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Martin Peitz*

Markus Reisinger**

*Department of Economics, University of Mannheim, 68131 Mannheim, Germany.
E-mail: martin.peitz@googlemail.com.
**Department of Economics, University of Munich, Kaulbachstr. 45, 80539 Munich, Germany.
E-mail:markus.reisinger@lrz.uni-muenchen.de

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Speaker: Prof. Dr. Urs Schweizer. · Department of Economics · University of Bonn · D-53113 Bonn,
Phone: +49(0228)739220 · Fax: +49(0228)739221

Indirect Taxation in Vertical Oligopoly*

Martin Peitz[†]
University of Mannheim

Markus Reisinger[‡]
University of Munich

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Abstract

This paper analyzes the effects of specific and ad valorem taxation in an industry with downstream and upstream oligopoly. We find that in the short run, i.e. when the number of firms in both markets is exogenous, the results concerning tax incidence tend to be qualitatively similar to models where the upstream market is perfectly competitive. However, both over- and undershifting are more pronounced, potentially to a very large extent. Instead, in the long run under endogenous entry and exit overshifting of both taxes is more likely to occur and is more pronounced under upstream oligopoly. As a result of this, a tax increase is more likely to be welfare reducing. We also demonstrate that downstream and upstream taxation are equivalent in the short run while this is not true for the ad valorem tax in the long run. We show that it is normally more efficient to tax downstream.

Keywords: Specific Tax, Ad Valorem Tax, Value-Added Tax, Tax Incidence, Tax Efficiency, Indirect Taxation, Imperfect Competition, Vertical Oligopoly.

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†Department of Economics, University of Mannheim, 68131 Mannheim, Germany. E-mail: martin.peitz@googlemail.com. Also affiliated to: CEPR, CESifo, ENCORE, and ZEW.

‡Department of Economics, University of Munich, Kaulbachstr. 45, 80539 Munich, Germany. E-mail: markus.reisinger@lrz.uni-muenchen.de

1 Introduction

Indirect taxes are a major source of revenue for governments. For example, in the EU 27 in 2007, the sum of revenue from commodity taxes and value-added taxes was 1,204.5 Bill. Euros which is around 12.5% of GDP in these countries.[1] In the U.S. the revenue from sales taxes are somewhat lower but they are still important, e.g. they account for 15% of total tax revenue on the state level.[2] Moreover, both commodity and value-added taxes have risen dramatically over the last decades.[3] It is also likely that indirect taxation may increase further in many countries to finance a planned reduction in income taxes. For example, in the U.S. in 2005 the Advisory Panel on Federal Tax Reform viewed a value-added tax as worthy of discussion and a recent report to Congress proposed it as new source of revenue.[4] Therefore, to understand and evaluate the effects of indirect taxation is important both for consumer surplus and overall welfare.

To make progress in this direction, the recent literature on indirect taxation compared the effects of unit and ad valorem taxes under oligopoly due to the fact that many industries exhibit an oligopolistic structure. This literature considers an oligopoly only in the downstream market. Such a partial analysis is justified if the input price does not change with a change in tax rates. This would appear to be a good approximation to real world industries if upstream markets are close to perfectly competitive. However, in many industries the upstream market is highly concentrated with few large manufacturing firms. Consider for example the markets for soft drinks and milk products (e.g. yoghurt or ice cream). Here, a few producers supply almost the whole market. Therefore, they enjoy market power vis-á-vis retailers. Retailers in turn are often highly concentrated in a region, thereby exerting market power vis-á-vis final consumers.[5]

[1] See Eurostat, Statistical books, Government finance statistics 1/2008.

[2] The exact revenue is $73.96 Bill. on a federal level and $106.07 Bill. on a state level. (Federal data stem from CBO, The Budget and Economic Outlook: Fiscal Years 2009 to 2018, January 2008, Table F-3, available at: http://www.cbo.gov/. State level data stem from U.S. Census Bureau, State Government Tax Collections: 2006, available at: http://www.census.gov/.)

[3] For example, in Germany the value-added tax increased from 11% in 1976 to 19% in 2007 (see www.oecd.org/ctp/taxdatabase) and e.g. the tax for unleaded gasoline increased from around 24.5 Eurocent per liter in 1985 to around 66.9 Eurocent in 2008 (see http://www.bwl-bote.de/20070723.htm).

[4] See J.M. Bickley, CRS Report for Congress, *Value-Added Tax: A new U.S. revenue source*, August 2006.

[5] Another striking example is the market for microprocessors in the personal computer industry. Almost all of the supply in this market stems from just two firms (Intel Corp. and AMD, Inc.). The downstream market consisting of computer manufacturers is in turn an oligopoly with around five firms controlling most of the market in many countries.

More importantly, products that are subject to a product specific tax in addition to the general value-added or sales tax typically have the feature that the upstream market is highly concentrated. Examples are alcoholic beverages, tobacco products, vehicles or gasoline.[6] For instance, consider the beer brewing industry and the associated retailing market. Both are currently highly concentrated in the U.S. and this concentration increased steadily over the last decades.[7] Beer is taxed both with a specific and an ad valorem tax in all U.S. states.[8] These taxes have been increased over the last years, especially in 1991 when the federal unit tax doubled. In light of these facts it is very unlikely that a change in the excise tax leads to a change only in the consumer price but leaves the input price constant. A similar development can be observed in the UK. Here the upstream market is very concentrated as well, e.g. in 2000, five national brewers produced 90% of beer volumes.[9] Due to mergers there are currently even fewer beer producers and Heineken, Carlsberg and Guinness control most of the market. Similar to the U.S., the excise duty in the UK increased by a large amount, namely from 3.1 pence per pint of beer in 1973 to 34.0 pence in 2008.[10,11]

To understand the effects of a change in indirect taxes, it is inevitable to take into account how these taxes affect the intermediate product price, which in turn can have profound consequences on the produced quantity and the final consumer price. The goal of this paper is to explicitly model the upstream market and take strategic interaction between firms in the upstream and the downstream market into account. Such models

[6]Historically, an excise was levied on these goods because they were seen as luxury goods. Nowadays, these taxes still exist for environmental and consumer protection reasons. A modern example of a luxury tax is Canada's tax of C$150 on each air conditioning unit in vehicles (see http://en.wikipedia.org/wiki/Taxation_in_Canada).

[7]Rojas (2008) reports that the number of mass-producers decreased from 350 in 1950 to 24 in 2000 making it one of the most concentrated industries in the United States. A similar pattern but with a somewhat larger number can be observed in the retailing industry.

[8]In 2003, the unit tax on beer has yield a revenue of $5.6 Bill. and the ad valorem tax on beer one of $16.8 Bill. See "The Tax Burden on the Brewing Industry", Report prepared by Global Insight, Inc. and The Parthenon Group (2005). In general the total spending on beer in 2003 was $78.1 Bill. where breweries contributed 34.72% and retailers 49.63%.

[9]See Mintel International Group Ltd., *Ales and Stouts* (2000).

[10]See British Beer and Pub Association Statistical Handbook 2007, updates by Institute for Alcohol Studies 2008, available at: http://www.ias.org.uk/.

[11]A similar market structure can be observed in the industry for tabacco products. For example, in the U.S. the Herfindahl index in the cigarette manufacturing industry in 1997 was 3100 as compared to an average of 91 for manufacturing industries (see U.S. Census Bureau, 1997 Economic Census, available at: http://www.census.gov/prod/ec97/m31s-cr.pdf). Excise tax increases were also large and increasing, e.g. in the years from 2000 until 2008 in the states New Jersey and Montana the excise duty per package increased by $1.775 and $1.52, respectively.

come under the name of *vertical oligopoly* and we will refer to such industries as *two-layer industries*. In particular, we are interested in comparing the effects in two-layer industries with those of perfect competition upstream to find out if models that take the input price as given may err, and if so, what the direction of this difference is and under which conditions this difference is particularly large. As far as we are aware of, this paper is the first contribution that looks into the effects of indirect taxation in vertical oligopoly.

In this set-up of vertical oligopoly it is also natural to go beyond previous analyzes and explicitly differentiate between upstream and downstream taxation. This distinction is important since taxes on the same product can be levied on different layers in production. For example, in the U.S. sales taxes are charged at the final consumer market while excise duties for alcoholic beverages and cigarettes are charged at the intermediate product market. Our analysis allows an answer to the question at which level taxation is more efficient.

To fix ideas, we consider an industry consisting of an imperfectly competitive upstream and downstream market, i.e. a *two-layer industry*. There is an ad valorem and a specific tax on the final product.[12] At each layer firms compete à la Cournot with homogeneous products: downstream firms set quantities non-cooperatively taking the input price as given. Taking the reactions of downstream firms into account, upstream firms set their quantities non-cooperatively. Quantity competition with homogeneous products appears to be the natural starting point for the analysis of indirect taxation in oligopoly. Indeed, most of the literature supposes Cournot competition in the downstream market as well.[13] This allows us to directly compare our results with previous work where the input price is assumed to be constant in the tax rate. We also note that the modern industrial organization literature has not come up with tractable models of vertical oligopoly where firms offer differentiated products.[14]

[12]For example, in the U.S. both forms of taxes are levied on several products including vehicles and gasoline while, for instance, in China beer and yellow spirits are taxed at a per unit rate while white spirits and other alcoholic drinks are taxed at a percentage rate (see http://en.wikipedia.org/wiki/Tax_system_in_China).

[13]The exceptions are the papers by Anderson et. al (2001a, 2001b). They find that for symmetric firms the implications of excise taxes under price competition are similar to those under quantity competition. Hamilton (2008) also considers price competition but with multi-product firms. He shows that the results in this case are markedly different to those under price or quantity competition with single-good firms.

[14]The problem is to find a satisfactory model to express how differentiated upstream firms compete when their consumers consist of a finite number of downstream firms that value their inputs in a different way and compete with each other.

In this set-up, we consider the short-run when the numbers of firms in the upstream and the downstream market are given and the long-run when the number of firms is endogenously determined through entry and exit at both layers. We are especially interested if and how our findings differ from an industry in which the upstream price is constant. We refer to such an industry as a *one-layer industry*.

We obtain the following results: We first demonstrate that in our setting in the short-run downstream taxation is equivalent to upstream taxation. We also show that an ad valorem tax is equivalent to a value-added tax. Thus, our analysis holds for a value-added tax as well. Our main result under short-run competition concerns over- and undershifting, i.e. if an increase in the tax rate is passed on to consumers by more or less than 100%. We obtain that the predictions on over- or undershifting of the two forms of taxes, specific and ad valorem, tend to be qualitatively similar in a two-layer industry to those in a one-layer industry, corroborating earlier results for one-layer industries. However, the parameter ranges for over- and undershifting do not necessarily coincide. In the case of two widely-used classes of demand functions, we show that both over- and undershifting are amplified in vertical oligopoly, possibly to a very large extent. This result is in line with empirical findings that even shortly after a tax increase consumer prices increase by more than 100% or even 200% of the tax increase.[15] The reason is that if overshifting occurs, the price-cost margin in the downstream market increases. In this case, firms in the upstream market have an incentive to participate in this increase and lower their quantity, and vice versa if undershifting occurs. Thus, the direction is the same as in a one-layer industry but the results are exacerbated. We also demonstrate that the difference between the results for the one- and two-layer industry increases if the upstream market becomes less competitive and (for most cases) if demand is more elastic.

In the long-run, i.e. with free entry, results in vertical oligopoly differ sharply from those with perfect competition upstream. Our main result under long-run competition is that overshifting is more likely and occurs to a larger degree. Thus, consumers are likely to bear a larger fraction of the tax burden. The intuition is that as a consequence of a tax

[15]For example, Rojas (2008) documents that in the first quarter after the tax increase in the beer industry in the U.S. in 1991 the price increase for different beer segments was from around 85% to 230% larger than the actual tax increase. Besley and Rosen (1999) also find a price increase for products like shampoo and bread that are around 100% and 240% larger than the tax rise. For further evidence on overshifting see also Harris (1987) on the U.S. cigarette industry, Karp and Perloff (1989) on the Japanese television industry and the references in the survey on tax incidence by Fullerton and Metcalf (2002).

increase, firms exit in the upstream market, thereby leading to an increase in the input price that is further shifted to consumers by firms at the downstream layer. As a result of this effect, the introduction of a commodity tax is less likely to be welfare enhancing in vertical oligopoly. Thus, governments considering an increase in commodity tax rates are likely to underestimate the negative effect that this change may have on consumer surplus and general welfare when ignoring the effects on input prices. We also find that this underestimation tends to be large if competition in the downstream and the upstream market is small and (for most cases) if the demand curve is relatively elastic. We further show that even in the long-run an ad valorem tax and a value-added tax are equivalent and, therefore, our results hold for a value-added tax as well. Yet, a downstream ad valorem tax is no longer equivalent to an upstream ad valorem tax in the long-run. We demonstrate that downstream ad valorem taxation dominates upstream ad valorem taxation from a welfare perspective although the latter may induce a smaller price increase for final consumers.

The existing literature on commodity taxation under imperfect competition starting with the papers by Kay and Keen (1983), Stern (1987) and Besley (1989) focussed almost exclusively on the final consumer market. The first systematic comparison between specific and ad valorem taxes in a Cournot oligopoly is provided by Delipalla and Keen (1992). They provide conditions for overshifting of the two taxes and show the superiority of ad valorem taxation compared to specific taxation both in the short and long run.[16] The downstream market in our analysis is modelled in the same way as in Delipalla and Keen (1992) but, as mentioned above, in contrast to their analysis we do not keep the input price fixed and instead allow for changes both in the input price and in the number of upstream firms. The only papers that explicitly take the upstream market into account are Konishi (1990), Hamilton (1999b) and Asker (2008): Konishi (1990) considers a model with an oligopolistic downstream market and several perfectly competitive upstream markets. He allows for entry and exit downstream and restricts the tax policy to consist of a lump-sum tax downstream and specific taxes on inputs and the final product. Konishi (1990)

[16]The model by Delipalla and Keen (1992) has been extended in several ways. For example, Skeath and Trandel (1994) provide conditions under which ad valorem taxation Pareto dominates specific taxation and Hamilton (1999a) generalizes the analysis to allow for comparisons between less familiar forms of taxation. Anderson et al. (2001a, 2001b) show under which conditions the results concerning tax incidence and efficiency of the Cournot model carry over to a differentiated Bertrand model and point out possible differences in the results. Hamilton (2009) develops a model with differentiated Bertrand competition in which firms can offer multiple products. He analyzes how the conclusions concerning overshifting and efficiency have to be modified compared to the single-product case.

shows that in this set-up the first-best can be achieved by a positive lump-sum tax and a specific final product subsidy while the input specific taxes are zero. If lump-sum taxes are not feasible, taxation and subsidization of inputs is optimal instead. Hamilton (1999b) compares tax efficiency between monopoly and monopsony. In particular, he compares the case of upstream monopoly and perfect downstream competition with the one of perfect competition upstream and downstream monopsony. He shows that in the latter case a specific tax welfare-dominates an ad valorem tax while the opposite holds true for the former case. Asker (2008) analyzes a market with a downstream monopolist and a fixed number of upstream producers that compete to serve the monopolist via a first-price auction.[17] He compares different forms of taxes (or subsidies) on the input price and finds that overshifting is likely to arise under very general demand conditions. In contrast to these papers we consider an industry with an oligopoly structure both upstream and downstream, analyze the comparison between specific, ad valorem and value-added taxes and compare the results to the ones obtained for perfect competition upstream.

The rest of the paper is organized as follows: The next Section sets out the model and characterizes the short-run equilibrium. In Section 3 we consider the effects of taxation in the short run. We analyze tax incidence under the two forms of indirect taxes and compare our results to the results that obtain in one-layer industries. In Section 4 we address the same issues in the long run, when firms can enter and exit the industry. We also analyze the efficiency of a tax policy and point out differences between taxation downstream and upstream. Section 5 concludes. All proofs are relegated to the Appendix.

2 The model and equilibrium characterization

We consider a two-layer industry with an upstream and a downstream market. In the upstream market $M \geq 1$ symmetric firms produce a homogeneous intermediate product at constant marginal costs $c > 0$. In the downstream market, the intermediate product constitutes an input and $N \geq 1$ symmetric firms transform the input in constant proportions into a homogeneous final product at constant marginal costs. To simplify the exposition, marginal costs of downstream firms are set equal to zero. Without loss of generality, downstream firms need one unit of input for one unit of output. For instance, if

[17] Producers differ with respect to their marginal costs and their fixed costs. In addition, fixed costs are uncertain and after realization, only firms with low enough fixed costs enter the bidding.

downstream firms are retailers, this naturally holds. In the short run, the number of firms is exogenous in the upstream and downstream market.[18] The inverse demand function for the final product is denoted by $p(Q)$, with $p'(Q) < 0$ and $Q = \sum_{i=1}^{n} q_i$. For simplicity, we assume that $p(Q)$ is thrice continuously differentiable and that $c > \lim_{Q\to\infty} p(Q)$. Some shape assumptions on $p(Q)$ are spelled out below.

Firms play a two-stage game in the short-run. At the first stage, upstream firms play a Cournot game, i.e. each firm chooses its profit-maximizing quantity taking its competitors' outputs as given. The market-clearing input price (from the point of view of downstream firms), denoted by r, is determined by equating the total amount of output supplied by the upstream firms with the demand of the downstream firms. At the second stage, downstream firms compete à la Cournot in the final product market taking the input price that was determined at the previous stage as given.[19] The equilibrium concept is subgame perfect Nash equilibrium.

There are two different forms of taxes. The first is a specific tax $t \geq 0$ per unit sold. The second is an ad valorem tax τ under which the downstream producer price is $(1-\tau)p(Q)$, where $\tau \in [0, 1]$.

Solving the game by backward induction we can write the profit function of downstream firm i as

$$\Pi_i(q_i, Q_{-i}) = q_i \left((1-\tau)p(Q) - r - t\right),$$

where $Q_{-i} = Q - q_i$. The resulting first-order condition of profit maximization is

$$\frac{\partial \Pi_i}{\partial q_i} = (1-\tau)p(Q) - r - t + (1-\tau)q_i p'(Q) = 0.$$

Noting that at a symmetric equilibrium we have $q_i = q = Q/N \ \forall i \in [1, ..., N]$, the first-order condition can be rewritten to get

$$r = (1-\tau)p(Q) - t + (1-\tau)\frac{Qp'(Q)}{N}. \tag{1}$$

[18]In Section 4, the game is modified in a straightforward way to allow for entry and exit.

[19]In line with much of the previous literature on vertical oligopolies, e.g. Greenhut and Ohta (1979), Salinger (1988) and Ghosh and Morita (2007), downstream firms do not have market power in the upstream market. This property approximately holds if the upstream sector serves a large number of independent downstream markets so that a quantity change by a firm in one of these downstream markets has a negligible effect on the price of its input. One example are a large number of local retail markets that are served by the upstream firms and whose demand is independent of one another. To make the property hold exactly, we have to assume that upstream firms sell to a continuum of (identical) downstream sectors. In the remainder of this paper we consider one such representative downstream sector.

We need to make sure that the solution to the first-order condition uniquely determines Q. Since $r \geq c > 0$, aggregate equilibrium quantity is finite for all $t \geq 0$, $\tau \in [0, 1]$ at any viable price of the intermediate product. We make the following assumption:

Assumption 1: $(N+1)p'(Q) + Qp''(Q) < 0$.

Assumption 1 is a key condition for existence and uniqueness of a Cournot equilibrium (see e.g. Vives (1999)): it guarantees that the solution to (1) is unique and, thus, that the inverse demand function faced by the upstream firms is unique as well. To make sure that the unique solution to (1) is indeed a maximizer for each downstream firm given $Q_{-i} = [(N-1)/N]Q$ we assume that $Q/N = \arg\max_{q_i} \Pi_i(q_i, [(N-1)/N]Q)$, where Q solves (1).[20]

Using (1) we can derive the profit function of upstream firm j. Denote the aggregate quantity produced by the upstream firms as X, where $X = \sum_{j=1}^{M} x_j$ and $X_{-j} = X - x_j$ and note that in equilibrium $Q = X$. Thus, the profit function of upstream firm j can be written as

$$\Pi_j^u(x_j, X_{-j}) = x_j \left((1-\tau) \left(p(X) + \frac{Xp'(X)}{N} \right) - t - c \right). \tag{2}$$

This yields the first-order condition

$$\frac{\partial \Pi_j^u}{\partial x_j} = (1-\tau) \left[p(X) + \frac{Xp'(X)}{N} + x_j p'(X) \frac{1+N}{N} + \frac{x_j X p''(X)}{N} \right] - t - c = 0.$$

Using the fact that in a symmetric equilibrium $Mx_j = Nq_i = Q$, we can conclude that aggregate quantity in this two-layer industry under specific and ad valorem taxation is given by

$$(1-\tau) \left[p(Q^\star) + \frac{Q^\star p'(Q^\star)}{N} + \frac{(1+N)Q^\star p'(Q^\star)}{NM} + \frac{(Q^\star)^2 p''(Q^\star)}{NM} \right] - t - c = 0. \tag{3}$$

In any interior equilibrium (3) has to be satisfied. To make sure that such a solution exists and is unique, we make the following assumption:

Assumption 2: $(N+1)(M+1)p'(Q) + (N+M+3)Qp''(Q) + Q^2 p'''(Q) < 0$.

Assumption 2 is the counterpart to Assumption 1 for the upstream market. To make the problem interesting we must also have that $\lim_{Q \to 0} (1-\tau)p(Q) - t > c$ because otherwise

[20] For a related discussion see Kolstad and Mathiesen (1987) and Gaudet and Salant (1991).

no firm would produce a positive quantity. Finally, we have to assume that each upstream firm's profits are in fact maximized at $Q^\star/M$ given $X_{-j} = [(M-1)/M]Q^\star$, i.e. $Q^\star/M = \arg\max_{x_j} \Pi_j^u(x_j, [(M-1)/M]Q^\star)$.

Lemma 1 *For any taxes t and τ such that $\lim_{Q\to 0}(1-\tau)p(Q) - t > c$, there exists a unique symmetric equilibrium with aggregate quantity $Q^\star > 0$ which is the solution to (3).*

In the following we occasionally consider two frequently used parametric specifications of demand to illustrate some of our results. The first specification is a generalization of the linear demand function while the second one exhibits the property of constant returns to scale (CES demand function).

Example 1 $p(Q) = 1 - Q^b$. *Assumptions 1 and 2 are satisfied for $b > 0$. Equilibrium final-product price and equilibrium quantity are given by*

$$p^* \equiv p(Q^*) = 1 - \frac{MN((1-\tau) - t - c)}{(1-\tau)(N+b)(M+b)} \tag{4}$$

and

$$Q^* = \left(\frac{MN((1-\tau) - t - c)}{(1-\tau)(N+b)(M+b)}\right)^{\frac{1}{b}}.$$

Example 2 $p(Q) = Q^{-b}$. *Here, assumptions 1 and 2 are satisfied for $\min\{M, N\} > b > 0$. Equilibrium final-product price and equilibrium quantity are given by*

$$p^* \equiv p(Q^*) = \frac{MN(t+c)}{(1-\tau)(N-b)(M-b)}$$

and

$$Q^* = \left(\frac{MN(t+c)}{(1-\tau)(N-b)(M-b)}\right)^{-\frac{1}{b}}. \tag{5}$$

Before starting with the analysis two important remarks are in order.

Remark 1 Value-added tax (VAT)

In our two-layer industry it is of particular interest to analyze the consequences of a VAT, especially given its prevalence in many countries. Here we point out that our following analysis for the ad valorem tax encompasses the case of a VAT since both taxes are equivalent. To see this suppose that the government sets a VAT of τ. The profit function of a downstream firm is then given by

$$\Pi_i(q_i, Q_{-i}) = q_i\left((p(Q) - r)(1-\tau) - t\right), \tag{6}$$

which after maximizing yields

$$r = p(Q) + \frac{Qp'(Q)}{N} - \frac{t}{1-\tau}. \tag{7}$$

The profit function of upstream firm j is then given by

$$\Pi_j^u(x_j, X_{-j}) = x_j \left((1-\tau) \left(p(X) + \frac{Xp'(X)}{N} \right) - t - c \right).$$

But this profit function is the same as (2). Therefore, the equilibrium quantity is the same under ad valorem and under VAT given by (3) and the comparative statics with respect to τ are also the same.[21,22]

Remark 2 Downstream vs. Upstream Taxation
We set up the model by restricting taxation to occur downstream. Yet, all of our results are valid if the commodity is taxed in the upstream market. To see this suppose that ad valorem and unit tax are charged in the upstream market. The profit function of a downstream firm i is then given by

$$\Pi_i(q_i, Q_{-i}) = q_i \left(p(Q) - r \right), \tag{8}$$

Maximizing and rearranging yields

$$r = p(Q) + \frac{Qp'(Q)}{N}. \tag{9}$$

The profit function of an upstream firm j then reads as

$$\Pi_j^u(x_j, X_{-j}) = x_j \left((1-\tau) \left(p(X) + \frac{Xp'(X)}{N} \right) - t - c \right).$$

But obviously this is the same profit function as with taxation in the downstream market. Therefore, the equilibrium Q is again given by (3) and the comparative static results on t and τ are the same as in our analysis.[23]

[21] Here we suppose that the upstream sector is the first layer in the production chain. Thus, a VAT is the same as a revenue tax in this sector. If the upstream sector was an intermediate stage in the production chain, the VAT would be only a profit tax and therefore would have no distorting effects. But since there is always a starting layer in the production chain, the equivalence of VAT and ad valorem tax holds also when considering more than just two layers in the production chain.

[22] The equivalence between ad valorem taxation and VAT carries over to a setting with downstream marginal costs different from zero (presuming that either the VAT does not apply to these costs (e.g. labor costs) or that the costs arise from other inputs whose net price is constant in VAT-rates).

[23] The equivalence is less general if we allow for positive marginal costs in the downstream market. Here, the equivalence holds only if the these downstream costs are due to some other input that is taxed at the same rate or if variable input costs can be deducted downstream.

3 Short-run effects of indirect taxation in vertical oligopoly

In this section we investigate the effects of unit and ad valorem taxes in the short run. We analyze under which conditions overshifting of each of the two taxes occurs. We are especially interested if and how these conditions might differ compared to an analysis which neglects strategic interaction in the upstream market and takes the upstream price as given. This corresponds to an industry in which the upstream market is perfectly competitive and firms have constant marginal costs. We also show under which circumstances these differences are especially pronounced. Furthermore, we briefly look at optimal taxes.

Tax Incidence

The study of tax incidence concerns the possibility of over- and undershifting. *Overshifting under the unit tax* occurs if and only if the retail price net of taxes (the producer price) increases with the unit tax, i.e., if $d(p^\star(1-\tau)-t)/dt > 0$ or, equivalently, $(1-\tau)\,(dp^\star/dt) > 1$. Similarly, *overshifting under the ad valorem tax* occurs if and only if $d\,(p^\star(1-\tau)-t)\,/d\tau = (dp^\star/d\tau)\,(1-\tau)-p^\star > 0$. Undershifting occurs if ">" is replaced by "<". To save on notation in the following we abbreviate $p(Q^\star)$ by p and $p'(Q^\star)$ by p'. Analogously, for higher order derivatives. To shorten expressions, we define $\epsilon = -Q^\star p'/p > 0$ as the elasticity of the inverse demand function, $E = -Q^\star p''/p'$ as the elasticity of the slope of the inverse demand function and $F = -Q^\star p'''/p''$ as the elasticity of the curvature of the inverse demand function.

Proposition 1 *(i) The specific tax leads to overshifting if and only if*

$$\frac{NM}{(N+1)(M+1)-E(N+M+3)+EF} > 1. \tag{10}$$

(ii) The ad valorem tax leads to overshifting if and only if

$$\frac{NM-\epsilon((N+M+1)-E)}{(N+1)(M+1)-E(N+M+3)+EF} > 1. \tag{11}$$

(iii) Overshifting of the ad valorem tax implies overshifting of the specific tax.

As Delipalla and Keen (1992) show, overshifting of the specific tax implies overshifting of the ad valorem tax when only considering the downstream market. As part (iii) of Proposition 1 demonstrates, this result carries over to vertical oligopoly.[24]

[24] An empirical study by Delipalla and O'Donnell (2001) on the European cigarette industry for the years

We now take a closer look at the overshifting results (i) and (ii) and compare the effects of excise taxes in an industry with perfect competition in the upstream market (one-layer industry) to the ones that occur in vertical oligopoly (two-layer industry).

We ask two questions: First, when does overshifting in a one-layer industry imply overshifting in our two-layer industry and when does the reverse implication hold? Second, how strong is the over- or undershifting? So we are interested in determining if there are different predictions concerning tax policy in industries where the market structure upstream is oligopolistic to those with perfect competition upstream. Thereby we also provide an answer to the question how, and to which extent, models err that, for simplicity, take the input price as given and therefore independent of the tax rate. In the following we denote the respective price, quantity, derivatives of the demand function and elasticities in a one-layer industry with a subscript o.

As established by Delipalla and Keen (1992), in a one-layer industry overshifting of the specific tax occurs if

$$\frac{N}{N+1-E_o} > 1 \tag{12}$$

or, equivalently, $E_o > 1$, while overshifting of the ad valorem tax occurs if

$$\frac{N-\epsilon_o}{N+1-E_o} > 1 \text{[25]} \tag{13}$$

or, equivalently, $E_o > 1+\epsilon_o$.[26] We can now compare these results with the condition for overshifting in a two-layer industry.

Proposition 2 *(i) Specific tax: If*

$$EF > E(M+N+3) - E_o(M+N+1),$$

then overshifting of the specific tax in a two-layer industry implies overshifting in a one-layer industry and vice versa.

(ii) Ad valorem tax: If

$$EF > E(M+N+3+\epsilon) - \left(1+\frac{\epsilon}{\epsilon_o}(E_o-1)\right)(M+N+1),$$

1982-1997 confirms this result. They demonstrate that an increase in the specific tax had a significantly larger effect on price than an increase in the ad valorem tax. On average, the ratio of specific tax effect to ad valorem tax effect is around 1.4.

[25] As can be checked from (10) and (11), both conditions are obtained in our model in the limit as $M \to \infty$ because the upstream market becomes perfectly competitive then.

[26] As in the two-layer industry, $\epsilon_o = Q_o^\star p_o'/p_o$ and $E_o = Q_o^\star p_o''/p_o'$, where p_o and its derivatives are evaluated at $Q_o^\star$.

then overshifting of the ad valorem tax in a two-layer industry implies overshifting in a one-layer industry and vice versa.

This result can be explained as follows: As can be seen from (12) and (13), when considering only the final consumer market, overshifting occurs if E_o is relatively large, i.e. the second derivative of the inverse demand curve is positive and sufficiently large. In a similar way, if an additional layer is taken into account, the price in this layer increases with a tax increase if the third derivative is positive and large enough. Since $EF = p'''/p'$, a large third derivative implies that EF is highly negative because $p' < 0$. As Proposition 2 shows, if EF is below a certain threshold, overshifting in a one-layer industry implies overshifting in a two-layer industry, i.e. overshifting is more likely to occur in a two-layer industry than in a one-layer industry. Thus, when comparing the two market structures, overshifting of taxes to final consumers is increased if the third derivative is sufficiently large, so that upstream oligopoly amplifies the overshifting effect.

The proposition shows that the overshifting result in an industry where the input price is fixed can be misleading when applied to an industry with imperfect competition in the upstream market. This is important when considering the effects of a change in the government's tax policy. A government that ignores the fact that the upstream market is imperfectly competitive and thus ignores the effect of a tax change on the input price, will make wrong predictions concerning the allocative effects of a change in tax policy.

To shed some further light on how the results differ between one- and two-layer industries we take a look at the parametric demand functions from Examples 1 and 2. Note that the class of demand functions given by Example 1 exhibits the property that cost shifters are passed on to consumers by less than unity, while the opposite holds true for the class of demand functions given by Example 2 (see e.g. Weyl (2008)). Thus, undershifting always occurs in the first class of demand functions and overshifting in the second class. When taking into account imperfect competition in the upstream market, the qualitative predictions concerning over- or undershifting remain valid in these two examples. However, as the next proposition shows, both become more pronounced in a two-layer industry.

Proposition 3 *(i) If $p(Q) = 1 - Q^b$, undershifting of both the unit tax and the ad valorem tax is larger in a two-layer industry than in a one-layer industry.*
(ii) If $p(Q) = 1/Q^b$, overshifting of both the unit tax and the ad valorem tax is larger in a two-layer industry than in a one-layer industry.

With the help of a numerical example we illustrate that differences can be substantial. Suppose that parameter values are $N = 5$, $M = 3$ and $b = 1$. Then, for $p(Q) = 1 - Q$ (special case of example 1) we obtain $(dp_o^\star/dt)(1-\tau) - 1 = -1/6$ in a one-layer industry while $(dp^\star/dt)(1-\tau)-1 = -3/8$ in a two-layer industry. Hence, the extent of undershifting is more than doubled after taking into account imperfect competition in the upstream market. With the demand specification $p(Q) = 1/Q$ (special case of example 2) we obtain $(dp_o^\star/dt)(1-\tau) - 1 = 1/4$ in a one-layer market while $(dp^\star/dt)(1-\tau) - 1 = 7/8$ in a two-layer market. Thus, ignoring imperfect competition in the upstream market would predict an overshifting of 25% while the correct number is 87.5%.

Note that in the classes of parametric demand functions given by Examples 1 and 2 the sign of the second and third derivative often point in the same direction and, if not, their difference is small.[27] So, if there is overshifting which implies that the price-cost margin at the downstream market increases, the equilibrium reaction of firms in the upstream market is to reduce their quantity so that their price-cost margins increase as well. The opposite holds true for undershifting. As a consequence, in our two examples the qualitative result concerning over- or undershifting in the downstream market is essentially robust to imperfect competition in the upstream market. However, taking into account imperfect competition in the upstream market amplifies the degree of over- or undershifting and is thus important for evaluating quantitative effects of tax changes on final product prices and consumer surplus.

It is of interest under which conditions the mistake that results from ignoring the downstream market is particularly large. This is important because when evaluating the effects of an increase in excise taxes for a particular industry, one needs to know if an estimation that concerns only the downstream market is sufficient in practice. Moreover, since the mistake can be large, one must distinguish between different industries to find out if a similar tax increase may have different consequences.

With the help of the two parametric families of demand functions we investigate the role of the elasticity parameter b and the number of firms in each layer, N and M: How does b, N and M affect the difference in the marginal response of price to the tax rate between a one- and a two-layer industry?

[27] They generally point in the same direction for the demand function $p(Q) = 1/Q^b$. For the function $p(Q) = 1 - Q^b$ this is true if $b > 2$ while for $b \leq 2$ the difference between the two is small.

Proposition 4 *(i) If $p(Q) = 1 - Q^b$, both*

$$\left|\frac{dp_o^\star}{dt} - \frac{dp^\star}{dt}\right| \quad \text{and} \quad \left|\frac{dp_o^\star}{d\tau} - \frac{dp^\star}{d\tau}\right|$$

are increasing in b if and only if $b < \sqrt{NM}$. Both are increasing in N and decreasing in M.

(ii) If $p(Q) = Q^{-b}$, both

$$\left|\frac{dp_o^\star}{dt} - \frac{dp^\star}{dt}\right| \quad \text{and} \quad \left|\frac{dp_o^\star}{d\tau} - \frac{dp^\star}{d\tau}\right|$$

are increasing in b, decreasing in N and decreasing in M.

Let us explain these comparative static results: For demand functions of the form $p(Q) = 1 - Q^b$, the difference between the effects in a one- and two-layer industry is maximal if b takes some intermediate value. The reason is that if b is either close to zero or very large, the demand function is very flat or steep and the effect of upstream oligopoly compared to perfect competition is only small, since the curvature of the demand function dominates all other effects. The second finding, namely that the difference between a one- and a two-layer market increases in N, appears to be surprising at first glance. If the downstream market becomes perfectly competitive, the difference is most pronounced. The intuition is that as long as the demand function is not perfectly inelastic (which is not the case here), undershifting gets the larger, the larger the number of downstream firms. This effect is more pronounced the larger the amount of undershifting. Since this amount is larger in a two-layer industry, the difference increases. Finally, the finding that the difference decreases in M is not surprising, since in the limit as $M \to \infty$ one- and two-layer industry are the same. Therefore, the result only establishes monotonicity, i.e. the difference is monotonically decreasing in M.

For demand functions of the form $p(Q) = Q^{-b}$, the result concerning the elasticity parameter b is similar to the case above. If b is close to zero, the elasticity of the demand function dominates all other effects and therefore the difference is small. Yet, here b is restricted to be smaller than the minimum of N and M to ensure a unique interior solution. Thus, it cannot become too large and for all admissible values of b we obtain that the difference is still increasing. With respect to an increase in N, overshifting becomes less and less pronounced, and this is even more so the higher the degree of overshifting. Since overshifting is larger in two-layer industries than in one-layer industries this difference decreases in N. The comparative static effects with respect to M are the same as in the other example.

To summarize the results of this subsection, we provide conditions on the shape of the demand curve such that the effects of specific and ad-valorem tax in a two-layer industry are qualitatively the same as those in a one-layer industry. As illustrated by two parametric examples, although the qualitative results are similar, the effects are amplified in a two-layer industry, possibly to a large extent. Thus, when neglecting the interaction of the final product market with the input market, a tax policy underestimates the effects of a tax change. In Section 4 we will show that in the long-run results in two-layer industries are markedly different from those in one-layer industries.

Tax Efficiency

We now shortly compare the two taxes with respect to their efficiency properties. What is the welfare maximizing tax policy if the government wants to raise a given tax revenue with the two indirect taxes? Here our answer is unambiguously in favor of using the ad valorem tax. In Appendix B we show that to raise a given amount of tax revenue it is optimal to do that exclusively with the ad valorem tax and to set the unit tax to zero. In other words, for a welfare-maximizing government the ad valorem tax is the more efficient one. This finding corroborates previous findings that were obtained in one-layer industries (see e.g. Delipalla and Keen (1992) or Anderson et al. (2001b)). It essentially shows that the structure of the optimal tax policy is robust to imperfect competition in the upstream market in the short run. The ad valorem tax distorts final output by less than the specific tax.

We can also consider what is the optimal corrective tax to restore the efficient outcome. Since imperfect competition leads to price above marginal costs, it is obvious that a policy of subsidization is optimal for both the unit and the ad valorem tax. It is easy to calculate that the optimal amount of the unit tax is

$$t = \frac{Q^\star(p'(1+N+M) + Q^\star p'')}{NM} < 0$$

given that the ad valorem tax is zero and, likewise, the optimal amount of the ad valorem tax is

$$\tau = \frac{Q^\star(p'(1+N+M) + Q^\star p'')}{cNM + Q^\star(p'(1+N+M) + Q^\star p'')} < 0,$$

given that the unit tax is zero.[28]

4 Long-run effects of indirect taxation in vertical oligopoly

In this section, we no longer treat the number of firms as given. Instead, there is a large number of firms, which can be active or inactive. Firms at both layers adjust via the process of entry and exit to the tax regime. To capture the result of this process of entry and exit the number of active firms (as sellers) is determined by the zero-profit condition in each layer. We refer to this as the long-run analysis. We denote the cost of being active as a seller in the downstream market by F_d and in the upstream market by F_u. The game now consist of three stages, and we solve for the subgame perfect Nash equilibrium of this game. At the first stage, each firm decides to be active in the upstream or in the downstream market or to be inactive. If a firm enters as a seller, it bears the respective fixed cost which is sunk at this stage. In line with most of the literature, we treat the numbers of firms in both layers as a continuous variable.[29] The second and third stages play out exactly as before. Given the number of entering firms, at the second stage, each upstream firm chooses its profit-maximizing quantity taking the quantity of its rivals as given, and, at the third stage, each downstream firm chooses its profit-maximizing quantity taking the quantity of its rivals as given.

The long-run equilibrium: Preliminaries

Since the last two stages are the same as in the short run analysis, the aggregate quantity for a given number of active firms $N^\star$ and $M^\star$ is, as above, given by

$$(1-\tau)\left[p(Q^\star)+\frac{Q^\star p'(Q^\star)}{N^\star}+\frac{(1+N^\star)Q^\star p'(Q^\star)}{N^\star M^\star}+\frac{(Q^\star)^2 p''(Q^\star)}{N^\star M^\star}\right]-t-c=0. \tag{14}$$

At the first stage, firms enter at both layers so that profits are zero, i.e., in a symmetric equilibrium $N^\star$ and $M^\star$ are jointly determined by the equations

$$\begin{aligned}\frac{Q^\star((1-\tau)p(Q^\star)-r-t)}{N^\star} &= F_d,\\ \frac{X^\star\left[(1-\tau)\left(p(X^\star)N^\star+X^\star p'(X^\star)\right)-N^\star(t+c)\right]}{M^\star N^\star} &= F_u,\end{aligned}$$

[28]The inequality follows from the fact that (3) must still hold at $p(Q)=c$. Therefore, the numerator is negative while the denominator is positive.

[29]See e.g. Besley (1989), Delipalla and Keen (1992), and Anderson et al. (2001a, 2001b). This not only simplifies the analysis but also avoids non-monotone comparative statics that would stem from the discreteness of the number of firms.

and

$$X^\star = Q^\star.$$

Using first-order conditions of profit maximization at stages 2 and 3, the first two equations can be rewritten as

$$-(1-\tau)\left(\frac{Q^\star}{N^\star}\right)^2 p'(Q^\star) \quad = \quad F_d \tag{15}$$

and

$$-(1-\tau)\frac{(Q^\star)^2}{N^\star(M^\star)^2}\left((N^\star+1)p'(Q^\star)+Q^\star p''(Q^\star)\right) \quad = \quad F_u. \tag{16}$$

The equilibrium $Q^\star$, $M^\star$, and $N^\star$ under free entry in both layers are characterized by the solution to the system of the three equations (14), (15) and (16).

Before starting the analysis let us make two remarks.

Remark 1B VAT
From Remark 1 we know that the aggregate quantity is the same for a VAT and an ad valorem tax of τ, given the number of firms is fixed. To demonstrate this equivalence in the long-run as well we have to show that the same number of firms enter upstream and downstream, i.e. that profits under both tax regimes are the same. Yet, this can be easily checked using Remark 1. Inserting (7) into (6) gives that the profit of a downstream firm under VAT is the same as in (15), the profit of a downstream firm under ad valorem taxation, given the number of upstream firms. But from Remark 1 we already know that the profit function of an upstream firm is the same under both forms of taxes. Thus, the number of entering upstream firms is also the same. As a consequence, ad valorem tax and value-added tax are equivalent in the long run as well.

Remark 2B Upstream vs. Downstream Taxation
As in Remark 1B, we have to check if the number of firms may differ when taxation is set upstream instead of downstream. Interestingly, here this is the case for the ad valorem tax but not for the unit tax. This can be seen from Remark 2. Using (8) and (9) yields that the profit of a downstream firm is given by

$$-\left(\frac{Q}{N}\right)^2 p'(Q) = F_d \tag{17}$$

instead of $-(1-\tau)\left(Q^\star/N^\star\right)^2 p'(Q^\star) = F_d$, which is the relevant equation under downstream taxation. The equations determining the number of upstream firms and the aggregate quantity are still given by (14) and (16). Thus, if $\tau = 0$, the equilibrium is the same. Therefore, setting a unit tax upstream or downstream is equivalent. But for $\tau > 0$ the profit function for downstream firms differ which leads to a different $N^\star$ and thereby also to a different $M^\star$ and $Q^\star$. We will provide a comparison between ad valorem taxation upstream and downstream concerning incidence and efficiency at the end of this Section.

Tax Incidence

We first analyze whether and under which conditions the specific or the ad valorem tax leads to overshifting. Note that now firms adjust their participation decision to changes in the tax rates.[30]

Proposition 5 *(i) Overshifting of the specific tax under free entry occurs if and only if*

$$\frac{4NM}{4NM - 2 - E^2 - E(1 + 2N + 2M) + 2EF} > 1. \tag{18}$$

(ii) Overshifting of the ad-valorem tax under free entry occurs if and only if

$$\frac{4NM - \epsilon(1 + 2N + 2M) + E\epsilon}{4NM - 2 - E^2 - E(1 + 2N + 2M) + 2EF} > 1. \tag{19}$$

(iii) Overshifting of the ad-valorem tax implies overshifting of the specific tax.

The result is similar to the one in the short run where overshifting of the ad valorem tax also implies overshifting of the specific tax. Again, it is of particular interest to compare these conditions with the ones in a one layer industry. From work by Besley (1989) and Delipalla and Keen (1992) we know that overshifting of the unit tax in a one-layer industry occurs if

$$\frac{2N_o}{2N_o - E_o} > 1 \tag{20}$$

or, equivalently, $E_o > 0$, where N_o denotes the equilibrium number of downstream firms in a one-layer industry. Likewise, overshifting of the ad valorem tax occurs if

$$\frac{2N_o - \epsilon_o}{2N_o - E_o} > 1 \tag{21}$$

or, equivalently, $E_o > \epsilon_o$.

[30] To abbreviate notation here and in the following we omit the superscript $\star$ at the equilibrium numbers of firms, $N^\star$ and $M^\star$.

Proposition 6 *(i) Specific tax: If*

$$EF > \frac{1}{2}(2 + E^2),$$

then overshifting of the specific tax in a two-layer industry implies overshifting in a one-layer industry and vice versa.

(ii) Ad valorem tax: If

$$EF > \frac{1}{2}\left[E(1 + 2N + 2M + E + \epsilon) + 2 - E_o\frac{\epsilon}{\epsilon_o}(1 + 2N + 2M)\right],$$

then overshifting of the ad valorem tax in a two-layer industry implies overshifting in a one-layer industry and vice versa.

At a first glance, these results may look similar to the ones of Proposition 2 concerning the short-run comparison between a one- and a two-layer industry. Yet, the results are very distinct. Let us explain this with the example of the specific tax. In the short run, the threshold value for EF is given by $E(M + N + 3) - E_o(M + N + 1)$. But the only ex ante information we have on E and E_o stems from Assumptions 1 and 2 and restricts p'' not to be too large. Thus, E and E_o are either negative or not too positive but we have no restriction on how they might differ. Thus, the sign of the threshold value can either be positive or negative. As we have shown for our two examples in Proposition 3, both over- and undershifting can be more pronounced in a two-layer market. Instead looking at the condition in the long run, it is easy to see that the threshold $1/2(2 + E^2)$ is strictly positive. We find that only if EF is large and positive, then overshifting in a two-layer industry implies overshifting in a one-layer industry. But if EF is not highly positive, the reverse holds true. As a consequence, overshifting in the long run occurs for a larger set of demand functions in a two-layer industry than in a one-layer industry. A similar observation can be made for the ad valorem tax; the effects are somewhat weaker but are qualitatively the same.[31]

The intuition behind the result is the following: A tax increase reduces profits of the downstream firms. Therefore, under free entry, fewer downstream firms enter. This leads to reduced supply in the final product market and therefore demand on the intermediate

[31] This can be seen in the easiest way by supposing that both $\epsilon_o \approx \epsilon$ and $E_o \approx E$. Then, the threshold value can be written as $1/2(2 + E^2 + \epsilon E)$. Thus, it consists of two positive and one indeterminate term. As a consequence, if EF is not overly large, overhifting occurs for a larger parameter range in a two-layer industry than in a one-layer industry (for a properly chosen parametric family of demand functions).

product market decreases as well. But this lowers the profits of the upstream firms. Thus, fewer of them enter. As a consequence, upstream firms have more market power and restrict their quantities further to keep the intermediate product price high. This in turn leads to higher final product prices. As a result, a model that neglects the reaction of input prices to a tax change normally underestimates the effects of a tax increase on the price increase in the long run. It may predict undershifting when indeed overshifting takes place or underestimates the extent of overshifting.

It is instructive to see how these effects play out in the two classes of demand function we introduced above. We can partly use the previous proposition to show the following result:

Proposition 7 *(i) If $p(Q) = 1 - Q^b$, then for both, the specific and the ad valorem tax, overshifting in a two-layer industry occurs for a larger set of parameters than in a one-layer industry.*
(ii) If $p(Q) = 1/Q^b$, then for both, the specific and the ad valorem tax, the extent of overshifting in a two-layer industry is larger than in a one-layer industry.

Hence, in contrast to the short run where the effects of both over - and under shifting are more pronounced in a two layer industry, in the long run overshifting is always underestimated when neglecting the upstream market.

As in the short-run, we can analyze under which conditions the difference between a one- and a two-layer industry is particularly large. Yet, this is more complicated in the long-run since the number of firms is endogenous in both industries. Solving explicitly for the number of firms in a two-layer industry is possible only for special cases of the demand functions since the zero profit conditions are often nonlinear in the number of firms. For that reason, we confine our analytical results to the simpler demand functions $p(Q) = 1 - Q$ and $p(Q) = 1/Q$, the special case of $b = 1$ in the demand functions of the Examples 1 and 2, respectively.[32]

Proposition 8 *(i) If $p(Q) = 1 - Q$, both*

$$\left|\frac{dp^\star}{dt} - \frac{dp_o^\star}{dt}\right| \quad \textit{and} \quad \left|\frac{dp^\star}{d\tau} - \frac{dp_o^\star}{d\tau}\right|$$

[32] In fact, in this case the second demand function allows for an explicit solution for the number of firms in both markets. With the first one, this is still impossible but we can apply the implicit function theorem.

increase in F_d and in F_u.

(ii) If $p(Q) = 1/Q$, both

$$\left|\frac{dp^\star}{dt} - \frac{dp_o^\star}{dt}\right| \quad \textit{and} \quad \left|\frac{dp^\star}{d\tau} - \frac{dp_o^\star}{d\tau}\right|$$

increase in F_d and in F_u.

Most of these results parallel those that were obtained in the short-run analysis and the intuition was already given there. The exception is that the difference in absolute values for the demand function $p(Q) = 1 - Q$ increases in F_d. In the short-run we obtained that this difference is increasing in N while here it is increasing in F_d and therefore decreasing in N. In the short run, this function implied undershifting in both industries. Instead, in the long-run for $b = 1$ overshifting occurs, and, as explained after Proposition 4, this becomes more pronounced the smaller is N.

Since for tractability reasons we restrict our analysis to the case in which $b = 1$ in both functions, we cannot obtain analytical comparative static results concerning b. Yet, we did simulations to draw some conclusions about the effects of a change in b. Here we find similar results to the ones obtained in the short-run, i.e. that for $p(Q) = 1 - Q^b$ the absolute difference between one- and two-layer industry is first increasing and then decreasing in b while for $p(Q) = 1/Q^b$ it is always increasing in b. Thus, short-run and long-run effects do not differ qualitatively in this respect.

To sum up, a tax policy that concentrates on the final products market is likely to err in the extent of the tax incidence. In the long-run, not only downstream firms but also upstream firms exit the market in response to a tax increase, and this has a detrimental effect on consumer surplus.

Tax Efficiency

As in the short run, we can analyze which tax is more efficient in the long-run. As shown in Appendix B, we find that the ad valorem tax is the more efficient tax in the long run as well. Thus, we confirm that the result obtained by Delipalla and Keen (1992) for a fixed intermediate product price also holds if price and entry in the intermediate product market are endogenous. In contrast to the short-run our result is less obvious here. The reason is the following: In a one-layer industry, there is socially excessive entry.[33] The ad

[33]The result of socially excessive entry in Cournot industries with a fixed intermediate good price has been shown by e.g. Mankiw and Whinston (1986) and Suzumura and Kiyono (1987).

valorem tax now has the advantage to reduce profits and thereby the number of firms to a larger extent than the specific tax. Since this saves entry costs, it is socially desirable. This is not necessarily true anymore once we allow for free entry in the upstream market. In this case, even in the absence of taxation, insufficient entry at the two layers may occur, as shown by Ghosh and Morita (2007). The reason is that the surplus that arises when an additional upstream firm enters is in part captured as profit by the downstream firms. Therefore, an upstream firm receives less than the total surplus as profit and so insufficient entry may occur. A similar reasoning holds for downstream entry. Since the ad valorem tax reduces profits by more than the unit tax, a shift from the unit to the ad valorem tax leads to less entry which can be detrimental to welfare. Yet, our result shows that this effect can never dominate. Although entry might be insufficient, the fact that the distortive effect of an ad valorem tax is smaller than the one of a specific tax dominates.

It is also of interest to analyze under which conditions the introduction of a positive tax rate increases welfare compared to a one-layer industry. Here we obtain the following result:

Proposition 9 *(i) Specific tax: If $EF > 1/2(2 + E^2)$, then an increase in welfare by the introduction of a small specific tax (conditional on $\tau = 0$) occurs in a two-layer industry only for a subset of demand functions compared to a one-layer industry and vice versa.*
(ii) Ad valorem tax: If $EF > \frac{1}{2}[E(1 + 2N + 2M + E + \epsilon) + 2 - E_o\epsilon/\epsilon_o(1 + 2N + 2M)]$, then an increase in welfare by the introduction of a small ad valorem tax (conditional on $t = 0$) occurs in a two-layer industry only for a subset of demand functions compared to a one-layer industry and vice versa.

The thresholds given in the proposition are the same as the thresholds in Proposition 6. The reason is that overshifting of the specific or the ad valorem tax implies that its introduction reduces welfare (given that the other tax is zero). This holds for both, a one- and a two-layer industry. Given our previous results, the proposition implies that the introduction of a specific or an ad valorem tax reduces welfare for a larger range of parameters when taking the upstream market into account. The intuition is also easy to grasp from the previous results. Since a reduction in overall quantity is more likely in a two-layer industry, the introduction of a tax is more likely to have a detrimental effect on welfare. The implication for policy makers when considering the introduction of such a tax in a particular industry is therefore to have a clear picture about the competitive environment in the upstream market. If the effects in the upstream market are ignored, the consequences after the introduction may turn out to be undesirable.

Taxation in the Upstream Market

As shown in Remark 2B, if the ad valorem tax is set upstream instead of downstream the profit of downstream firms is no longer directly affected by τ and, therefore, the equilibrium allocation is different. In this subsection we compare now ad valorem taxation upstream with ad valorem taxation downstream. This is, in general, not an easy task since the equilibrium number of firms is endogenous and different in the two regimes. Still, we can draw some conclusions. We start by looking at a price change in response to the introduction of a small ad valorem tax.

Proposition 10 *At $\tau = 0$, the introduction of a small ad valorem tax in the downstream market increases $p^\star$ by more than the introduction of an ad valorem tax in the upstream market.*

The intuition is clear: if an ad valorem tax is introduced downstream, the profit of downstream firms decreases by more than if the tax is introduced upstream, since the profit is directly affected by the tax. Since an upstream firm's profit is given by the same formula in both regimes, the consequence is that the quantity decrease resulting from downstream taxation is larger. Thus, the price increase is larger as well.

Note that the result of Proposition 10 is also valid for a perfectly competitive upstream market since the zero profit condition in the downstream market is still different under both scenarios, independent of the degree of competition upstream. Yet, as the next proposition shows for analytically tractable cases of our examples, the effects are weaker if the upstream market is perfectly competitive.

Proposition 11 *For the demand functions $p(Q) = 1 - Q$ and $p(Q) = 1/Q$ the difference between the price increase resulting from an ad valorem tax downstream versus upstream is larger in a two-layer industry than in a one-layer industry.*

Thus, when the upstream market becomes perfectly competitive the difference between upstream and downstream taxation is smaller (although not eliminated). The intuition is that a competitive upstream market dampens a price increase in general but this effect becomes more important the larger the price increase.

Finally, we analyze if ad valorem taxation upstream or downstream is more efficient. To do so, we restrict attention to the demand function $p(Q) = 1/Q$ because it allows us to solve for the equilibrium number of firms at both layers, and thus to compare welfare under the two regimes analytically.

Proposition 12 *Suppose that $p(Q) = 1/Q$. Then for any upstream ad valorem tax, there exists a downstream ad valorem tax that is more efficient.*

This result shows that although the price increase is larger when the ad valorem tax is charged downstream than upstream, welfare under a downstream ad valorem tax is higher. The intuition behind this result is the following: As explained above, under upstream taxation more downstream firms enter. Thus, aggregate entry costs are higher. Moreover, the tax revenue of upstream taxation is smaller if the tax rates were the same, since in the upstream market the tax revenue is a fraction of the aggregate quantity times the upstream price $r(Q)$. In contrast, under downstream taxation the tax revenue is a fraction of the aggregate quantity times the downstream price $p(Q) > r(Q)$. Of course, to induce the same aggregate quantity, the downstream ad valorem tax rate must be lower than the upstream one. However, this effect is smaller than the ones described above.

For tractability reasons, we could only provide the welfare result for the function $p(Q) = 1/Q$. To check if the result also holds for more general functions, we did numerical calculations for the CES demand functions with $b \neq 1$ and also for the function $p(Q) = 1 - Q^b$. We found in all our calculations that for any upstream ad valorem tax a more efficient downstream ad valorem tax exists. Thus, our result seems to hold for more general demand functions than just the one we can provide an analytical result for.

5 Conclusion

This paper analyzed the effects of indirect taxation in an industry with upstream oligopoly, and, in particular, compared the results to previous models that presume a perfectly competitive upstream market. We first demonstrated that, in our setting, in the short-run a VAT is equivalent to an ad valorem tax and that upstream and downstream taxation are also equivalent. We then showed that the results concerning over- or undershifting are qualitatively similar in a one- and in a two-layer industry but can be much larger in a two-layer industry. This is different in the long-run. There, overshifting is more likely in a two-layer industry and is also more pronounced. Thus, final consumers bear more of the tax burden when explicitly taking into account entry and exit in the upstream market. This also makes the introduction of any tax less likely to be welfare enhancing. Finally, we showed that in the long-run ad valorem taxation downstream is different to ad valorem taxation upstream, and proved in a specific setting that the first one is welfare superior.

A possible limitation of our analysis is that we only considered the case of Cournot competition in both layers. As mentioned in the introduction, the reason is that the industrial organization literature has only started to analyze alternative models of competition in the context of vertical oligopoly, namely Cournot oligopoly with supply constraints and price competition with differentiated products. However, without having worked out the mathematics of these alternative models, our conjecture is that our qualitative findings with respect to overshifting in the long-run are confirmed. The reason is the following: as a tax increase leads to lower profits of firms, exit will happen in the upstream and the downstream market of the industry. Thus, the input price in the downstream market rises and overshifting becomes more likely and/or occurs to a larger extent in a two-layer industry compared to a one-layer industry. This effect is independent of the specifics of the model of imperfect competition. Similarly, the superiority of introducing an ad valorem tax downstream instead of upstream relies on the savings of entry costs and the higher tax revenue that is due to the fact that the downstream price is larger than the intermediate product price. Again, this intuition does not rely on the specific model of imperfect competition.

6 Appendix

6.1 Appendix A

Proof of Lemma 1

Due to the boundary properties any solution must be an interior solution and by continuity, there must exist a solution $Q^\star$ to (3). Uniqueness of a symmetric solution follows because Assumption 2 implies that the term in square brackets in (3) is monotone in Q. By assumption, each firm's profits are maximized given the choices of the other firms. ■

Proof of Proposition 1

By the implicit function theorem, differentiating (3) with respect to Q and t and noting that $dp^\star/dt = p'(dQ^\star/dt)$ yields after rearranging

$$\frac{dp^\star}{dt} = \frac{NM}{(1-\tau)\left[(N+1)(M+1) - E(N+M+3) + EF\right]}. \tag{22}$$

Overshifting occurs if $(1-\tau)\left(dp^\star/dt\right) > 1$. Inserting (22) then yields (10).

Overshifting of the ad valorem tax occurs if $(dp^\star/d\tau)(1-\tau) - p^\star > 0$. Calculating $d\left(p^\star(1-\tau)\right)/d\tau$ and rearranging then yields the result in (11).

Finally, to show that overshifting of the ad valorem tax implies overshifting of the specific tax, note that, by Assumption 1, $N + M + 1 - E > 0$. As a consequence, the left-hand side of (10) is strictly larger than the one of (11) which implies the result. ■

Proof of Proposition 2

On (i): After rewriting (10) we get that overshifting of the specific tax in a two-layer industry occurs if[34]

$$p'(M+N+1) + Qp''(M+N+3) + Q^2p''' > 0, \tag{23}$$

while from (12) we can deduce that overshifting in a one-layer industry occurs if $p'_o + Q_o p''_o > 0$. Multiplying the last inequality by $(M+N+1)p'/p'_o > 0$ and rewriting it yields $(M+N+1)p'(1-E_o) > 0$. Subtracting the left hand side of the last inequality from the

[34]To safe on notation in the following we omit the superscript $\star$ at Q and Q_o. As mentioned above, p and all derivatives of p are evaluated at the equilibrium $Q^\star$. Likewise, p_o and all derivatives of p_o are evaluated at the equilibrium $Q_o^\star$.

left hand side of (23) yields

$$p'\left(E(F-(M+N+3))+E_o(M+N+1)\right).$$

If this expression is negative and there is nevertheless overshifting in the two-layer industry, this must imply that $p'_o + Q_o p''_o > 0$, i.e. overshifting in a one-layer industry. Rearranging $p'\left(E(F-(M+N+3))+E_o(M+N+1)\right) < 0$ yields $EF > E(M+N+3) - E_o(M+N+1)$, which yields the desired result. Reversely, in the same way we can show that for $EF < E(M+N+3) - E_o(M+N+1)$ overshifting of the unit tax in a one-layer market implies overshifting in a two-layer market.

On (ii): From (13) we know that overshifting of the ad valorem tax in a one-layer industry occurs if $p'_o(Q_o p'_o - p_o) - Q_o p_o p''_o > 0$ while from (11) overshifting in a two-layer industry occurs if

$$p'(M+N+1)(Qp'-p) - Qpp''(M+N+3) + Q^2(p'p''-pp''') > 0.$$

Now multiplying the left hand side of the first of these inequalities by $(M+N+1)(p'/p'_o)^2 Q/Q_o > 0$ and subtracting it from the left hand side of the second inequality yields, after rearranging,

$$-p'(M+N+1)p - Qpp''(M+N+3) + Q^2(p'p''-pp''') + (M+N+1)p_o\left(\frac{Q}{Q_o}\frac{(p')^2}{p'_o} + Qp''_o\frac{(p')^2}{(p'_o)^2}\right).$$

If this expression is negative but there is overshifting in the two-layer industry, this implies necessarily overshifting in a one-layer industry. After dividing last expression by $pp' < 0$ and rearranging we find that this is case if $EF > E_t(M+N+3+\epsilon) - \left(1+\epsilon/\epsilon_o(E_o-1)\right)(M+N+1)$. Correspondingly, the reverse holds true if $EF < E(M+N+3+\epsilon) - \left(1+\epsilon/\epsilon_o(E_o-1)\right)(M+N+1)$. ■

Proof of Proposition 3

On (i): If $p(Q) = 1 - Q^b$, the equilibrium price in a two-layer industry is from (4) given by

$$p^\star = 1 - \frac{MN((1-\tau)-t-c)}{(1-\tau)(N+b)(M+b)}.$$

Hence, the equilibrium price in a one layer market p_o is

$$p_o^\star = \lim_{M\to\infty}\left(1 - \frac{MN((1-\tau)-t-c)}{(N+b)(M+b)}\right) = 1 - \frac{N((1-\tau)-t-c)}{(1-\tau)(N+b)}$$

Differentiating both expression with respect to t and substracting the resulting derivative in the two-layer industry from the one in a one-layer industry gives

$$\frac{dp_o^\star}{dt} - \frac{dp^\star}{dt} = \frac{N}{(1-\tau)(N+b)}\left(1 - \frac{M}{M+b}\right) > 0. \tag{24}$$

Therefore, undershifting in a two-layer industry takes place to a larger extent. Conducting the same analysis for the ad valorem tax, we get

$$\frac{dp_o^\star}{d\tau} - \frac{dp^\star}{d\tau} = \frac{N(t+c)}{(1-\tau)(N+b)}\left(1 - \frac{M}{M+b}\right) > 0. \tag{25}$$

On (ii): If $p(Q) = 1/Q^b$, we have

$$p_o^\star = \frac{N(t+c)}{(1-\tau)(N-b)} \quad \text{and} \quad p^\star = \frac{MN(t+c)}{(1-\tau)(N-b)(M-b)}.$$

Consequently,

$$\frac{dp_o^\star}{dt} - \frac{dp^\star}{dt} = \frac{N}{(1-\tau)(N-b)}\left(1 - \frac{M}{M-b}\right) < 0 \tag{26}$$

and

$$\frac{dp_o^\star}{d\tau} - \frac{dp^\star}{d\tau} = \frac{N(t+c)}{(1-\tau)(N-b)}\left(1 - \frac{M}{M-b}\right) < 0. \tag{27}$$

■

Proof of Proposition 4

On (i): From (24) and (25) it is evident that the sign of the derivatives of $dp_o^\star/dt - dp^\star/dt$ and $dp_o^\star/d\tau - dp^\star/d\tau$ with respect to b, N, and M is the same since $(dp_o^\star/dt - dp^\star/dt)\,(t+c) = dp_o^\star/d\tau - dp^\star/d\tau$. Differentiating with respect to b then yields

$$\frac{\partial\left(dp_o^\star/dt - dp^\star/dt\right)}{\partial b} = \frac{N(MN-b^2)}{(1-\tau)(N+b)^2(M+b)^2}.$$

Since $dp_o^\star/dt - dp^\star/dt > 0$, the absolute value of the difference is increasing in b as long as $b < \sqrt{NM}$.

Differentiating with respect to N and M yields

$$\frac{\partial\left(dp_o^\star/dt - dp^\star/dt\right)}{\partial N} = \frac{b^2}{(1-\tau)(N+b)^2(M+b)} > 0$$

and

$$\frac{\partial\left(dp_o^\star/dt - dp^\star/dt\right)}{\partial M} = -\frac{Nb}{(1-\tau)(N+b)(M+b)^2} < 0.$$

On (ii): As above, from (26) and (27) it is evident that $(dp_o^\star/dt - dp^\star/dt)(t+c) = dp_o^\star/d\tau - dp^\star/d\tau$ and therefore the sign of the derivatives of $dp_o^\star/dt - dp^\star/dt$ and $dp_o/d\tau - dp_t/d\tau$ with respect to b, N, and M is the same. Differentiating with respect to b then yields

$$\frac{\partial\left(dp_o^\star/dt - dp^\star/dt\right)}{\partial b} = -\frac{N(NM-b^2)}{(1-\tau)(N-b)^2(M-b)^2}.$$

Since $dp_o^\star/dt - dp^\star/dt < 0$, the absolute value of the difference is increasing in b as long as $b < \sqrt{NM}$. But because $b < \min\{N, M\}$, we know that $b < \sqrt{NM}$ for all admissible values of b.

Differentiating with respect to N and M yields

$$\frac{\partial\left(dp_o^\star/dt - dp^\star/dt\right)}{\partial N} = \frac{b^2}{(1-\tau)(N-b)^2(M-b)} > 0$$

and

$$\frac{\partial\left(dp_o^\star/dt - dp^\star/dt\right)}{\partial M} = \frac{Nb}{(1-\tau)(N-b)(M-b)^2} > 0.$$

Since $dp_o^\star/dt - dp^\star/dt < 0$, the absolute value of the difference is decreasing both in N and M. ■

Proof of Proposition 5

Totally differentiating (14), (15) and (16) and solving the resulting system of equations for $dq^\star/dt$, $dN^\star/dt$ and $dM^\star/dt$ yields

$$\frac{dq^\star}{dt} = -\frac{2QMp''}{(1-\tau)\left[(4NM-2)(p')^2 - Q^2(p'')^2 + p'(p''Q(1+2N+2M)+2Q^2p''')\right]},$$

$$\frac{dN^\star}{dt} = \frac{2M(2p'+Qp'')}{(1-\tau)Q\left[(4NM-2)(p')^2 - Q^2(p'')^2 + p'(p''Q(1+2N+2M)+2Q^2p''')\right]}, \tag{28}$$

and

$$\frac{dM^\star}{dt} = \frac{M^2N\left(-2(1+2N)(p')^2 - p'Q(p''(2N+5)+2Qp''') + Q^2(p'')^2\right)}{(1-\tau)Q} \times \frac{1}{\left[(4NM-2)(p')^2 - Q^2(p'')^2 + p'(p''Q(1+2N+2M)+2Q^2p''')\right]}. \tag{29}$$

From this we can calculate $dp^\star/dt$ to get

$$\frac{dp^\star}{dt} = p'\frac{dQ^\star}{dt} = p'\left(\frac{dq^\star}{dt}N + \frac{dN^\star}{dt}q\right)$$

$$= \frac{4NM(p')^2}{(1-\tau)\left[(4NM-2)(p')^2 - Q^2(p'')^2 + p'(p''Q(1+2N+2M)+2Q^2p''')\right]} > 0 \tag{30}$$

which, after using that $Q = Nq$ and inserting the respective expressions for E and F, simplifies to

$$\frac{dp^\star}{dt} = \frac{4NM}{(1-\tau)\left(4NM - 2 - E^2 - E(1+2N+2M) + 2EF\right)} > 0. \tag{31}$$

Since overshifting occurs if $(dp^\star/dt)(1-\tau) > 1$, (18) follows.

The expression for overshifting of the ad valorem tax can be calculated in exactly the same way to get

$$\frac{dq^\star}{d\tau} = -\frac{Q\left((1-2NM)(p')^2 - p'p'''Q^2 + 2pp''NM + (p'')^2Q^2\right)}{(1-\tau)N\left[(4NM-2)(p')^2 - Q^2(p'')^2 + p'(p''Q(1+2N+2M) + 2Q^2p''')\right]},$$

$$\frac{dN^\star}{d\tau} = \frac{N\left(2Q(1+M+N-MN)(p')^2 + p'(p''Q^2 + 4pM - p'''Q^3) + (p'')^2Q^3 + 2QMpp''\right)}{(1-\tau)Q\left[(4NM-2)(p')^2 - Q^2(p'')^2 + p'(p''Q(1+2N+2M) + 2Q^2p''')\right]} \tag{32}$$

and

$$\frac{dM^\star}{d\tau} = \frac{\phi}{(1-\tau)} \times \frac{1}{\left[(4NM-2)(p')^2 - Q^2(p'')^2 + p'(p''Q(1+2N+2M) + 2Q^2p''')\right]} \tag{33}$$

with

$$\begin{aligned}
\phi \equiv\ & (p')^3(1+2N)(MN - M - N - 1) \\
& +(p')^2\left(p''(MN - 2M - 3N - 2M)Q - p'''Q^2M - 2pM(1+2N)\right) \\
& -p'\left(-(p'')^2Q^2(M-1) + p''pMN(2N+5) + 2p'''pQNM\right) + p''QNpM.
\end{aligned}$$

Using these expressions we obtain

$$\frac{dp^\star}{d\tau} = \frac{(p')^2\left(p'Q(1+2N+2M) + 4pM + p''Q^2\right)}{(1-\tau)\left[(4NM-2)(p')^2 - Q^2(p'')^2 + p'(p''Q(1+2N+2M) + 2Q^2p''')\right]} > 0. \tag{34}$$

Since overshifting occurs if $(dp^\star/d\tau)(1-\tau) > p^\star$, we obtain, after rearranging,

$$\frac{(p')^2\left(4NMp + Qp'(2M+2N+1) + p''Q^2\right)}{p\left[(4NM-2)(p')^2 - Q^2(p'')^2 + p'(p''Q(1+2N+2M) + 2Q^2p''')\right]} > 1$$

which yields (19).

Finally, comparing the left hand side of (18) with the left hand side of (19) yields that the latter is smaller than the former if $\epsilon\left(-(1+2N+2M) + E\right) < 0$ which is fulfilled since $\epsilon > 0$ and $p'(1+2N+2M) + Qp'' < 0$ by Assumption 1. This shows that overshifting of the ad valorem tax implies overshifting of the specific tax. ■

Proof of Proposition 6

On (i): Rewriting (18) yields that overshifting in a two-layer industry occurs if

$$2(p')^2 + Q^2(p'')^2 - p'\left(p''Q(1+2N+2M) + 2Q^2p'''\right) > 0. \tag{35}$$

From (20) overshifting in a one-layer industry occurs if $p''_o > 0$ which, after multiplying by $-p'Qp''/p''_o(1+2N+2M) > 0$, can be written as $-p'Qp''(1+2N+2M) > 0$. Now subtracting the left hand side of the last inequality from the left hand side of (35) yields after rearranging $2(p')^2 + Q^2(p'')^2 - 2Q^2p'p'''$. If this expression is smaller than zero but there is still overshifting in the two-layer industry, this necessarily implies that there is over shifting in the one-layer industry. Rearranging the expression $2(p')^2+Q^2(p'')^2-2Q^2p'p''' < 0$ yields that this is the case if $EF > 1/2(2+E^2)$. Proceeding in the same way to identify the condition under which overshifting in a one-layer market implies overshifting in a two layer market gives $EF < 1/2(2+E^2)$.

On (ii): From (21) overshifting occurs in a one-layer industry if $p''_o p_o - (p'_o)^2 > 0$ while from (19) overshifting occurs in a two-layer industry if

$$((p')^3 - pp'p'')Q(1+2N+2M) + (p')^2(2p+Q^2p'') - 2pp'p'''Q^2 + p(p'')^2Q^2 > 0. \tag{36}$$

Multiplying the first inequality by $-(1+2N+2M)Q(p')^3/(p'_o)^2 > 0$ yields

$$-(1+2N+2M)Q\frac{(p')^2}{(p'_o)^2}p'p''_o p_o + (1+2N+2M)Q(p')^3 > 0.$$

Now subtracting the left hand side of the last inequality from the left hand side of (36), dividing by $(pp')^2 > 0$ and rearranging gives

$$\left[E(1+2N+2M+E+\epsilon) + 2 - E_o\frac{\epsilon}{\epsilon_o}(1+2N+2M)\right] - 2EF.$$

If this expression is negative, overshifting of the ad valorem tax in a two-layer industry implies overshifting in a one-layer industry. Rearranging yields that this is the case if

$$EF > \frac{1}{2}\left[E(1+2N+2M+E+\epsilon) + 2 - E_o\frac{\epsilon}{\epsilon_o}(1+2N+2M)\right].$$

Correspondingly the reverse holds true if the inequality is reversed. ■

Proof of Proposition 7

On (i): We start with the specific tax. Inserting the demand function $p(Q) = 1 - Q^b$ into (18) and (20), one can easily show that there exists for both the one- and the two-layer industry, respectively, a unique threshold of b below which overshifting occurs. From the last Proposition we know that if $EF < 1/2(2+E^2)$, then overshifting in a one-layer industry implies overshifting in a two-layer industry. With $p(Q) = 1 - Q^b$ this is equivalent to the statement that if we have $EF < 1/2(2 + E^2)$ at the threshold for the one-layer industry, then overshifting in a two-layer industry occurs for a larger set of b than in a one-layer industry. Reformulating this condition to

$$\frac{Q^2 p'''}{p'} < \frac{1}{2}\left(2 + \left(\frac{Qp''}{p'}\right)^2\right)$$

and inserting the specific demand function yields $(b-1)(b-2) < 1 + (b-1)^2/2$ or $b^2 - 4b + 2 < 0$. From (20), we know that overshifting in a one-layer industry occurs if $p_o'' > 0$. Thus, the threshold of b in a one-layer industry is $b = 1$. But at $b = 1$ the inequality $b^2 - 4b + 2 < 0$ is fulfilled. This implies that overshifting in a two-layer industry occurs also for values of $b > 1$ which proves the result.

Now let us turn to the ad valorem tax. As before, one can easily show from (19) and (21) that in case of $p(Q) = 1 - Q^b$ there exists a unique threshold of b below which overshifting occurs both in a one- and in a two-layer industry, respectively. The last Proposition showed that for

$$EF < \frac{1}{2}\left[E(1 + 2N + 2M + E + \epsilon) + 2 - E_o\frac{\epsilon}{\epsilon_o}(1 + 2N + 2M)\right]$$

overshifting in a one-layer industry implies overshifting in a two-layer industry. Rewriting this condition yields

$$(p')^2\left(2 + Q^2\frac{p''}{p}\right) - p'p''Q(1+2N+2M) - 2p'p'''Q^2 + (p'')^2Q^2 + Q(1+2N+2M)\frac{p'}{p}\frac{(p')^2}{(p_o')^2}p_o''p_o > 0.$$

Inserting the respective demand function and rearranging yields that the sign of the left hand side of the last inequality is given by

$$\operatorname{sign}\left\{Q_o^b\left[(1-3b)Q^b + 2(1-b)(N+M) + b(3-b)\right] - Q^b(1-b)(1+2N+2M)\right\}.$$

Rearranging this yields

$$\operatorname{sign}\left\{Q_o^b\left[(1-3b)Q^b + b(3-b)\right] - Q^b(1-b) + 2(Q_o^b - Q^b)(1-b)(2N+2M)\right\}.$$

We know that $Q_o^b > Q^b$. Moreover, overshifting in a one-layer industry occurs only if $E_o > \epsilon_o$ which implies for the respective demand function that $b < 1 - Q_o^b$. Therefore, $b < 1$ in the relevant parameter range and so the last term of the above expression is positive. Since we want to show that overshifting occurs for a larger range of b in a two-layer industry than in a one-layer industry, this is equivalent to show that the sign of the last expression is positive at $b = 1 - Q_o^b$. Inserting $b = 1 - Q_o^b$ into $Q_o^b \left[(1-3b)Q^b + b(3-b)\right] - Q^b(1-b)$ and rearranging yields $Q_o^b(1-Q_o^b)(2+Q_o^b-3Q^b)$. But since $Q_o^b < 1$ and $Q_o^b > Q^b$, this term is strictly positive, which proves the result.

On (ii): With this demand function we cannot use the results obtained in the last Proposition because there is overshifting both in a one-layer and in a two-layer industry. We therefore continue by directly comparing the price changes induced by a tax change. Again, we start with the specific tax. Inserting the demand function and its derivatives into $dp^\star/dt$ and $dp_o^\star/dt$ yields

$$\frac{dp^\star}{dt} = \frac{4NM}{(1-\tau)(4NM - 2(1+b)(N+M) + b(3+b))} \quad \text{and} \quad \frac{dp_o^\star}{dt} = \frac{2N_o}{(1-\tau)(2N_o - 1 - b)}.$$

Subtracting the latter from the former and rearranging gives

$$\text{sign}\left\{\frac{dp^\star}{dt} - \frac{dp_o^\star}{dt}\right\} = \text{sign}\left\{2\left(N_o(N+M) - NM - N_o b\right) + b\left(2N_o(N+M) - 2NM - N_o b\right)\right\}.$$

To show that the last expression is positive, we can use the following fact: We know that the aggregate quantity in a one-layer industry is larger than in a two-layer industry since it is strictly increasing in the number of upstream firms. This implies that $Q_o^b > Q^b$ or, from (5), that $(N_o - b)/N_o > (N-b)(M-b)/(NM)$ and therefore

$$N_o(N+M) - NM - bN_o > 0.$$

But this obviously implies that the sign $\{dp^\star/dt - dp_o^\star/dt\}$ is positive.

Proceeding in the same way for the ad valorem tax yields

$$\text{sign}\left\{\frac{dp^\star}{d\tau} - \frac{dp_o^\star}{d\tau}\right\} =$$

$$= \text{sign}\left\{Q_o^b(2N-b)(2M-b)(2N_o-1-b) - Q^b(2N_o-b)(4NM - 2(1+b)(N+M) + b(3+b)\right\}.$$

The multiplier of Q_o^b is positive while the multiplier of Q^b is negative. We also know that $Q_o^b > Q^b$. Thus, via setting $Q_o^b = Q^b$ we underestimate the value of the whole expression.

Nevertheless doing so and simplifying yields that the sign of the expression is given by the sign of

$$2\left(N_o(N+M)-NM-bN_o\right)+b^2.$$

This is strictly positive because the fact that $Q_o^b > Q^b$ implies that $N_o(N+M)-NM-bN_o > 0$, as demonstrated above. ■

Proof of Proposition 8

On (i): With the demand function $p(Q)=1-Q$ the zero profit conditions can be written as

$$\frac{M^2(1-\tau-t-c)^2}{(1-\tau)(N+1)^2(M+1)^2}=F_d \quad \text{and} \quad \frac{N(N+1)(1-\tau-t-c)^2}{(1-\tau)(N+1)^2(M+1)^2}=F_u.$$

Totally differentiating both equations and setting $dF_u=0$ yields

$$\frac{dM}{dF_d}=-\frac{(M+1)^3(N+1)^2(1-\tau)}{2\psi} \quad \text{and} \quad \frac{dN}{dF_d}=-\frac{N(M+1)^2(N+1)^3(1-\tau)}{\psi}, \tag{37}$$

while totally differentiating and setting $dF_d=0$ yields

$$\frac{dM}{dF_u}=-\frac{M(M+1)^3(N+1)(1-\tau)}{\psi} \quad \text{and} \quad \frac{dN}{dF_u}=-\frac{(M+1)^2(N+1)^2(1-\tau)}{\psi}, \tag{38}$$

with $\psi=(1-\tau-t-c)^2M(2NM-1)$.

Now inserting the demand function and its respective derivatives in $dp^\star/dt$ given by (31) gives

$$\frac{dp^\star}{dt}=\frac{2NM}{(1-\tau)(2NM-1)}.$$

Differentiating this with respect to N and M and using (37) and (38) yields

$$\frac{d\left(\frac{dp^\star}{dt}\right)}{dF_d}=\frac{(M+1)^2(N+1)^2N(2NM+3M+1)}{(1-\tau-t-c)^2(2NM-1)^3M}>0 \tag{39}$$

and

$$\frac{d\left(\frac{dp^\star}{dt}\right)}{dF_u}=\frac{2(M+1)^2(N+1)M(NM+2N+1)}{(1-\tau-t-c)^2(2NM-1)^3}>0. \tag{40}$$

In a one-layer industry $dp_o^\star/dt$ is given by

$$\frac{2N_op_o'}{(1-\tau)(2N_op_o'+Q_op_o'')}.$$

Inserting the respective derivatives of the demand function simplifies the expression to $dp_o^\star/dt=1/(1-\tau)$. Thus, it does not change with F_d. As a consequence, from (39) and

(40), we know that $dp^\star/dt - dp_o^\star/dt$ is increasing in F_d and F_u. It remains to show that the absolute value of this difference is increasing in F_d and F_u. We thus have to show that the difference always takes positive values, $dp^\star/dt - dp_o^\star/dt > 0$. To do so, remember from (20) that overshifting in a one-layer market occurs if $b \leq 1$ and from Proposition 7 that overshifting in a two-layer market occurs for a larger range of b than in a one-layer market. Since in this case $b = 1$, it is indeed the case that $dp^\star/dt > dp_o^\star/dt$.

The proof for the ad valorem tax proceeds exactly in the same way and is therefore omitted.

On (ii): With the function $p(Q) = 1/Q$ we can explicitly solve for N and M. From (15) and (16) we get

$$N = \sqrt{\frac{1-\tau}{F_d}} \quad \text{and} \quad M = \sqrt{\frac{1-\tau-\sqrt{F_d(1-\tau)}}{F_u}}. \tag{41}$$

Since $N \geq 1$ and $M \geq 1$, we must have that $F_d < 1-\tau$ and $F_u < 1-\tau_d - \sqrt{F_d(1-\tau_d)}$.

First, inserting the respective derivatives of the function into $dp^\star/dt$ and simplifying yields

$$\frac{dp^\star}{dt} = \frac{NM}{(1-\tau)(N-1)(M-1)}.$$

Then, inserting the respective values for N and M gives

$$\frac{dp^\star}{dt} = \frac{\sqrt{(1-\tau)F_d}\sqrt{F_u(1-\tau-\sqrt{(1-\tau)F_d})}}{(1-\tau)\left(F_u - \sqrt{F_u(1-\tau-\sqrt{(1-\tau)F_d})}\right)\left(F_d - \sqrt{(1-\tau)F_d}\right)}.$$

Following the above steps for a one-layer industry gives that $N_o = \sqrt{(1-\tau)/F_d}$ and $dp_o^\star/dt = N_o/((1-\tau)(N_o-1))$. Therefore,

$$\frac{dp_o^\star}{dt} = \frac{\sqrt{(1-\tau)F_d}}{(1-\tau)(F_d - \sqrt{(1-\tau)F_d})}.$$

Thus, we get that

$$\frac{dp^\star}{dt} - \frac{dp_o^\star}{dt} = \frac{\sqrt{(1-\tau)F_d}F_u}{(1-\tau)\left(F_u - \sqrt{F_u(1-\tau-\sqrt{(1-\tau)F_d})}\right)\left(F_d - \sqrt{(1-\tau)F_d}\right)}.$$

Differentiating this with respect to F_d yields

$$\frac{F_dF_u^2\left(3(1-\tau) - \sqrt{(1-\tau)F_d} - \sqrt{F_u(1-\tau-\sqrt{1-\tau F_d})}\right)}{4\sqrt{F_u(1-\tau-\sqrt{(1-\tau)F_d})}\left(F_d - \sqrt{(1-\tau)F_d}\right)^2\sqrt{(1-\tau)F_d}\left(F_u - \sqrt{F_u(1-\tau-\sqrt{(1-\tau)F_d})}\right)^2}.$$

One can easily check that this expression is strictly positive since $F_d < 1-\tau$ and $F_u < 1-\tau-\sqrt{F_d(1-\tau)}$.

Differentiating $dp^\star/dt - dp_o^\star/dt$ with respect to F_u yields

$$\frac{F_u\left(\sqrt{(1-\tau)F_d}(1-\tau-\sqrt{(1-\tau)F_d})\right)}{2\sqrt{F_u(1-\tau-\sqrt{(1-\tau)F_d})}\left(F_d-\sqrt{(1-\tau)F_d}\right)\left(F_u-\sqrt{F_u(1-\tau-\sqrt{(1-\tau)F_d})}\right)^2},$$

which is also strictly positive because of the two restrictions above.

Again, the proof for the ad valorem tax proceeds in the same way and is therefore omitted. ■

Proof of Proposition 9

Since profits are zero in the long run, welfare W can be written as the sum of consumer surplus and tax revenue, i.e.

$$W = \int_0^Q p(s)ds - pQ + tQ + \tau pQ.$$

Differentiating with respect to t yields

$$\frac{\partial W}{\partial t} = p\frac{\partial Q}{\partial t} - p'\frac{\partial Q}{\partial t}Q - p\frac{\partial Q}{\partial t} + Q + t\frac{\partial Q}{\partial t} + \tau p'\frac{\partial Q}{\partial t}Q + \tau p\frac{\partial Q}{\partial t}.$$

Evaluating this derivative at $t = \tau = 0$ gives

$$\frac{\partial W}{\partial t} = Q\left(1 - \frac{\partial p^\star}{\partial t}\right).$$

Thus, it is evident that welfare increases in t at $t = \tau = 0$ exactly in the case of no overshifting.

In the same way we can calculate the derivative with respect to τ to get

$$\frac{\partial W}{\partial \tau} = Q\left(p^\star - \frac{\partial p^\star}{\partial \tau}\right).$$

So again welfare increases in τ at $t = \tau = 0$ exactly in the case of no overshifting. This holds for both the one- and the two-layer industry.

The statement in the proposition then simply follows by combining the results of Propositions 5 and 6. ■

Proof of Proposition 10

Calculating $dp^\star/d\tau$ for the case of upstream taxation in the same way as in the Proof of Proposition 4 but now using (14), (16) and (17) yields

$$\frac{dp^\star}{d\tau} = 2\frac{(p')^2\left(p'Q(1+N+2M)+2pM+p''Q^2\right)}{(1-\tau)\left[(4NM-2)(p')^2-Q^2(p'')^2+p'(p''Q(1+2N+2M)+2Q^2p''')\right]}. \quad (42)$$

At $\tau = 0$, the equilibrium number of firms is the same independent at which layer taxation is introduced. Subtracting (42) from (34), i.e. $dp^\star/d\tau$ in case of downstream taxation, we obtain

$$\lim_{\tau\to 0} -\frac{Q(p')^2\left(p'(1+2M)+p''Q\right)}{\left[(4NM-2)(p')^2+Q^2(p'')^2-p'p''Q(1+2N+2M)+2Q^2p'''\right]}, \quad (43)$$

which is the difference in price changes between downstream and upstream taxation. Assumption 2 implies $p'(M+1)+p''Q<0$ and so the numerator of the fraction in (43) is negative. The denominator is the same as the one in (34) and since $dp^\star/d\tau>0$ it is positive. Therefore, (43) is positive which yields the stated result. ■

Proof of Proposition 11

We start with the function $p(Q) = 1-Q$. Inserting the respective derivatives of this demand function into (43) yields that the difference between a downstream and an upstream ad valorem tax in a two-layer industry is $Q(2M+1)/2(2MN-1)$. Similarly, in a one-layer market, where $M_o\to\infty$, we get that this difference is given by $Q_o/2N_o$. Subtracting the latter from the former and plugging in the equilibrium values for Q and Q_o yields that the sign of the resulting expression is determined by the sign of

$$1+2M^2(N_o-N)+M(N_o+1-2N). \quad (44)$$

Now using the zero profit condition for the downstream market in a one- and a two-layer industry we get that N_o and N are given by

$$N_o = \frac{(1-t-c)}{\sqrt{F_d}}-1 \quad \text{and} \quad N = \frac{M(1-t-c)}{\sqrt{F_d}(M+1)}-1.$$

Inserting this into (44) and simplifying gives $1+2M+M(1-t-c)/\sqrt{F_d}>0$. Thus, the difference is positive.

Now we turn to the function $p(Q) = 1/Q$. Again inserting the respective derivatives of this demand function together with the expressions for N and M given by (41) into (43) yields

$$\frac{\sqrt{F_d}\left(2\sqrt{1-\sqrt{F_d}}-\sqrt{F_u}\right)}{4Q(1-\sqrt{F_d})\left(\sqrt{1-\sqrt{F_d}}-\sqrt{F_u}\right)}, \tag{45}$$

which is the difference between a downstream and an upstream ad valorem tax in a two-layer industry. For a one-layer industry we get, by first letting $M_o \to \infty$ and then repeating the same steps as above, that the difference is given by

$$\frac{\sqrt{F_d}}{2Q_o(1-\sqrt{F_d})}. \tag{46}$$

We can now analyze the difference between (45) and (46). To do so we first simplify this difference by setting $Q_o = Q$. Since we know that in fact $Q_o > Q$, by doing so we only underestimate the value of the difference. Using $Q_o = Q$, we get that this difference is at least given by

$$\frac{\sqrt{F_d}\sqrt{F_u}}{4Q\left(\sqrt{1-\sqrt{F_d}}-\sqrt{F_u}\right)(1-\sqrt{F_d})} > 0,$$

where the inequality follows from the restrictions on F_d and F_u stated in the proof of Proposition 8. Therefore, the difference is again positive. ■

Proof of Proposition 12

We start with downstream taxation and denote the tax rate there by τ_d. We know from the proof of Proposition 8 that

$$N = \sqrt{\frac{1-\tau_d}{F_d}} \quad \text{and} \quad M = \sqrt{\frac{1-\tau_d-\sqrt{F_d(1-\tau_d)}}{F_u}}.$$

It follows that aggregate quantity can then be written as

$$Q = \frac{(1-\tau_d)(F_d-\sqrt{F_d(1-\tau_d)})(F_u-\sqrt{F_u(1-\tau_d-\sqrt{F_d(1-\tau_d)})})}{\sqrt{1-\tau_d-\sqrt{F_d(1-\tau_d)}}\sqrt{F_d(1-\tau_d)}(t+c)}.$$

Proceeding in the same way for upstream taxation at a rate of τ_u yields an aggregate quantity of

$$Q = \frac{(1-\tau_u)(1-\sqrt{F_d})(F_u-\sqrt{F_u(1-\tau_u)(1-\sqrt{F_d})})}{\sqrt{F_u(1-\tau_u)(1-\sqrt{F_d}}(t+c)}.$$

Now setting the tax rates in such a way that aggregate quantity is equal under both regimes gives

$$\tau_u = (1-F_d)^{-1}\times$$

$$\times\left(\sqrt{F_d\,(1-\tau_d)}+\tau_d-1/2F_u-1/2\sqrt{\xi}+\sqrt{F_u\left(1-\tau_d-\sqrt{F_d\,(1-\tau_d)}\right)}-F_d+\right. \tag{47}$$

$$\left.+\sqrt{F_d\,(1-\tau_d)}\sqrt{F_d}+\tau_d\sqrt{F_d}-\left(1/2F_u+1/2\sqrt{\xi}\right)\sqrt{F_d}+\sqrt{F_u\left(1-\tau_d-\sqrt{F_d\,(1-\tau_d)}\right)}\sqrt{F_d}-\sqrt{F_d}\right),$$

with $\xi = F_u^2 - 4F_u\tau_d - 4\sqrt{F_d(1-\tau_d)}F_u + 4F_u - 4\sqrt{F_u(1-\tau_d-\sqrt{F_d(1-\tau_d)})}F_u$.

Since these tax rates imply that aggregate quantity is the same under both regimes, any differences in welfare in both regimes can only stem from differences in the tax revenue and from differences in the entry costs. Calculating $\tau p(Q)Q - F_dN - F_uM$ for the case of downstream taxation yields

$$\tau_d - \sqrt{F_u(1-\tau_d-\sqrt{F_d(1-\tau_d)})} - \sqrt{F_d(1-\tau_d)}. \tag{48}$$

Repeating this for the case of upstream taxation we get

$$\tau r(Q)Q - F_dN - F_uM = \tau(p(Q)+p(Q)'Q/N)Q - F_dN - F_uM$$

$$= \tau_u - (1+\tau_u)\sqrt{F_d} - \sqrt{F_u(1-\tau_u)(1-\sqrt{F_d})}.$$

Now inserting τ_u from (47) into the last equation and comparing it with (48) shows that the resulting welfare under downstream taxation is always higher than under upstream taxation. ■

6.2 Appendix B

Proposition *If the specific tax is restricted to be non-negative, then it is optimal both in the short-run and in the long-run to raise a given tax revenue only via the ad valorem tax and set the specific tax equal to zero.*

Proof. We start with the short run. Consider a tax pair $\{t_a, \tau_a\}$, with $t_a > 0$ and $\tau_a < 1$. With this tax pair the overall quantity is implicitly given by

$$(1-\tau_a)\left[p + \frac{p'Q(M+N+1)}{MN} + \frac{p''Q^2}{MN}\right] - c - t_a = 0.$$

Now consider another pair $\{t_b, \tau_b\}$, with $t_b = 0$. This new pair gives the same aggregate quantity as the old one if

$$(1-\tau_a)\xi - c - t_a = (1-\tau_b)\xi - c$$

or, equivalently,

$$\tau_b = \frac{\xi \tau_a + t_a}{\xi},$$

where $\xi = p + \left[p' Q (M + N + 1) + p'' Q^2 \right] / MN > 0$. But the new pair $\{t_b, \tau_b\}$ gives a strictly higher tax revenue than the old pair since

$$Qp\tau_a + Qt_a - \tau_b Q = Q \left(p \left(\tau_a - \frac{\xi \tau_a + t_a}{\xi} \right) + t_a \right) = Qt_a \left(\frac{\xi - p}{\xi} \right) < 0, \qquad (49)$$

where the last inequality follows from the fact that

$$\frac{p' Q (M + N + 1)}{MN} + \frac{p'' Q^2}{MN} < 0$$

and therefore $\xi < p$.

Now we turn to the long run. Consider a shift from the ad valorem tax to the specific tax such that aggregate quantity and therefore the price stay constant,

$$\frac{dp^\star}{dt} - \alpha \frac{dp^\star}{d\tau} = 0.$$

Using (30) and (34) to calculate α we get

$$\alpha = \frac{4NM}{p' Q (2M + 2N + 1) + 4pNM + p'' Q^2}.$$

Since aggregate quantity does not change, consumer surplus is unchanged, and the change in welfare consists of the change in the tax revenue and in the fixed entry costs, i.e.

$$dW = \frac{dT}{dt} - \alpha \frac{dT}{d\tau} - F_d \left(\frac{dN}{dt} - \alpha \frac{dN}{d\tau} \right) - F_u \left(\frac{dM}{dt} - \alpha \frac{dM}{d\tau} \right).$$

Since $T = Q(\tau p + t)$ and p and q do not change, we get that

$$\frac{dT}{dt} - \alpha \frac{dT}{d\tau} = Q(1 - \alpha p) = Q \left(\frac{p' Q (2M + 2N + 1) + p'' Q^2}{p' Q (2M + 2N + 1) + 4pNM + p'' Q^2} \right).$$

From equations (28), (29), (32) and (33) of the Proof of Proposition 5 we get

$$\frac{dN}{dt} - \alpha \frac{dN}{d\tau} = \frac{2N^2 M}{\left(p' Q (2M + 2N + 1) + 4pNM + p'' Q^2 \right) (1 - \tau)} > 0$$

and

$$\frac{dM}{dt} - \alpha \frac{dM}{d\tau} = \frac{NM^2 \left(p'(1 + 2N) + Qp'' \right)}{\left(p'(N + 1) + Qp'' \right) \left(p' Q (2M + 2N + 1) + 4pNM + p'' Q^2 \right) (1 - \tau)} > 0.$$

Inserting the last three equations into dW, eliminating F_d and F_u by using equations (15) and (16), and solving the resulting expression yields

$$dW = \frac{2Q(p'Q(2M + 2N + 1) + QNp'')}{p'Q(2M + 2N + 1) + 4pNM + p''Q^2} < 0.$$

■

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