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Separating Equilibria with Imperfect Certification

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Abstract

Viscusi (1978) shows how, in markets with quality uncertainty, *perfect* certification results in separation from top down due to an unraveling process similar to Akerlof (1970). De and Nabar (1991) argue that *imperfect* certification prevents unraveling so that equilibria with full separation do not exist. This note shows that, if one considers the buyers' buying decision explicitly, a separating equilibrium with imperfect certification does exist.

Keywords: certification, unraveling, separating equilibrium
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1 Introduction

Viscusi (1978) shows how, in markets with quality uncertainty, *perfect* certification results in separation from top down due to an unraveling process similar to Akerlof (1970). De and Nabar (1991) argue that *imperfect* certification prevents unraveling so that equilibria with full separation do not exist. The authors, subsequently, conclude in their abstract that

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imperfect certification results in "equilibrium outcomes which are very different from those implied by models of perfect certification". This conclusion casts doubts on the robustness of theoretical work on certification that, for reasons of tractability, assumes perfect certification technologies (e.g., Biglaiser 1993, Lizerri 1999, Albano and Lizzeri 2001, Strausz 2005, Faure–Grimaud et al. 2009, Stahl and Strausz 2010). This note shows that, if one considers the buyers' buying decision explicitly, a separating equilibrium with imperfect certification does exist. As a consequence, the results from certification models that are based on perfect certification are more robust to imperfections in the certification technology than the results of De and Nabar (1991) suggest.

2 Setup

To demonstrate my argument, I follow exactly the setup of De and Nabar (1991), who consider a market for a perishable commodity with n potential sellers of one unit of the commodity. The quality (t) of the product could be either type A or type B.[1] Under full information, type A and type B products would be valued as V_A and V_B respectively by each of the set of potential buyers, I, where $V_A > V_B > 0$. A fraction $\pi \in (0, 1)$ of all suppliers is endowed with type A product. Each supplier knows the true quality level of his product. In an asymmetrically informed market, a supplier's decision (d), where $d \in D = \{C, U\}$, could be either to offer his product for sale uncertified ($d = U$) or get the quality level of the good certified ($d = C$) for an exogenously fixed cost, $K < V_A - V_B$, prior to offering it for sale.[2] The certification process results in a rank (r), where $r \in R = \{1, 2\}$, for the product which could be either 1 or 2. The certification process is such that a type A product gets a ranking of 1 with probability q_A while a type B product gets a ranking of 1 with probability q_B with $1 \geq q_A > q_B \geq 0$. Certification is perfect when $q_A = 1$ and $q_B = 0$. The buyers, $i \in I$, who are all alike, do not observe the true quality level of any product before purchase. They do, however, know the distribution of types (π) and the characteristics of the certification

[1] As in De and Nabar (1991), I consider only two types, but results extend to more than two types.

[2] Clearly, for $K > V_A - V_B$ certification is too costly to be helpful.

process (q_A, q_B). In addition, they observe each supplier's decision d and, for those suppliers with $d = C$, also the certifier's ranking r.

De and Nabar introduce $P(U)$ as the price of an uncertified good and $P(C,r)$ as the price of a good that is certified with rank r. They assume that "all economic agents are risk-neutral and all markets are competitive" and I follow their idea that, "in a competitive market, the price the buyers pay for a product would equal the value they expect from it. In other words, in such a market, $P(C,r) = V_A\rho(A|C,r) + V_B\rho(B|C,r)$ and $P(U) = V_A\rho(A|U) + V_B\rho(B|U)$, where ρ indicates market posterior beliefs".

De and Nabar, subsequently, treat supply and demand in a reduced form and, implicitly, assume that when the price for a product equals the value that buyers expect from it, all goods on the market are bought. It is here that I extend the analysis and explicitly introduce $\alpha(P,U)$ as the probability that buyers buy an uncertified good with a price P and $\alpha(P,C,r)$ as the probability that an uncertified good is sold when the price is P.

Because a supplier with quality t has probability q_t of obtaining rank $r = 1$, the expected revenue for a supplier of quality t who certifies is

$$W(t,C,P) = q_t P(C,1)\alpha(P(C,1),C,1) + (1-q_t)P(C,2)\alpha(P(C,2),C,2) - K.$$

His revenue from selling the good uncertified is $W(t,U,P) = P(U)\alpha(P(U),U)$. A comparison to De and Nabar confirms that the extension incorporates their framework for $\alpha(P(C,1),C,1) = \alpha(P(C,2),C,2) = \alpha(P(U),U) = 1$.

Based on Kreps and Wilson (1982), De and Nabar define a Nash Sequential Equilibrium (NSE), as an ordered triple (d_t^*, P^*, ρ) satisfying the three conditions supplier rationality, buyer competitiveness, belief consistency. Because I consider the buyers' buying behavior explicitly, I need to extend these conditions by buyer rationality. Hence, I define a Nash Sequential Equilibrium (NSE) as a combination $\gamma^* = (d_t^*, \alpha^*, P^*, \rho^*)$ that satisfies:

(I') supplier rationality: $d_t^* \in \text{argmax}_d\{W(t,d,P)\}$, $d \in [0,1]$, $t \in T$.

(I") buyer rationality: $\alpha^*(P,U) \in \text{argmax}_\alpha \alpha[V_A\rho(A|U) + V_B\rho(B|U) - P]$ and $\alpha(P,C,r) \in \text{argmax}_\alpha \alpha[V_A\rho(A|C,r) + V_B\rho(B|C,r) - P]$, $t \in T$, $r \in \{0,1\}$.

(II) buyer competitiveness: $P^*(C,r) = \sum V_t \rho(t|C,r)$, $P^*(U) = \sum V_t \rho(t|U)$.

(III) belief consistency: If C or U is an equilibrium strategy, the posterior beliefs of each buyer i, $\rho^*(t|C,r)$ or $\rho^*(t|U)$, are determined by Bayes rule. If C or U is an off-equilibrium strategy, then $\rho^*(t|C,r), \rho^*(t|U) \in [0,1]$.

3 Results

In this note, I am interested in the existence of a separating equilibrium in which only sellers with the high quality A certify, i.e., $(d_A, d_B) = (C, U)$. Viscusi (1978) already noted the existence of such an equilibrium when certification is perfect but not too costly. With perfect certification, the sellers' separation strategies (C, U) together with the belief consistency requirement (III) pin down the beliefs $\rho^v(A|C,1) \equiv 1$ and $\rho^v(A|U) \equiv 0$, but leave the belief $\rho^v(A|C,2)$ undetermined. Hence, $\rho^v(A|C,2) \equiv 0$ is consistent with the belief consistency requirement. Due to the buyer competitiveness requirement (II), the belief structure $\rho^v(.|.)$ implies $P^v(U) \equiv V_B$, $P^v(C,1) \equiv V_A$, and $P^v(C,2) \equiv V_B$. Given these prices, the separating strategy (d^s_A, d^s_B) is consistent with supplier rationality requirement (III). Hence, in the framework of De and Nabar (1991) there exists a separating equilibrium when certification is perfect. With appropriate choices of $\alpha(.,.)$, the separation equilibrium also exists in the extended framework.

Proposition 1 *(Viscusi 1978) With perfect certification* $(q_A, q_B) = (1, 0)$ *there exists an equilibrium that sustains the separating strategy* $(d_A, d_B) = (C, U)$.

Proof: It is straightforward to check that the combination $\gamma^v \equiv (d^v_t, \alpha^v, P^v, \rho^v)$ with $\alpha^v(P,U) \equiv 1$ for $P \leq V_B$ and $\alpha(P,U) \equiv 0$ for $P > V_B$, $\alpha(P,C,1) \equiv 1$ for $P \leq V_A$ and $\alpha(P,U) \equiv 0$ for $P > V_A$, $\alpha(P,C,2) \equiv 1$ for $P \leq V_B$ and $\alpha(P,U) \equiv 0$ for $P > V_B$ satisfies the conditions (I'), (I"), (II), and (III) and is therefore an NSE. Q.E.D.

De and Nabar (1991) argue that there does not exist a separation equilibrium when certification is imperfect. I concentrate on the case $q_A < 1$ and $q_B \in [0, q_A)$, where the argument of De and Nabar is clearest. In this case, the belief consistency condition (III) implies not only that $\rho^*(A|C,1) = 1$ and $\rho^*(B|U) = 0$, but also that $\rho^*(A|C,2) = 1$. Hence, the buyer competitiveness condition (II) implies $P^*(C,1) = P^*(C,2) = V_A$ and $P^*(U) = V_B$. Because De and Nabar, effectively, assume $\alpha(P(U),U) = \alpha(P(C,1),C,r) = \alpha(P(C,2),C,r) = 1$, we have $W(A,C,P) = W(B,C,P) = V_A - K$ and $W(A,U,P) = W(B,U,P) = V_B$. Due to $K < V_A - V_B$, it follows that also the seller of quality q_B has a strict incentive to certify. This leads to the following result.

Proposition 2 *(De and Nabar 1991) For any $q_A < 1$, there does not exist an equilibrium with $\alpha(P(U),U) = \alpha(P(C,1),C,r) = \alpha(P(C,2),C,r) = 1$ that sustains the separating strategy $(d_A, d_B) = (C,U)$.*

Proposition 2 suggests that the existence of a separating equilibrium crucially depends on perfect certification and is not robust to imperfections, however small, in the certification technology. Yet, because the buyers at the prices $P^*(C,1)$, $P^*(C,2)$ and $P^*(U)$ are actually indifferent, the buying behavior $\alpha(P(U),U) = \alpha(P(C,1),C,r) = 1$ and $\alpha(P(C,2),C,r) = 0$ is also consistent with buyer rationality (I'). In this case, we have $W(A,C,P) = q_a V_A - K$ and $W(B,C,P) = q_B V_A - K$ so that we have a separating equilibrium when $q_B V_A - V_B < K < q_A V_A - V_B$. If we define $\bar{q} \equiv (K + V_B)/V_A$ so that, due to $K < V_A - V_B$, we have $\bar{q} < 1$, we obtain the following result.

Proposition 3 *For imperfect certification (q_A, q_B) with $q_B < \bar{q}$ and $q_A > \bar{q}$ there exists an equilibrium with $\alpha(P(U),U) = \alpha(P(C,1),C,r) = 1$ and $\alpha(P(C,2),C,r) = 0$ that sustains the separating strategy $(d_A, d_B) = (C,U)$.*

Proof: From $q_B < \bar{q}$ and $q_A > \bar{q}$ it follows that $q_B V_A - V_B < K < q_A V_A - V_B$ so that under the buyers' buying behavior α and prices $P^*(C,1)$, $P^*(C,2)$ and $P^*(U)$, the seller of quality V_A has a strict incentive to certify, whereas the seller of quality V_B has a strict incentive not to certify. To complete the proof, the buyers' rationality condition pins down the buying behavior α for all remaining prices. In particular, $\alpha(P,U) = 1$ for $P < V_B$ and $\alpha(P,U) = 0$ for $P > V_B$, $\alpha(P,C,1) = 1$ for $P < V_A$ and $\alpha(P,U) = 0$ for $P > V_A$, $\alpha(P,C,2) = 1$ for $P < V_A$ and $\alpha(P,U) = 0$ for $P > V_A$. Q.E.D.

4 Conclusion

When buyers and sellers are price takers, a separation equilibrium (C, U) exists when the imperfection is not too extreme. In particular, an equilibrium exists when the certification technology is close to the perfect one $(q_A, q_B) = (1, 0)$. In this sense, the separation equilibrium of Proposition 1 is robust to imperfections in the certification technology.

The robustness result does not only hold when buyers and sellers are price takers, but also when, as in Stahl and Strausz (2010), the privately informed seller sets prices. In this case, the seller's price is a signal about his quality and the buyer's belief will depend on it. The competitive prices $P^*(C, 1) = P^*(C, 2) = V_A$ and $P^*(U) = V_B$ of Proposition 2 and 3 can be sustained with the out–off–equilibrium belief that buyers interpret any other price as an indication that quality is q_B.

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