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Investments and the Holdup Problem in a Matching Market

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Abstract

This paper studies investment incentives in the steady state of a dynamic bilateral matching market. Because of search frictions, both parties in a match are partially locked–in when they bargain over the joint surplus from their sunk investments. The associated holdup problem depends on market conditions and is more important for the long side of the market. In the case of investments in homogenous capital only the agents on the short side acquire ownership of capital. There is always underinvestment on both sides of the market. But when market frictions become negligible, the equilibrium investment levels tend towards the first–best.

Keywords: Holdup Problem, Matching Market, Investments; *JEL Classification No.:* C78, D23, D92 .

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1 Introduction

This paper studies investment incentives in a bilateral matching market with search frictions. The agents on both sides of the market make their investment decisions before entering the matching process. In a match they negotiate about sharing the joint surplus from their sunk investments. Since switching to an alternative bargaining partner involves search costs, both parties in a match are partially locked–in. This generates a holdup problem for the agents' investment decisions at the market entry stage. We investigate how the resulting underinvestment effect depends on market conditions. It turns out that the holdup problem is more important for the long side of the matching market. Therefore, these agents invest less than the agents on the short side of the market, even when the investments of all agents are equally productive. Indeed, in the case of investments in homogenous capital only the agents on the short side of the market acquire ownership of capital.

In our setting, the productivity of investments is independent of the trading partners' identity. Nonetheless, in a match the traders are partially locked–in because they face search frictions. The level these frictions determines the degree to which investments effectively become relation-specific in a particular match. The agents discount future utilities and so their outside options decrease with the expected length of time to achieve a transaction with an alternative bargaining partner. Thus the agents' discount rate reflects the degree of asset specificity in a match. Following Rubinstein and Wolinsky (1984), we can study a 'frictionless market' by considering the equilibrium outcome when the time cost associated with the matching process becomes negligible. In this limit, the parties in a match are no longer locked-in and the holdup problem disappears, because switching to another trading partner is costless. We show that in such a 'frictionless market' the agents' investments at the market entry stage are indeed efficient in the sense that they maximize the joint surplus from a match.

The holdup problem in its classical formulation refers to relationship–specific investments in environments with incomplete contracts.[1] The trading partners make investments that have little value outside the relationship. These investments are observed by the partners but are not verifiable in court. Therefore contracts are incomplete, and the partners negotiate the division of the surplus ex post at a stage where the investments are

[1]See e.g. Grout (1984), Williamson (1985), Hart and Moore (1988). Schmitz (2001) provides a survey on the holdup problem and incomplete contracts.

sunk. Accordingly, they will not be able to appropriate the full marginal benefit of their investment, which typically leads to underinvestment in specific assets.[2] In this paper, the agents cannot contract over the levels of investments because investments are made before entering the matching process and meeting a trading partner. As emphasized by Acemoglu and Shimer (1999), this seems relevant in many situations. For instance, in a labor market firms invest before hiring workers and workers acquire human capital before finding a job.

The holdup problem in our context may provide an explanation of asset ownership that differs from the theory of property rights as developed by Grossman and Hart (1986) and Hart and Moore (1990).[3] In their theory, the residual control right associated with the ownership of physical assets determines the parties' outside options in ex post bargaining and, therefore, affects their ex ante incentives for human capital investments. Before making these investments, agents can contractually reallocate property rights. The resulting cooperative allocation of asset ownership then minimizes the inefficiencies generated by the holdup problem.[4] In our model the agents acquire physical assets through their ex ante investments. Since investment incentives depend on the nature of the matching process, the non–cooperative allocation of asset ownership is determined by market conditions. Consider for example a labor market where unemployed workers are randomly matched with entrepreneurs, who seek to fill vacancies. As Samuelson (1957) notes, in a competitive Walrasian market it remains unclear whether workers or entrepreneurs become the owners of capital. But we show that only the entrepreneurs will invest in homogenous capital inputs whenever there are more unemployed workers than vacancies. In this situation the holdup problem is more serious for the workers than for the entrepreneurs. Therefore the entrepreneurs can realize a higher marginal return on their investment and only they become capitalists.

Our analysis relies on the steady state equilibrium of a dynamic matching market, similar to the one used by Rubinstein and Wolinsky (1984) and Gale (1987). On each

[2] While some authors argue that ex post negotiations necessarily lead to inefficient investments (Hart and Moore (1988), Che and Hausch (1999)), others have identified contractual devices and environments that induce first–best investments even with incomplete contracts (Chung (1991), Aghion, Dewatripont and Rey (1994), Nöldeke and Schmidt (1995), Edlin and Reichelstein (1996), Rogerson (1992), Che and Sakovics (2004), and Evans (2008)).

[3] Hart (2009) presents a different explanation of asset ownership in which the driving force is payoff uncertainty, rather than the non–contractibility of investments.

[4] Gans (2005) modifies this approach by studying a non–cooperative market for asset ownership.

side of the market there is a constant number of active agents who remain in the market until they have traded. Their matching probabilities reflect the relative number of active agents. At each point in time, there is a constant flow of potential market entrants. In the steady state, the number of agents who enter is identical to the number of agents who exit after trade. We combine the market entry stage with the agents' investment decisions. Each agent who enters the market selects his investment as a best response against the equilibrium choices of the other agents. Our assumptions on the productivity of investments include the different categories that have been considered in the literature on the holdup problem. The model applies for instance to a buyer–seller market in which the sellers make 'cooperative investments' in product quality to increase the buyers' valuation of the good. We also allow for 'selfish investments' as for example the acquisition of human capital by workers in a labor market. Further, our analysis applies not only to 'one–sided investments' but also to 'two–sided investments'. In the latter case, the investments on both sides of the markets may be substitutes as well as complements.

This paper relates to a few articles that study investment incentives in matching environments. Acemoglu and Shimer (1999) consider a labor market in which firms make one–sided investments before matching workers. They show that ex post bargaining over wages will always induce inefficiencies: As long as workers have some bargaining power, firms underinvest because more capital–intensive production leads to higher wages. If, however, workers have no bargaining power, then there is excessive entry of firms. These inefficiencies can be prevented when the firms can commit to posting wages ex ante. Similarly, Felli and Roberts (2001) show that Bertrand competition may resolve the holdup problem of match specific investments.

Cole, Mailath and Postlewaite (2001) present a two-sided matching model in which buyers and sellers make complementary investments prior to matching. As in our model, the agents' outside options in a match are determined endogenously by their investments. But there are no search frictions in their model. Cole, Mailath and Postlewaite consider stable matchings and show that efficient investment decisions can always be sustained in equilibrium. Inefficient equilibria typically also exist, but they are 'constrained' efficient.

In Ramey and Watson (2001) non–contractible investment decisions are made *after* two agents are matched and start a relationship. At the beginning of their relation the agents non–cooperatively choose long–term investments. In each period they simultaneously select ongoing efforts that determine whether their cooperation remains productive. Since market frictions shape effort incentives, a frictionless market can actually minimize welfare. In certain cases there exists an optimal positive level of frictions that leads to first–best investments.

The remainder of this paper is organized as follows. Section 2 describes our model. Section 3 derives the steady–state equilibrium. In Section 4 we analyze how market conditions affect investment incentives in the presence of the holdup problem. Section 5 concludes.

2 The Model

Our model considers the steady state of a random matching market with two types of agents. In each period, new agents of both types arrive and decide about entering the market. When entering the market, an agent also decides about his investment. This investment increases the surplus available from a match with an agent of the other type. After an agent enters the market, he is matched with an agent of the other type by a stochastic process. In such a match, the two agents bargain about sharing their joint surplus. If they reach an agreement, they leave the market. Otherwise, both enter the matching process again.

More specifically, we consider the following environment. Time is discrete and is indexed by the integers. Since we consider a steady state, all variables remain constant over time. There are two types of agents, indexed $i = A, B$, who are risk–neutral and discount future payoffs by the common discount factor $\delta \in (0, 1)$. All agents of a particular type are identical. In each period t, there is a mass of $\bar{M}_i > 0$ of agents of type i considering entering the market. If an agent does not enter, he disappears and obtains utility zero. We denote by $h_i \in [0, 1]$ the fraction of i–type agents that enter the market. This means that in each period $h_i\bar{M}_i$ agents of type i join those who are already active in the matching market.

An agent of type i who decides to enter the market chooses his investment $I_i \geq 0$. As we explain below, the investments (I_A, I_B) determine the surplus that can be realized when two agents meet in the market. In addition, each agent who enters the market has to pay a (small) fixed cost $f > 0$. After spending $I_i + f$ in period t, the agent participates in the matching process in period $t + 1$.

The steady state mass of traders of type i who are actively searching for a match in the market is denoted as M_i. The number M_i is endogenously determined by the flows of agents who enter and exit the matching process. In each period, each agent of type i is matched with at most one agent of type $j \neq i$. The probability of an A–type agent to meet a B–type agent is denoted as $\alpha \in [0,1]$. Analogously, a B–type is matched with an A–type with probability $\beta \in [0,1]$. The probabilities α and β depend on the matching technology and the numbers of searching agents. The total number of matches in each period is assumed to be a strictly increasing and linearly homogenous function of (M_A, M_B).[5] Therefore, the individual matching probability for type i depends only on the ratio $m_i \equiv M_i/(M_A + M_B)$. In what follows, we assume that $\alpha = \alpha(m_A)$ and $\beta = \beta(m_B)$ satisfy the following conditions:

Assumption 1 *(i) The matching probabilities $\alpha(m_A)$ and $\beta(m_B)$ are continuous and non–increasing for all $m_i \in [0,1]$. Moreover, they satisfy $\alpha(m_A) > 0$ for all $m_A \in [0,1)$, $\beta(m_B) > 0$ for all $m_B \in [0,1)$, and $\alpha(1) = \beta(1) = 0$. (ii) For all $(M_A, M_B) \in \mathbb{R}^2_{++}$,*

$$\alpha(m_A)\, M_A = \beta(m_B)\, M_B. \tag{1}$$

By part (i) of this assumption, for a given number M_i type i is less likely to be matched with a type j if the number M_j of agents on the other side of the market decreases. Indeed, in the limit $M_j \to 0$ the probability to meet type j approaches zero. Part (ii) is a simple consistency condition for bilateral matchings. Since each agent is matched with at most one agent of the other type, the total number of matches must be identical for both types of agents. A simple example for such a matching process is the *efficient* matching technology. With this technology, $M_i \leq M_j$ implies that all agents of type i are matched so that $M_j - M_i$ agents of type $j \neq i$ remain unmatched. Accordingly,

$$\alpha = \min\left[\frac{M_B}{M_A}, 1\right] = \min\left[\frac{1-m_A}{m_A}, 1\right], \quad \beta = \min\left[\frac{M_A}{M_B}, 1\right] = \min\left[\frac{1-m_B}{m_B}, 1\right]. \tag{2}$$

When matched in period t, agent A and B negotiate about sharing the surplus from their investments. If they disagree, both enter the matching process again in period $t+1$. We denote by V_i type i's expected payoff from searching for a partner. This means that, when bargaining in period t, the agents' outside options are δV_A and δV_B, respectively. Of course, V_A and V_B are endogenously determined in equilibrium. As will become clear in the next section, in particular an agent's expected payoff from entering the matching

[5] These properties of the matching function are usually assumed in the theoretical and empirical literature on search markets, see e.g. Petrongolo and Pissarides (2001).

market depends on his own investment and the investments of the other agents in the market.

If two matched agents agree to cooperate, their (gross) payoffs depend on their investments and are given by $U_A(I_A, I_B)$ and $U_B(I_A, I_B)$, respectively. We denote by

$$S(I_A, I_B) \equiv U_A(I_A, I_B) + U_B(I_A, I_B) \tag{3}$$

the available joint (gross) surplus in a match. As long as $S(I_A, I_B) > \delta(V_A + V_B)$, both agents can gain by splitting the surplus. We describe a partition of the surplus by a transfer τ from agent A to agent B. If $\tau < 0$, this indicates a payment from agent B to agent A. For simplicity, we use the Nash bargaining solution do describe the outcome of bargaining over τ.[6] Since the agents' outside options are δV_A and δV_B, this means that they agree on the transfer

$$\tau^* = \text{argmax}_\tau \left[U_A(I_A, I_B) - \tau - \delta V_A\right] \left[U_B(I_A, I_B) + \tau - \delta V_B\right] \tag{4}$$

Thus $\tau^* = [U_A(I_A, I_B) - \delta V_A - U_B(I_A, I_B) + \delta V_B]/2$ and the bargaining payoffs are

$$\begin{aligned} U_A(I_A, I_B) - \tau^* &= \left[S(I_A, I_B) + \delta V_A - \delta V_B\right]/2, \\ U_B(I_A, I_B) + \tau^* &= \left[S(I_A, I_B) - \delta V_A + \delta V_B\right]/2. \end{aligned} \tag{5}$$

After agreeing upon τ^* the agents in a match realize the payoffs in (5) and then leave the market.

The literature on the holdup problem distinguishes between *selfish* and *cooperative* investments. Purely selfish investments directly benefit only the investor so that $\partial U_i/\partial I_i > 0$ and $\partial U_j/\partial I_i = 0$. For instance, if the A–type agents are sellers and the B–type agents are buyers, a selfish investment by the seller reduces his production costs. In contrast, purely cooperative investments directly benefit only the other party so that $\partial U_i/\partial I_i = 0$ and $\partial U_j/\partial I_i > 0$. An example is the seller's investment in quality, which increases the buyer's value of the good.

A large part of the literature further focuses on *one–sided* investments. In this case only one side of the market is able to increase the surplus S by its investment so that $\partial S/\partial I_i = 0$ for some $i \in \{A, B\}$. In contrast, with *two–sided* investments both types of agents can make productive investments. An example is a labor market where entrepreneurs invest in factory equipment and workers invest in human capital before searching for a match.

[6] Our results do not rely on the specific properties of the Nash bargaining solution. The same results could be obtained with other cooperative or non–cooperative bargaining rules.

The following assumptions on $S(\cdot,\cdot)$ cover the above categories of investments as special cases.

Assumption 2 *(i)* $S(0,0) > 2f$. *(ii)* $S(\cdot,\cdot)$ *is strictly concave and continuously differentiable on* $\mathbb{R}^2_+$. *(iii)* $S(\cdot,\cdot)$ *satisfies*

$$\frac{\partial S(I_A, I_B)}{\partial I_i} \geq 0, \quad \max_i \frac{\partial S(0,0)}{\partial I_i} > 1, \quad \lim_{I_i \to \infty} \frac{\partial S(I_A, I_B)}{\partial I_i} < 1, \tag{6}$$

for $i = 1, 2$.

Part (i) of this assumption ensures that, for δ sufficiently large, the gains from trade exceed the market entry costs. By parts (ii) and (iii),

$$(\hat{I}_A, \hat{I}_B) \equiv \operatorname{argmax}_{(I_A,I_B)} S(I_A, I_B) - I_A - I_B \tag{7}$$

is uniquely defined and satisfies $\hat{I}_A + \hat{I}_B > 0$. Equation (7) describes the efficient investments in the limit $\delta \to 1$, where the time interval between investing (I_A, I_B) and realizing the surplus $S(I_A, I_B)$ plays no role.

3 Steady State Equilibrium

In equilibrium, new agents enter the market if this yields non–negative expected payoffs. We consider a symmetric equilibrium in which all agents of type i chose the same level of investment I_i at the entry stage. In a steady state, the number of new entrants is equal to the number of agents who exit after successful match.

We first derive the expected payoffs from being active in the market. Let (I_A^*, I_B^*) denote the agents' equilibrium investments. Thus, when a pair of agents is matched in the market, they bargain about a division of the joint surplus $S(I_A^*, I_B^*)$. At the beginning of period t, an agent of type A expects to be matched with probability α. In a steady state equilibrium, all matches induce an agreement on the transfer τ^*, which by (4) divides the joint surplus according to the Nash bargaining solution.[7] Thus, agent A's payoff from bargaining in a match is given by $U_A(I_A^*, I_B^*) - \tau^*$ in (5). With probability $1 - \alpha$ he

[7]We show that the condition $S > \delta(V_A + V_B)$ for an agreement is automatically satisfied in equilibrium at the end of this paragraph.

remains unmatched and continues searching for a match in $t+1$. Therefore, his expected payoff from participating in the matching process is

$$V_A^* = \alpha \left[S(I_A^*, I_B^*) + \delta V_A^* - \delta V_B^* \right] /2 + (1-\alpha)\delta V_A^*. \tag{8}$$

Analogously, we obtain for an agent of type B

$$V_B^* = \beta \left[S(I_A^*, I_B^*) - \delta V_A^* + \delta V_B^* \right] /2 + (1-\beta)\delta V_B^*. \tag{9}$$

Solving equations (8) and (9) yields

$$V_A^* = \frac{\alpha S(I_A^*, I_B^*)}{2(1-\delta) + \delta(\alpha+\beta)}, \quad V_B^* = \frac{\beta S(I_A^*, I_B^*)}{2(1-\delta) + \delta(\alpha+\beta)} \tag{10}$$

Note that (10) implies $S(I_A^*, I_B^*) > \delta(V_A^* + V_B^*)$. Since in a steady state equilibrium the available net surplus is positive in a match, efficient bargaining always induces an agreement according to the Nash bargaining solution described by (4).

Using (10) we can now describe the agents' market entry decisions. To ensure that the market operates and entry takes place, it is required that investing I_i^* and paying the entry cost f does not generate a negative payoff for either type i. Therefore, it must be the case that

$$\delta V_A^* \geq I_A^* + f, \quad \delta V_B^* \geq I_B^* + f. \tag{11}$$

If the inequality in (11) is strict for some type i, then entry actually generates a positive profit. In this case, all $\bar{M}_i$ agents will enter the market so that $h_i = 1$. This is reflected by the following equilibrium condition:

$$(1-h_A^*)(\delta V_A^* - I_A^* - f) = 0, \quad (1-h_B^*)(\delta V_B^* - I_B^* - f) = 0. \tag{12}$$

Note that, by (11) and (12), $h_i^* \in (0,1)$ implies $\delta V_i^* = I_i^* + f$. Thus, if not all agents of type i enter the market, this type must be indifferent between entering and obtaining zero utility from disappearing.

The entry conditions and the matching technology determine the steady state numbers (M_A, M_B) of agents who are searching for a partner. In each period, αM_A agents of type A and βM_B agents of type B are matched and leave the market. In the steady state, the number of agents who leave and exit the market must be identical so that (M_A, M_B) remains constant. This is ensured by the following condition:

$$\alpha \left(\frac{M_A^*}{M_A^* + M_B^*} \right) M_A^* = h_A^* \bar{M}_A, \quad \beta \left(\frac{M_B^*}{M_A^* + M_B^*} \right) M_B^* = h_B^* \bar{M}_B. \tag{13}$$

Finally, the agents' equilibrium investments (I_A^*, I_B^*) have to be consistent with maximization of expected payoffs at the entry stage. If the investments of all agents in the market are given by (I_A^*, I_B^*), no single agent of type i should gain from deviating to $I_i \neq I_i^*$. Consider a single agent of type A who invests I_A in a situation where the investments of all other agents in the market are given by (I_A^*, I_B^*). We denote this agent's expected payoff from participating in the matching process by $\tilde{V}_A$. In a match with an agent of type B, the available gross surplus is $S(I_A, I_B^*)$, and the two parties' payoffs from disagreement are $(\delta\tilde{V}_A, \delta V_B^*)$. Note that type B's disagreement payoff is δV_B^*, as defined in (10), because he expects the other side in all future matches to have chosen the equilibrium investment I_A^*.

With probability α the A–type agent is matched and, as long as $S(I_A, I_B^*) + \delta\tilde{V}_A - \delta V_B^* \geq 0$, an agreement is reached according to the Nash bargaining solution in (5). With probability $1 - \alpha$ he remains unmatched and continues searching in the next period. Therefore, $\tilde{V}_A$ is determined as

$$\tilde{V}_A = \alpha \max\left[0,\ S(I_A, I_B^*) + \delta\tilde{V}_A - \delta V_B^*\right]/2 + (1-\alpha)\delta\tilde{V}_A. \tag{14}$$

Thus $\tilde{V}_A$ depends on I_A and (I_A^*, I_B^*) according to

$$\tilde{V}_A(I_A|I_A^*, I_B^*) \;\; \equiv \;\; \max\left[0, \frac{\alpha[S(I_A, I_B^*) - \delta V_B^*]}{2(1-\delta) + \delta\alpha}\right]. \tag{15}$$

Analogously, we obtain for a single agent of type B who enters the market with the investment I_B that

$$\tilde{V}_B(I_B|I_A^*, I_B^*) \;\; \equiv \;\; \max\left[0, \frac{\beta[S(I_A^*, I_B) - \delta V_A^*]}{2(1-\delta) + \delta\beta}\right]. \tag{16}$$

The following equilibrium conditions ensure that investing I_i^* is a best response for each single agent of type i to the investments (I_A^*, I_B^*) of all other agents in the market:

$$\begin{aligned} I_A^* &= \operatorname{argmax}_{I_A} \delta\tilde{V}_A(I_A|I_A^*, I_B^*) - I_A, \\ I_B^* &= \operatorname{argmax}_{I_B} \delta\tilde{V}_B(I_B|I_A^*, I_B^*) - I_B. \end{aligned} \tag{17}$$

Note that (15) and (16) are consistent with (10) as $\tilde{V}_i(I_i^*|I_A^*, I_B^*) = V_i^*$.

In a steady state equilibrium, the variables

$$E^* = \{h_i^*, M_i^*, V_i^*, I_i^*\}_{i=A,B} \tag{18}$$

satisfy conditions (10) – (13) and (17). To focus on outcomes where the matching market actually operates, we require in addition that $h_A^* > 0$ and $h_B^* > 0$. This eliminates equilibria with coordination failure, in which agents of type i do not enter because no agent of type $j \neq i$ is active in the market.

We first show that our assumptions on the matching probabilities, $\alpha(\cdot)$ and $\beta(\cdot)$, and the surplus function $S(\cdot,\cdot)$ guarantee the existence of an equilibrium:

Proposition 1 *Let δ be sufficiently large. Then there exists an equilibrium E^*, and $I_A^* + I_B^* > 0$ in any equilibrium E^*.*

Existence of an equilibrium requires the discount factor δ to be sufficiently large because otherwise the expected present value from a match would not cover the market entry cost f. Indeed, the number of agents who enter the search market must be identical for both types. This is so because after a bilateral match both parties leave the market and are replaced by new entrants so that M_A^* and M_B^* remain constant. If there are more agents of type i than of type j considering entering the market, this implies that some of the $\bar{M}_i$ agents must refrain from entering. Of course, this will be optimal for type i only if the profit from entering is zero. Therefore, competition at the market entry stage leads to the following observation:

Proposition 2 *Let $\bar{M}_i > \bar{M}_j$. Then $\delta V_i^* = I_i^* + f$ and $h_i^* < h_j^*$ in any equilibrium E^*.*

In a centralized Walrasian market, a zero profit condition is ensured by the adjustment of prices. In contrast, the zero profit condition in Proposition 2 relies on the adjustment of the matching probabilities α and β. When outside the steady state more agents of type i than of type j enter the market, then the ratio M_i/M_j of agent types in the matching process increases over time. By Assumption 1 this lowers the matching probability of type i and raises the matching probability of type j. Thereby, type i's expected payoff V_i from searching and his profit from entering the market are reduced.

Of course, the zero profit condition in Proposition 2 depends not only on the matching probabilities but also on the equilibrium investments. Therefore, one cannot conclude that $\bar{M}_i > \bar{M}_j$ at the entry stage implies that also $M_i^* > M_j^*$ for the numbers of active agents in the market. As the following Proposition shows this is the case only if the surplus from the efficient investments in (7) is large enough in relation to type i's investment.

Proposition 3 *Let $\bar{M}_i > \bar{M}_j$ and let $(\hat{I}_A, \hat{I}_B)$ denote the investments defined in (7). Then*

$$S(\hat{I}_A, \hat{I}_B) > 2\,\hat{I}_i + 2\,f, \tag{19}$$

implies that, for δ sufficiently large, $M_i^* > M_j^*$ and $V_i^* < V_j^*$ in any equilibrium E^*. If the inequality in (19) is reversed, then $M_i^* < M_j^*$ and $V_i^* > V_j^*$ for δ sufficiently large.

Notice that by Assumption 2 and (7), (19) is necessarily satisfied at least for one type $i \in \{A, B\}$. As the proof of Proposition 3 shows, condition (19) implies that, for δ sufficiently large, the equilibrium investment I_i^* of the i–type agents is relatively small. These agents, therefore, break–even at the market entry stage when they get less than half of the available surplus in a match. As the division of the surplus is determined by the matching probabilities, the zero profit condition in Proposition 2 thus implies $M_i^* > M_j^*$ so that type i is less likely to be matched than type j.

As a special case, condition (19) applies in markets with one–sided investments where one type i does not invest so that $\hat{I}_i = 0$. Consider, for example, a labor market where each entrepreneur can employ one worker. Only the entrepreneurs invest in capital and workers have no investment opportunities. If workers outnumber entrepreneurs at the market entry stage, then by Proposition 3 also in the matching market the number of workers exceeds the number of available jobs. Therefore, even with efficient matching as in (2), some workers remain unemployed in each period. Moreover, as in Shaked and Sutton (1984), unemployment is involuntary because the workers who are selected for a job are better off than those who have to continue looking for employment.

4 Investments and the Holdup Problem

The agents' investment incentives depend on the expected bargaining outcome, because investments take place before the division of the surplus is determined. This creates a holdup problem and so the equilibrium will be inefficient. The extent of this inefficiency is related to the agents' matching probabilities, which affect their outside options in a match. As the matching probabilities reflect the numbers of active agents in the market, these numbers are important for the equilibrium level of investments.

We first look at the case where the investments of type A and B are equally productive because that they are perfect substitutes. The investments (I_A, I_B) are *perfect substitutes* if the joint surplus in a match depends only on the sum $I_A + I_B$ of the investments so that S can be written as $S(I_A + I_B)$. Any difference in the equilibrium investments I_A^* and I_B^* is then related to the frictions of the matching process and does not depend on asymmetries in productivity.[8]

Proposition 4 *Let the agents' investments be perfect substitutes. Then $M_i^* > M_j^*$ implies $I_i^* = 0$ in any equilibrium E^*.*

In the case of perfectly substitutable investments only the short side of the matching market makes an investment and the other side does not invest at all. This happens because the agents on the short side of the market have a higher matching probability. Therefore, they can expect to realize the returns from their investment earlier than the agents on the long side. This also increases the value of their outside options in a match and so they can appropriate a larger share of the marginal surplus from their investment. Consequently, investments are zero on the long side of the market, because with perfect substitutes only the agent–type with the largest marginal private benefit will invest.

Proposition 4 may resolve Samuelson's (1957) puzzle of why in a capitalist economy workers do not invest in capital. Indeed, in a competitive economy it does not matter whether capital hires labor or labor hires capital. Capital ownership is indeterminate because firm revenues are distributed according to marginal productivity independently of the investor's identity. Therefore, both sides of the labor market have the same investment incentives. This is not true, however, in our framework where market frictions affect private returns from investments. Applied to a labor market with workers seeking for vacancies offered by entrepreneurs, Proposition 4 implies that only the entrepreneurs invest in homogenous capital inputs when there are fewer vacancies than workers.

The insight that the short side of the market invests more than the long side can be generalized to situations in which the investments (I_A, I_B) are *symmetric* in the sense that $S(I', I'') = S(I'', I')$ for all $(I', I'') \in \mathbb{R}_+^2$. This means that the available surplus does not depend on the investor's type: The investments (I_A, I_B) generate the same surplus as if type A would invest the amount I_B and type B the amount I_A. This property of the surplus function includes perfectly substitutable investments as a special case; but it allows also for imperfectly substitutable or complementary investments. The same intuition as for Proposition 4 explains the following result:

[8]Notice that the following result remains valid if the investments are not perfect but sufficiently close substitutes.

Proposition 5 *Let the agents' investments be symmetric. Then in any equilibrium E^*, $M_i^* > M_j^*$ implies $I_i^* < I_j^*$ whenever $I_A^* + I_B^* > 0$.*

Suppose for instance that each agent can raise his productivity in a match by investing in human capital. If these investments are equally efficient for both types, then in a perfectly competitive market all agents would acquire the same level of skills. Proposition 5 shows that our model does not have this efficiency property. Matching frictions and ex post bargaining induce differential rates of skill acquisition as the long side of the market invests less in human capital than the short side.

The inefficiency of the equilibrium investments arises because search frictions generate a holdup problem. Both parties in a match are partially locked–in as searching for another partner would delay an agreement. Therefore, the sunk investments are to some extent match–specific. As the bargaining outcome divides the available surplus, each party can only appropriate a share of the marginal return from its investment. This creates insufficient investment incentives at the market entry stage. Indeed, by (17) each agent i selects his investment I_i to maximize his net benefit $\delta\tilde{V}_i - I_i$ from entering the market. He fails to take into account that his investment also increases the other market side's benefit from searching for a match.

The socially efficient investments would internalize this externality by maximizing the joint payoff $\delta(V_A^* + V_B^*) - I_A - I_B$ from entering the market. By (10) this joint payoff depends on the investments (I_A, I_B) according to

$$W(I_A, I_B) \equiv \frac{\delta(\alpha+\beta)}{2(1-\delta)+\delta(\alpha+\beta)} S(I_A, I_B) - I_A - I_B. \tag{20}$$

In the limit $\delta \to 1$, the joint payoff W is maximized by the investments $(\hat{I}_A, \hat{I}_B)$ in (7).

Proposition 6 *In any equilibrium E^*, underinvestment occurs because there exists an $(\epsilon_A, \epsilon_B) \in \mathbb{R}_+^2$ such that*

$$W(I_A^* + \epsilon_A, I_B^* + \epsilon_B) > W(I_A^*, I_B^*) \tag{21}$$

whenever $I_A^* + I_B^* > 0$. Moreover, (21) holds for some $(\epsilon_A, \epsilon_B) \in \mathbb{R}_{++}^2$ if $(I_A^*, I_B^*) \in \mathbb{R}_{++}^2$.

Because the agents share the surplus but not the cost of their ex ante investments, this leads to underinvestment. Indeed, in a market with two–sided investments both sides of the markets underinvest independently of whether the investments are substitutes or complements. But, as Proposition 5 indicates, the holdup problem is more severe on the long side of the market.

Search frictions arise in our model because the matching process consumes time and the agents discount future utilities. Following Rubinstein and Wolinsky (1984), we can remove these frictions and approximate a 'frictionless market' by considering the limiting equilibrium as the discount factor δ approaches one. The following result shows that the holdup problem and the underinvestment effect disappear when the market becomes frictionless.

Proposition 7 *For a given discount factor δ, let $E^*(\delta)$ denote an equilibrium. Then in the limit $\delta \to 1$ the equilibrium investments become efficient, because*

$$\lim_{\delta \to 1} (I_A^*(\delta), I_B^*(\delta)) = (\hat{I}_A, \hat{I}_B), \tag{22}$$

where $(\hat{I}_A, \hat{I}_B)$ is defined in (7).

The inefficiency of the equilibrium investments becomes negligible as $\delta \to 1$. In this limit, it follows from (10) that type A receives a share $\alpha/(\alpha+\beta)$ and type B a share $\beta/(\alpha+\beta)$ of the surplus $S(I_A^*, I_B^*)$. But surplus sharing does not prevent agent i from appropriating the full *marginal* return from his investment. Indeed, if he invests the amount I_i, then by (15) and (16) his share of the surplus is equal to $S(I_i, I_j^*) - V_j^*$ in the limit $\delta \to 1$. This means that in a match with an agent of type $j \neq i$, agent i has to concede the fixed amount V_j^* to type j and he himself receives the entire surplus in excess of V_j^*. Therefore, ex post bargaining does not distort investment decisions in a frictionless market. Effectively, in such a market there is no longer a lock–in effect in a match because searching for a new partner is costless.

For the case of two–sided investments, Proposition 7 also shows that there is no coordination failure in the frictionless market, even when the agents' investments are complements. In the limit $\delta \to 1$, (15) – (17) imply that the investments (I_A^*, I_B^*) are determined by

$$\begin{aligned} S(I_A^*, I_B^*) - I_A^* &\geq S(I_A, I_B^*) - I_A \quad \text{for all} \quad I_A \geq 0, \\ S(I_A^*, I_B^*) - I_B^* &\geq S(I_A^*, I_B) - I_B \quad \text{for all} \quad I_B \geq 0. \end{aligned} \tag{23}$$

Thus (I_A^*, I_B^*) may be viewed as the Nash equilibrium of a simultaneous game between type A and B. Perhaps surprisingly, this equilibrium coincides with the efficient investments

$(\hat{I}_A, \hat{I}_B)$ in (7). Important for this observation are the concavity and differentiability properties of the function $S(\cdot, \cdot)$ under Assumption 2. These properties ensure that the first–order conditions implied by (23) are identical to the necessary and sufficient first–order conditions for the optimization problem in (7). Without concavity of $S(\cdot, \cdot)$ it may happen that the investments (I_A^*, I_B^*) defined by (23) are merely locally rather than globally efficient.

A major question in the literature on decentralized trading in matching markets concerns the relation to the competitive Walrasian equilibrium when search frictions become negligible.[9] In our model, the steady state equilibrium approaches the perfectly competitive outcome in the frictionless limit $\delta \to 1$. Indeed, by Proposition 2 all surplus at the market entry stage goes to the type of agent that is present in smaller numbers, and the other type earns zero profit. This is the Walrasian outcome of a market where the agents on each side of the market are homogenous. Moreover, Proposition 7 shows that for $\delta \to 1$ the agents' investments maximize the available surplus. This is the well–known efficiency property of perfect competition.

5 Concluding Remarks

We have analyzed how search frictions affect investment incentives in a decentralized matching market. These frictions generate a holdup problem and lead to underinvestment. The importance of this problem depends on the matching process and differs for both sides of the market. This can explain why investments and the ownership of assets are concentrated on one side of the market. We have also shown that the holdup problem disappears when search frictions tend to zero. The outcome of a frictionless decentralized market has the same properties as the Walrasian equilibrium of a centralized market.

These results have been derived under standard neoclassical assumptions. The matching technology of our model is homogenous of degree one so that the number of matches is proportional to the number of agents active in the market. Further, the surplus in a match is assumed to be a concave function of the agents' investments. These assumptions seem important in particular for the efficiency properties of the market outcome with negligible search frictions. Our analysis indicates that with increasing returns one cannot rule out coordination failures in markets with two–sided investments.

[9] See e.g. Rubinstein and Wolinsky (1984) and Gale (1987). More recent contributions include De Fraja and Sakovics (2001), Moreno and Wooders (2001) and Satterthwaite and Shneyerov (2007).

6 Appendix

Proof of Proposition 1: By Assumption 2 (i), $S(0,0) > 2\,f$. In what follows, we assume that δ is large enough so that

$$\frac{\delta(\alpha+\beta)S(0,0)}{2(1-\delta)+\delta(\alpha+\beta)} > 2\,f. \tag{24}$$

It is easy to see that then for all (I_A^*, I_B^*) satisfying (17)

$$\delta(V_A^*+V_B^*) = \frac{\delta(\alpha+\beta)S(I_A^*,I_B^*)}{2(1-\delta)+\delta(\alpha+\beta)} > I_A^*+I_B^*+2\,f. \tag{25}$$

Define

$$H \equiv \{I_A, I_B | S(I_A,I_B) - I_A - I_B \geq S(0,0)\}\,. \tag{26}$$

By Assumption 2 (ii), H is a compact and convex set.

We first consider the case $\bar{M}_A \geq \bar{M}_B$. Let

$$h_A^* = \bar{M}_B/\bar{M}_A, \quad h_B^* = 1. \tag{27}$$

Note that this implies $h_A^*\bar{M}_A = h_B^*\bar{M}_B$. Therefore, condition (13) is consistent with Assumption 1 (ii).

Let $M \equiv \{m = (m_A, m_B) \in \mathbb{R}_+^2 \,|\, m_A + m_B = 1\}$ and define the correspondence $F\colon H \to M$ by

$$F(I_A,I_B) \equiv \left\{ m \in M \,|\, \frac{\delta\alpha(m_A)S(I_A,I_B)}{2(1-\delta)+\delta[\alpha(m_A)+\beta(m_B)]} = I_A + f \right\}. \tag{28}$$

It is easily verified that Assumption 1 (i) and continuity of $S(\cdot)$ ensure that $F(\cdot)$ is a non–empty, convex-valued and uhc correspondence. Next, define the correspondences

$$\begin{aligned} G_A(m_A, I_B) &\equiv \underset{I_A}{\text{argmax}}\ \frac{\delta\alpha(m_A)S(I_A,I_B)}{2(1-\delta)+\delta\alpha(m_A)} - I_A, \\ G_B(m_B, I_A) &\equiv \underset{I_B}{\text{argmax}}\ \frac{\delta\beta(m_B)S(I_A,I_B)}{2(1-\delta)+\delta\beta(m_B)} - I_B. \end{aligned} \tag{29}$$

Let $G \equiv G_A \times G_B$. By Assumption 2 (ii) and continuity of $\alpha(\cdot)$ and $\beta(\cdot)$, $G\colon H \times M \to H$ is a non–empty, convex-valued and uhc correspondence.

By the properties of F and G, Kakutani's fixed point theorem ensures that the correspondence $F \times G\colon H \times M \to H \times M$ has a fixed point $(m_A^*, m_B^*, I_A^*, I_B^*)$. Given (m_A^*, m_B^*) and (h_A^*, h_B^*) in (27), we obtain from equilibrium condition (13)

$$M_A^* = h_A^*\bar{M}_A/\alpha\,(m_A^*)\,, \quad M_B^* = h_B^*\bar{M}_B/\beta\,(m_B^*)\,. \tag{30}$$

Also, given $\alpha = \alpha(m_A^*), \beta = \beta(m_B^*)$ and (I_A^*, I_B^*), equilibrium condition (10) determines (V_A^*, V_B^*).

To prove existence of an equilibrium, it thus remains to show that equilibrium conditions (11), (12) and (17) are satisfied. Since $m^* \in F(I_A^*, I_B^*)$, (10) and (28) imply that $\delta V_A^* = I_A^* + f$. By (29), therefore $\delta V_B^* \geq I_B^* + f$. This proves that (11) holds. As $\delta V_A^* = I_A^* + f$ and $h_B^* = 1$, also (12) is satisfied. Finally, (17) holds because $(I_A^*, I_B^*) \in G(m^*, I_A^*, I_B^*)$. This completes the proof of existence of E^* for the case $\bar{M}_A \geq \bar{M}_B$. An analogous argument applies to the case $\bar{M}_B \geq \bar{M}_A$.

To show that $I_A^* + I_B^* > 0$, suppose to the contrary that $I_A^* = I_B^* = 0$. Then by (15)–(17) the following first–order conditions must hold:

$$\frac{\delta\alpha}{2(1-\delta)+\delta\alpha}\frac{\partial S(0,0)}{\partial I_A} \leq 1, \quad \frac{\delta\beta}{2(1-\delta)+\delta\beta}\frac{\partial S(0,0)}{\partial I_B} \leq 1. \tag{31}$$

But for δ sufficiently close to unity this yields a contradiction to Assumption 2 (iii). Q.E.D.

Proof of Proposition 2: Suppose that $\bar{M}_i > \bar{M}_j$ and $h_i^* \geq h_j^*$. Then $h_i^* \bar{M}_i > h_j^* \bar{M}_j$. This yields a contradiction because $h_i^* \bar{M}_i = h_j^* \bar{M}_j$ in any equilibrium by (1) and (13). This proves that $h_i^* < h_j^* \leq 1$. By (12) this implies $\delta V_i^* = I_i^* + f$. Q.E.D.

Proof of Proposition 3: If (19) holds, then it follows from the proof of Proposition 7 below that, for δ sufficiently large, the equilibrium investments satisfy

$$S(I_A^*, I_B^*) > 2\, I_i^* + 2\, f. \tag{32}$$

Let, for example, $\bar{M}_A > \bar{M}_B$. Suppose to the contrary that $M_A^* \leq M_B^*$. Then (1) implies that $\alpha \geq \beta$. Therefore, by (10) and (32)

$$\lim_{\delta \to 1} \delta V_A^* \geq \frac{S(I_A^*, I_B^*)}{2} > I_A^* + f. \tag{33}$$

For δ sufficiently large this yields a contradiction to Proposition 2. Thus, $M_A^* > M_B^*$. This implies $\alpha < \beta$ by (1). Therefore, by (10), $V_A^* < V_B^*$. If the inequality in (19) is reversed, an analogous argument shows that $\bar{M}_A > \bar{M}_B$ implies $M_A^* < M_B^*$ and $V_A^* > V_B^*$ for δ sufficiently large. Q.E.D.

Proof of Proposition 4: Let $M_A^* > M_B^*$. Suppose that $I_A^* > 0$. Then (15)–(17) imply

$$\frac{\delta\alpha}{2(1-\delta)+\delta\alpha}\frac{\partial S(I_A^*, I_B^*)}{\partial I_A} = 1, \quad \frac{\delta\beta}{2(1-\delta)+\delta\beta}\frac{\partial S(I_A^*, I_B^*)}{\partial I_B} \leq 1. \tag{34}$$

Since the investments are perfect substitutes, $\partial S(I_A^*, I_B^*)/\partial I_A = \partial S(I_A^*, I_B^*)/\partial I_B$. Therefore (34) implies that $\alpha \geq \beta$. But by (1) this implies that $M_A^* \leq M_B^*$, a contradiction. This proves that $M_A^* > M_B^*$ implies $I_A^* = 0$. An analogous argument for the case $M_B^* > M_A^*$ completes the proof. Q.E.D.

Proof of Proposition 5: By (15)–(17), the equilibrium investments (I_A^*, I_B^*) satisfy

$$\frac{\partial S(I_A^*, I_B^*)}{\partial I_A} \leq \frac{2(1-\delta)+\delta\alpha}{\delta\alpha}, \quad \frac{\partial S(I_A^*, I_B^*)}{\partial I_B} \leq \frac{2(1-\delta)+\delta\beta}{\delta\beta}, \tag{35}$$

with the equality holding if $I_i^* > 0$.

We first show that $I_A^* \neq I_B^*$ whenever $I_A^* + I_B^* > 0$ and $M_i^* \neq M_j^*$. Suppose the contrary, i.e. $I_A^* = I_B^* > 0$. Then the equalities have to hold in (35). Further, by (1), $M_i^* \neq M_j^*$ implies $\alpha \neq \beta$. Therefore (35) yields $\partial S(I_A^*, I_B^*)/\partial I_A \neq \partial S(I_A^*, I_B^*)/\partial I_B$. But this yields a contradiction because $I \equiv I_A^* = I_B^*$ implies

$$\begin{aligned} \frac{\partial S(I_A^*, I_B^*)}{\partial I_A} &= \lim_{\Delta \to 0} \frac{S(I+\Delta, I) - S(I,I)}{\Delta} \\ = \lim_{\Delta \to 0} \frac{S(I, I+\Delta) - S(I,I)}{\Delta} &= \frac{\partial S(I_A^*, I_B^*)}{\partial I_B}, \end{aligned} \tag{36}$$

where the second equality holds by symmetry of (I_A, I_B). This proves that that $I_A^* \neq I_B^*$ whenever $I_A^* + I_B^* > 0$ and $M_i^* \neq M_j^*$.

In what follows, let $M_i^* \neq M_j^*$. It is easily verified that by (35)

$$(I_A^*, I_B^*) = \underset{(I_A, I_B)}{\operatorname{argmax}}\ S(I_A, I_B) - \frac{2(1-\delta)+\delta\alpha}{\delta\alpha} I_A - \frac{2(1-\delta)+\delta\beta}{\delta\beta} I_B. \tag{37}$$

By Assumption 2 (ii), (I_A^*, I_B^*) is unique. Because $I_A^* \neq I_B^*$, (37) therefore implies

$$\begin{aligned} S(I_A^*, I_B^*) - \frac{2(1-\delta)+\delta\alpha}{\delta\alpha} I_A^* - \frac{2(1-\delta)+\delta\beta}{\delta\beta} I_B^* &> \\ S(I_B^*, I_A^*) - \frac{2(1-\delta)+\delta\alpha}{\delta\alpha} I_B^* - \frac{2(1-\delta)+\delta\beta}{\delta\beta} I_A^* & \end{aligned} \tag{38}$$

whenever $I_A^* + I_B^* > 0$. By symmetry of the investments, $S(I_A^*, I_B^*) = S(I_B^*, I_A^*)$ so that (38) simplifies to

$$\left[\frac{2(1-\delta)+\delta\alpha}{\delta\alpha} - \frac{2(1-\delta)+\delta\beta}{\delta\beta}\right][I_B^* - I_A^*] > 0 \tag{39}$$

whenever $I_A^* + I_B^* > 0$.

Suppose that $M_A^* > M_B^*$. Then (1) implies that $\alpha < \beta$. Therefore $I_A^* < I_B^*$ by (39). An analogous argument shows that $M_B^* > M_A^*$ implies that $I_B^* < I_A^*$ whenever $I_A^* + I_B^* > 0$. Q.E.D.

Proof of Proposition 6: Let, for example, $I_A^* > 0$. Then by (17), the following first–order condition must hold

$$\frac{\delta\alpha}{2(1-\delta)+\delta\alpha}\frac{\partial S(I_A^*, I_B^*)}{\partial I_A} = 1. \tag{40}$$

By (20) one has

$$\frac{\partial W(I_A^*, I_B^*)}{\partial I_A} = \frac{\delta(\alpha+\beta)}{2(1-\delta)+\delta(\alpha+\beta)}\frac{\partial S(I_A^*, I_B^*)}{\partial I_A} - 1. \tag{41}$$

Thus, by (40),

$$\frac{\partial W(I_A^*, I_B^*)}{\partial I_A} = \frac{\delta(\alpha+\beta)}{2(1-\delta)+\delta(\alpha+\beta)}\frac{2(1-\delta)+\delta\alpha}{\delta\alpha} - 1 > 0. \tag{42}$$

This proves that $W(I_A^* + \epsilon_A, I_B^*) > W(I_A^*, I_B^*)$ for some $\epsilon_A > 0$. If $(I_A^* + I_B^*) \in \mathbb{R}^2_{++}$, then the above argument implies that $\partial W(I_A^* + \epsilon, I_B^* + \epsilon)/\partial\epsilon > 0$ for ϵ sufficiently small. This proves the second statement of the Proposition. Q.E.D.

Proof of Proposition 7: By (7), the efficient investments $(\hat{I}_A, \hat{I}_B)$ are given by the necessary and sufficient first–order conditions

$$\frac{\partial S(\hat{I}_A, \hat{I}_B)}{\partial I_A} \leq 1, \quad \frac{\partial S(\hat{I}_A, \hat{I}_B)}{\partial I_B} \leq 1, \tag{43}$$

with the equality holding if $\hat{I}_i > 0$. By (15)–(17), the equilibrium investments (I_A^*, I_B^*) satisfy

$$\frac{\delta\alpha}{2(1-\delta)+\delta\alpha}\frac{\partial S(I_A^*, I_B^*)}{\partial I_A} \leq 1, \quad \frac{\delta\beta}{2(1-\delta)+\delta\beta}\frac{\partial S(I_A^*, I_B^*)}{\partial I_B} \leq 1, \tag{44}$$

with the equality holding if $I_i^* > 0$. As (44) becomes identical to (43) when $\delta \to 1$, this proves the proposition. Q.E.D.

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