better. Rather than restricting to full or no information revelation, the entrepreneur may induce the investor to reveal her information partially. Given the nature of the problem, the optimal way to do this is to reveal the good state only with a small probability α_h. This leaves the entrepreneur still uninformed about the actual state of demand when he receives a message θ_l, because this message is also sent with positive probability when the state of demand is θ_h. Indeed, the probability α_h of revealing the high state is kept low enough so that, when the entrepreneur forms his rational expectation ν_l after receiving the message θ_l, he still has enough incentives to provide an adequate level of effort.

Finally, we address the qualifier of Proposition 3 that it is optimal to ensure the participation the investor in both states of demand. The following proposition confirms the intuition that this is the case when the ex ante probability, ν, is small enough. In this case, it is relatively unlikely that the state of demand is high so that a contract that is only accepted in this state yields the entrepreneur rather little in expected terms.

Proposition 4 *There exists some $\bar{\nu} > 0$ such that for $\nu < \bar{\nu}$ the optimal contract induces participation of the investor in both states θ_l and θ_h.*[5]

Hence, for ν small enough, the contracts of Proposition 3 are indeed optimal.

6 Conclusion

When investors possess private information about (some aspects of) an entrepreneur's project, the question arises whether the investor should reveal this information. This paper shows that, in general, such revelation is not optimal, because it exacerbates the classical moral hazard problem in corporate finance. Since the entrepreneur hold less than 100 percent of the residual claim, while bearing the entire cost of profit enhancing activities, an undersupply of effort occurs (Jensen and Meckling (1976)). A revelation of the investor's private information may exacerbate this problem and, hence, it is suboptimal to reveal all information.

Indeed, this paper demonstrates how optimal contracts carefully calibrate the amount of information revelation that they induce to mitigate problems of moral hazard. Depending on the severeness of the moral hazard problem, optimal contracts induce either partial, full, or no revelation of information.

[5] For the range $I \in (\bar{I}_1, \bar{I}_2)$ the cutoff value $\bar{\nu}$ may be obtained analytically as $\frac{\theta_l \theta_h ([(4\theta_h - 3\theta_l)\sqrt{1-4I/(\theta_h\theta_l)} - 4\theta_h + 5\theta_l - 2I/\theta_h]}{\theta_l\theta_h\left[\sqrt{\theta_h^2 - 4I} - 4\theta_h + 5\theta_l + 3(\theta_h - \theta_l)\sqrt{1-4I/(\theta_h\theta_l)}\right] + 2I(\theta_h - 2\theta_l)}$.

Appendix

Proof of Proposition 2: Implementability of α is equivalent to the existence a combination (R_l, R_h) that satisfies (9) and (10). We show that condition (11) is both necessary and sufficient for the existence of such a pair. From (10) it follows $I \leq (1-R_l)R_l\theta_l(\theta_h\nu_l(\alpha)+(1-\nu_l)\theta_l) \leq (\theta_l(\theta_h\nu_l(\alpha)+(1-\nu_l)\theta_l))/4$. Hence, whenever (11) is violated, then (10) is violated for any R_l. Consequently, (11) represents a necessary condition for implementation.

Sufficiency follows from the observation that when (11) holds, then for $R_l = 1/2$ inequality (10) holds. Moreover, since $\nu_h(\alpha) \geq \nu_l(\alpha)$ it follows that for $R_l = 1/2$ one may find an $R_h \in [0, 1/2]$ such that (9) holds.

To derive the optimal combination (R_l^*, R_h^*) that implements α we first establish that, given some fixed R_l, the entrepreneur's utility is decreasing in R_h. This follows from a substitution of (7) and (6) into $V(\Gamma)$, as this yields

$$\frac{dV(\Gamma)}{dR_h} = -\frac{(1-R_h)(\alpha_h\theta_h\nu+(1-\nu)(1-\alpha_l)\theta_l)^2}{\alpha_h\nu+(1-\alpha_l)(1-\nu)} \leq 0 \qquad (12)$$

Moreover, since

$$\frac{dV(\Gamma)}{dR_l} = -\frac{(1-R_l)((1-\alpha_h)\theta_h\nu+(1-\nu)\alpha_l\theta_l)^2}{(1-\alpha_h)\nu+\alpha_l(1-\nu)} \leq 0, \qquad (13)$$

it follows that, given some R_h, the entrepreneur's utility is also decreasing in R_l.

From (12) it follows after solving (10) with respect to R_h that, whenever R_l^* is optimal then $R_h^* = \tilde{R}_h(R_l^*)$ is optimal, where

$$\tilde{R}_h(R_l) \equiv \frac{1}{2} - \sqrt{\frac{(1-\nu_h(\alpha))\theta_l+\nu_h(\alpha)\theta_h-4(1-R_l)R_l((1-\nu_l(\alpha))\theta_l+\nu_l(\alpha)\theta_h)}{4(\nu_h(\alpha)\theta_h+(1-\nu_h(\alpha))\theta_l)}}.$$

Now suppose $R_l^* \in (1/2, 1]$ is optimal, then $R_h^* = \tilde{R}_h(R_l^*)$ is optimal. However, since $\tilde{R}_h(R_l) = \tilde{R}_h(1-R_l)$, also the combination $(\hat{R}_l, R_h^*)$, with $\hat{R}_l \equiv 1 - R_l^*$, satisfies the adverse selection constraint (9). Moreover, $\hat{R}_l$ satisfies the individual rationality constraint (10) whenever R_l^* does. Hence, also $(\hat{R}_l, R_h^*)$ implements the reporting strategy α. But since $\hat{R}_l < R_l^*$ it follows from (13) that $(\hat{R}_l, R_h^*)$ yields a higher utility such that $R_l^* > 1/2$ cannot be optimal.

Hence, $R_l^* \leq 1/2$. But for $R_l \leq 1/2$, the function $\tilde{R}_h(R_l)$ is increasing, since

$$\begin{aligned} \frac{\partial \tilde{R}_h}{\partial R_l} &= \frac{1-2R_l}{\sqrt{\nu_h(\alpha)\theta_h + (1-\nu_h(\alpha))\theta_l}} \times \\ & \frac{\nu_l(\alpha)\theta_h + (1-\nu_l(\alpha))\theta_l}{\sqrt{(1-\nu_h(\alpha))\theta_l + \nu_h(\alpha)\theta_h - 4(1-R_l)R_l((1-\nu_l(\alpha))\theta_l + \nu_l(\alpha)\theta_h)}} \end{aligned}$$

is non–negative for $R_l \leq 1/2$. Hence, as R_l decreases also $\tilde{R}_h(R_l)$ decreases and from (12) and (13) it follows that the entrepreneur's utility increases. Consequently, the optimal combination $(R_l, \tilde{R}_h(R_l))$ is the lowest value R_l such that the individual rationality constraint (10) is still satisfied. That is,

$$R_l^* = \frac{1}{2} - \frac{\sqrt{\nu_l(\alpha)\theta_h + (1-\nu_l(\alpha))\theta_l - 4I/\theta_l}}{2\sqrt{\nu_l(\alpha)\theta_h + (1-\nu_l(\alpha))\theta_l}}$$

and

$$R_h^* = \frac{1}{2} - \frac{\sqrt{\nu_h(\alpha)\theta_h + (1-\nu_h(\alpha))\theta_l - 4I/\theta_l}}{2\sqrt{\theta_h\nu_h(\alpha) + (1-\nu_h(\alpha))\theta_l}}.$$

. Q.E.D.

Proof of Proposition 3: Solving $R_h^*(\alpha)$ and $R_l^*(\alpha)$ with respect to α_1 and α_2 yields

$$\alpha_h^*(R_h, R_l) = \frac{((1-R_h)R_h\theta_l^2 - I)((1-R_l)R_l\theta_l\bar{\theta} - I)}{\nu(R_h - R_l)(1-R_h-R_l)(\theta_h-\theta_l)\theta_l I} \tag{14}$$

$$\alpha_l^*(R_h, R_l) = \frac{((1-R_l)R_l\theta_l\theta_h - I)((1-R_h)R_h\theta_l\bar{\theta} - I)}{(1-\nu)(R_h - R_l)(1-R_h-R_l)(\theta_h-\theta_l)\theta_l I} \tag{15}$$

Substitution into $V(\Gamma)$ yields

$$\hat{V}(R_h, R_l) \equiv \frac{((1-R_h)(1-R_l)(R_h+R_l)\theta_l\bar{\theta} - I)I}{2R_hR_l(1-R_h-R_l)\theta_l^2}. \tag{16}$$

Hence, the optimal contract is found by maximizing $V(\Gamma)$ over the domains

$$R_h \in D_h \equiv [\underline{D}_h, \overline{D}_h] \equiv \left[\frac{1}{2}\left(1-\sqrt{\frac{\theta_h\theta_l - 4I}{\theta_h\theta_l}}\right), \frac{1}{2}\left(1-\sqrt{\frac{\bar{\theta}\theta_l - 4I}{\bar{\theta}\theta_l}}\right)\right]$$

and

$$R_l \in D_l \equiv [\underline{D}_l, \overline{D}_l] \equiv \left[\frac{1}{2}\left(1-\sqrt{\frac{\bar{\theta}\theta_l - 4I}{\bar{\theta}\theta_l}}\right), \frac{1}{2}\left(1-\sqrt{\frac{\theta_l^2 - 4I}{\theta_l^2}}\right)\right].$$

The second order derivative is

$$\frac{d^2\hat{V}(R_h, R_l)}{dR_h^2} = \frac{((3/2R_h - (1-R_l))^2 + 3R_h^2/4)((1-R_l)R_l\theta_l\bar{\theta} - I)I}{(R_h^3 R_l(1-R_h-R_l)^3\theta_l^2)} \geq 0$$

where the inequality follows, because $(1-R_l)R_l\theta_l\bar{\theta} \geq I$ for all $R_l \in D_l$. Consequently, $\hat{V}(R_h, R_l)$ is convex in R_h so that it does not have an interior maximum. I.e., the optimal value of R_h is either $\underline{D}_h$ or $\overline{D}_h$Note that, by (14) and (15), the candidate $R_h = \overline{D}_h$ implies the full pooling solution $a_h = 1$ and $a_l = 0$. Yet, since $R_h = \underline{D}_h$ and $R_l = \underline{D}_l$ also implies the full pooling solution (with $a_h = 0$ and $a_l = 1$), any payoff attainable with $R_h = R_2^*$ is also attainable under $R_h = R_1^*$. Consequently, we may discard the candidate $\overline{D}_h$ and concentrate on $\underline{D}_h$.

Taking the first order condition of $\hat{V}(\underline{D}_h, R_l)$ with respect to R_l yields

$$R_l^* = \frac{1}{4}\left(1+\sqrt{1-4I/(\theta_h\theta_l)}\right).$$

It satisfies the second order condition, as

$$\frac{\partial^2 V}{\partial R_l^2}(\underline{D}_h, R_l^*) = -\frac{512(1-\nu)(\theta_h-\theta_l)I^2}{\theta_l\theta_h\left(1-\sqrt{1-4I/(\theta_h\theta_l)}\right)\left(1+\sqrt{1-4I/(\theta_h\theta_l)}\right)^4} < 0.$$

Hence, R_l^* is optimal whenever, it lies in the domain D_l. Straightforward calculations yield

$$R_l^* \geq \underline{D}_l \Leftrightarrow I \geq \bar{I}_1 \text{ and } R_l^* \leq \overline{D}_l \Leftrightarrow I \leq \bar{I}_2.$$

(To see that $\bar{I}_2 > \bar{I}_1$ note that $Sign[\bar{I}_2 - \bar{I}_1] = Sign[8(2-\nu)\theta_h^2 + 3(1-\nu)\theta_l^2 - \theta_h\theta_l(5 + 11(1-\nu))]$. The sign of the last expression is positive if and only if $\nu < 1 + (\theta_h(8\theta_h - 5\theta_l))/((8\theta_h - 3\theta_l)(\theta_h - \theta_l))$ which holds for any $\nu \in [0,1]$.)
Q.E.D.

Proof of Proposition 4

For $I \in [\bar{I}_1, \bar{I}_2)$ it follows from Proposition 3 that the entrepreneur's optimal payoff from ensuring the participation of both types of investors is

$$V^* = \theta_h(\theta_h - 5(1-\nu)(\theta_h - \theta_l)) + \frac{1}{4}(\theta_h + 3(1-\nu)(\theta_h - \theta_l))\sqrt{1 - 4I/(\theta_h\theta_l)} - \frac{I\bar{\theta}}{2\theta_l}.$$

For $I > [\bar{I}_2, \theta_l\bar{\theta}/4]$ it follows from Proposition 3 that the entrepreneur's optimal payoff from ensuring the participation of both types of investors is

$$V^* = \frac{1}{4}\left(\bar{\theta}^2 + \bar{\theta}\sqrt{\bar{\theta}^2 - 4I\bar{\theta}/\theta_l} - 2I\bar{\theta}/\theta_l\right).$$

The optimal contract when there is only participation of the θ_h investor coincides with the optimal contract in Proposition 1, because in any such Perfect Bayesian equilibrium the Bayes' consistent belief, ν^e, of the entrepreneur after an acceptance of the contract is 1. Consequently, the payoff associated with this contract is

$$V^h \equiv \frac{1}{4}\left(\theta_h^2 + \theta_h\sqrt{\theta_h^2 - 4I} - 2I\right)\nu.$$

For $\nu = 0$ it holds $V^h = 0 < V^*$. Since V^h and V^* are continuous in ν it follows that $V^* > V^h$ for $\nu > 0$ small enough.

Q.E.D.

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