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Signaling an Outside Option

Susanne Ohlendorf*
Patrick Schmitz**

* City University of Bonn, Adenauerallee 24-42, 53113 Bonn
** University of Cologne, Albertus-Magnus-Platz, 50923 Cologne.

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Speaker: Prof. Dr. Urs Schweizer. · Department of Economics · University of Bonn · D-53113 Bonn,
Phone: +49(0228)739220 · Fax: +49(0228)739221

Signaling an Outside Option

Susanne Ohlendorf*and Patrick W. Schmitz†

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Abstract

We consider the case of an upstream seller who works to improve an asset that has been specialized to a downstream buyer's needs. The buyer then makes a take it or leave it offer to the seller about how the future surplus should be split. We assume that the seller from the outset has private information about the fraction of the surplus that he can realize on his own, and show that this leads to higher investment compared to the complete information case. This positive effect on investment is countervailed by the occurrence of inefficient separations, which result when the buyer mistakenly tries to call the seller's bluff with a low offer.

JEL classification: D23, D82

Keywords: signaling, relationship-specific investment, incomplete contracts, outside options

1 Introduction

In many industries firms rely on the relationship-specific investments of their suppliers, yet the subcontractors are small firms compared to their customers, and potential customers are few. If the bargaining power lies entirely with the customer, how are the necessary investments induced in this environment in which the customer dictates the rules and suing for payment is unthinkable, i.e., how do the firms overcome a potential hold-up problem?[1] First, it seems vital for small suppliers to have many potential customers such that in case of separation they can make up for the loss by dealing with others. Second, the customer will likely not

*Wirtschaftspolitische Abteilung, Department of Economics, University of Bonn, Adenauerallee 24-42, 53113 Bonn, Germany. Email: sohlendo@uni-bonn. Financial support from *Deutsche Forschungsgemeinschaft* through SFB-TR 15 is gratefully acknowledged. We would like to thank Andrew Daughety, Paul Heidhues, Jennifer Reinganum, Urs Schweizer, and seminar participants in Munich, Mannheim and Bonn for helpful comments.

†CEPR and University of Cologne, Albertus Magnus Platz, 50923 Cologne. Email: patrick.schmitz@uni-koeln.de

[1]For early work on the hold-up problem see Williamson (1975) and (1979), Grout (1984), and Hart and Moore (1988).

have enough information about the position of the supplier in the market to fully exploit her superior bargaining position.

In this paper, we explore the consequences of asymmetric information about the supplier's opportunities outside the relationship. We show that the resulting information rents may stimulate innovation if the buyer has no other way to commit to adequately reward the supplier's investment. Moreover, there is a signaling motive in the investment choice. If the best alternative use of the relationship-specific asset is private information to the supplier, the buyer will try to deduce this outside option from the level of investment. If the supplier is very reluctant to invest, it seems likely that he fears to be held-up because of a low outside option, and the buyer will then indeed make a low offer. If instead the supplier is very eager to invest, the buyer may belief that the private value from the investment is high, hence she has to make a high offer. Now the possibility arises that a supplier with a low outside option mimics the type with the high outside option and invests more. This effect may mitigate the hold-up problem and lead to higher investment.

The baseline model that we use in this paper is a simplified version of the property rights model developed by Grossman and Hart (1986) and Hart and Moore (1990)[2]. An upstream seller invests into an asset, which can generate a return within the relationship with a downstream buyer. It is not possible to write detailed long-term contracts, instead the buyer later makes a take-it-or-leave-it offer to the seller and thereby determines how the return to investment is shared. We assume that the seller owns the asset, so that he can threaten to use it for a different purpose. His bargaining position then depends on how specific the asset is to the relationship with the buyer.

An investment is called general if there is a competitive market that fully appreciates the generated return, while a specific investment loses a large part of its value if it is used outside a particular relationship.[3] For example, subcontractors that produce an input for a downstream firm will have some incentives to innovate if they are granted ownership of the asset that they work to improve, or a legal title to the innovation that they develop. As another example, consider an employee who may increase her human capital in the safe knowledge that it cannot be taken away from her and has a value for other employers as well. Nevertheless, if there is a large discrepancy between the asset's value in the current relationship and the next best alternative, investment incentives may be diluted for fear of opportunistic behavior of the other party.

Many investments in machines or human capital are a mix of specific and general investment,

[2]See also Hart (1995), Farrell and Gibbons (1995), Chiu (1998), and de Meza and Lockwood (1998).

[3]In the terminology of Klein, Crawford and Alchian (1978), general investments create no appropriable rents, while the quasi-rents that are generated by specific investments can partly be appropriated by the other party in the relationship. See Becker (1964) for the implications of general and specific investment in the context of human capital acquisition.

where the degree of specificity determines the severity of the hold-up problem. In fact, how specific an investment is also depends on characteristics of the investing party, e.g., on its access to the market for the asset or its ability to transform the asset to general use; and moreover these characteristics are likely to be unobservable. For a worker who has the entrepreneurial ability to use his training to start his own business, all training might be considered general. In contrast, for a worker who has to rely on finding a job in a similar business the specificity of the investment depends on his cost of switching jobs. Entrepreneurial ability and cost of switching jobs are certainly private information to the employee, and similarly, for a seller of a good (like a house), his own consumption value of the good is usually not known to a potential buyer. In this paper, we make the assumption that the degree of specificity, as captured by the best alternative use of the investment, is private information to the seller.

We find that this game, in which the seller tries to signal a high outside option with specific investment, has an essentially unique equilibrium. All perfect Bayesian equilibria lead to the same payoffs and distribution of investments. If the seller's outside option is known to be relatively low compared to the value of the investment to the buyer, all types of sellers invest the same amount. They choose the investment level that the type with the maximum outside option would also choose under symmetric information. Clearly, in such a pooling equilibrium investments and joint surplus are higher than in the case with complete information.

In general, the equilibrium is a hybrid, or semi-pooling, equilibrium. There is a cut-off type such that all sellers with a lower outside option pool on this type's strategy. This cut-off type, and all higher ones, mix between their own and all higher types' complete information investments. All these types hence separate in the sense that they choose different strategies. Because of the randomization, however, a chosen investment does not give away the type ex post. An observed investment could have been chosen by any type who would invest less under complete information. While the information asymmetry leads to higher investment, this effect is traded against the inefficiency generated by the non-investing party trying to appropriate part of the information rents. How the joint surplus compares to the case with complete information therefore depends on the parameters of the model.

That in our model relationship-specific investment can be used as a signal for an outside option distinguishes this paper from the rest of the literature. The idea that private information about outside options can lead to rents that foster investment has been addressed before, e.g. in Malcomson (1997) and Sloof (2008). In these papers, the outside options are realized after investment decisions have been made. Although signaling plays no role in such a model, the models yield similar qualitative predictions: in comparison to the standard hold-up model, which excludes all ex post frictions and focuses on inefficient preparations, there are now greater inefficiencies ex post and less ex ante. In particular, investment levels can be too high relative to their later use. A characteristic of the signaling model, in comparison, is a "bluffing" element that leads to an equilibrium in mixed strategies.[4]

Signaling models by now have a long tradition in economics, starting with Spence (1973), who models education as a wasteful signal of productivity. The general idea is that it can be possible to reveal private information like productivity or quality by means of signals (like education, warranties, or high prices), if the cost of the signal differs across types. In contrast, in the outside option signaling game the cost of investment depends only indirectly on types. Since all types of sellers have the same cost of investment, types only matter if the uninformed party makes low offers. In particular, different types of sellers would choose different levels of investment if information was symmetric, while in the original Spence model the wasteful signal would then not be used at all.[5] More related to the present paper is recent work on signaling that assumes productive investment and shows that signaling can lead to higher investment and even to a Pareto improvement. This includes Hermalin (1998), in which a leader may signal a worthwhile project by exerting high effort, and Daughety and Reinganum (2009), in which a signaling motive helps a team to overcome a free-riding problem.

The remainder of the paper is organized as follows. In Section 2, the outside option signaling game is introduced. In Section 3, we first go through the special case of two possible types in order to illustrate the kind of equilibria that we find in the general case of a finite type space, which we analyze in Section 4. In Section 5 we look at the case that all types are drawn from an interval of the real line. We analyze both these cases because it is much more natural to think about the problem using a finite type space, but the solution has a more tractable form in the limit of an atom-less distribution. We also discuss how changes in the timing or information structure would change the outcome of the game; in particular we analyze a version with commitment in Section 6. Proofs not given in the text can be found in an appendix.

2 The model

The model describes an interaction between a downstream buyer-manufacturer and an up-stream supplier who has to tailor his production processes to the needs of the buyer.[6] In the game with complete information, the seller chooses an investment $i \in \mathrm{I}$, at cost $c(i)$, to improve the value of an asset/good to be traded. If seller and buyer work together, they can generate a value of $v(i)$, while the value of the good or asset to the seller without the buyer is only the

[4]This outcome of an equilibrium in mixed strategies due to a commitment problem is reminiscent of equilibria in hold-up problems with asymmetric information as studied in Gul (2001) and Gonzales (2004).

[5]That by definition signaling cannot lead to too little education changes, however, if one allows education to be productive (see Weiss (1983)).

[6]As explained in the introduction, the model is very abstract and therefore fits a variety of settings, including an employer-employee relationship.

fraction $\theta v(i), \theta \in \Theta \subset [0,1]$.[7] The buyer observes the investment and the value of the asset and makes an offer about how to share the surplus with the seller. If the seller rejects the offer, he gets $\theta v(i)$ from taking his outside option, while the buyer is left with zero. If the seller accepts, they split the generated surplus as proposed by the buyer.

Throughout the paper, we make the following assumptions:

Assumption 1. *We assume that* $\mathrm{I} = \mathbb{R}_+$, *that the functions* v *and* c *are differentiable, increasing, and concave resp. strictly convex. Furthermore* $v(0) \geq 0$, $c(0) = 0$, $c'(0) = 0$, *and* $\lim_{i\to\infty} c'(i) = \infty$.

The buyer has no way to commit to a particular reaction or to write a contract that conditions on i or $v(i)$ or that specifies a particular bargaining game. Instead she makes a take-it-or-leave-it offer to the seller after having observed the investment. If θ is the type of the buyer, i the seller's investment, $o \in [0,1]$ the buyer's offer, expressed as a share of the surplus, and $a \in \{0,1\}$ the acceptance decision of the seller, then the seller's payoff is given by

$$(ao + (1-a)\theta)v(i) - c(i)$$

and the buyer's payoff by

$$a(1-o)v(i).$$

This game can be easily solved by backward induction. The seller will accept all offers $o > \theta$, and since the buyer can always offer a little bit more, we assume that the seller accepts all offers $o \geq \theta$[8]. The buyer will offer a share θ of the realized surplus, which the seller will accept, leaving him a profit of $\theta v(i) - c(i)$ from investment i. In anticipation of this return to investment the seller invests

$$i^c(\theta) = \arg\max \theta v(i) - c(i),$$

which given Assumption 1 always exists and is unique. Therefore also the inverse of i^c exists, which we denote by $\theta^c : i^c(\Theta) \to \Theta$. The seller's payoff under complete information, in dependence on the outside option θ, is denoted by

$$u^c(\theta) = \max_i \theta v(i) - c(i).$$

Note that the derivative of u^c is equal to $v \circ i^c$, and in particular, u^c is increasing and strictly convex.[9]

[7]There does not need to be a deterministic relationship between the value and the investment. As long as the downstream party can observe the investment and the value, the analysis would extend with some notational changes to the case that $v(i)$ represents the expected value of investment i.

[8]This holds for all types except $\theta = 1$. Since the buyer makes no profit on this type, it does not matter whether we assume that this type rejects or accepts an offer of 1.

[9]We could alternatively make this, or other conditions from which it follows, our assumption. That is, investment decisions can be allowed to be multi-dimensional or discrete as long as the optimal investment levels lead to an increasing and strictly convex function u^c.

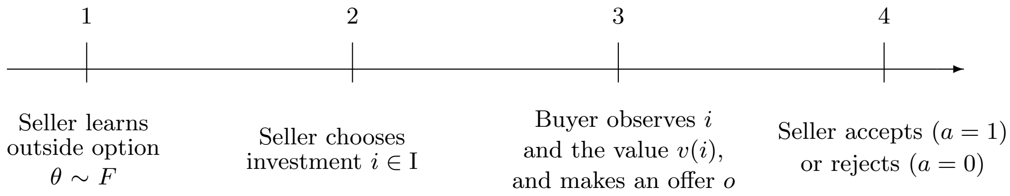

Figure 1: Timeline of the outside option signaling game.

In the game with incomplete information, θ is private information of the upstream seller. The sequence of events is illustrated in Figure 1. We assume that first the seller learns his type θ, which is drawn from a type space $\Theta \subset [0, 1]$ according to a distribution function F. We have to make assumptions on F (like log-concavity), but these will be introduced separately in later sections. The buyer only knows the distribution of the outside option, but not the realized value. She observes the seller's investment, forms beliefs about the outside option and then makes a take-it-or-leave-it offer that is optimal for her given her updated beliefs about the acceptance threshold of the seller. We are interested in perfect Bayesian equilibria of this game, and in any such equilibrium a seller of type θ will accept an offer if and only if it is greater than the outside option. We therefore fix this acceptance decision, the same as in the game with perfect information, as the outcome of the subgame following the buyer's offer. In the remainder of the paper we then deal with the following payoff functions: if the seller is of type θ and invests i, and the buyer makes an offer o, then the seller gets $\max(\theta, o)v(i) - c(i)$ and the buyer gets $(1 - o)v(i)$ if $\theta \leq o,$ and 0 else.

A strategy of the seller specifies an investment for each type, possibly using a randomization device to mix over a set of investments. A strategy of the seller is a function $Q : \Theta \times \mathrm{I} \rightarrow [0, 1]$ such that $Q(.|\theta) = Q(\theta, .)$ is the distribution of investments that a type θ chooses. A strategy for the buyer maps investments into a share of the surplus that she offers to the seller, where she as well may randomize over a set of offers. While a pure strategy is given by a function from investments I to offers in $[0, 1]$, we write a mixed strategy as a function $P : \mathrm{I} \times [0, 1] \rightarrow [0, 1]$, where $P_i(o) = P(i, o)$ is the probability that the buyer's offer, when observing investment i, is less or equal to o.

If the buyer's strategy is given by P, the seller's expected profit from choosing investment i is

$$U(P, i, \theta) = v(i) \int \max(\theta, o) dP_i(o) - c(i),$$

and given a strategy Q of the seller, the buyer's expected payoff from the pure strategy $o : \mathrm{I} \rightarrow [0, 1]$ is

$$V(Q, o) = \int \int_{[\theta \leq o(i)]} (1 - o(i)) v(i) dQ(i|\theta) dF(\theta).$$

3 Example: Two types

In this section, we illustrate the effects that are at work in the model for the case that there are only two possible types, $0 < \theta_L < \theta_H < 1$. Let f_L denote the probability that the outside option is low, and $f_H = 1 - f_L$ the probability that it is high. The analysis of the general model in the next section involves some technicalities that are absent in this special case, which nevertheless conveys much of the intuition.

We start with the buyer's offer decision. It is clear that offering any share greater than θ_H ensures acceptance, and among those offers θ_H is the most profitable for the buyer. Any offer strictly lower than θ_L is sure to be rejected, and is thus weakly dominated by offering θ_L. Offers inbetween θ_L and θ_H lead to only the low type accepting, with θ_L being the cheapest with this outcome. Therefore, the buyer essentially chooses between offers θ_L and θ_H according to her beliefs. She will offer θ_L if she beliefs that the probability of a low outside option is greater than $\frac{1-\theta_H}{1-\theta_L}$.

Next, put yourself in the shoes of a high type seller. This seller knows that for any investment i he will get $\theta_H v(i)$ ex post, given that it is never optimal for the buyer to offer more than θ_H. Therefore, he invests $i_H = \arg\max \theta_H v(i) - c(i)$. His payoff is $u^c(\theta_H)$, which is not surprising as there is no incentive to mimic lower types in this game. Now that we know the strategy of the high type in any possible equilibrium, it is clear that a seller with a low outside option will reveal his type if he invests any amount different from i_H. Therefore, this type faces a choice between i_L, which yields a payoff of $u^c(\theta_L)$, and i_H. Since $u^c(\theta_H) > u^c(\theta_L)$, a separating equilibrium cannot exist. The best the low type can hope for is to pool with the high type and get $u^c(\theta_H)$. This will happen if the buyer indeed has an incentive to make a generous offer in case both types invest high with probability one. If it holds that $f_L \leq \frac{1-\theta_H}{1-\theta_L}$ then the unique equilibrium of the game is the pooling equilibrium.

If the pooling equilibrium does not exist, the only possibility left is a hyprid, or semi-pooling, equilibrium in which the low type mixes between the low and the high investment. For the low type to be indifferent between low and high investment, the probability of offer θ_L following investment i_H must be such that the low type's payoff from choosing i_H is equal to $u^c(\theta_L)$. The probability that has this property is

$$p_{HL} = \frac{u^c(\theta_H) - u^c(\theta_L)}{(\theta_H - \theta_L)v(i_H)}.$$

To make the buyer indifferent between the high and the low offer, the low type seller has to choose the low investment with probability

$$q_{LL} = \frac{(1-\theta_H) - f_L(1-\theta_H)}{(\theta_H - \theta_L)f_L}.$$

This equilibrium exists if and only if the pooling equilibrium does not exist. This insight, that depending on the distribution there is either a pooling equilibrium or an equilibrium with mixed strategies and partial pooling, remains valid in the general case.

Note that the pooling equilibrium becomes more likely the higher the fraction of high types is, and the closer together the two types are. It is also true that increasing the high type's value, or even increasing the high and the low value by an equal amount, can turn a pooling equilibrium into a semi-pooling equilibrium and thereby decrease the ex ante expected payoff of the seller. Interpreting the outside option as the fraction of the asset that he owns, this means that giving the seller more property rights can sometimes be detrimental to his payoff.

4 Finite type space

In this section, we assume that $\Theta = \{\theta_1, ..., \theta_H\}$ with $0 \leq \theta_1 < \theta_2 < ... < \theta_H < 1$.[10] We shortcut $i^c(\theta_k) =: i_k$. Let (P, Q) be a perfect Bayesian equilibrium of the outside option signaling game. In the following, we will derive properties of (P, Q), in order to eventually arrive at a characterization of all equilibrium outcomes. Let I^* be the set of investments that are chosen with positive probability in the equilibrium (P, Q), and let $\Theta^*(i)$ denote the set of all types that choose $i \in \mathrm{I}^*$ with positive probability. We denote by $u^*(\theta)$ the equilibrium payoff received by a seller of type θ, i.e., with this notation we have for all $i \in \mathrm{I}^*$ and $\theta \in \Theta^*(i)$ that $u^*(\theta) = U(P, i, \theta)$.

Note that $u^*(\theta) \geq u^c(\theta)$, because a type θ can always guarantee himself the payoff $u^c(\theta)$ independent of the buyer, by investing $i^c(\theta)$ and taking his outside option. Similarly, because the seller's payoff is weakly increasing in θ for all offers and investments, $U(P, i, \theta)$ and $u^*(\theta)$ are weakly increasing in θ. A higher type could always play a lower type's strategy and get at least the same payoff as that type.

In the following, we will first show that if an investment i may occur at all in equilibrium, then it is chosen with positive probability by the type $\theta^c(i)$ that chooses i under symmetric information, and by none of the higher types. Then, in Lemma 2, we show that investing i is optimal for all lower types, i.e. those between θ_1 and $\theta^c(i)$. Finally, in Prop. 5 we will answer the question which investments will be chosen in equilibrium. The reader who is not interested in the proofs may skip the lemmas leading to Prop. 5 which contains the main result of this section.

When the buyer observes an investment $i \in \mathrm{I}^*$, she updates that the seller must have an outside option in $\Theta^*(i)$. The share she offers will therefore also lie in $\Theta^*(i) \subset \{\theta_1, ..., \theta_H\}$, and it will never be more than the highest possible type would accept, i.e. the offer is not higher than $\theta_m = \max \Theta^*(i)$. The profit received by type θ_m from choosing i is therefore equal to $\theta_m v(i) - c(i)$, which would be strictly smaller than $u^c(\theta_m)$ if $i \neq i_m$. Therefore $i = i_m$, which means that if an investment i occurs in the signaling equilibrium, then $\theta^c(i)$ is the highest

[10] The assumption $\theta_H < 1$ is made only for simplicity. We could easily add types $\theta \geq 1$ who would always invest $i^c(\theta)$ and get no acceptable offer from the buyer. That is, a type $\theta \geq 1$ seller would neither mimic other types nor be mimicked himself.

type to choose this investment. In particular, only investments $i_k, k = 1, ..., H$ can occur in equilibrium.

We will sometimes use the one-to-one relationship between θ_k and i_k and express everything in types. This highlights that in this model types are distinguished by their investment in the complete information case. We can also identify the buyer's offer with the type that just accepts it, and then write the equilibrium strategies P and Q as matrices. An entry p_{kl} in the matrix P stands for the probability of offer θ_l when investment i_k is observed, and an entry q_{kl} in Q is the probability of type k investing i_l, or "mimicking" type l. Since we have shown that in any equilibrium the mixed strategy of type θ_k has support $\{i_k, ..., i_H\}$ and the buyer's random offer following investment i_k takes on values in $\{\theta_1, ..., \theta_k\}$, equilibrium strategies P and Q are triangular matrices. Equilibrium conditions for strategies (P, Q) in matrix form then look as follows:

(i) $q_{kl} > 0$ implies that

$$l \in \arg\max_m v(i_m) \sum_{j=1}^{m} p_{mj} \max(\theta_j, \theta_k) - c(i_m),$$

(ii) for each l with $i_l \in \mathrm{I}^*$, $p_{lj} > 0$ implies that

$$j \in \arg\max_m (1 - \theta_m) \sum_{k=1}^{m} f_k q_{kl}.$$

We will show next that the set of best responses to P of a given type θ_k includes all investments that are greater or equal than i_k and are chosen at all in the equilibrium. In other words, if an investment i_k is chosen at all, then it is optimal for every type smaller or equal to the corresponding type θ_k.

Lemma 2. *For all $i_k \in \mathrm{I}^*$ it holds that $U(P, i_k, \theta) = u^*(\theta)$ for all $\theta = \theta_1, ..., \theta_k$.*

Proof. We know already that $U(P, i_k, \theta_k) = u^*(\theta_k)$. First, we show that the equality also holds for the lowest type, i.e. that $U(P, i_k, \theta_1) = u^*(\theta_1)$. To this end, let θ_l be the lowest type with this property, i.e., $U(P, i_k, \theta_l) = u^*(\theta_l)$ and $U(P, i_k, \theta) < u^*(\theta)$ for all $\theta < \theta_l$. Since no type below θ_l chooses i_k, the offer following it cannot be lower than θ_l. Type l's expected payoff then does not depend on him being type θ_l, but every lower type would get the same payoff when investing i_k :

$$U(P, i_k, \theta_l) = v(i_k) \int o \, dP_{i_k}(o) - c(i_k) = U(P, i_k, \theta) \text{ for all } \theta \leq \theta_l.$$

Payoff monotonicity then implies that $U(P, i_k, \theta) = u^*(\theta)$ for any type $\theta \leq \theta_l$, hence $l = 1$.

Second, we show that for a seller of type θ_l the investments that are best responses to P can be found by maximizing $P_i(\theta_{l-1})v(i)$ over all $i \in \mathrm{I}^*$, where we define $P_i(\theta_0) = 0$. More precisely, the claim is

$$\arg\max_{i\in\mathrm{I}^*} U(P,i,\theta_l) = \arg\max_{i\in\mathrm{I}^*} P_i(\theta_{l-1})v(i) \subset \arg\max_{i\in\mathrm{I}^*} U(P,i,\theta_{l-1}). \tag{1}$$

The claim verifies the lemma, since it implies that

$$i_k \in \arg\max_{i\in\mathrm{I}^*} U(P,i,\theta_k) \subset ... \subset \arg\max_{i\in\mathrm{I}^*} U(P,i,\theta_1).$$

It remains to prove the claim, which we will do by induction. Since we know that $U(P,i,\theta_1) = u^*(\theta_1)$ for all $i \in \mathrm{I}^*$, it holds for $l = 1$ for the appropriate definitions. Assume the claim is true for type $l-1 \geq 1$. For all $i \in \mathrm{I}^*$ with $u^*(\theta_{l-1}) = U(P,i,\theta_{l-1})$ type θ_l's payoff is

$$U(P,i,\theta_l) = u^*(\theta_{l-1}) + (\theta_l - \theta_{l-1})P_i(\theta_{l-1})v(i). \tag{2}$$

while for any $i' \in \mathrm{I}^*$ with $U(P,i',\theta_{l-1}) < u^*(\theta_{l-1})$ it holds that

$$U(P,i',\theta_l) < u^*(\theta_{l-1}) + (\theta_l - \theta_{l-1})P_{i'}(\theta_{l-1})v(i'). \tag{3}$$

Using the induction hypothesis, we have that for any such i and i'

$$P_{i'}(\theta_{l-1})v(i') = P_{i'}(\theta_{l-2})v(i') \leq P_i(\theta_{l-2})v(i) \leq P_i(\theta_{l-1})v(i),$$

hence we have shown that $U(P,i',\theta_l) < U(P,i,\theta_l)$. The remainder of the claim follows easily. □

To summarize, we have shown so far that in any equilibrium, while there may be investments that do not occur at all, every investment that does occur is chosen by the type that would invest the same amount with symmetric information. Furthermore, all lower types' payoff from choosing this investment equals their equilibrium payoff. In order to be consistent with this structure, the buyer's strategy must induce all these indifferences. This observation gives rise to the following lemma.

Lemma 3. *For all k and $i_m \in \mathrm{I}^*$ with $m > k$ it holds that*

$$P_{i_m}(\theta_k)v(i_m) = \frac{u^*(\theta_{k+1}) - u^*(\theta_k)}{\theta_{k+1} - \theta_k}. \tag{4}$$

Moreover, for all $i_m, i_k \in \mathrm{I}^*$ with $m \geq k$ it holds that $p_{mk} > 0$.

Proof. The first claim follows from the proof of Lemma 2, because there we had that for all $i_m \in \mathrm{I}^*$ with $m > k$ it holds that

$$u^*(\theta_{k+1}) = u^*(\theta_k) + (\theta_{k+1} - \theta_k)P_{i_m}(\theta_k)v(i_m).$$

To show the last claim of the lemma, note first that for any type θ_k with $i_k \in \mathrm{I}^*$ it must be true that $p_{kk} > 0$, because else $U(P,i_k,\theta_{k-1})$ is too low: if $p_{kk} = 0$, this payoff is equal to

$$U(P,i_k,\theta_{k-1}) = ((1-p_{kk})\theta_{k-1} + p_{kk}\theta_k)v(i_k) - c(i_k) = \theta_{k-1}v(i_k) - c(i_k) < u^c(\theta_{k-1}).$$

Second, assume that for $m > k$ as in the lemma we have $p_{mk} = 0$. Then

$$0 = P_{i_m}(\theta_k)v(i_m) - P_{i_m}(\theta_{k-1})v(i_m) = \frac{u^*(\theta_{k+1}) - u^c(\theta_k)}{\theta_{k+1} - \theta_k} - \frac{u^c(\theta_k) - u^*(\theta_{k-1})}{\theta_k - \theta_{k-1}},$$

and hence

$$u^c(\theta_k) = u^*(\theta_{k+1})\frac{\theta_k - \theta_{k-1}}{\theta_{k+1} - \theta_{k-1}} + u^*(\theta_{k-1})\frac{\theta_{k+1} - \theta_k}{\theta_{k+1} - \theta_{k-1}}.$$

As mentioned before, the function u^c is strictly convex. Therefore, and because

$$\theta_k = \theta_{k+1}\frac{\theta_k - \theta_{k-1}}{\theta_{k+1} - \theta_{k-1}} + \theta_{k-1}\frac{\theta_{k+1} - \theta_k}{\theta_{k+1} - \theta_{k-1}},$$

we have that

$$u^c(\theta_k) < u^c(\theta_{k+1})\frac{\theta_k - \theta_{k-1}}{\theta_{k+1} - \theta_{k-1}} + u^c(\theta_{k-1})\frac{\theta_{k+1} - \theta_k}{\theta_{k+1} - \theta_{k-1}}.$$

Hence, $p_{mk} > 0$. □

Now that we have some idea about the offers that the buyer must be willing to make, we turn to a description of the buyer's behavior, in order to pin down the seller's equilibrium strategy. The details can be found in the proof of the following proposition that describes the structure of an equilibrium. But first we need more notation and an assumption:

Assumption 4. *Let $R(\theta) := (1-\theta)F(\theta)$ and $\bar{k} := \min\{k : \; R(\theta_k) > R(\theta_{k+1})\}$.*[11] *We assume that R is strictly concave on $\{\theta_{\bar{k}}, ..., \theta_H\}$.*

This assumption implies that $\theta_{\bar{k}}$ is a maximizer of the function R. To understand the role of R, assume for a moment that all types choose the same investment i. Then $R(\theta)$ describes the buyer's expected share of the surplus $v(i)$ if she makes a take it or leave it offer of θ. The maximum $\theta_{\bar{k}}$ of this function is the offer that she would make in a pooling equilibrium. Can a pooling equilibrium exist? Since the highest type θ_H chooses i_H in any equilibrium, if all types pool on the same investment, this must be i_H. It follows that there is such a pooling equilibrium if and only if $\theta_{\bar{k}} = \theta_H$. This suggests that complete pooling is only possible for types lower than $\theta_{\bar{k}}$, and since a separating type could easily be mimicked by a lower type, equilibria must typically be in mixed strategies.

Proposition 5. *If Assumption 4 holds, then any perfect Bayesian equilibrium of the signaling game must have the following form: No investment below $i_{\bar{k}}$ is chosen. A type θ_k with $k \geq \bar{k}$ mixes between all investments in $\{i_k, ..., i_H\}$, with expected payoff equal to $u^c(\theta_k)$. All types θ_k with $k \leq \bar{k}$ mix over $\{i_{\bar{k}}, ..., i_H\}$ with payoff $u^c(\theta_{\bar{k}})$. When observing investment i_k, the buyer mixes between offers in $\{\theta_{\bar{k}}, ..., \theta_k\}$, and her expected payoff from any such offer is $(1-\theta_k)v(i_k)$.*

Proof. See the Appendix. □

[11] Let $\theta_{H+1} = 1$.

All equilibria of the outside option signaling game lead to the same payoffs. Refinements to pin down beliefs following zero probability events are not needed for this result. This is unusual for a signaling game and is due to the special struture of this game, in which equilibrium relationship-specific investment is in fact not a good signal for a high outside option. The types that pool never reveal their outside options, and the others do not improve their payoff in the signaling compared to what they could get independent of the buyer. Because the buyer's offers only matter to a limited extent, beliefs also do not matter as much as in other signaling games.

From all the indifference conditions that have to be met in an equilibrium we are able to obtain an equilibrium candidate. Combining Prop. 5 and Lemma 3 yields for all $k \geq \bar{k}$ and $m > k$

$$P_{i_m}(\theta_k) = \frac{u^c(\theta_{k+1}) - u^c(\theta_k)}{(\theta_{k+1} - \theta_k)v(i_m)} \quad \text{and } P_{i_k}(\theta_k) = 1, \tag{5}$$

as well as for $k < \bar{k}$

$$P_{i_m}(\theta_k) = 0. \tag{6}$$

The equilibrium conditions for the seller's strategy are

$$(1-\theta_l)\sum_{j=1}^{l} f_j q_{jk} = (1-\theta_k)\sum_{j=1}^{k} f_j q_{jk} \quad \text{for all } k \geq l \geq \bar{k} \tag{7}$$

and

$$(1-\theta_l)\sum_{j=1}^{l} f_j q_{jk} \leq (1-\theta_k)\sum_{j=1}^{k} f_j q_{jk} \quad \text{for all } l < \bar{k}. \tag{8}$$

Due to the definition of $\bar{k}$, the latter condition can be fulfilled by defining

$$q_{jk} = q_{\bar{k}k} \qquad \text{for all } j < \bar{k}. \tag{9}$$

Let us further define $\lambda_k := \frac{f_k(1-\theta_k)(1-\theta_{k-1})}{\theta_k - \theta_{k-1}}$ and $\lambda_{H+1} := 0$. Possible values for the q_{jk} are:

$$q_{\bar{k}k} = \frac{\lambda_k - \lambda_{k+1}}{R(\theta_{\bar{k}})} \qquad \text{for all } k > \bar{k} \tag{10}$$

$$q_{\bar{k}\bar{k}} = 1 - \frac{\lambda_{\bar{k}+1}}{R(\theta_{\bar{k}})} \tag{11}$$

$$q_{jk} = \frac{\lambda_k - \lambda_{k+1}}{\lambda_j} \qquad \text{for all } k \geq j > \bar{k} \tag{12}$$

Proposition 6. *The strategies described in equations (5), (6), (9), (10), (11) and (12) form an equilibrium of the outside option signaling game.*

Proof. See the Appendix. □

An example with three types

We look at an example with three types to illustrate the different kinds of equilibrium and the uniqueness issue. First, since $R(\theta)$ is the buyer's expected share of the value if all types choose the same investment and the buyer offers θ, pooling on the investment i_3 is an equilibrium if and only if $(1-\theta_3) = \max_\theta R(\theta)$. We write this equilibrium in the matrix form described at the beginning of this section:

$$\mathrm{Q} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}, \ \mathrm{P} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Note that beliefs out of equilibrium, i.e. after observing an investment $i \neq i_3$, are not pinned down uniquely. Consequently also the first two rows in P are not uniquely determined.

In case $(1-\theta_2)F(\theta_2) = \max_\theta (1-\theta)F(\theta)$ an equilibrium is of the following form:

$$\mathrm{Q} = \begin{pmatrix} 0 & q_{12} & 1-q_{12} \\ 0 & q_{22} & 1-q_{22} \\ 0 & 0 & 1 \end{pmatrix}, \ \mathrm{P} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & p_{32} & 1-p_{32} \end{pmatrix}$$

Again, the first row of P does not have to be the unit vector. To see how the structure of Q translates into the condition for R, let $\mu_2 := q_{22}f_2 + q_{12}f_1$ be the probability of i_2 being chosen, which here is the same as the probability of any lower investment being chosen. The conditions for the buyer are

- $(1-\theta_3)(1-\mu_2) = (1-\theta_2)(F(\theta_2)-\mu_2)$ which is equivalent to $\mu_2 = \frac{R(\theta_2)-R(\theta_3)}{\theta_3-\theta_2}$. This expression is always less or equal to 1, and it is nonnegative iff $R(\theta_2) \geq R(\theta_3)$.
- $(1-\theta_2)(F(\theta_2)-\mu_2) \geq (1-\theta_1)(F(\theta_1)-q_{12}f_1)$ which is equivalent to $q_{12}f_1 \geq \frac{R(\theta_1)-R(\theta_2)}{1-\theta_1} + \frac{(1-\theta_2)\mu_2}{(1-\theta_1)}$
- $(1-\theta_2)\mu_2 \geq (1-\theta_1)q_{12}f_1$ which is equivalent to $q_{12}f_1 \leq \frac{(1-\theta_2)}{(1-\theta_1)}\mu_2$

Obviously, the last two conditions can only be fulfilled if $R(\theta_1) \leq R(\theta_2)$. If this holds, the solutions are $q_{12} = \frac{(1-\theta_2)\mu_2}{R(\theta_1)} - \Delta$ for any $0 \leq \Delta \leq \frac{R(\theta_2)-R(\theta_1)}{R(\theta_1)}$. Thus, in this case the solution is typically not unique. If we make the restriction $q_{12} = q_{22}$, the last two conditions, which state that the buyer prefers offering θ_2 to offering θ_1, read

- $q_{12}f_1 \geq \frac{R(\theta_1)-R(\theta_2)}{1-\theta_1} + \frac{R(\theta_2)q_{12}}{(1-\theta_1)} \iff 1 \geq q_{12}$
- $f_1 \leq \frac{(1-\theta_2)}{(1-\theta_1)}F(\theta_2) \iff R(\theta_1) \leq R(\theta_2)$

That is, we immediately have a solution, given by $q_{12} = q_{22} = \frac{R(\theta_2)-R(\theta_3)}{F(\theta_2)(\theta_3-\theta_2)}$. This is not surprising, because here the pooling condition (R increasing) holds up to θ_2. The proposed equilibrium in Prop. 6 also uses this fact. The buyer's expected profit does not depend on the values of q_{12} and q_{22}, only on μ_2, which is uniquely defined.

Last, if $(1-\theta_1)F(\theta_1) = \max_\theta(1-\theta)F(\theta)$, then the equilibrium is unique:

$$\mathrm{Q} = \begin{pmatrix} q_{11} & q_{12} & 1-q_{11}-q_{12} \\ 0 & q_{22} & 1-q_{22} \\ 0 & 0 & 1 \end{pmatrix}, \mathrm{P} = \begin{pmatrix} 1 & 0 & 0 \\ p_{21} & 1-p_{21} & 0 \\ p_{31} & p_{32} & 1-p_{31}-p_{32} \end{pmatrix}$$

For the values of the entries, see Proposition 6. The expressions may become complex, that is why we look at a continuous strategy space in the next section.

We know from Prop. 5 that a strategy of the form

$$\mathrm{Q} = \begin{pmatrix} q_{11} & 0 & 1-q_{11} \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

cannot be part of an equilibrium. This can be checked explicitly here, showing that for this to be an equilibrium it must be true that $R(\theta_1) = \max_\theta R(\theta)$ and R convex, contradicting our assumption that R is concave. While it may well be possible to relax this assumption and still say something about the resulting equilibria, we do not address this question in this paper.

5 Continuum type space

The expressions for the equilibrium strategies will have a simpler form in this section, which treats the continuum type space as the limit case. Hence, in this section Θ is an interval, $\Theta = [\theta_L, \theta_H]$. We assume that F is an atomless distribution on Θ with density $f > 0$, for which the derivative f' exists.

Assumption 7. *F is log-concave.*

Analogous to the previous section, we define $\bar{\theta} = \theta_H$ if $R'(\theta) \geq 0$ on Θ, and else

$$\bar{\theta} = \inf\{\theta \in \Theta : \; R'(\theta) < 0\}, \tag{13}$$

and have

Lemma 8. *Given Assumption 7, R is weakly increasing on $[\theta_L, \bar{\theta}]$, concave on $[\bar{\theta}, \theta_H]$, and maximized at $\bar{\theta}$.*

See the appendix for the proof.

Proposition 9. *Given Assumption 7, an equilibrium of the signaling game is given by*

$$P_i(\theta) = \begin{cases} 0 & \theta < \bar{\theta} \\ \frac{v(i^c(\theta))}{v(i)} & \bar{\theta} \leq \theta \leq \theta^c(i) \\ 1 & \theta \geq \theta^c(i) \end{cases} \tag{14}$$

and $Q(i|\theta) = Q(i|\bar{\theta})$ *for all* $\theta < \bar{\theta}$, *and for all* $\theta \geq \bar{\theta}$

$$(15) \qquad Q(i|\theta) = \begin{cases} 0 & i < i^c(\theta) \\ 1 - \frac{(1-\theta^c(i))^2 f(\theta^c(i))}{(1-\theta)^2 f(\theta)} & i^c(\theta) \leq i < i^c(\theta_H) \\ 1 & i = i^c(\theta_H) \end{cases}$$

The proof is straightforward and therefore omitted. While this result does not say that the described equilibrium is the unique outcome of the game, it can easily be shown that this equilibrium is the limit of the equilibrium found in the previous section, and therefore the only reasonable equilbrium. It is the limit in the following sense: For any partition $\theta_L < \theta_1 < ...\theta_n < \theta_H$ of the interval $[\theta_L, \theta_H]$, one can define a discrete distribution by $f_k = F(\theta_k) - F(\theta_{k-1}), k = 2, ..., n$. For this distribution take the equilibrium described in Prop. 6, and interpret the equilibrium strategies as step functions on $[\theta_L, \theta_H] \times I$ resp. $I \times [\theta_L, \theta_H]$. These step functions converge against the functions defined in Proposition 9 as the partition becomes finer.

5.1 Surplus Comparison

In the following paragraphs, we compare different timings and information regimes with respect to the payoff that is generated for the seller and the buyer as well as the joint surplus. In some applications as for example the mobility of a worker, it seems realistic that the worker knows his mobility but the employer never learns it until it is too late. Alternatively, it may be the case that the worker learns his outside option only after making the firm-specific investment. In a market setting, it may be that alternative offers are known to both sides from the start, or that both sides learn of them after investment decisions have been made. We always evaluate payoffs and surplus with respect to the distribution F, and this is also how the expectations in the following expressions should be understood.

First we look at the case of complete information. In this case the outside option is common knowledge from the start. The seller's expected profit is $E[u^c(\theta)]$ and the buyer obtains $E[(1-\theta)v(i^c(\theta))]$. The expected joint surplus is $E[S(i^c(\theta))]$, with $S := v - c$.

In constrast, if the outside option becomes common knowledge only after the investment is sunk, and is not known before to any party, the expected social surplus is $S(i^c(E[\theta]))$. If we assume that $S(i^c(\theta))$ is concave function in θ (eg. $v''' - c''' \leq 0$) then this surplus is higher than the one before. The seller gets $u^c(E[\theta])$ and is therefore worse off than in the complete information case, because he cannot prepare for his outside option. The buyer is better off with $(1 - E[\theta])v(i^c(E[\theta]))$, capturing the quasi-rent from low types who invest too much.

A third possible timing and information structure of the game is that the seller, and only the seller, learns the outside option later. In this case, there is no signaling motive. The buyer makes an offer of $\bar{\theta}$ and the seller invests $i^c(E[\theta \vee \bar{\theta}])$. While the investment is higher than in the two cases above, it is not always put to its best use, as all types above $\bar{\theta}$ reject the offer. The seller gets $u^c(E[\bar{\theta} \vee \theta])$ which is more than in the previous case, as he enjoys some informations rents. The buyer gets $R(\bar{\theta})v(i^c(E[\theta \vee \bar{\theta}]))$.

Finally, in the signaling equilibrium (Prop. 9), a seller with outside option θ gets $\max(u^c(\theta), u^c(\bar{\theta}))$, i.e. the seller's expected profit is

$$F(\bar{\theta})u^c(\bar{\theta}) + \int_{\bar{\theta}}^{\theta_H} u^c(\theta)dF(\theta).$$

To find the buyer's surplus in the signaling equilibrium, note first that $-R''(\hat{\theta})$ is the probability density of investment on $[\bar{\theta}, \theta_H)$. Therefore, the buyer's expected payoff is

$$\int_{\bar{\theta}}^{\theta_H} -R''(\hat{\theta})(1-\hat{\theta})v(i^c(\hat{\theta}))d\hat{\theta} + (1-\theta_H)^2 f(\theta_H)v(i(\theta_H))$$

We see that the seller has an incentive to learn the outside option early, because in the signaling equilibrium his expected payoff is $E[u^c(\theta \vee \bar{\theta})] > u^c(E[\theta \vee \bar{\theta}])$. Since of all the possible regimes the seller is best of in the signalling equilibrium, if he could influence the timing or information distribution, then he would do so in the direction of the signalling structure.

If sellers came from two distinguishable groups, such that the distribution of outside options for one group first order dominates the distribution of the other group, then all sellers in the group with higher outside options are better off if the cut-off value is also shifted to the right. Higher outside options have the effect that lower types can hide behind the better average bargaining position in their group. In the cases where the cut-off value is decreased, these types are worse off.

The effects of a FOSD shift in the distribution of types on the buyer's profit is ambiguous. While the presence of higher types leads to higher investment, it also means that the buyer has to make higher offers. This reflects the nonmonotonic relationship between the buyer's profit and the type of the seller that arises in a hold-up problem even under symmetric information: low types generate low surplus and high types get a large share of the pie, so that typically the buyer prefers sellers of intermediate bargaining power. In the next section we consider the case that the investment decision is contractible. It is clear that in that case, lower seller types are unambiguously better for the buyer.

6 Buyer-optimal contract if investment is verifiable

In the game that is studied in the main part of this paper, all the buyer can do is to make a take it or leave it offer based on her updated beliefs. This is optimal for her from an ex post perspective, but not necessarily from an ex ante perspective. In this section we shall explore the consequences of full commitment and ask what would happen if the buyer could offer a binding contract conditional on investment before the seller moves. We maintain the assumption that the seller's type is not observable, and characterize the optimal screening contract.[12]

Proposition 10. *If the buyer can offer a contract conditional on the investment decision, the outcome involves investment of $i^c(1)$ and inefficient separation for types $\theta \geq \bar{\theta}$, which take the outside option with probability $\frac{v(i^c(\theta))}{v(i^c(1))}$. Each seller type is left with the same payoff as in the case without commitment.*

Proof. We use the revelation principle and let a general contract be a map from types into outcomes that satisfies the incentive compatibility constraints of each type of seller telling the truth. In addition, the buyer has to take into account that the seller can go for his outside option, then getting a payoff of $\theta v(i)$ after having invested an amount i, or $u^c(\theta)$ ex ante. All that matters for truth telling and participation of the seller is his expected payoff, and the buyer in addition cares for the surplus created by the contract. Therefore, it is sufficient to focus on contracts of the form $(t(\theta), i(\theta), x(\theta))$, where $t(\theta)$ is an up-front payment from the seller to the buyer that an announced type θ is required to make, $i(\theta)$ is the required investment, and $x(\theta)$ the probability of separation. With probability $1 - x(\theta)$, buyer and seller collaborate and the seller gets the whole ex post surplus $v(i(\theta))$. There is no loss of generality in assuming this form of contracts, because all payoff transfers from the seller to the buyer can be handled by the up-front payment $t(\theta)$. Given such a contract, the expected payoff to a seller of type θ who pretends to be of type $\tilde{\theta}$ is

$$(1 - x(\tilde{\theta}))v(i(\tilde{\theta})) + x(\tilde{\theta})\theta v(i(\tilde{\theta})) - c(i(\tilde{\theta})) - t(\tilde{\theta}).$$

A truth-telling seller creates the joint surplus $S(i(\theta)) - x(\theta)(1 - \theta)v(i(\theta))$, and gets $u_S(\theta) = S(i(\theta)) - x(\theta)(1 - \theta)v(i(\theta)) - t(\theta)$ for himself. The buyer's optimization problem is the following:

$$\max \int_{\theta_L}^{\theta_H} t(y) dF(y),$$

subject to the incentive compatibility constraint

$$\text{(IC)} \qquad u_S(\theta) \geq u_S(\tilde{\theta}) + (\theta - \tilde{\theta})x(\tilde{\theta})v(i(\tilde{\theta}))$$

and the ex ante participation constraint

$$\text{(IR)} \qquad u_S(\theta) \geq u^c(\theta),$$

which have to hold for all $\theta, \tilde{\theta} \in [\theta_L, \theta_H]$.

It may seem intuitive that an optimal contract should specify the investment $i^c(1)$ for all types, because seller types differ only with respect to the outside option. The screening device is the probability of separation, not the investment. However, in order to separate the seller's

[12] Adverse selection problems with type-dependent reservation utilities have been addressed before (Moore (1985), Jullien (2000)), but is not a special case of these results.

types this probability must be positive. Therefore, it is not obvious that $i(\theta) = i^c(1)$, because $i^c(1)$ is not the optimal preparation for every type (which would be $i^c(1-(1-\theta)x(\theta))$). In particular, so far the formulation also allows for some types not participating and choosing a contract of the form $x = 1, i = i^c(\theta), t = 0$.

To see that the optimal contract will nevertheless specify $i(\theta) = i^c(1)$, consider any contract $(t(\theta), i(\theta), x(\theta))$. The contract $(\tilde{t}(\theta), \tilde{i}(\theta), \tilde{x}(\theta))$ defined by

$$\tilde{t}(\theta) = t(\theta) + S(i^c(1)) - S(i(\theta)) \geq t(\theta),$$

$\tilde{i}(\theta) = i^c(1)$, and

$$\tilde{x}(\theta) = x(\theta)\frac{v(i(\theta))}{v(i^c(1))} \in [0,1]$$

leads to the same IC and PC constraints and higher expected profit for the buyer. In particular, this means that excluding types entirely is generally not a good idea for the buyer.

For any $x : [\theta_L, \theta_H] \to [0,1]$ that is part of an IC contract, if $x(\tilde{\theta}) = 0$ for some type $\tilde{\theta}$, then we know that lower types pool on this type, i.e. $u_S(\tilde{\theta}) = u_S(\theta)$ for all types $\theta \leq \tilde{\theta}$. In the buyer's optimal contract it will then hold that $x(\theta) = 0$ and $t(\theta) = S(i^c(1)) - u^c(\tilde{\theta})$ for all $\theta \leq \tilde{\theta}$. We therefore now take a threshold $\theta^0 \in \Theta$ as given and replace the IC constraints by the requirement that x is nondecreasing and

$$u_S(\theta) = v(i^c(1)) \int_{\theta_0}^{\theta} x(y)dy + u^c(\theta^0).$$

We define $X^0 := \{x : [\theta^0, \theta_H] \to (0,1], \text{ nondecreasing}\}$. Following the standard method of finding an optimal screening contract we write the problem as

$$\max_{x \in X^0} S(i^c(1)) - u^c(\theta^0) - \int_{\theta^0}^{\theta_H} (R'(\theta)+1)x(\theta)v(i^c(1))d\theta$$

s.t. $\int_{\theta^0}^{\theta} x(y) - \frac{v(i^c(y))}{v(i^c(1))}dy \geq 0$.

Because $R'(\theta)+1 \geq 0$, $x(\theta)$ must be as small as possible. This suggests that IR should bind everywhere, which we will indeed show next. First, because the objective function can also be written as

$$S(i^c(1)) - u_S(\theta_H) - \int_{\theta^0}^{\theta_H} R'(\theta)x(\theta)v(i^c(1))d\theta$$

it is clear that $\theta^0 \geq \bar{\theta}$. Furthermore, for the part that depends on x we can use integration by parts to get

$$\begin{aligned}
&u_S(\theta_H) + \int_{\theta^0}^{\theta_H} R'(\theta)x(\theta)v(i^c(1))d\theta \\
=&(1-\theta_H)f(\theta_H)u_S(\theta_H) - R'(\theta^0)u^c(\theta^0) - \int_{\theta^0}^{\theta_H} R''(\theta)u_S(\theta)d\theta \\
\geq&(1-\theta_H)f(\theta_H)u^c(\theta_H) - R'(\theta^0)u^c(\theta^0) - \int_{\theta^0}^{\theta_H} R''(\theta)u^c(\theta)d\theta
\end{aligned}$$

This shows that the objective function is maximized if IR is binding everwhere. For this to be true, the buyer would have to set

$$x(\theta) = \frac{v(i^c(\theta))}{v(i^c(1))},$$

which is indeed increasing, hence must be the solution to the optimization problem. Finally, we find the optimal θ_0: Solving

$$\max_{\theta_0} S(i^c(1)) - u^c(\theta_0) - \int_{\theta_0}^{\theta_H} (R'(\theta) + 1)v(i^c(\theta))d\theta$$

yields $\bar{\theta}$ as the optimal cut-off value. □

To summarize, the buyer promises a contract over the full surplus $v(i^c(1))$ with some probability, in exchange for an up-front payment. The seller can choose among a menu of contracts consisting of combinations of separation probabilities and up-front payments

$$\left(\frac{v(i)}{v(i^c(1))}, S(i^c(1)) - S(i)\right), \quad i \in [i^c(\bar{\theta}), i^c(\theta_H)],$$

or trade for sure and pay $S(i^c(1)) - u^c(\bar{\theta})$ up-front.

The separation probability for a given type is never higher than the separation probability of that type in the signaling equilibrium. A seller of type θ receives the same payoff, $\max(u^c(\theta), u^c(\bar{\theta}))$. The optimal contract induces higher investment now that i is verifiable and a hold-up problem does not exist. Interestingly, for types higher than $\bar{\theta}$, there is in fact overinvestment relative to the investment's later use. The buyer's expected payoff is of course higher than in the case without commitment, as is the expected surplus, which amounts to $S(i^c(1)) - \int_{\bar{\theta}}^{\theta_H} v(i^c(\theta))(1-\theta)dF(\theta)$, while in the signaling equilibrium it is $\int\int S(i)dQ(i|\theta)dF(\theta) - \int_{\bar{\theta}}^{\theta_H} v(i^c(\theta))(1-\theta)dF(\theta)$.

7 Conclusion

In the present paper, we have introduced private information about the reservation value in a simple property rights model.The simplicity of the model allows us to fully characterize the resulting equilibrium payoffs, which are uniquely determined. The equilibrium involves pooling up to a certain type of outside option, such that all lower types get the same payoff and because they accept all offers in equilibrium, these types are not distinguishable, even ex post. Higher types follow a mixed strategy and on average obtain the same payoff as with complete information. The seller has to mix between the investments because there is a strong force against a separating equilibrium in this model: if only high types choose a certain investment and get high offers, they will be mimicked by lower types.

In the outside option signaling game, there is a gap between the chosen investment and the investment that would result if the seller would get the full return to his investment. We have shown that this gap vanishes if investment is verifiable. This gap would also shrink if the seller had greater bargaining power than in the game that was analyzed. For example, if the bargaining game was modeled as the seller making a take-it-or-leave-it offer with probability α and the buyer only with probability $1 - \alpha$, then a higher α would increase the surplus and the seller's payoff. Since there is more investment on average, the buyer's payoff can be non-monotonic in α. Although it is standard in principal-agent models to assume take-it-or-leave-it offers by the principal, it would of course be interesting to allow for more complex bargaining games at the ex post stage. One game that should leave the results unchanged is repeated offers by the buyer, but if both players can make offers, results are likely to change and become difficult to obtain (c.f. Skryzpazc (2004)).

There are a couple of other extensions of the model that present themselves. One interesting task for future work is to allow the payoff that the buyer gets when the seller takes the outside option to be dependent on the seller's type. This would admit a greater set of applications, in particular the interpretation of the outside option as sueing for payment, with private information about the probability of winning.[13] Another possible extension is the case of pure rent-seeking, in which the investment increases the outside value but is of little use inside the relationship. Investment can still be used as a signal for profitable outside opportunities, but higher investment is no longer more efficient.

[13] See Chonné and Linnemer (2008) for a related model in the context of pretrial bargaining and investment in trial preparation.

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Proofs

Proof of Proposition 5. Let $i_k \in \mathrm{I}^*$. When observing i_k, the buyer's expected profit from offering θ_l is

$$(1-\theta_l)\frac{\sum_{j=1}^{l} f_j q_{jk}}{\sum_{j=1}^{k} f_j q_{jk}}.$$

We know from Lemma 3 that to be consistent with the seller's behavior, the buyer, when observing i_k, has to offer all θ_j, $i_j \in \mathrm{I}^*, j \leq k$ with positive probability. She will offer θ_k if

$$\sum_{j=1}^{k} f_j q_{jk}(1-\theta_k) \geq \sum_{j=1}^{l} f_j q_{jk}(1-\theta_l) \text{ for all } l,$$

and θ_l if

$$\sum_{j=1}^{k} f_j q_{jk}(1-\theta_k) = \sum_{j=1}^{l} f_j q_{jk}(1-\theta_l).$$

As a first step, we write down all inequalities that define the buyer's behavior in an equilibrium (P, Q). Denote by

$$K := \{k : \ i_k \in \mathrm{I}^* \backslash \{i_H\}\}$$

all chosen investments that are strictly smaller than i_H. We treat H separately because we have to account for the fact that Q is a stochastic matrix, i.e., that the row entries add up to one. For all $j, l \leq k, l, k \in K$ the following inequalites must hold:

$$\begin{aligned}
\sum_{i=1}^{j} f_i(\theta_k-\theta_j)q_{ik} + \sum_{i=j+1}^{k} f_i(\theta_k-1)q_{ik} &\leq 0 \\
-\left(\sum_{i=1}^{l} f_i(\theta_k-\theta_l)q_{ik} + \sum_{i=l+1}^{k} f_i(\theta_k-1)q_{ik}\right) &\leq 0 \\
-q_{jk} &\leq 0
\end{aligned}$$

as well as (straightforward calculation) for all $l < H, i \in K$

$$\begin{aligned}
R(\theta_H)-R(\theta_l) &\geq \sum_{j=1}^{l}\sum_{j\leq k\in K} f_j(\theta_l-\theta_H)q_{jk} + \sum_{j=l+1}^{H-1}\sum_{k\in K} f_j(1-\theta_H)q_{jk} \\
R(\theta_i)-R(\theta_H) &\geq \sum_{j=1}^{i}\sum_{j\leq k\in K} f_j(\theta_H-\theta_i)q_{jk} + \sum_{j=k+1}^{H-1}\sum_{j\leq k\in K} f_j(\theta_H-1)q_{jk} \\
1 &\geq \sum_{j\leq l\in K} q_{ji}
\end{aligned}$$

We are going to treat the variables we are looking for as one big vector, denoted by q. The entries in q are indexed by $jk, 1 \leq j \leq k, k \in K$. Similarly, we define a vector μ^{jk} by $\mu_{ik}^{jk} = f_i(\theta_k-\theta_j)$ for all $i \leq j$ and $\mu_{ik}^{jk} = f_i(\theta_k-1)$ for all $i > j$ and zero else. Furthermore, define a vector μ^l by $\mu_{jk}^l = f_j(\theta_l-\theta_H)$ for all $j \leq l$ and $\mu_{jk}^l = f_j(1-\theta_H)$ for all $j > l$.

Last, let 1^j denote a vector with $1^j_{jk} = 1$ for $j \leq k \in K$ and 0 else; and let e^{jk} be a vector with $e^{jk}_{jk} = 1$ and 0 else.

Our inequalities now read

$$\begin{array}{rcll} -e^{jk}q & \leq & 0 & 1 \leq j \leq k, k \in K \\ 1^j q & \leq & 1 & j = 1, ..., H-1 \\ \mu^{jk}q & \leq & 0 & \text{for all } k \in K, j < k \text{ and } \geq 0 \text{ for } j \in K \\ \mu^l q & \leq & R(\theta_H) - R(\theta_l) & \text{for all } l < H \text{ and } \geq 0 \text{ for } l \in K \end{array}$$

As the second step, we find a system of inequalities that is an alternative of this system, i.e. that has a solution if and only if this one has none. We use Theorem 22.1 of Rockafellar (1970) to get the following alternative system:

(i) $\sum_{j=1}^{H-1} \beta_j + \sum_{l=1}^{H-1} \delta_l (R(\theta_H) - R(\theta_l)) < 0$

(ii) $\sum_{j=1}^{H-1} 1^j \beta_j + \sum_{jk} \mu^{jk} \gamma_{jk} + \sum_{l=1}^{H-1} \mu^l \delta_l \geq 0$

where we are looking for coefficients $\beta_j \geq 0$, $j = 1, ..H-1$, γ_{jk} (≥ 0 if $j \notin K$), δ_l, (≥ 0 if $l \notin K$). For the analysis, it is convenient to write the second equation as an equation in each coefficient jk with $k \in K$ and $j \leq k$

$$\beta_j + \sum_{i=1}^{j-1} \gamma_{ik} f_j(\theta_k - 1) + \sum_{i=j}^{k-1} \gamma_{ik} f_j(\theta_k - \theta_i) + \sum_{l=1}^{j-1} \delta_l f_j(1 - \theta_H) + \sum_{l=j}^{H-1} \delta_l f_j(\theta_l - \theta_H) \geq 0$$

Let $\hat{k} = \min K$. We claim that $\bar{k} = \hat{k}$ and first show that $R(\theta_l) \leq R(\theta_{\hat{k}})$ for $l < \hat{k}$. Assume not. Then there is a solution with $\delta_l = \gamma_{lk} = 1$ and $\delta_{\hat{k}} = \gamma_{\hat{k}k} = -1$ and all other coefficients equal to zero: The first inequality is obviously satisfied, and for the second, since $k \geq \hat{k} > l$ always holds, there are only three cases to distinguish, $j > \hat{k}$, $l < j \leq \hat{k}$, and $j \leq l$.

Similarly, one can show that $R(\theta_{\hat{k}+1}) \leq R(\theta_{\hat{k}})$ is also necessary, because else there is a solution with $\delta_{\hat{k}+1} = \gamma_{\hat{k}+1k} = 1$ and $\delta_{\hat{k}} = \gamma_{\hat{k}k} = -1$. The easy case distinctions are again left to the reader. Hence, $\hat{k} = \bar{k}$. Note that we could have shown more generally that $K \subset \{k \text{ with } R(\theta_k) \geq R(\theta_{k+1})\}$.

Next we show that K is an interval. Assume to the contrary that there is a gap in K, i.e, there exist $l < m < h$ with $m \notin K$, $l = \max\{k \in K, k \leq m\}$ and $h = \min\{k \in K, k \geq m\}$. There is a $\lambda \in (0,1)$ with $(1-\lambda)\theta_h + \lambda\theta_l = \theta_m$. Define $\delta_l = \gamma_{lk} = -\lambda, \delta_m = \gamma_{mk} = 1, \delta_h = \gamma_{hk} = -(1-\lambda)$ for all relevant $k \in K$. Then the first condition holds because R is concave on K: $\lambda R(\theta_l) + (1-\lambda)R(\theta_h) - R(\theta_m) < 0$. That the second condition always holds with equality is seen immediately if $k \leq l$, for which this condition takes the form $\theta_m - \theta_H - \lambda(\theta_h - \theta_H) - (1-\lambda)(\theta_l - \theta_H) = 0$. For the remaining case $k \geq h$ there has to be again a case distinction regarding j, each case leading to the same result. Thus concavity of R implies that there are no gaps in chosen investment, $K = \{\bar{k}, ..., H-1\}$.

□

Proof of Prop. 6. First, we check that the strategies fulfill equation 7. For $k > \bar{k}$:

$$\begin{aligned}
(1-\theta_l)\sum_{j=1}^{l} f_j q_{jk} &= (1-\theta_l)\left(\sum_{j=1}^{\bar{k}} f_j \frac{\lambda_k - \lambda_{k+1}}{R(\theta_{\bar{k}})} + \sum_{j=\bar{k}+1}^{l} \frac{\lambda_k - \lambda_{k+1}}{\lambda_j}\right) \\
&= (1-\theta_l)\left(\frac{(\lambda_k - \lambda_{k+1})}{1-\theta_{\bar{k}}} + \sum_{j=\bar{k}+1}^{l}\left(\frac{(\lambda_k - \lambda_{k+1})}{1-\theta_j} - \frac{(\lambda_k - \lambda_{k+1})}{1-\theta_{j-1}}\right)\right) \\
&= \lambda_k - \lambda_{k+1},
\end{aligned}$$

which is independent of l. Similarly for $k = \bar{k}$.

Next, note that

$$\frac{R(\theta_k) - R(\theta_{k-1})}{\theta_k - \theta_{k-1}} = \frac{f_k(1-\theta_{k-1})}{(\theta_k - \theta_{k-1})} - F(\theta_k) = \frac{f_k(1-\theta_k)}{(\theta_k - \theta_{k-1})} - F(\theta_{k-1})$$

and therefore

$$\lambda_k - \lambda_{k+1} = (1-\theta_k)\left(\frac{R(\theta_k) - R(\theta_{k-1})}{\theta_k - \theta_{k-1}} - \frac{R(\theta_{k+1}) - R(\theta_k)}{\theta_{k+1} - \theta_k}\right) \geq 0.$$

Also,

$$R(\theta_{\bar{k}}) \geq \lambda_{\bar{k}+1} \Leftrightarrow (\theta_{\bar{k}+1} - \theta_{\bar{k}})F(\theta_{\bar{k}}) \geq f_{\bar{k}+1}(1-\theta_{\bar{k}+1}) \Leftrightarrow R(\theta_{\bar{k}}) \geq R(\theta_{\bar{k}+1}).$$

These conditions imply that all $q_{jk} \geq 0$. We still need to show that they add up to one:

$$\begin{aligned}
\sum_{k=j}^{H} q_{jk} &= \sum_{k=j}^{H} \frac{\lambda_k - \lambda_{k+1}}{\lambda_j} = 1 \qquad\qquad \text{for all } j > \bar{k} \\
\sum_{k=\bar{k}}^{H} q_{j\bar{k}} &= 1 - \frac{\lambda_{\bar{k}+1}}{R(\theta_{\bar{k}})} + \sum_{k=\bar{k}+1}^{H} \frac{\lambda_k - \lambda_{k+1}}{R(\theta_{\bar{k}})} = 1
\end{aligned}$$

Here we have that all low types follow the same strategy. If such a restriction is not imposed, there may be more possible values for the strategies. □

Proof of Lemma 8. To show that F log-concave (which is implied by f log-concave) is sufficient for this property of R, we will show first that

$$R''(\theta) \geq 0 \Rightarrow R'(\theta) > 0.$$

The second derivative of R is

$$R''(\theta) = (1-\theta)f'(\theta) - 2f(\theta)$$

such that $R''(\theta) \geq 0$ implies that $f'(\theta) > 0$ and

$$(1-\theta) \geq \frac{2f(\theta)}{f'(\theta)}.$$

Hence,

$$R'(\theta) = (1-\theta)f(\theta) - F(\theta) \geq \frac{2f(\theta)^2 - F(\theta)f'(\theta)}{f'(\theta)} \geq \frac{f(\theta)^2}{f'(\theta)} > 0.$$

From the definition of $\bar{\theta}$, where we have a local maximum, the claim easily follows.

$\square$