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Complementary Patents and Market Structure

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ABSTRACT: Many high technology goods are based on standards that require access to several patents that are owned by different IP holders. We investigate the royalties chosen by IP holders under different market structures. Vertical integration of an IP holder and a downstream producer solves the double mark-up problem between these firms. Nevertheless, it may raise royalty rates and reduce output as compared to non-integration. Horizontal integration of IP holders (or a patent pool) solves the complements problem but not the double mark-up problem. Vertical integration discourages entry and reduces innovation incentives, while horizontal integration always encourages entry and innovation.

Keywords: IP rights, complementary patents, standards, licensing, patent pool, vertical integration.

JEL Classification Codes: L15, O31, L24, O32, K11.

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1. Introduction

Many high technology products are based on technological standards that require the use of multiple essential patents owned by different IP holders. By definition an essential patent is strictly necessary for the standard, either because it is legally impossible or prohibitively expensive to do without it.[1] Thus, if a downstream firm wants to produce goods that are based on the standard it requires access to each of the essential patents. All the patents are perfect complements. Therefore each of the upstream IP holders has monopoly power over the downstream market. This "patent thicket" (Shapiro, 2001) gives rise to a *complements problem*: each patent holder does not internalize the negative external effect on the revenues of the other patent holders when setting his royalties, so the sum of all royalties will be inefficiently high. In addition, there is a *vertical double marginalization problem* if firms on the downstream market have market power. These externalities affect not only the prices charged downstream, they also affect the incentives to enter the downstream market with new product varieties and to develop new technology that improves the quality of the standard.

Firms have used different strategies to deal with these externalities. Many standard setting organisations require their members to charge "reasonable and non-discriminatory" (RAND) royalties.[2] There seems to be a consensus that RAND commitments prevent outright refusal to license and exclusive licensing, but any additional constraints implied by RAND, in particular concerning royalties, are controversial. As Swanson and Baumol (2005) point out: „*It is widely acknowledged that, in fact, there are no generally agreed tests to determine whether a particular license does or does not satisfy a RAND commitment*". Thus, a reference to RAND hardly restricts the pricing policies of patent holders.

In some industries patent holders horizontally integrated, either by merging or by forming a patent pool that licenses all patents as a package at a single royalty. Patent pools have been perceived as a device for collusion by anti-trust authorities for many decades. This perception has changed within the last decade when the U.S. Department of Justice approved the MPEG-2 patent pool in 1997 and two DVD patent pools shortly thereafter. However, the

[1] A patent is "legally essential" for a standard if the standard cannot be implemented without infringing the patent. It is "commercially essential" if it is prohibitively expensive to implement the standard without the patent, even if this is technologically feasible. See Layne-Farrar and Lerner (2008, p. 9). In reality it is not always obvious whether a patent is essential or not. Patent holders have a strong incentive to overstate the importance of their IP rights. Furthermore, it is often unclear whether a patent will survive if it is challenged in court. For a more detailed discussion of these problems see Lemley and Shapiro (2007) and Dewatripont and Legros (2008). In this paper we do not consider these problems and assume that it is common knowledge which patents are in fact essential.

[2] In Europe, most SSOs require royalties to be "fair" in addition.

patent pool has to consist only of "blocking" patents, i.e. that all patents are perfect complements that are essential to the standard.[3]

In other industries firms vertically integrated. For example, in the mobile phone industry some firms such as Nokia or Sony Ericson not only own essential patents to the WCDMA standard, they also produce handsets on the downstream market. However, on the same market there are also firms that own essential patents without producing handsets (e.g. Qualcomm), and firms who produce handsets but do not own essential patents (e.g. Panasonic). Similarly, most of the DVD patent holders (Phillips, Sony, Toshiba, etc.) also produce DVD players and DVDs.

In this paper we discuss the effects of different market structures on upstream royalties, downstream prices, entry decisions and incentives to innovate. Our model of the downstream market is very general and allows for all kinds of downstream market interaction (competition in prices, quantities, product differentiation, advertising, etc.) as long as a weak regularity condition is satisfied. As a base line we consider a market structure in which upstream and downstream firms are non-integrated and where linear royalties have to be used upstream. Then we ask how the market outcome changes if some (or all) upstream firms vertically integrate with some downstream firms. It turns out that even though vertical integration partially solves the vertical double mark-up problem it may result in higher royalties and less production on the downstream market than non-integration. This is due to the fact that a vertically integrated firm has an incentive to raise its royalty rate in order to raise its rivals' cost. In contrast, horizontal integration of upstream firms (either by merging or by forming a patent pool) is always beneficial. Furthermore, if the number of downstream firms is sufficiently large, horizontal integration outperforms vertical integration.

We also consider the use of two-part tariffs. It is well known that two-part tariffs can be used to solve the double mark-up problem in a vertical relationship of two firms that both have market power. We show that it can also be used to solve the complements problem (together with the double mark-up problem) under all market structures. This is particularly simple if all upstream firms are horizontally integrated. If firms are non-integrated there exists a symmetric pure strategy equilibrium in which all upstream firms charge two-part tariffs that solve the complements and the double mark-up problem. If firms are vertically integrated, however, this equilibrium fails to exist if there are sufficiently many firms. In this case

[3] The Department of Justice and the Federal Trade Commission have softened this stance in their joint report on antitrust and IP issued April 2007 (http://www.ftc.gov/opa/2007/04/ipreport.shtm). Now they acknowledge that including substitute patents need not be anti-competitive. Patent pools will be reviewed according to the rule of reason in the future. See Layne-Farrar and Lerner (2008) and Gilbert (2002) for more on the history of patent pools and the shift of US policy.

efficiency can be achieved in an asymmetric equilibrium with the awkward property that one firm monopolizes the downstream market but makes zero profits, while all the other vertically integrated firms do not produce downstream but extract all the monopoly profits from the producing firm with their fixed fees. Thus, it seems far less likely that firms manage to coordinate on the efficient equilibrium under vertical integration than under horizontal integration.

Perhaps even more important than the effects of market structure on prices are the effects on entry and innovation. We show that vertically integrated firms have an incentive to discriminate against entrants on the downstream market in order to raise their rival's cost which is not the case for a horizontally integrated or a non-integrated upstream firm. Even if a firm enters with an entirely new product that requires the standard but does not compete against the other goods on the downstream market, a horizontally integrated firm will charge lower royalties and induce more entry than vertically integrated and non-integrated firms because the latter firms cannot coordinate their royalties upstream.

Finally we consider the incentives of an upstream firm to innovate and invest in an improvement of the standard. This improvement may reduce downstream production costs, it may make the products based on the standard more valuable to consumers, or it may open the door to new applications. No matter what the benefits of the innovation are, the incentives to innovate are smaller the more firms there are on the upstream market. The reason is that the innovator requires access to all the other patents in the standard. The more IP holders there are, the smaller are the profits that can be generated with any given innovation and the more reluctant the incumbent IP holders are to include an additional essential patent in the standard. Thus, horizontal integration on the upstream market is an important instrument to stimulate innovation. This is an additional argument in favor of the current shift in US competition policy to permit patent pools for complementary patents.

Our paper is closely related to the growing literature on patent pools and complementary patents. Shapiro (2001) discusses the case of patents that are perfect complements and argues that patent pools and cross licensing agreements can be a solution to the complements problem. Lerner and Tirole (2004) argue that it is often not obvious whether patents are complements or substitutes. They show that patent pools that are based on complementary patents are welfare increasing, while patents that include substitutes reduce competition and welfare. Furthermore, if patents are complements, patent pools will allow for independent licensing, while patent pools that include substitutes will not do so. This is confirmed empirically by Lerner, Stojwas and Tirole (2006). They propose independent licensing as a screening device to be used by anti-trust authorities to distinguish between welfare increasing and welfare reducing patent pools. Aoki and Nagaoka (2004) consider the free rider problem that arises in the process of the formation of a patent pool. Each upstream firm benefits if other firms join the pool and reduce their royalties, but it may be profitable for each firm to stay out. None of these papers considers the effects of vertical integration nor do they analyse the effects on entry and innovation.

Layne-Farrar and Lerner (2008) provide an empirical investigation of the different sharing rules employed in modern patent pools and the factors that affect the decision of an IP holder to join a patent pool. They find that vertically integrated firms are more likely to join a pool and that IP holders with more valuable patents are less likely to join if the pool shares profits proportional to the number of essential patents.

Layne-Farrar, Padilla and Schmalensee (2006) discuss potential methods for assessing whether licensing terms are “fair, reasonable and non-discriminatory” (FRAND). They argue that patents that make a greater contribution to the value of the standard should be allowed to charge higher royalties. Gilbert and Katz (2007) analyze different sharing rules in patent pools and their impact on the incentives to develop new technology. In our model, all upstream firms are symmetric, so the sharing rule is trivially the equal split.

Our paper is also related to the literature on raising rivals’ costs strategies by vertically integrated firms. Salop and Sheffman (1983, 1987) consider a dominant firm that can affect marginal and average costs of a competitive fringe. They show that the dominant firm will raise its rivals’ cost in order to either foreclose the market or to induce competitors to raise their prices and to relax competition. Ordover, Saloner and Salop (1990) consider a two-stage duopoly model with price competition and differentiated products. In their model there is a foreclosure effect only if downstream firms compete in prices. Goods produced upstream are perfect substitutes. In our model upstream goods are perfect complements and our results hold for any form of downstream competition. Kim (2004) analyses a model similar to ours, but he only considers Cournot competition with linear demand on the downstream market and he does not analyze the implications for entry and innovation.

The remainder of the paper is organized as follows. In Section 2 we set up a very general model of a vertically structured industry in which all upstream goods are perfect complements. Section 3 restricts attention to linear royalties and compares a market structure where all firms are non-integrated to market structures where some firms are vertically or horizontally integrated. Section 4 allows for two-part tariffs. In Section 5 we discuss the effect of different market structures on entry on the same or another (unrelated) downstream market. In Section 6 we consider the incentives of upstream firms to innovate. Section 7 concludes and discusses the application of the model to other industries with complementary inputs such as rail or electricity networks.

2. The Model

Consider an industry with an upstream and a downstream market. The crucial feature of the upstream market is that the goods offered upstream are perfect complements, all of which are required for downstream production. This is the case in many high technology industries with direct or indirect network externalities in which firms agreed to technological standards to make sure that their products can interoperate or that they are compatible to complementary products. For example, the GSM or WCDMA standards on the telecommunication market guarantee that different handsets can communicate with each other, and the BlueRay and HD-DVD standards ensure that high definition video discs are compatible with DVD players produced by different companies. Typically, a standard requires access to a number of complementary patents that are often owned by different IP holders.

On the upstream market there are m firms, indexed by $u = 1,...,m$. Each upstream firm owns one essential patent. The costs for developing patents are sunk. Upstream firms license their patents at non-discriminatory, linear royalties r_u. Linear royalties are frequently used in many industries because of their simplicity and their risk-sharing properties (see Section 3). In Section 4 we also consider the case where upstream firms can charge two-part tariffs and show that our main qualitative arguments are unaffected.

On the downstream market there are n symmetric firms, indexed by $d = 1,...,n$. The focus of the analysis is on the royalties charged on the upstream market. Therefore, we want to keep the downstream market as general as possible. We assume that each downstream firm chooses an action vector $x_d \in X_d \subseteq \mathbb{R}^K$ that affects the quantities of the (possibly differentiated) goods that the firm itself and its competitors sell on the downstream market.[4] In the simplest interpretation each firm chooses its quantity q_d directly, so $x_d = q_d$ and $X_d = \mathbb{R}_0^+$. However, downstream firms may also decide on price, advertising, marketing,

[4] See Höffler and Kranz (2007).

sabotage or other business strategies. In this case x_d is a vector with $x_d \in X_d$ where X_d is a subset of some multi-dimensional Euclidean space.

The production of one unit of each of the downstream goods requires full access to all patents. Thus, if downstream firms produce q_d, *d=1,...,n*, then each upstream firm sells $Q = \sum_{d=1}^{n} q_d$ licenses. In the basic model we assume that all fixed costs are sunk and that downstream firms are symmetric and all incur the same marginal production cost k. In addition, each downstream firm has to pay linear royalties $r = \sum_{u=1}^{n} r_u$ for each unit of production. Thus, total marginal cost is $c_d = k + \sum_{u=1}^{n} r_u$.

The time structure of the game is as follows: At stage 1, all upstream firms set their royalty rates simultaneously. At stage 2, all downstream firms observe the royalty rates and choose $x_d \in X_d$ simultaneously. The quantity sold by each firm and the total quantity are functions of the action profile $x = (x_1, ..., x_n)$. The following regularity assumption is needed to make sure that we can do comparative statics in order to compare different royalty structures.

Assumption 1: *For any vector of royalties* $\vec{r} = (r_1, ..., r_m)$ *and any corresponding vector of marginal costs* $\vec{c} = (c_1, ..., c_n)$ *there exists a <u>unique</u> pure strategy Nash equilibrium* $x^*(\vec{r})$ *of the downstream market game at stage 2 that gives rise to quantities* $\left(q_1\left(x^*\left(\vec{r}\right)\right), ..., q_n\left(x^*\left(\vec{r}\right)\right)\right)$ *with* $Q\left(x^*\left(\vec{r}\right)\right) = \sum_{d=1}^{n} q_d\left(x^*\left(\vec{r}\right)\right)$. *If firm d's marginal cost* c_d *increases, its equilibrium quantity* q_d *decreases. Total equilibrium quantity* Q *is continuous and decreasing in the marginal cost* c_d *of each firm* d, $d \in \{1, ..., n\}$.

Example 1 in the appendix shows that Assumption 1 is satisfied in a Cournot model with *n* firms under a mild condition on the demand function. The assumption that the equilibrium production level of each firm is a decreasing function of its own marginal cost, and that total production decreases as well is very natural and holds much more generally. Dixit (1986) shows that it is satisfied in duopoly models of price and quantity competition with very general demand functions, and in oligopoly models with homogenous goods for both Bertrand and Cournot competition.

If there are multiple equilibria in the downstream market game a comparative static analysis is possible only with respect to the set of equilibria. Some of our results continue to hold in this case, but the analysis is messy and not insightful. Therefore, we restrict attention to the case of a unique downstream equilibrium.

We do not derive downstream demand from the preferences of rational consumers and we do not model oligopolistic interaction explicitly because we want to keep the downstream market as general as possible. Therefore, we cannot make any explicit welfare statements. As a point of reference we will compare the market outcome under different market structures to the outcome that would obtain if there were no contracting problems and firms could solve the complements and the double mark-up problem perfectly, i.e. if all upstream and all downstream firms could agree on a set of royalties that maximize total industry profits. This is called the "full integration outcome". It will turn out that in all the cases we consider the market outcome involves higher royalties and lower total quantities than this full integration benchmark. Almost all models of oligopoly imply that in this case an increase in total quantity Q is associated with an increase of consumer surplus. Therefore, we will often say that an increase of Q "tends to increase" consumer surplus and social welfare.

For simplicity we suppress the reference to the action profile x^* and use $q_d\left(\vec{r}\right)=q_d\left(x^*\left(\vec{r}\right)\right)$ and $Q\left(\vec{r}\right)=Q\left(x^*\left(\vec{r}\right)\right)$ in the following.

3. Linear Royalties under Different Market Structures

In this section we characterize the royalties that obtain under different market structures. Upstream firms are restricted to use non-discriminatory, linear royalties. Linear royalties are predominantly used in practice because of their risk-sharing properties. If downstream demand is uncertain, a linear royalty shares the risk between upstream and downstream firms, while a fixed fee imposes all the risk on downstream firms. In fact, Layne-Farrar and Lerner (2008, p. 10) report that linear royalties were used by all the patent pools they investigated. We compare the case of non-integration, where all upstream and downstream firms are owned separately and set royalties and downstream prices independently, to the cases of vertical integration, where some (or all) upstream firms are vertically integrated with some downstream firms, and horizontal integration on the upstream market, where several or all patents are owned by one firm. An alternative interpretation of horizontal integration is a patent pool that jointly licenses all patents in a bundle.

3.1 Non-integration

At stage 1 each upstream firm maximizes $\Pi_u = r_u Q(r_u, r_{-u})$. Note that $c_d = k_d + \sum_{u=1}^{m} r_u$. Thus, by Assumption 1, Q is a continuous function of r_u and depends on $r = \sum_{u=1}^{n} r_u$ only. The following standard assumption[5] is required to guarantee the existence of a unique equilibrium in the royalty setting game.

Assumption 2: *The marginal revenue of upstream firm u from increasing its royalty r_u does not increase if other firms increase their royalty rates, i.e.*

$$\frac{\partial^2 \Pi_u}{\partial r_u \partial r_j} = \frac{\partial Q}{\partial r} + r_u \frac{\partial^2 Q}{\partial r^2} < 0 \qquad (1)$$

Proposition 1: *There exists a unique pure strategy equilibrium in the royalty setting game at stage 1.*

<u>Proof:</u> See Appendix.

In equilibrium each upstream firm maximizes its profits $\Pi_u = r_u Q(r_u, r_{-u})$. Because all firms are symmetric they all charge the same royalty rate r_u^{NI} that is fully characterized by the first order condition

$$\frac{\partial \Pi_u}{\partial r_u} = \underbrace{Q}_{>0} + \underbrace{\frac{\partial Q}{\partial r}}_{<0} \cdot \underbrace{r_u^{NI}}_{>0} = 0 \qquad (2)$$

where the superscript *NI* stands for “Non-integration”.

As a reference point, suppose that all upstream and all downstream firms can agree on a set of royalties that maximize total industry profits, but they cannot restrict the actions chosen on the

[5] A similar assumption is required in any Cournot game to guarantee existence of a pure strategy equilibrium. See Novshek (1985) and Shapiro (1989).

downstream market. This is called the "full integration" outcome or benchmark. Total industry profits are given by

$$\Pi = \underbrace{r_u Q(r_u, r_{-u})}_{\substack{\text{upstream profit} \\ \text{of firm } u}} + \underbrace{\sum_{j \neq u} r_j Q(r_u, r_{-u})}_{\substack{\text{profits of all other} \\ \text{upstream firms } j \neq u}} + \underbrace{\sum_{d=1}^{m} \Pi_d (r_u, r_{-u})}_{\text{downstream profits}} \tag{3}$$

Comparing the objective function (2) of a single upstream firm to total profits (3), we see that each upstream firm does not take into account the impact of its own royalty rate on the profits of all other upstream firms, nor on the profits of all downstream firms nor on consumer surplus.

Proposition 2: *In equilibrium royalties are too high as compared to the royalties in the full integration benchmark. By increasing its royalty rate firm u exerts two negative externalities:*

- *by reducing total quantity Q it reduces the profits of the other upstream firms (complements effect)*
- *by raising the total royalty burden it reduces the profits of the downstream firms (double mark-up effect)*

Proof: The first order conditions for the maximization of total industry profits require for all $u = 1, \ldots, n$:

$$\frac{\partial \Pi}{\partial r_u} = \underbrace{Q(r^{FI})}_{>0} + \underbrace{\frac{\partial Q(r^{FI})}{\partial r} \cdot r_u^{FI}}_{<0} + \underbrace{\sum_{v \neq u} \frac{\partial Q(r^{FI})}{\partial r} \cdot r_v^{FI}}_{\substack{<0 \\ \text{complements effect}}} + \underbrace{\sum_{d=1}^{m} \frac{\partial \Pi_d}{\partial r_u}}_{\substack{<0 \\ \text{double mark-up effect}}} = 0 \tag{4}$$

where $r^{FI} = \sum_{u=1}^{n} r_u^{FI}$ and the superscript "*FI*" stands for "Full Integration". Note that for total industry profits only the sum of royalties matters, while the distribution across upstream firms is irrelevant. Therefore, we impose w.l.o.g. $r_1^{FI} = \ldots = r_m^{FI}$. If all upstream firms choose the optimal royalties under full integration total quantity is $Q(r^{FI})$. Comparing (4) to (2) it is straightforward to see that the first derivative of each firm's profit function would be strictly positive. Thus, this cannot be an equilibrium. Each firm would have an incentive to increase its royalty r_u until the FOC is satisfied. Hence, $r_u^{NI} > r_u^{FI}$. *Q.E.D.*

The complements effect has first been observed by Cournot (1838, Chapter 9). It stems from the fact that the goods produced by the upstream firms are perfect complements that are sold by independent firms. The double mark-up effect is due to the vertical chain of producers that all have market power. Upstream firms have a monopoly on their patents that are essential inputs for downstream firms that also have market power and impose an additional mark-up when they sell to consumers.

3.2 Vertical Integration

Suppose that l upstream firms and l downstream firms vertically integrate, one upstream firm with one downstream firm each. Thus, we now have l vertically integrated firms, m-l non-integrated upstream firms, and n-l non-integrated downstream firms. At the first stage the non-integrated upstream firms and the upstream divisions of the vertically integrated firms set linear royalties r_i, $i = 1,...,m$. At the second stage the non-integrated downstream firms and the downstream divisions of the vertically integrated firms choose action vectors x_d giving rise to quantities q_d, $d = 1,...,n$, on the downstream market. By Assumption 1 there exists a unique Nash equilibrium in the Cournot game at stage 2. Furthermore, Assumption 1 implies:

Corollary 1: *If firms charge the same royalties under non-integration and under vertical integration, then the total quantity produced is larger the more upstream firms are vertically integrated.*

Proof: Under vertical integration a firm does not have to pay royalties to its own upstream division. Therefore, if royalties are the same, the marginal costs of each vertically integrated firm are lower than the cost of a non-integrated firm:

$$c_i^{NI} = k + \sum_{j=1}^{n} r_j > k + \sum_{j \neq i} r_j = c_i^{VI} \tag{5}$$

By Assumption 1, total equilibrium quantity Q increases if the marginal cost of one firm decreases. Therefore, Q increases if upstream firms vertically integrate. *Q.E.D.*

The result suggests that vertical integration is beneficial because it raises total quantity. However, this need not be the case. Corollary 1 assumes that royalty rates are the same under non-integration and vertical integration. This could be the case if prices on the upstream market are regulated by the same price cap that is binding under both market structures. However, if firms are not constrained in their royalties they will choose different royalty rates under different market structures.

What royalties will be chosen under vertical integration? When a vertically integrated firm sets its royalty rate, it internalizes the effect on the profits of its own downstream division. Thus, vertical integration solves the double mark-up problem within each firm. However, there are still three negative externalities:

1. The well known double mark-up problem across firms remains, because firm i does not take into account the effect of its own royalty r_i on firm j's downstream profit.
2. Furthermore, a vertically integrated firm does not internalize the effect of its royalty rate on the upstream profits of the other firms. Thus, vertical integration does not solve the complements problem.
3. Finally, there is a new externality that does not exist under non-integration nor under horizontal integration. This is the "raising one's rivals' costs" effect: The higher the royalty charged by firm i the higher are the costs of the other firms active on the downstream market, while firm i's costs are unaffected. This induces firm i to raise its royalties in order to raise its rivals' costs.

Because all of these externalities are negative, royalties in a vertically integrated industry are too high.

Proposition 3: *Royalties chosen if all upstream firms are vertically integrated are larger than the royalties in the full integration benchmark.*

<u>Proof:</u> See Appendix.

The more interesting question is whether vertical integration is superior to non-integration. Perhaps surprisingly this is not necessarily the case. Vertically integrated firms may charge higher royalties and induce a less efficient market outcome than non-integrated firms.

At stage 1, vertically integrated firm v chooses royalty r_v in order to maximize the sum of profits in its upstream and downstream division:[6]

$$\Pi_v = \underbrace{r_v Q^{VI}\left(\vec{r}\right)}_{upstream\_profit} + \underbrace{q_v\left(\vec{r}\right)\cdot\left[P\left(Q^{VI}\left(\vec{r}\right)\right)-c_v\left(\vec{r}\right)\right]}_{downstream\_profit} \tag{6}$$

Differentiating with respect to r_v the FOC for the optimal r_v requires:

$$\frac{\partial\Pi_v}{\partial r_v} = \underbrace{Q^{VI} + \frac{\partial Q^{VI}}{\partial r_v}\cdot r_v}_{\substack{\text{same as under}\\ \text{non-integration}}} + \underbrace{\left[\frac{\partial P}{\partial Q}\cdot\frac{\partial Q^{VI}}{\partial r_v}-1\right]\cdot q_v^{VI}}_{\substack{<0\\ \text{internalization of}\\ \text{vertical double mark-up}}} + \underbrace{\left[P-k-\sum_{j=1}^{n} r_j\right]\cdot\frac{\partial q_v^{VI}}{\partial r_v}}_{\substack{>0\\ \text{raising one's rivals' costs}}} \tag{7}$$

The first two terms correspond to the FOC under non-integration: An increase in r_v raises revenues per unit of output, but it reduces the quantity of output. The last two terms reflect the effect of an increase of r_v on downstream profits and have no analogue under non-integration.

Consider the third term first: By increasing its royalty rate r_v firm v raises the costs of all downstream firms which increases the market price. However, it also increases the cost of its own downstream division, so profits of the downstream division are reduced. Because firm v internalizes this vertical double mark-up problem it has an incentive to moderate its royalty rate as compared to a non-integrated upstream firm.

However, there is a forth effect that works in the opposite direction: By raising its royalty rate firm v increases the marginal costs of its downstream competitors $i \neq v$. Thus, in the downstream continuation equilibrium the quantities chosen by all other firms are reduced while the quantity of firm v goes up, so firm v receives the mark-up, $P-c_i$, on a larger quantity. Thus, the forth term gives an additional incentive to raise royalties as compared to a non-integrated upstream firm.

This "raising one's rivals' cost effect" implies that each vertically integrated firm has an incentive to raise its royalty rate in order to improve its own market position to the detriment of its rivals. However, there is a prisoners' dilemma. In equilibrium all vertically integrated firms choose the same royalty, nobody has a competitive advantage, and everybody would be better off if all firms could jointly reduce their royalties.

[6] Note that the royalty income from its own downstream division is a cost of the downstream division and thus cancels out in the profit function. However, it will be convenient to keep these two terms separate in order to facilitate the comparison of the first order conditions.

Proposition 4: *Vertically integrated firms may choose higher or lower royalties than non-integrated upstream firms.*

In the Appendix we offer two simple examples showing that the net effect can go in both directions.

Even if vertical integration yields higher royalties than non-integration it may still yield a more efficient market outcome because vertically integrated firms are not distorted by the royalties that they pay to themselves. However, for the case of a Cournot model with linear demand Kim (2004, p. 245) shows that if the number of vertically integrated firms is not too large, then vertical integration yields a total quantity that is smaller than the total quantity produced under non-integration. Thus, vertical integration may reduce total output, total industry profit and social welfare.

3.3 Horizontal Integration

We now consider the possibility that some upstream firms merge and integrate horizontally. The integrated firm bundles its IP rights and licenses them at a joint royalty rate on the downstream market.

Proposition 5: *As the number of upstream firms decreases, total equilibrium royalties are reduced and total quantity sold on the downstream market increases.*

Proof: See Appendix.

Proposition 5 shows that - in contrast to the case of vertical integration - a horizontal merger unambiguously reduces royalties and increases total industry profits. Furthermore, it increases the total quantity of production and thus improves efficiency. Hence, horizontal integration is always more profitable and more efficient than non-integration. However, under horizontal integration royalties are still higher than the royalty rate that maximizes total industry profits.

Proposition 6: *If all upstream firms are horizontally integrated there is no complements effect and no raising one's rivals' cost effect, but the double mark-up effect remains.*

Therefore, the royalty charged by the upstream firm is too high and downstream quantity is too low as compared to the royalty charged in the full integration benchmark..

Proof: See Appendix.

The royalty rate chosen by a horizontally integrated upstream firm is larger than the royalty in the full integration benchmark because of the double mark-up problem. However, the more competition there is on the downstream market, the smaller is the mark-up. In the limit, if downstream competition becomes fully competitive, the double mark-up problem disappears and the royalty chosen by a horizontally integrated upstream firm approaches the royalty rate that maximizes total industry profits.

This result suggests that if the downstream market is sufficiently competitive, horizontal integration outperforms vertical integration. Indeed in the example of a Cournot model with linear demand three downstream firms are already sufficient to render horizontal integration superior to vertical integration:

Example 2: *Suppose that there is Cournot competition downstream and that the demand function is linear. If there are more than two firms on the downstream market, horizontal integration yields a higher output on the downstream market than vertical integration.*

Proof: see Appendix.

4. Two-part Tariffs

So far we assumed that upstream firms are restricted to use linear royalties which is the prevalent case in reality. However, firms could also use two-part tariffs. It is well known that two part-tariffs can be used to solve the double mark-up problem. If firms are horizontally integrated or form a patent pool it is very simple (and a dominant strategy) to implement the full integration outcome: set the linear royalty such that downstream firms are induced to charge the monopoly price and choose the fixed fee such that it extracts all downstream profits.

In this section we show that if firms are not horizontally integrated they can still use two-part tariffs to solve the complements problem and to implement the fully integrated outcome. However, in contrast to the case of horizontal integration this requires coordination among the IP holders. We will show that this can be difficult and is more likely to happen when firms are non-integrated than when they are vertically integrated.

Proposition 7: *Suppose that all firms are non-integrated. If upstream firms are restricted to use non-discriminatory two-part tariffs there exists a symmetric pure strategy equilibrium in which upstream firms charge linear royalties that implement the full integration outcome and extract all profits via the fixed fees from downstream firms.*

Proof : Note first that in any equilibrium all downstream profits must be extracted. If this was not the case each upstream firm would have an incentive to further raise the fixed fee of its royalties. Consider now an equilibrium candidate where the royalties are sufficiently small that all downstream firms want to produce. Suppose that the sum of all linear royalties is larger (smaller) than the royalty rate that implements the full integration benchmark. In this case each firm has an incentive to lower (raise) its own linear royalty. This increases total industry profit. Hence, by raising the fixed fee of its royalty scheme at the same time, the firm would be better off. Thus, the only symmetric subgame perfect equilibrium in which all downstream firms produce must have identical linear royalties for all upstream firms that sum up to r^{FI}. *Q.E.D.*

Note that if upstream firms can also use discriminatory royalties the equilibrium breaks down. In this case a deviating upstream firm can raise its fixed fee for $n-1$ downstream firms to infinity, so that only one downstream firm survives and serves the downstream market as a monopolist. If the deviating upstream firm raises its fixed fee for this remaining downstream firm so that it extracts all the monopolist's profits, the deviation is profitable. Note further that there are other symmetric pure strategy equilibria as well. For example, it is always an equilibrium that all upstream firms charge fixed and/or linear royalties that are so high that no downstream firm wants to license.

Proposition 8: *Suppose that all m upstream firms are vertically integrated. There exists an* $\overline{m}$ *such if* $m \geq \overline{m}$ *there does not exist a symmetric pure strategy subgame perfect equilibrium that implements the full integration outcome. However, there always exists an asymmetric subgame perfect equilibrium in which one firm serves the entire downstream market at the monopoly price but makes zero profit. The other firms set the linear royalties equal to 0 and extract all the profits of the downstream monopolist through their fixed fees.*

Proof: See Appendix.

With vertical integration and sufficiently many upstream firms the symmetric equilibrium breaks down. The reason is that a vertically integrated firm de facto discriminates in favour of its own downstream division even if it charges all firms the same royalties. This is because the downstream division of a vertically integrated firm is not affected by the royalty charged by its own company. Thus, a vertically integrated firm could raise its fixed and/or linear royalty to a prohibitive level and thus exclude all other firms from the downstream market. If the number of VI firms is sufficiently large, so that the share of total profits accruing to each firm in a symmetric equilibrium is sufficiently small, such a deviation becomes profitable. In this case a symmetric pure strategy equilibrium that implements the full integration outcome fails to exist which makes the coordination problem much more difficult. To be sure, there are asymmetric pure strategy equilibria that implement the monopoly outcome, but these equilibria are asymmetric and awkward: One firm monopolizes the downstream market, but this firm makes zero profits and all the rents go to the upstream firms that are not active downstream. Because nobody wants to be the zero profit monopolist, it seems very difficult to coordinate on such an equilibrium.

To summarize: Two part-tariffs can be used to increase total industry profit. This tends to increase social welfare because total quantity increases. For a horizontally integrated firm (or a patent pool) it is a dominant strategy to set royalties that implement the full integration outcome. With non-integrated firms there exists a symmetric pure strategy equilibrium that implements this outcome, but the equilibrium is not unique. If firms are vertically integrated a symmetric equilibrium implementing the monopoly outcome fails to exist if the number of VI firms is sufficiently large. In this case there are only asymmetric equilibria with an uneven distribution of profits. Thus, with horizontal integration it seems more likely that a welfare improvement will be implemented than under non-integration which in turn outperforms vertical integration.

5. Entry on the Downstream Market

What are the effects of vertical and horizontal integration on market entry and innovation? In this section we consider the cases of entry on the same downstream market and of entry on a separate market that also requires the upstream goods as essential inputs.

5.1. Entry on the same downstream market: Suppose that a potential entrant considers entering the downstream market. The entrant can produce with marginal cost k_e and has to incur an entry cost $K > 0$. Whether entry is profitable depends on how royalties and the final price on the downstream market will react to an additional competitor downstream. More competition on the downstream market reduces the downstream mark-up and thus reduces the double mark-up problem which benefits upstream firms. Under non-integration upstream firms may increase their royalties, but they have no incentive to discriminate against the entrant or to squeeze him out of the market.

The same holds for a horizontally integrated upstream firm. Furthermore, under horizontal integration the complements problem disappears, so total royalties are lower than under non-integration which makes entry more likely.

In contrast, under vertical integration the "raising one's rivals' costs" effect induces the vertically integrated firms to discriminate against non-integrated downstream competitors. In the example of a linear Cournot model in which all downstream firms have identical marginal costs ($k_e = k$), it can be shown that royalties charged by the vertically integrated firms are so high that no independent downstream firm can enter the market even if the entry cost is 0.[7] If the entrant has an efficiency advantage over existing firms ($k_e < k$), the vertically integrated firms face a trade-off. On the one hand, the low cost competitor reduces their downstream market shares and profits. On the other hand, the low cost competitor lowers the downstream price and thereby extends the downstream market, which benefits their upstream profits. Comparing a vertically integrated to a non-integrated or horizontally integrated upstream firm, the latter benefit from the extension of the downstream market but do not suffer a reduction of downstream profits. Thus, a vertically integrated firm tends to charge higher royalties to an entrant than the non- or horizontally integrated firm which makes the probability of entry less likely.

[7] See Kim (2004, Theorem 1).

5.2. Entry on a separate downstream market: Consider now the case of an independent company that comes up with an idea to use the upstream patents for a new product that is sold on a new separate market where this firm is a monopolist. Suppose that the monopoly profit that the new firm can make on this market is $\Pi^e(r)$ with $\frac{\partial \Pi^e}{\partial r} < 0$, where r denotes the total royalties to be paid upstream. However, in order to develop the new product and to enter the market the firm has to incur a sunk cost of $K > 0$. Suppose that K is private information of the entrant. The suppliers of the essential inputs only know that K is drawn from $\left[\underline{K}, \overline{K}\right]$, $\underline{K} > 0$, according to cdf $G(K)$.

Suppose that the suppliers of the essential inputs cannot commit to royalties before entry occurs and are restricted to use linear royalties. Note that the new market is independent of the original downstream market, so it does not make a difference whether the IP holders are vertically integrated or non-integrated. Because all IP holders set their royalties independently we get:

Proposition 9: *Suppose that upstream firms set linear royalties after entry occurred, and suppose that Assumption 2 is satisfied on the new market. Vertically integrated and non-integrated upstream firms charge the same royalties that are higher than the royalties of a horizontally integrated firm. The larger the number of vertically integrated or non-integrated upstream firms, the higher are the royalties and the smaller is the probability that entry will occur.*

Proof: See Appendix.

The reason is that the vertically integrated or non-integrated upstream firms suffer from the complements problem. However, Proposition 9 assumes that upstream firms cannot commit ex ante to the royalties they will charge after entry occurs and that they are restricted to use linear royalties. What happens if these assumptions are relaxed?

With two-part tariffs all upstream firms will set the linear part of the royalty equal to zero. Recall that there is only one downstream firm on the new market, so linear royalties of zero will induce the monopoly outcome and the monopoly profit $\Pi^e(0)$. In addition, each upstream firm will charge a fixed fee in order to extract as much of the monopoly profit as possible. The problem is that upstream firms do not know the entry costs *K* of the entrant. If the sum of all fixed royalties is larger than *K*, the entrant will not enter.

If all upstream firms are horizontally integrated, the horizontally integrated firm will charge a fixed fee that maximizes

$$\Pi^{HI} = F \cdot prob(\Pi^e(0) - F) \geq K) = F \cdot G(\Pi^e(0) - F) = F \cdot H(F) \tag{8}$$

where $H(F) \equiv G(\Pi^e(0) - F)$ (so $H(F)$ is the probability of entry given the total fixed fee F). Let us compare this to the fixed fees that are chosen by vertically (or non-)integrated upstream firms. Each of them maximizes

$$\Pi_v^{NI} = F_v \cdot prob\left(\Pi^e(0) - \sum_{u=1}^{m} F_u^{NI} \geq K\right) = F_v \cdot H\left(\sum_{u=1}^{m} F_u^{NI}\right) \tag{9}$$

Note that without horizontal integration an upstream firm $v \in \{1,...,m\}$ does not take into account the loss of expected profits of the other upstream firms if the potential entrant does not enter the market because of an increase of the fixed fee of firm v. Because of this externality, the total fixed fee will be higher the larger the number of upstream firms. To show this formally we need the following regularity condition:

Assumption 3: $-H'(F) + F \cdot H''(F) < 0$ *for all* $F < \Pi^e(0)$.

Assumption 3 implies that the maximization problem of each non-integrated upstream firm is strictly concave.

Proposition 10: *Suppose that upstream firms can use two-part tariffs and can commit to their royalties before market entry occurs. Given Assumption 3 there exists a unique pure strategy equilibrium for any number of upstream firms. The total fixed fee is lowest and the probability of entry is largest if all upstream firms are horizontally integrated. The larger the number of independent upstream firms, the larger is the total fixed royalty and the smaller is the probability of entry.*

<u>Proof:</u> See Appendix.

Thus, Proposition 10 confirms the result of Proposition 9 that a horizontally integrated upstream firm (or a patent pool) facilitates entry as compared to a market with several vertically integrated (or non-integrated) firms.

6. Innovation on the Upstream Market

What are the implications of different market structures on the incentives to innovate and to come up with new technologies on the upstream market? Suppose that a company has an idea for an innovation that improves the quality of the technology. This may be an additional feature that makes it possible to use the technology for new applications, to reduce the cost to employ the technology in downstream production or to raise the benefits of consumers from using the downstream product. Innovation can be interpreted as entry on the upstream market. However, while the entrant on the downstream market produces a substitute to the products of the other downstream firms, the entrant on the upstream market produces a complement to the other upstream goods.

To develop the innovation and to protect it by a patent the innovator has to incur an investment cost $I > 0$. The innovation can be used only if the existing upstream firms include it in the standard. The innovation raises consumers' willingness to pay and/or lowers production costs. This raises the profits that can be made on upstream and downstream markets. By how much profits increase depends on the market structure. The analysis of the preceding sections suggests that if linear royalties have to be used additional profits will be higher under horizontal integration than under non-integration. It will also be higher under horizontal integration than under vertical integration if the number of downstream producers is sufficiently large. In this section we do not model explicitly how different market structures affect the profits that can be derived from the innovation. Instead we assume that if the innovative patent is owned by a horizontally integrated firm the profits of this firm will increase from Π to $\Pi + \Delta$. If however, there are m independent upstream firms initially, and if the number of independent upstream firms increases to $m+1$ because the innovation is included in the standard then total upstream profits change from $\Pi(m)$ to $\Pi(m+1) + \Delta(m+1)$. By Propositions 5 and 6 we know that total royalties are increasing with m and that for all $m \geq 1$ total royalties are higher than the royalty a monopolist would choose. Therefore we must have $\Pi(m) > \Pi(m+1)$ and $\Delta(m) > \Delta(m+1)$.

Suppose first that one of the existing upstream firms comes up with the idea for the innovation. If it innovates total profits on the upstream market increase by $\Delta(m)$, $\Delta(m)/m$ of which accrues to the innovator. Thus, we get:

Proposition 11: *An existing upstream firm will innovate if and only if* $I \leq \Delta(m)/m$. *The smaller the number of upstream firms, the larger is* $\Delta(m)/m$ *and the larger are the incentives to innovate.*

Suppose now that the potential innovator is a new company that does not own any other patents that are essential to the standard. Furthermore, the company has to develop the innovation and to incur the (sunk) investment cost I before negotiating on the terms of including the innovation in the standard.

If the standard is controlled by a horizontally integrated company the analysis is straightforward. If $\Delta > 0$ the two parties will agree that the horizontally integrated firm buys the innovation and includes it in the standard. Assuming Nash bargaining they will split the surplus equally, so the innovator receives $\frac{\Delta}{2}$. Thus, the investment in the innovation will be undertaken if and only if $I < \frac{\Delta}{2}$.

Consider now the case with $m > 1$ independent upstream firms. It does not matter whether these firms are vertically integrated or not. In principle, there are two ways how the innovation can be included in the standard. First, the innovator could join the standard as an independent firm, so the number of upstream firms increases to $m+1$. Second, one of the upstream firms could acquire the patent from the innovator.

Suppose that the innovator joins the standard and becomes an additional independent firm on the upstream market. For this he needs the consent of all m upstream firms. Without the innovator each upstream firm's profit is $\frac{\Pi(m)}{m}$. With the innovator the profit of each firm on the upstream market is $\frac{\Pi(m+1)+\Delta(m+1)}{m+1}$. Thus, the innovation will be included in the standard if and only if maximizes

$$\Delta(m+1) > \underline{\Delta}(m) = \underbrace{\frac{\Pi(m)}{m}}_{>0} + \underbrace{\Pi(m) - \Pi(m+1)}_{>0} > 0 \tag{10}$$

Thus, the additional profit generated by the innovation if there are $m+1$ upstream firms must be larger than the average profit of each firm without the innovation $\left(\frac{\Pi(m)}{m}\right)$ plus the reduction in total profits that is due to having one additional firm (and thus one additional complements problem) on the upstream market $(\Pi(m)-\Pi(m+1))$. If the innovation is included in the standard, the innovator gets $\frac{1}{m+1}$ of total profits. Thus, the innovation is profitable if and only if $I<\bar{I}(m)=\frac{\Pi(m+1)+\Delta(m+1)}{m+1}$.

If these conditions are not satisfied, the innovation could still be included in the standard if one of upstream firms acquires the patent. If it does so, the innovation raises total profits from $\Pi(m)$ to $\Pi(m)+\Delta(m)$. Thus, the profit of the firm that acquired the patent increases by $\frac{\Delta(m)}{m}-P$, where P is the price to be paid to the innovator. Assuming Nash bargaining, $P=\frac{\Delta(m)}{2\cdot m}$. Thus, the investment will be undertaken only if $I<\frac{\Delta(m)}{2\cdot m}$. Note, however, that this is an asymmetric equilibrium and that the profits of all other firms increase by $\frac{\Delta(m)}{m}$ because they benefit from the innovation without having to pay for it. Thus, there is a free rider problem where each firm prefers the other firms to acquire the patent. It turns out that there does not exist a symmetric equilibrium with a positive probability of innovation. A symmetric equilibrium would have to be a mixed strategy equilibrium where each firm acquires the patent with positive probability. However, in a mixed strategy equilibrium each firm has to be indifferent between acquiring and not acquiring. Thus, the surplus to be shared with the innovator is zero. But if the price for the innovation is zero, the innovation will not be undertaken.

These results are summarized in the following proposition:

Proposition 12: Suppose that an independent innovator can develop an innovation at cost I.

a) If a horizontally integrated firm controls the upstream market, any innovation that is value increasing is included in the standard. The innovation will be undertaken iff $I<\frac{\Delta}{2}$.

b) If m firms are active on the upstream market the innovation will be included in the standard and the innovator will become firm $m+1$ on the upstream market if the value of the innovation is sufficiently large and the investment cost is sufficiently small, i.e. if $\Delta(m+1) > \underline{\Delta}(m) = \underbrace{\frac{\Pi(m)}{m}}_{>0} + \underbrace{\Pi(m) - \Pi(m+1)}_{>0} > 0$ and $I < \bar{I}(m) = \frac{\Pi(m+1) + \Delta(m+1)}{m+1}$. As $m \to \infty$, $\underline{\Delta}(m)$ and $\underline{I}(m)$ go to 0.

c) If m firms are active on the upstream market and these conditions are not satisfied there also exists an asymmetric pure strategy equilibrium where one of the upstream firms buys any innovation that is value increasing at price $P = \frac{\Delta(m)}{2 \cdot m}$. In this case the innovation is developed iff $I \leq \frac{\Delta(m)}{2 \cdot m}$.

Thus, we find that innovation becomes more likely the fewer firms there are on the upstream market. The incentives to innovate are maximized if all firms on the upstream market are horizontally integrated or, equivalently, if all upstream firms form a patent pool.

7. Conclusions

If different IP holders own complementary patents that are all essential to a standard, several externalities arise that affect their pricing decisions. In a general model of downstream competition we have shown that horizontal integration has positive effects on total output and tends to increase social welfare, while the effects of vertical integration are ambiguous or negative. Horizontal integration eliminates the complements effect and induces lower prices and higher quantities on the downstream market. Furthermore, a horizontally integrated firm benefits from downstream market entry and encourages innovation upstream. Vertical integration, on the other hand, solves the double-mark-up problem between the two merging firms, but it gives rise to a raising one's rivals' cost effect. The net effect may increase prices and reduce output and social welfare. Furthermore, vertically integrated firms compete against new market entrants and want to discriminate against them. Finally, vertical integration does not affect the problem that an innovator needs permission by all upstream IP holders to join the standard, so it does not encourage innovation. These results suggest that the current shift in US competition policy to permit patent pools for complementary patents is beneficial. At the same time it suggests that vertical integration can have ambiguous effect and should be seen with caution.

Our model also applies to industries that require access to a physical network such as electricity, railways or fixed-line telecommunications, if the network is split up in separate parts that complement each other. For example, there are often regional monopolies that own separate parts of the electricity grid, of the railway lines or of the telecommunications infrastructure. If a downstream firms wants to offer services that are based on the network it often need access to the entire network. In this case the different parts of the network are perfect complements and the results of this paper apply. However, there are also cases where upstream goods are imperfect complements. These cases are considerably more complicated and are an important topic for future research.

Appendix

Example 1: Suppose that firms compete in quantities on the downstream market and assume that the following regularity condition holds:

Condition 1: *There exists* $\overline{Q} < \infty$ *such that* $P\left(\overline{Q}\right) = 0$. *For all* $Q \in \left[0, \overline{Q}\right]$, $P(Q)$ *is continuous, twice continuously differentiable and strictly decreasing with*

$$P'(Q) + QP''(Q) < 0 \tag{11}$$

Note that (1) is equivalent to the assumption that each firm's marginal revenue is declining in the aggregate output of all other firms, i.e.

$$\frac{\partial^2 \Pi_d}{\partial q_d \partial q_{-d}} = P'(Q) + q_d P''(Q) \quad \forall q_d \le Q, \quad Q \le \overline{Q} \tag{12}$$

where $q_{-d} = q_1 + ... + q_{d-1} + q_{d+1} + ... + q_n$.[8]

Proposition 0: *Given Condition 1 there exists a* <u>*unique*</u> *pure strategy Nash equilibrium of the Cournot game at stage 2. If the marginal cost of firm* d *increases, its equilibrium quantity* q_d *decreases. Total equilibrium quantity* Q *is continuous and decreasing in the marginal cost* c_d *of each firm* d, $d \in \{1, ..., n\}$.

Proof: Novshek (1985) shows that Condition 1 implies the existence of a pure strategy equilibrium in the Cournot game. To see that the equilibrium is unique, note that given Condition 1 each firm's profit function is globally concave, so equilibrium quantities are characterized by first order conditions

$$\frac{\partial \Pi_d}{\partial q_d} = P(Q) + q_d P'(Q) - c_d \le 0 \tag{13}$$

Suppose that there are two equilibria with corresponding output vectors $(q_1^A, ..., q_n^A)$ and $(q_1^B, ..., q_n^B)$. Let $Q^A = \sum_{d=1}^{n} q_d^A$ and $Q^B = \sum_{d=1}^{n} q_d^B$, and suppose wlog that $Q^A > Q^B$. This implies

[8] To see that the (2) implies (1), simply set $q_d = Q$. To see that (1) implies (2), suppose first that $P''(Q) \le 0$. In this case both conditions are surely satisfied. So suppose that $P''(Q) > 0$. In this case $q_d \le Q$ implies $q_d P''(Q) \le QP''(Q)$. By (1) we have $QP''(Q) < -P'(Q)$. Therefore, we must have $q_d P''(Q) < -P'(Q)$ as well which is equivalent to (2).

that $P(Q^A) < P(Q^B)$. Comparing the FOCs for q_d in the two equilibria and using the fact that $P(Q^A) < P(Q^B)$ we have $q_d^A P'(Q^A) > q_d^B P'(Q^B)$. Summing up over all d this implies $Q^A P'(Q^A) \geq Q^B P'(Q^B)$. However, $P'(Q) + QP''(Q) < 0$ implies

$$\int_{Q^B}^{Q^A} [P'(Q) + QP''(Q)]dQ = \left[QP'(Q)\right]_{Q^B}^{Q^A} = Q^A P'(Q^A) - Q^B P'(Q^B) < 0$$

a contradiction.

Dixit (1986, p. 120) shows that firm *d*'s equilibrium quantity decreases if its marginal costs increases. Finally, we have to show that total quantity is a continuous and decreasing function of c_d for all $d \in \{1,...,n\}$. For all firms with $q_d > 0$, (3) has to hold with equality. Summing up (3) over all $d = 1,...,n$ with $q_d > 0$ yields

$$mP(Q) + QP'(Q) - \sum_{d=1}^{n} c_d = 0 .$$

Using the implicit function theorem we get that $Q(c_d)$ is continuously differentiable with

$$\frac{\partial Q}{\partial c_d} = -\frac{-1}{mP'(Q) + P'(Q) + QP''(Q)} < 0 .$$

The strict inequality is implied by Condition 1. *Q.E.D.*

<u>Proof of Proposition 1:</u> Assumption 2 implies that the profit function of each upstream firm is globally concave in r_u,

$$\frac{\partial^2 \Pi}{\partial r_u^2} = \frac{\partial Q}{\partial r} + \frac{\partial Q}{\partial r} + r_u \frac{\partial^2 Q}{\partial r^2} < \frac{\partial Q}{\partial r} + r_u \frac{\partial^2 Q}{\partial r^2} < 0$$

Thus, by the well known existence proof for concave games (Debreu, 1952), a pure strategy equilibrium exists. In equilibrium the FOCs

$$\frac{\partial \Pi_u}{\partial r_u} = Q + r_u \frac{\partial Q}{\partial r} = 0 \tag{14}$$

must be satisfied. Suppose that there are two equilibria with corresponding royalty vectors $(r_1^A,...,r_m^A)$ and $(r_1^B,...,r_m^B)$. Let $r^A = \sum_{u=1}^{m} r_u^A$ and $r^B = \sum_{u=1}^{m} r_u^B$, and suppose wlog that $r^A > r^B$. This implies that $Q(r^A) < Q(r^B)$. Comparing the FOCs for r_u in the two equilibria we have $r_u^A Q'(r^A) > r_u^B Q'(r^B)$. Summing up over all *u* this implies $r^A Q'(r^A) \geq r^B Q'(r^B)$. However, Assumption 2 implies $\frac{\partial Q}{\partial r} + r\frac{\partial^2 Q}{\partial r^2} < 0$ which in turn implies

$$\int_{r^B}^{r^A} [Q'(r) + rQ''(r)]dQ = \left[rQ'(r)\right]_{r^B}^{r^A} = r^A Q'(r^A) - r^B Q'(r^B) < 0$$

a contradiction. Thus, $r_A = r_B$. Symmetry implies $r_1^A = ... = r_m^A$ and $r^B = ... = r_m^B$. Thus, equilibrium royalties are unique. *Q.E.D.*

Proof of Proposition 3: A vertically integrated firm v, $v \in \{1,...,l\}$ chooses its royalty rate to maximize $\Pi_v^V = \underbrace{r_v Q^{VI}(\vec{r})}_{upstream\_profit} + \underbrace{q_v(\vec{r}) \cdot \left[P\left(Q^{VI}(\vec{r})\right) - c_v(\vec{r})\right]}_{downstream\_profit}$. The FOC for this maximization problem is

$$\frac{\partial \Pi_v^V}{\partial r_v} = \underbrace{Q^{VI} + \frac{\partial Q^{VI}}{\partial r_v} \cdot r_v}_{\text{same as under non-integration}} + \underbrace{\left[\frac{\partial P}{\partial Q} \cdot \frac{\partial Q^{VI}}{\partial r_v} - 1\right] \cdot q_v^{VI}}_{\substack{<0 \\ \text{internalization of} \\ \text{vertical double mark-up}}} + \underbrace{\left[P - k - \sum_{j=1}^{n} r_j\right] \cdot \frac{\partial q_v^{VI}}{\partial r_v}}_{\substack{>0 \\ \text{raising one's rivals' costs}}} \tag{15}$$

A non-integrated upstream firm u, $u \in \{l+1,...,m\}$, chooses its royalty rate to maximize $\Pi_u^U = r_u Q^{VI}(r_u, r_{-u})$. Its FOC is given by

$$\frac{\partial \Pi_u^U}{\partial r_u} = Q^{VI} + \frac{\partial Q^{VI}}{\partial r_u} \cdot r_u \tag{16}$$

The first order condition for the maximization of total industry profits can be rewritten as:

$$\frac{\partial \Pi}{\partial r_v} = \underbrace{Q(r^{FI})}_{>0} + \underbrace{\frac{\partial Q(r^{FI})}{\partial r} \cdot r_v^{FI}}_{<0} + \underbrace{\left[\frac{\partial P}{\partial Q} \cdot \frac{\partial Q^{VI}}{\partial r_v} - 1\right] \cdot q_v^{VI} + \left[P - k - \sum_{j=1}^{n} r_j\right] \cdot \frac{\partial q_v^{VI}}{\partial r_v}}_{\text{double margin effect on firm } v} + \underbrace{\sum_{d \neq v} \frac{\partial \Pi_d}{\partial r_u}}_{\substack{<0 \\ \text{double margin effect} \\ \text{on other firms}}} = 0$$

(17)

Suppose all firms on the upstream market choose the optimal royalties under full integration r_u^{FI}, so total downstream quantity is $Q(r^{FI})$. Then we have

$$\begin{aligned}
\frac{\partial \Pi_v^V}{\partial r_v} &= Q(r^{FB}) + \frac{\partial Q(r^{FB})}{\partial r_v} \cdot r_v^{FB} + \left[\frac{\partial P}{\partial Q} \cdot \frac{\partial Q(r^{FB})}{\partial r_v} - 1\right] \cdot q_v(r^{FB}) + \left[P - k - \sum_{j=1}^{n} r_j\right] \cdot \frac{\partial q_v(r^{FB})}{\partial r_v} \\
&> \frac{\partial \Pi_v^V}{\partial r_v} \underbrace{+\sum_{j \neq v} \frac{\partial \Pi_j}{\partial r_v}}_{\substack{<0 \\ \text{complements effect}}} + \underbrace{\sum_{d \neq v} \frac{\partial \Pi_d}{\partial r_u}}_{\substack{<0 \\ \text{double margin effect} \\ \text{on other dowstream firms}}} = \frac{\partial \Pi}{\partial r_v} = 0 \quad \forall v \in \{1,...,l\}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \Pi_u^U}{\partial r_u} &= Q(r^{FB}) + \frac{\partial Q(r^{FB})}{\partial r_u} \cdot r_u^{FB} \\
&> \frac{\partial \Pi_u^U}{\partial r_u} \underbrace{+\sum_{j \neq u} \frac{\partial \Pi_j}{\partial r_u}}_{\substack{<0 \\ \text{complements effect}}} + \underbrace{\sum_{d=1}^{n} \frac{\partial \Pi_d}{\partial r_u}}_{\substack{<0 \\ \text{double margin effect} \\ \text{on all dowstream firms}}} = \frac{\partial \Pi}{\partial r_v} = 0 \quad \forall u \in \{l+1,...,m\}
\end{aligned}$$

Thus, the derivative of each firm's profit function at r^{FB} is strictly positive, so each firm has an incentive to further increase its royalty rate. *Q.E.D.*

Proof of Proposition 4: We give two examples to show that the effect can go in both directions. Suppose that $m = n$, i.e. all firms are vertically integrated, all firms are symmetric and compete in quantities downstream. If the demand function is linear, it is easy to compute that

$$r_u^{VI} = \frac{(a-k)\cdot(n+3)}{n^2+4n-1} > \frac{a-k}{n+1} = r_u^{NI}.$$

Thus, with linear demand firms charge higher royalties when they are vertically integrated than when they are not integrated.

If the demand function is given by $P = Q^{-\frac{1}{\eta}}$ (constant price elasticity of demand equal to $\eta < 0$) and if $|\eta| > n$ it can be shown that

$$r_i^{VI} = \frac{k\left(n^2\eta - n + 2\eta n - 1 - \eta\right)}{n\left(2n\eta^2 + n - n^2\eta - \eta n - \eta^2\right)} < \frac{k}{\eta - n} = r_i^{NI},$$

so royalties charged by vertically integrated firms are smaller than under non-integration.[9]

Q.E.D.

Proof of Proposition 5: In equilibrium, the first order condition $\frac{\partial \Pi_u}{\partial r_u} = Q + \frac{\partial Q}{\partial r} \cdot r_u = 0$ has to hold for all upstream firms $u = 1,...,m$. Consider two upstream markets with m^1 and m^2 firms respectively, $m^1 > m^2$. Summing up (1.4) over all firms we get

[9] If $|\eta| \le n$ firms want to raise royalties to infinity, so an equilibrium does not exist.

$$m^1 \cdot Q(r^1) + r^1 \cdot \frac{\partial Q(r^1)}{\partial r} = 0$$

$$m^2 \cdot Q(r^2) + r^2 \cdot \frac{\partial Q(r^2)}{\partial r} = 0$$

where $r^i = \sum_{u=1}^{m} r_u^i$, $i \in \{1,2\}$. Substracting the second equation from the first we have

$$m^1 \cdot Q(r^1) - m^2 \cdot Q(r^2) + r^1 \cdot \frac{\partial Q(r^1)}{\partial r} - r^2 \cdot \frac{\partial Q(r^2)}{\partial r} = 0$$

Suppose that $r^2 \geq r^1$. Assumption 2 is equivalent to the assumption that $\frac{\partial Q}{\partial r} + r\frac{\partial^2 Q}{\partial r^2} < 0$ which implies

$$\int_{r^1}^{r^2} [Q'(r) + rQ''(r)]dr = \left[rQ'(r)\right]_{r^1}^{r^2} = r^2 Q'(r^2) - r^1 Q'(r^1) \leq 0$$

Thus, we must have

$$m^1 \cdot Q(r^1) - m^2 \cdot Q(r^2) \leq 0$$

However, because $m^2 < m^1$ this implies $Q(r^2) > Q(r^1)$ which implies $r^2 < r^1$, a contradiction. Thus, we must have $r^1 > r^2$. Note that $r^1 > r^2$ implies that all downstream firms have lower costs with m^2 than with m^1 upstream firms, so downstream prices are lower and the quantity sold on the downstream market is higher if the number of upstream firms decreases. *Q.E.D.*

<u>Proof of Proposition 6:</u> A horizontally chooses its royalty rate to maximize $\Pi^H = r\,Q^{HI}(r)$. The FOC for this maximization problem is

$$\frac{\partial \Pi^H}{\partial r} = Q^{HI} + \frac{\partial Q^{HI}}{\partial r} \cdot r^H \tag{18}$$

The first order condition for the maximization of total industry profits can be rewritten as:

$$\frac{\partial \Pi}{\partial r} = \underbrace{Q(r^{FI}) + \frac{\partial Q(r^{FI})}{\partial r} \cdot r^{FI}}_{\text{same as under non-integration}} + \underbrace{\sum_{d=1}^{n} \frac{\partial \Pi_d}{\partial r}}_{\substack{<0 \\ \text{internalize double margin effect} \\ \text{on downstream firms}}} = 0 \tag{19}$$

Suppose the horizontally integrated upstream firm chooses the optimal royalty rate under full integration r^{FI}, so total downstream quantity is $Q(r^{FI})$. Then we have

as:

$$\frac{\partial \Pi^H}{\partial r} = Q(r^{FI}) + \frac{\partial Q(r^{FI})}{\partial r} \cdot r^{FI} = \frac{\partial \Pi}{\partial r} - \underbrace{\sum_{d=1}^{n} \frac{\partial \Pi_d}{\partial r}}_{<0} > 0 \tag{20}$$

Hence, the horizontally integrated firm will choose a royalty that is larger than r^{FI} and total quantity on the downstream market will be lower. *Q.E.D.*

Example 2: If all upstream firms are horizontally integrated, total quantity is given by $Q^{HI}(n) = \dfrac{n(a-c)}{2b(n+1)}$. On the other hand, if all m upstream firms are vertically integrated they will set royalties that exclude all independent downstream firms from the market. In this case total quantity is given by $Q^{VI} = \dfrac{2m(a-c)}{b(m(m+3)+(m+1))}$. Note that Q^{HI} is strictly increasing in n while Q^{VI} is strictly decreasing in m. For *n=3* total quantity is higher under HI than total quantity under VI for *m=2*. *Q.E.D.*

Proof of Proposition 8: Note first that in any pure strategy subgame perfect equilibrium all downstream profits must be extracted via the fixed fees. Thus all downstream firms (or downstream divisions of vertically integrated firms) are just indifferent whether or not to produce downstream. Consider an equilibrium candidate where all vertically integrated choose identical linear royalties the sum of which induces the full integration outcome on the downstream market and identical fixed fees the sum of which extracts all profits of the downstream firms/divisions. Let $p^M(k)$ denote the price chosen by a fully integrated monopolist with marginal cost k and let $\Pi^M(k)$ denote his monopoly profit on the downstream market. If the symmetric linear royalties r implement the monopoly price downstream, it must be the case that $(n-1)\cdot r + k < p^M(k)$. Note that each firm makes a profit of $\dfrac{1}{m}\cdot\Pi^M(k)$ in the candidate equilibrium.

Consider now the following deviation of firm 1. It raises its fixed fee to infinity, so that no other firm can afford to license its patent. Thus, firm 1 becomes a monopolist on the downstream market. It has to pay fixed fees to the other firms that are equal to its downstream profit if it had not deviated. Note that this is bounded above by $\dfrac{1}{m}\cdot\Pi^M(k)$. On the other hand, it now monopolizes the downstream market, so it will make at least the profit of a monopolist with marginal cost of $(n-1)\cdot r + k < p^M(k)$. These profits are bounded below by the profits of a monopolist with marginal cost $p^M(k)$ (note that $p^M(k)$ is independent of n). Denote the profit of a monopolist on the downstream market with marginal cost $p^M(k)$ as $\Pi^M\left(p^M(k)\right)=\underline{\Pi}>0$.

Hence, a deviation is profitable if $\underline{\Pi}-\frac{1}{m}\Pi^M(\mathrm{k})>\frac{1}{m}\Pi^M(\mathrm{k}) \quad \Leftrightarrow \quad \underline{\Pi}>\frac{2}{m}\Pi^M(\mathrm{k})$. Note that for all k, $\frac{2}{m}\Pi^M(k)$ goes to zero as m goes to infinity. Thus, there exists an $\overline{m}$ such that for all $m \geq \underline{m}$ the deviation is profitable. Thus, if $m \geq \underline{m}$ there does not exist a symmetric pure strategy subgame perfect equilibrium that implements the monopoly outcome.

Consider now asymmetric pure strategy equilibria. Suppose that only one firm serves the downstream market. In equilibrium this firm must make zero profits. Otherwise the other firms would have an incentive to raise the fixed fees of their royalties. With a monopolist downstream profits are maximized if all upstream firms charge linear royalties of zero and fixed fees equal to the monopoly profit divided by *n-1*. Given these royalties, no firm has an incentive to deviate, so this is indeed a subgame perfect pure strategy equilibrium. In this equilibrium the monopolist serving the downstream market makes zero profits, while all the other firms share the monopoly profit. *Q.E.D.*

Proof of Proposition 9: By Proposition 5 we know that total royalties increase as the number of independent upstream suppliers increases. Thus, total royalties are larger under non-integration or vertical integration than under horizontal integration while $\Pi^e(r)$ is smaller. The probability that entry occurs is $prob(\Pi^E(r)\geq K)=1-prob(\Pi^E(r)\geq K)=1-G(\Pi^E(r))$. The cdf $G(\cdot)$ is strictly increaing, whilie $\Pi^e(r)$ is strictly decreasing. Hence, the probability of entry is higher under horizontal integration. *Q.E.D.*

Proof of Proposition 10: Consider an upstream market with m non-integrated firms. To simplify notation denote $H(F)=G(\Pi^0(0)-F)$. So $H(F)$ is the probability of entry given the total fixed fee F. Thus, each upstream firm maximizes $\Pi_u^{NI}=F_u\cdot H(F)$. In equilibrium, the first order condition

$$\frac{\partial\Pi_u^{NI}}{\partial F_u}=H\left(\sum_{v=1}^{m}F_v^{NI}\right)+F_u^{NI}\cdot H'\left(\sum_{v=1}^{m}F_v^{NI}\right)=0$$

has to hold for all upstream firms $u = 1,...,m$. Assumption 3 is equivalent to the assumption that $\frac{\partial H(F)}{\partial F} + F_u \frac{\partial^2 H(F)}{\partial F^2} < 0 \quad \forall F_u \leq F$ which implies that the second order condition is globally satisfied, so (x) characterizes a unique pure strategy equilibrium. Consider two upstream markets with m^1 and m^2 firms respectively, $m^1 > m^2$. Summing up (x) over all firms we get

$$m^1 \cdot H(F^1) + F^1 \cdot H'(F^1) = 0$$
$$m^2 \cdot H(F^2) + F^2 \cdot H'(F^2) = 0$$

where $F^i = \sum_{u=1}^{m} F_u^i$, $i \in \{1,2\}$. Substracting the second equation from the first we have

$$F^1 \cdot H(F^1) - F^2 \cdot H(F^2) + F^1 \cdot \frac{\partial H(F^1)}{\partial F} - F^2 \cdot \frac{\partial H(F^1)}{\partial F} = 0$$

Suppose that $F^2 \geq F^1$. Assumption 3 implies

$$\int_{F^1}^{F^2} [H'(F) + F \cdot H''(F)]dF = \left[F \cdot H'(F)\right]_{F^1}^{F^2} = F^2 H'(F^2) - F^1 H'(F^1) \leq 0$$

Thus, we must have

$$m^1 \cdot H(F^1) - m^2 \cdot H(F^2) \leq 0$$

However, because $m^2 < m^1$ this implies $H(F^2) > H(F^1)$ which implies $F^2 < F^1$, a contradiction. Thus, we must have $F^1 > F^2$. *Q.E.D.*

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