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Abstract

We analyze the optimal design of rank-order tournaments with heterogeneous workers. If tournament prizes do not differ between the workers (uniform prizes), as in the previous tournament literature, the outcome will be inefficient. In the case of limited liability, the employer may benefit from implementing more than first-best effort. We show that the employer can use individual prizes that satisfy a self-commitment condition and induce efficient incentives at the same time, thus solving a fundamental dilemma in tournament theory. Individual prizes exhibit two major advantages – they allow the extraction of worker rents and the adjustment of individual incentives, which will be important for the employer if he cannot rely on handicaps.

Key Words: heterogeneous workers, limited liability, rank-order tournaments, self commitment.

JEL Classification: J33, M12, M52.

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1 Introduction

Rank-order tournaments where workers compete for given prizes or the distribution of a fixed amount of bonuses are frequently used in practice. For example, managers face relative compensation schemes (Gibbons and Murphy, 1990; Eriksson 1999), workers compete for job-promotion in corporate hierarchies (Baker et al. 1994), salesmen are compensated according to relative performance (Murphy et al. 2004), and workers compete for higher shares in bonus-pool arrangements (see Kanemoto and MacLeod 1991, 1992 for Japanese and Rajan and Reichelstein 2006 for US firms). Basically, rank-order tournaments always occur when relative worker performance is linked to monetary consequences. Thus, forced-ranking systems also belong to the class of tournament compensation schemes. Here, supervisors rate their subordinates according to relative performance given a fixed distribution of different grades that can be assigned to the workers. Boyle (2001) points out that about 25 percent of the Fortune 500 companies apply forced-ranking systems to generate incentives (e.g., Cisco Systems, Intel, Sun Microsystems, Conoco, General Electric, Enron). Forced-ranking systems can even be used to create a kind of dismissal tournament. The most prominent advocate of this idea is the former General Electric CEO Jack Welch who constituted that the least 10 percent of the rated workers must leave General Electric each year.[1]

Whereas most of the tournament models analyze competition between homogeneous workers, the case of heterogeneous contestants seems to be the more relevant one in practice.[2] Our paper focuses on the optimal design of tournaments between two heterogeneous workers. In the first part of the paper, we follow the previous tournament literature by analyzing a setting where the employer chooses uniform winner and loser prizes for the workers,

[1] Note that a similar system has been used by Enron. Here, workers are rated on a scale of 1 to 5 and those belonging to grade 5 were typically fired within the next half of the year.

[2] "Most contests in this world are among unequal contestants" (O'Keeffe et al. 1984, p. 42).

that is winner and loser prizes do not depend on the identity of the respective winner or loser. Note that this implies the sum of tournament prizes to be fixed. In turn, the employer cannot save wage costs by misrepresenting the performance signal about the tournament winner and the tournament scheme even works if the performance signal is not verifiable to a third party. This is the important self-commitment property of tournaments highlighted by Malcomson (1984, 1986). Despite this property we show that even under unlimited liability a tournament with uniform prizes does not lead to first-best efforts. This implementation would be too costly for the employer as the more able worker still earns a positive rent. If workers are protected by limited liability, the employer may benefit from implementing more than first best efforts. He can use the worker competition to elicit high effort levels and, at the same time, to decrease the workers' rents. The employer will be interested to behave in this way, if heterogeneity between the workers is not too large because otherwise creating incentives would again be rather expensive for him.

The case of uniform prizes identifies a fundamental dilemma of tournament theory: on the one hand, a tournament with uniform prizes satisfies Malcomson's self-commitment constraint. On the other hand, it does not lead to first-best efforts so that there is an efficiency loss. In the second part of the paper, we show that tournaments with individual prizes that differ among workers (e.g., if worker A is declared tournament winner he may receive another prize than worker B in case of winning) can solve this dilemma. In particular, our results show that under unlimited liability the employer sets individual tournament prizes that (1) exhibit Malcomson's self-commitment property and (2) implement efficient effort levels. The additional self-commitment condition renders impossible for the employer to adjust individual incentives in the tournament. Nevertheless, the employer benefits from individual prizes because they can be used to extract rents from the workers. Under unlimited liability, this is all he needs to achieve efficiency. Under limited liability, we show that individual tournament prizes will work even better if the self-commitment condition can be relaxed due to employer reputation, for example. Then the prizes can also be used as a substitute for handicaps to adjust individual incentives so that the heterogeneous competition becomes less uneven.

Our paper is related to those few tournament models that also address the problem of heterogeneous workers. The seminal paper by Lazear and Rosen (1981) was the first one that points to the inefficient outcome of tournaments between heterogeneous workers.[3] If the employer can observe the workers' abilities and use handicaps, he will be able to restore efficient incentives. In our model, there only exists a binary signal on the relative performance of the two workers, which makes the use of handicaps impossible. O'Keeffe et al. (1984) and Bull et al. (1987) distinguish two alternatives of modeling heterogeneous contestants. On the on hand, workers' production functions may be additive in effort and individual ability. On the other hand, workers may have identical production functions but different cost functions. We adopt the first approach with performance being additive in effort and ability.[4] This setting allows a separate analysis of the two major advantages of individual tournaments prizes – extraction of worker rents and adjusting individual incentives. Note, however, that both settings do not fundamentally differ. Even if workers have different cost functions, the employer can implement the first-best solution by using individual prizes that satisfy the self-commitment constraint.

The paper is organized as follows. Section 2 introduces the model. In Section 3, we derive the optimal tournament contract under uniform prizes. The case of individual prizes is addressed by Section 4. In Subsection 4.1 the employer must satisfy a self-commitment constraint. The optimal tournament contract without this constraint is derived in Subsection 4.2. Section 5 concludes.

[3] See also McLaughlin (1988, pp. 243-247).

[4] This approach is also used by Meyer and Vickers (1997) and Höffler and Sliwka (2003), for example.

2 The Model

Two risk neutral workers A and B are hired by a risk neutral employer E. The two workers differ in their abilities a_A and a_B that influence relative performance. Let $\Delta a = a_A - a_B$ denote the workers' ability difference and let w.l.o.g. worker A be the more able one so that $\Delta a > 0$.[5] E cannot directly observe performance but receives an unverifiable relative performance signal s that can take two possible values. We have either $s = s_A$ indicating that worker A has performed better than B or $s = s_B$ indicating the opposite. Note that such unverifiable, ordinal binary signal renders both impossible the use of explicit incentives (e.g., based on piece rates) and the use of individual handicaps. E can only rely on a simple tournament in which prizes are allocated according to realized rank. The signal structure can be characterized as follows:

$$s = \begin{cases} s_A & \text{if} \quad e_A - e_B + \Delta a > \varepsilon \\ s_B & \text{if} \quad e_A - e_B + \Delta a < \varepsilon. \end{cases} \tag{1}$$

Here, e_i $(i = A, B)$ describes the effort choice of worker i, and ε an exogenous random term (e.g., measurement error) with density $g(\varepsilon)$ and cumulative distribution function $G(\varepsilon)$. The density $g(\varepsilon)$ is assumed to be unimodal and symmetric around zero.[6] Intuitively, the higher worker i's effort choice the more likely the employer will receive the signal $s = s_i$. In addition, worker B is less likely to win the tournament due to his ability deficit Δa. Effort e_i entails costs on worker i $(i = A, B)$ that are described in monetary terms by the function $c(e_i)$ with $c'(e_i), c''(e_i), c'''(e_i) > 0, \forall e_i > 0$, and $c'(0) = c(0) = 0$. We assume that each worker has a zero reservation value and is wealth-constrained so that his liability is limited to $\bar{w}$.

Employer E is the organizer of the rank-order tournament. In the following, we will discuss two different designs $D \in \{UP, IP\}$. On the one hand, E can fix *uniform prizes* so that the tournament winner receives w_1 and the

[5] Alternatively, we can think of Δa as worker A's lead compared to worker B.

[6] The assumption of a unimodal distribution is not unusual in tournament models; see, e.g., Dixit (1987), Drago et al. (1996), Hvide (2002), Chen (2003).

loser $w_2 < w_1$ irrespective of which worker has performed best ($D = UP$). On the other hand, E can choose *individual prizes* for the winner and the loser of the tournament so that worker A (B) receives w_{1A} (w_{1B}) if $s = s_A$ ($s = s_B$) but only w_{2A} (w_{2B}) if $s = s_B$ ($s = s_A$) with $w_{2i} < w_{1i}$ ($i = A, B$) ($D = IP$). Because of the limited-liability assumption, which has been introduced in the last paragraph, the loser prizes w_2, w_{2A} and w_{2B} are restricted to values equal or larger than $-\bar{w}$. Note that under uniform prizes the important self-commitment property of tournaments – emphasized by Malcomson (1984, 1986) – immediately applies whereas under individual prizes the employer may be confronted with a credibility problem: if tournament prizes are individually different E may save labor costs by claiming that worker i is the winner although $s = s_j$ ($i, j = A, B$; $i \neq j$). This credibility problem of the design $D = IP$ will be discussed in details in the following sections. E maximizes profits π that consist of the sum of the workers' efforts, $e_A + e_B$, minus tournament prizes.

As a benchmark, we can calculate the workers' first-best efforts. In case of efficient production, both workers would maximize $e_A + e_B - c(e_A) - c(e_B)$ so that each worker's first-best effort e^{FB} is described by

$$1 = c'\left(e^{FB}\right). \tag{2}$$

Finally, to assure existence of pure-strategy equilibria in the tournament we assume that[7]

$$\sup_{\Delta e} \Delta w \cdot |g'(\Delta e + \Delta a)| < \inf_{e>0} c''(e) \tag{3}$$

with $\Delta e := e_A - e_B$, and Δw denoting the spread between winner and loser prize under the respective tournament design.

[7] For a similar condition see Schöttner (2007).

3 Tournaments with Uniform Prizes

We start by considering the standard case of uniform tournament prizes. Under $D = UP$, the employer has no incentive to misrepresent the observed performance signal s as in any case the worker who is declared winner receives w_1 whereas the loser gets w_2. Hence, Malcomson's self-commitment property of tournaments with unverifiable signals is satisfied.

The optimal tournament contract (w_1^*, w_2^*) under $D = UP$ results from solving a two-stage game where E fixes the tournament prizes at stage 1 and workers observe the prizes and simultaneously choose efforts at stage 2. We work backwards, starting with the tournament competition between workers A and B. Worker A (B) is declared winner (loser) of the tournament with probability prob$\{s = s_A\} = G(e_A - e_B + \Delta a)$ according to (1). With probability $1 - G(e_A - e_B + \Delta a)$ worker B (A) becomes the winner (loser). Worker A maximizes

$$EU_A(e_A) = w_2 + \Delta w G(e_A - e_B + \Delta a) - c(e_A)$$

and worker B

$$EU_B(e_B) = w_2 + \Delta w [1 - G(e_A - e_B + \Delta a)] - c(e_B)$$

with $\Delta w := w_1 - w_2$ denoting the prize spread. The equilibrium (e_A^*, e_B^*) is described by the first-order conditions[8]

$$c'(e_A^*) = \Delta w g(e_A^* - e_B^* + \Delta a) = c'(e_B^*).$$

Hence, the equilibrium is symmetric with $(e_A^*, e_B^*) = (e^*, e^*)$ given by

$$\Delta w g(\Delta a) = c'(e^*). \tag{4}$$

Equation (4) shows that equilibrium effort e^* will be larger the higher the

[8] Recall that (3) guarantees existence.

prize spread and the smaller the degree of heterogeneity between the workers (measured by Δa).[9]

At stage 1, E anticipates the workers' behavior characterized by (4) and chooses optimal prizes that maximize $\pi(w_1, w_2) = 2e^* - w_1 - w_2$. Besides the incentive constraint (4), E has to take notice of the limited-liability constraint

$$w_2 \geq -\bar{w} \tag{5}$$

and the participation constraints of the workers. Note that the participation constraint for worker A can be neglected because he unambiguously earns a positive rent: we have $EU_A(e^*) > EU_B(e^*) \geq 0$ due to $\Delta a > 0$. Hence, only worker B's constraint matters:

$$w_2 + \Delta w \left[1 - G(\Delta a)\right] - c(e^*) \geq 0. \tag{6}$$

To solve E's optimization problem we set up the Lagrange function

$$\begin{aligned} L(e^*, w_1, w_2) &= 2e^* - w_1 - w_2 + \lambda_1 \cdot \left[w_2 + \Delta w \left[1 - G(\Delta a)\right] - c(e^*)\right] \\ &\quad + \lambda_2 \cdot \left[\Delta w g(\Delta a) - c'(e^*)\right] + \lambda_3 \cdot \left[w_2 + \bar{w}\right] \end{aligned}$$

with $\lambda_1, \lambda_3 \geq 0$ and λ_2 as multipliers. The optimality conditions with respect to e^*, w_1, and w_2 yield

$$\frac{\partial L}{\partial e^*} = 2 - \lambda_1 c'(e^*) - \lambda_2 c''(e^*) = 0 \tag{7}$$

$$\frac{\partial L}{\partial w_1} = -1 + \lambda_1 \left[1 - G(\Delta a)\right] + \lambda_2 g(\Delta a) = 0 \tag{8}$$

$$\frac{\partial L}{\partial w_2} = -1 + \lambda_1 G(\Delta a) - \lambda_2 g(\Delta a) + \lambda_3 = 0. \tag{9}$$

Conditions (8) and (9) together give $\lambda_1 + \lambda_3 = 2$, indicating that either the limited-liability constraint or the participation constraint or both are binding. This yields three different cases that are presented in the following

[9] Note that the last result follows from $g(\cdot)$ having a unique mode at zero.

proposition:[10]

Proposition 1 *There exist two cutoff-values* $\bar{w}_1$ *and* $\bar{w}_2$ *for* $\bar{w}$ *with* $\bar{w}_1 > \bar{w}_2$ *so that the following results hold: If* $\bar{w} > \bar{w}_1$*, then only the participation constraint (6) is binding and* E *implements the effort level* $e_1^* < e^{FB}$ *with*

$$c''(e_1^*) = 2g(\Delta a)\frac{1 - c'(e_1^*)}{2G(\Delta a) - 1}. \tag{10}$$

If $\bar{w}_1 > \bar{w} > \bar{w}_2$*, then both participation constraint (6) and limited-liability constraint (5) are binding and* E *implements effort* e_2^* *with*

$$c(e_2^*) = \frac{1 - G(\Delta a)}{g(\Delta a)}c'(e_2^*) - \bar{w}. \tag{11}$$

If $\bar{w} < \bar{w}_2$*, then only the limited-liability constraint (5) is binding and* E *implements effort* e_3^* *with* $c''(e_3^*) = 2g(\Delta a)$*. We have either* $e^{FB} > e_1^* > e_2^* > e_3^*$ *or* $e_1^* < e_2^* < e_3^*$*. In the latter case,* e_2^* *and* e_3^* *may be larger than* e^{FB}*. Particularly, if* $c'(c''^{-1}(2g(\Delta a))) > 1$*, then* $e_3^* > e^{FB}$*.*

Proof. See Appendix A. ■

While the less able worker will only earn a rent if his initial wealth $\bar{w}$ is sufficiently small, the more able worker always realizes a positive rent. Therefore, E does typically not implement first-best effort. Most interestingly, in those cases where the limited-liability constraint is binding the employer may implement more than first-best effort. To understand this result, note that the strength of incentives entirely depends on the winner prize if the limited-liability constraint binds. A higher winner prize in turn affects the workers' rents in three ways. First, it naturally increases these rents since the wage payments to the workers increase. Second, a worker chooses a higher effort which further increases his rent as otherwise the worker would have stuck to the initial effort. Finally, a worker's rent is reduced since his

[10] $c''^{-1}(\cdot)$ denotes the inverse of the second derivative $c''(\cdot)$.

opponent increases effort, too. If the third effect dominates, the workers' rents are decreasing in the strength of incentives. Then E may gain from implementing an inefficiently high effort just to reduce the workers' rents. Note that in a single-worker context the third effect disappears. This implies that the worker's rent always increases in the strength of incentives if the limited-liability constraint binds. Therefore, the employer never implements an inefficiently high effort in the single-worker case.

Consider, for example, the case of implementing $e^* = e_3^*$ in Proposition 1, where only the limited-liability constraint (5) is binding. Conditions (4) and (5) lead to the optimal tournament prizes

$$w_2^* = -\bar{w} \quad \text{and} \quad w_1^* = \frac{c'(e_3^*)}{g(\Delta a)} - \bar{w}.$$

Inserting into the expression for worker B's rent yields

$$R_B = w_2 + \Delta w [1 - G(\Delta a)] - c(e_3^*) = -\bar{w} + \frac{c'(e_3^*)}{g(\Delta a)} [1 - G(\Delta a)] - c(e_3^*).$$

From Proposition 1 we know that implementing $e_3^* > e^{FB}$ implies $e_3^* > e_1^*$. The proof of the proposition in Appendix A shows that in this case we have

$$\frac{\partial R_B}{\partial e_3^*} = \frac{c''(e_3^*)}{g(\Delta a)} [1 - G(\Delta a)] - c'(e_3^*) < 0.$$

The last result formalizes our argument: if workers earn positive rents it may pay for the employer to use very high-powered incentives for implementing inefficiently high efforts. The employer's labor costs for such policy are not too large: since the participation constraint is not binding, the employer need not compensate the workers for the additional disutility of effort, which is convex. The employer only has to pay higher tournament prizes, which are linear in money. The excessive costs of extra effort, $c(e_3^*) - c\left(e^{FB}\right)$, only reduce the workers' rents.

Finally, note that $e_3^* > e^{FB}$ can only be satisfied if $g(\Delta a)$ is sufficiently large and, therefore, Δa sufficiently small.[11] In other words, only if the degree of heterogeneity between the two workers is not too large and hence competition not too uneven, the employer may prefer to create incentives in excess of first-best effort. For sufficiently small values of Δa the incentives are quite high as Δa determines the workers' marginal winning probability in equilibrium (see equation (4)). In this case, creating incentives is not too expensive for the employer. We can show that the condition $e_3^* > e^{FB} \Leftrightarrow c'\left(c''^{-1}\left(2g\left(\Delta a\right)\right)\right) > 1$ in Proposition 1 can be satisfied without violating the existence condition (3). For this purpose, we have to specify the cost function and the probability distribution. Let the workers' cost function be exponential and described by $c\left(e_i\right) = \exp\left\{c \cdot e_i\right\} - 1$ with $c > 0$ $(i = A, B)$.[12] The noise term is assumed to be normally distributed with $\varepsilon \sim N\left(0, \sigma^2\right)$. For this parameterized setting, the condition $e_3^* > e^{FB}$ becomes[13]

$$\ln\left(\frac{2}{c\sqrt{2\pi\sigma^2}}\right) > \frac{\Delta a^2}{2\sigma^2}. \tag{12}$$

Obviously, Δa has to be sufficiently small to make this inequality hold. For the given cost function and the given distribution, the existence condition (3) becomes

$$\frac{2}{c\sigma^2\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\right\} < c^2. \tag{13}$$

We can easily check that conditions (12) and (13) are satisfied at the same time for a range of parameter constellations (e.g., for $c = 1.5$, $\sigma = 0.5$, $\Delta a = 0.01$).

To sum up, on the one hand we have seen that under uniform tournament prizes ($D = UP$) the employer typically does not implement efficient effort levels – if the limited-liability constraint is not binding E will implement inefficiently small efforts, otherwise optimal efforts are either smaller

[11] Recall that the density $g\left(\cdot\right)$ has a unique mode at zero, implying $g'\left(\Delta a\right) < 0$ as $\Delta a > 0$.

[12] Exponential cost functions are also used elsewhere in the literature. See, for example, Tadelis (2002).

[13] The calculations for the following conditions are relegated to Appendix B.

or larger than first-best effort. On the other hand, the tournament design $D = UP$ exhibits Malcomson's important self-commitment property so that tournament incentives work even under unverifiable performance signals.

4 Tournaments with Individual Prizes

In this section, we consider the tournament design $D = IP$, where the winner prize and the loser prize depend on the identity of winner and loser, respectively. E pays the winner prize w_{1A} (w_{1B}) if worker A (B) is declared winner of the tournament whereas the loser receives w_{2A} (w_{2B}) if $s = s_B$ ($s = s_A$) with $w_{2i} < w_{1i}$ ($i = A, B$). Note that the identity of the declared winner is verifiable, but E can misrepresent the unverifiable performance signal s to save labor costs. This problem could be eliminated if the sum of winner prize and loser prize is the same irrespective of who is declared winner of the tournament. In the following, we will derive the optimal tournament contract under this additional self-commitment constraint and without the constraint.

4.1 Optimal Tournament Contract with Self-Commitment Constraint

If the sum of loser and winner prize is fixed, we have the condition

$$w_{1A} + w_{2B} = w_{1B} + w_{2A} \tag{14}$$

as the employer's *self-commitment constraint*. Note that this can be transformed into $w_{1A} - w_{2A} = w_{1B} - w_{2B} =: \Delta\tilde{w}$. Although absolute prizes may differ between contestants, the fixing of the total payroll implies that the prize difference is the same for A and B. This means that both workers still have the same incentive to win the tournament and choose the same effort $\tilde{e}$

characterized by the incentive constraint (4):

$$\Delta\tilde{w}g\left(\Delta a\right) = c'\left(\tilde{e}\right).$$

At stage 1, E maximizes $\tilde{\pi} = 2\tilde{e} - w_{1A} - w_{2B} = 2\tilde{e} - \Delta\tilde{w} - w_{2A} - w_{2B}$ by implementing an optimal effort level $\tilde{e}^*$. Doing this, he has to consider the incentive constraint, the limited-liability constraints and the participation constraints of the workers. Now both participation constraints are relevant since wages for the workers may differ. To obtain clear and concise results we restrict our attention to the two polar cases where either both participation constraints or both limited-liability constraints are binding. The remaining intermediate cases could be derived in a similar way as in the previous section.

Proposition 2 *Let $\bar{w} \geq \frac{G(\Delta a)}{g(\Delta a)} - c(e^{FB})$. Then, both workers' participation constraints are binding and the employer implements the first-best effort, $\tilde{e}^* = e^{FB}$.*

Proof. See Appendix C. ■

Proposition 2 shows that the employer may implement the first-best solution by using individual prizes even if the total prize sum is fixed and handicaps are not feasible. This differs strongly from the findings in Proposition 1 where the first-best solution was not attainable. The intuition behind Proposition 2 is that E can fully extract the rent of worker A by reducing both his loser and his winner prize. Then, E receives the complete surplus to be produced and implements the effort that maximizes this surplus, i.e. e^{FB}. Note, however, that this is only feasible if the workers are so wealthy that A's limited-liability constraint is not violated. This requires $\bar{w} \geq \frac{G(\Delta a)}{g(\Delta a)} - c(e^{FB})$.

At first sight, the result of Proposition 2 seems to be rather special since first-best effort of each worker is the same and the equilibrium at the tournament stage is symmetric due to the uniform prize spread $\Delta\tilde{w}$. However, we can show that introducing individual cost functions $c_i\left(e_i\right)$ $(i = A, B)$, which lead to asymmetric equilibria in the rank-order tournament, and individual reservation values $\bar{u}_i$ $(i = A, B)$ will not qualitatively change the finding of Proposition 2:

Corollary 1 *Let the workers A and B have different cost functions* $c_A(e_A)$ *and* $c_B(e_B)$*, and different reservation values* $\bar{u}_A \neq \bar{u}_B$*. If the workers are sufficiently wealthy, the optimal tournament contract will still implement first-best effort for each worker.*

Proof. See Appendix D. ∎

The result of this corollary is important since it solves a fundamental dilemma of tournament theory: on the one hand, previous models solely rely on uniform prizes so that the self-commitment property of tournaments applies; on the other hand, mixed tournaments between heterogeneous contestants lead to an inefficient outcome under uniform prizes.[14] The result of Corollary 1 points out that the employer gets rid of this problem by using individual tournament prizes that both satisfy the self-commitment condition (14) and implement efficient effort levels for both workers. As the employer receives total efficiency gains he will optimally choose this tournament contract.

Now, we turn to the other polar case. If workers are not very wealthy both limited-liability constraints will be binding. Then individual prizes do not perform better than uniform ones.

Proposition 3 *Let* $\bar{w} \leq \frac{(1-G(\Delta a))c'(\tilde{e}^*)}{g(\Delta a)} - c(\tilde{e}^*)$*. Then both workers' limited-liability constraints are binding and* $w_{1A}^* = w_{1B}^*$, $w_{2A}^* = w_{2B}^* = -\bar{w}$*. The employer implements effort* $\tilde{e}^*$ *defined by* $c''(\tilde{e}^*) = 2g(\Delta a)$*.*

Proof. See Appendix C. ∎

Proposition 3 shows that individual prizes lead to the same outcome as uniform ones if both limited-liability constraints bind. Here, E cannot reduce

[14] See Proposition 1. On the inefficiency of mixed tournaments see also Lazear and Rosen (1981) and McLaughlin (1988, pp. 243-247).

worker A's wages since lowering the loser prize is prevented by the worker's limited wealth. Accordingly, the employer is not able to extract even a small part of A's rent. Note that the cutoff for $\bar{w}$ in Proposition 3 is identical with $\bar{w}_2$ in Proposition 1 (see Appendix A). Hence, if the workers' initial wealth is smaller than $\bar{w}_2$ the employer will be indifferent between uniform and individual prizes as both tournament designs yield exactly the same (inefficient) effort level. In fact, optimal individual prizes are the same for both contestants, i.e. we have a uniform prize structure.

In all intermediate cases where not both participation constraints or both limited-liability constraints bind, we obtain a contract that lies between the two polar cases of Propositions 2 and 3. In particular, individual prizes help the employer to increase his profit compared to uniform ones, while the first-best solution is not implemented. The proof is by solving the Kuhn-Tucker conditions derived in Appendix C. As it is very similar to the procedure in the proof of Proposition 1, we leave it out.

To summarize the findings of this subsection, individual prizes that satisfy the self-commitment property (14) (weakly) dominate uniform prizes from the employer's viewpoint. The self-commitment property implies that under tournament design $D = IP$ each worker faces the same prize spread so that individual prizes cannot be used to make the tournament competition less uneven by using individually adjusted incentives. In other words, because of the additional condition (14) individually tournament prizes cannot serve as a substitute for handicaps, which were impossible in the given setting. Nevertheless, the employer prefers the design $D = IP$ to $D = UP$ since individual prizes will be useful to extract rents from the workers if they are sufficiently wealthy.

4.2 Optimal Tournament Contract without Self-Commitment Constraint

In this subsection, we relax the restriction (14) that the total payroll is fixed. This implies that incentives and efforts may now differ between contestants.

This assumption may be justified if the workers stay for more than one period within the firm so that an opportunistic decision by the employer would trigger a punishment in terms of lower future efforts by the workers. E may then not want to misrepresent the tournament outcome, as otherwise the workers would lose trust in him.[15]

Again, we restrict our attention to the two polar cases where either the workers' initial wealth is very high or very low. Building on the analysis of the previous subsection, the former case is straightforward to solve: we have seen that the first-best solution is implemented if the players are sufficiently wealthy. Of course, this result continues to hold since we have removed a constraint from the optimization problem and, hence, make it easier to achieve the efficient solution.

The solution to the latter case, which will be discussed in the remainder of this subsection, is not straightforward. To simplify notation let the workers initial wealth be limited to $\bar{w} = 0$ so that the limited-liability constraints are $w_{2A} \geq 0$ and $w_{2B} \geq 0$. Note that in this case we can ignore the workers' participation constraints: by accepting the contract and choosing zero effort each worker can ensure himself a non-negative expected income, which is at least as large as his zero reservation value. Of course, the workers may even increase their expected incomes by choosing strictly positive effort levels. Altogether, under the limited-liability constraints $w_{2A} \geq 0$ and $w_{2B} \geq 0$ both workers will always accept the offered contract and realize nonnegative rents.

Without self-commitment constraint (14), each worker i ($i = A, B$) faces an individual prize spread $\Delta w_i := w_{1i} - w_{2i}$ where Δw_A may be different from Δw_B. At stage 2, the workers now maximize

$$\begin{aligned} EU_A(e_A) &= w_{2A} + \Delta w_A G(e_A - e_B + \Delta a) - c(e_A) \\ \text{and} \quad EU_B(e_B) &= w_{2B} + \Delta w_B [1 - G(e_A - e_B + \Delta a)] - c(e_B) \end{aligned}$$

[15]See MacLeod (2003) for very similar arguments.

leading to the first-order conditions that characterize the equilibrium (e_A^*, e_B^*):

$$F_1 := \Delta w_A g\left(\Delta a + e_A^* - e_B^*\right) - c'\left(e_A^*\right) = 0 \tag{15}$$

and $$F_2 := \Delta w_B g\left(\Delta a + e_A^* - e_B^*\right) - c'\left(e_B^*\right) = 0. \tag{16}$$

Obviously, if $\Delta w_A \neq \Delta w_B$ we will no longer have a symmetric equilibrium in the tournament game. Still, we can analyze the workers' interaction with respect to changes in the two prize spreads Δw_A and Δw_B. By implicitly differentiating the system of equations (15) and (16) we obtain

$$\begin{aligned} |J| &= \begin{vmatrix} \frac{\partial F_1}{\partial e_A^*} & \frac{\partial F_1}{\partial e_B^*} \\ \frac{\partial F_2}{\partial e_A^*} & \frac{\partial F_2}{\partial e_B^*} \end{vmatrix} = \begin{vmatrix} EU_A''\left(e_A^*\right) & -\Delta w_A g'\left(\Delta a + e_A^* - e_B^*\right) \\ \Delta w_B g'\left(\Delta a + e_A^* - e_B^*\right) & EU_B''\left(e_B^*\right) \end{vmatrix} \\ &= EU_A''\left(e_A^*\right) \cdot EU_B''\left(e_B^*\right) + \Delta w_A \Delta w_B \left[g'\left(\Delta a + e_A^* - e_B^*\right)\right]^2 > 0 \end{aligned}$$

for the Jacobian determinant with $EU_A''\left(e_A^*\right) = \Delta w_A g'\left(\Delta a + e_A^* - e_B^*\right) - c''\left(e_A^*\right) < 0$ and $EU_B''\left(e_B^*\right) = -\Delta w_B g'\left(\Delta a + e_A^* - e_B^*\right) - c''\left(e_B^*\right) < 0$ due to (3), and

$$\begin{aligned} \frac{\partial e_A^*}{\partial \Delta w_A} &= \frac{1}{|J|} \begin{vmatrix} -\frac{\partial F_1}{\partial \Delta w_A} & \frac{\partial F_1}{\partial e_B^*} \\ -\frac{\partial F_2}{\partial \Delta w_A} & \frac{\partial F_2}{\partial e_B^*} \end{vmatrix} = \frac{-g \cdot \left(-\Delta w_B g' - c''\left(e_B^*\right)\right)}{|J|} > 0 \\ \frac{\partial e_A^*}{\partial \Delta w_B} &= \frac{1}{|J|} \begin{vmatrix} -\frac{\partial F_1}{\partial \Delta w_B} & \frac{\partial F_1}{\partial e_B^*} \\ -\frac{\partial F_2}{\partial \Delta w_B} & \frac{\partial F_2}{\partial e_B^*} \end{vmatrix} = -\frac{g \cdot \Delta w_A \cdot g'}{|J|} \\ \frac{\partial e_B^*}{\partial \Delta w_A} &= \frac{1}{|J|} \begin{vmatrix} \frac{\partial F_1}{\partial e_A^*} & -\frac{\partial F_1}{\partial \Delta w_A} \\ \frac{\partial F_2}{\partial e_A^*} & -\frac{\partial F_2}{\partial \Delta w_A} \end{vmatrix} = \frac{g \cdot \Delta w_B \cdot g'}{|J|} \\ \frac{\partial e_B^*}{\partial \Delta w_B} &= \frac{1}{|J|} \begin{vmatrix} \frac{\partial F_1}{\partial e_A^*} & -\frac{\partial F_1}{\partial \Delta w_B} \\ \frac{\partial F_2}{\partial e_A^*} & -\frac{\partial F_2}{\partial \Delta w_B} \end{vmatrix} = \frac{-g \cdot \left(\Delta w_A g' - c''\left(e_A^*\right)\right)}{|J|} > 0 \end{aligned}$$

for the comparative statics with $g := g\left(\Delta a + e_A^* - e_B^*\right)$ and $g' := g'(\Delta a + e_A^* - e_B^*)$. Therefore, each worker's equilibrium effort increases in his own prize spread whereas a worker's reaction to an increase in his opponent's prize spread depends on whether this increase makes competition more or less uneven. Consider, for example, the case of $\partial e_A/\partial \Delta w_B$. If $\Delta a + e_A > e_B$ we are at the right-hand side of the probability distribution where we have $g' < 0$ because of g's unique mode at zero. In this situation, initially worker A is the stronger player. If now worker B's prize spread increases, competition will become less uneven. Consequently, B increases his effort and A increases his effort as well: $\partial e_A/\partial \Delta w_B > 0$.

At stage 1, E maximizes his expected profit

$$\begin{aligned}\pi &= e_A^* + e_B^* - G\left(\Delta a + e_A^* - e_B^*\right)\left(w_{1A} + w_{2B}\right) \\ &\quad - \left[1 - G\left(\Delta a + e_A^* - e_B^*\right)\right]\left(w_{1B} + w_{2A}\right) \\ &= e_A^* + e_B^* + \left(\Delta w_B - \Delta w_A\right) G\left(\Delta a + e_A^* - e_B^*\right) - w_{1B} - w_{2A}.\end{aligned}$$

Recall that $e_i^* = e_i^*\left(\Delta w_i, \Delta w_j\right)$ $(i = A, B)$. When deriving the optimal tournament prizes, first consider w_{1A} and w_{2A}. In the optimum, we must have that $\partial\pi/\partial w_{1A} \leq 0$ and $\partial\pi/\partial w_{2A} \leq 0$. Since $\partial\Delta w_A/\partial w_{1A} = -\partial\Delta w_A/\partial w_{2A}$, from E's objective function we obtain

$$\frac{\partial\pi}{\partial w_{2A}} = -\frac{\partial\pi}{\partial w_{1A}} - 1.$$

E will always choose an interior solution for the winner prize (that is $w_{1A}^* > 0$) since zero incentives cannot be optimal because of $c'(0) = 0$. Hence, $\partial\pi/\partial w_{1A} = 0$, which implies $\partial\pi/\partial w_{2A} = -1 < 0$ and, therefore, a corner solution for the loser prize: $w_{2A}^* = 0$. Analogously, we get $w_{2B}^* = 0$ and $w_{1B}^* > 0$. This is intuitive. If E wants to lower the incentives of a worker, it is always cheaper to decrease the winner prize than to increase the loser prize. Therefore, the two loser prizes are set equal to the lowest possible level.

By inserting $w_{2A}^* = w_{2B}^* = 0$, E's objective function boils down to

$$\pi = e_A^* + e_B^* + (w_{1B} - w_{1A})\, G(\Delta a + \Delta e^*) - w_{1B} \quad \text{where } \Delta e^* := e_A^* - e_B^*$$
$$\text{with } \frac{\partial \Delta e^*}{\partial w_{1A}} = \frac{g \cdot c''(e_B)}{|J|} > 0 \text{ and } \frac{\partial \Delta e^*}{\partial w_{1B}} = -\frac{g \cdot c''(e_A)}{|J|} < 0.$$

The first-order conditions for the optimal winner prizes are

$$\begin{aligned}
\frac{\partial \pi}{\partial w_{1A}} &= \frac{\partial e_A^*}{\partial w_{1A}} + \frac{\partial e_B^*}{\partial w_{1A}} + (w_{1B} - w_{1A}) \frac{\partial \Delta e^*}{\partial w_{1A}} \cdot g - G = 0 \\
\frac{\partial \pi}{\partial w_{1B}} &= \frac{\partial e_A^*}{\partial w_{1B}} + \frac{\partial e_B^*}{\partial w_{1B}} + (w_{1B} - w_{1A}) \frac{\partial \Delta e^*}{\partial w_{1B}} \cdot g - (1 - G) = 0
\end{aligned}$$

with $G := G(\Delta a + \Delta e^*)$. These conditions can be simplified to

$$\begin{aligned}
\frac{\partial \pi}{\partial w_{1A}} &= g \cdot \left(\frac{2w_{1B}g' + c''(e_B)(1 + (w_{1B} - w_{1A})\, g)}{|J|} - \frac{G}{g} \right) = 0 \\
\frac{\partial \pi}{\partial w_{1B}} &= g \cdot \left(\frac{-2w_{1A}g' + c''(e_A)(1 - (w_{1B} - w_{1A})\, g)}{|J|} - \frac{1 - G}{g} \right) = 0.
\end{aligned}$$

Inspection of the last two equations leads to the following result:

Proposition 4 *If $\bar{w} = 0$ and E does not have to consider the self-commitment constraint the optimal prizes are $w_{2A}^* = w_{2B}^* = 0$ and $w_{1B}^* > w_{1A}^* > 0$, implying $e_A^* < e_B^*$.*

Proof. The claim $w_{2A}^* = w_{2B}^* = 0$ has already been proved. To prove that $w_{1B}^* > w_{1A}^*$ it suffices to show that

$$\left. \frac{\partial \pi}{\partial w_{1B}} \right|_{w_{1A} = w_{1B}} > \left. \frac{\partial \pi}{\partial w_{1A}} \right|_{w_{1A} = w_{1B}}, \forall w_{1A} = w_{1B} \tag{17}$$

Since $w_{1A} = w_{1B} =: w$ implies $e_i^* = e_j^* =: \check{e}^*$ we have

$$\begin{aligned}
\left. \frac{\partial \pi}{\partial w_{1A}} \right|_{w_{1A} = w_{1B}} &= g(\Delta a) \left(\frac{2wg'(\Delta a) + c''(\check{e}^*)}{|J|} - \frac{G(\Delta a)}{g(\Delta a)} \right) \\
\left. \frac{\partial \pi}{\partial w_{1B}} \right|_{w_{1A} = w_{1B}} &= g(\Delta a) \left(\frac{-2wg'(\Delta a) + c''(\check{e}^*)}{|J|} - \frac{1 - G(\Delta a)}{g(\Delta a)} \right).
\end{aligned}$$

Because of $g'(\Delta a) < 0$ and $G(\Delta a) > \frac{1}{2}$ inequality (17) is satisfied. $e_A^* < e_B^*$ immediately follows from the optimal tournament prizes together with (15) and (16). ■

The intuition for the result of Proposition 4 is the following: the smaller $\Delta a + e_A - e_B$ the larger will be the workers' marginal winning probability $g(\Delta a + e_A - e_B)$ and, hence, the larger will be overall efforts because competition becomes less uneven.[16] Inducing $e_A^* < e_B^*$ via $w_{1B}^* > w_{1A}^*$ exactly serves this purpose. Therefore, without self-commitment constraint, individual tournament prizes are used by employer E as a substitute for handicaps to make competition more even.

To sum up, the analysis of the tournament design $D = IP$ has shown that the use of individual tournament prizes has two major advantages for the employer. First, individual prizes can be used to extract rents from the workers when the employer has to satisfy a self-commitment constraint. Second, if this constraint is skipped the employer will further use individual prizes to adjust individual incentives so that competition becomes more balanced.

5 Conclusion

In this paper, we have shown that individual tournament prizes dominate uniform ones. If the employer has to satisfy an additional self-commitment condition, individual prizes will be helpful for extracting rents from the workers. If the employer does not have to care for the self-commitment property of the announced tournament prizes, individual prizes exhibit a further advantage. Now they can be used as a substitute for handicaps when adjusting individual incentives in order to make the tournament competition more even.

The case of individual prizes without self-commitment constraint could be supplemented by deriving optimal self-enforcing agreements between the workers and the employer within an infinitely repeated game. On the on

[16] This can be directly seen from (15) and (16).

hand, if the employer is not sufficiently patient he might renege on the implicit contract. Reneging can be profitable for the employer since optimal individual tournament prizes differ so that the employer can realize a short-term advantage by claiming a wrong tournament winner and saving labor costs. The larger tournament prizes and, in particular, the larger the differences between the prizes the stronger will be the employer's incentive to renege on the contract. On the other hand, infinite repetition of the basic tournament game at stage 2 may provide the opportunity for the workers to form a stable collusion, thus reducing overall efforts. High tournament prizes might help to destabilize the collusion. The optimal tournament contract must address both problems.

Appendix

Appendix A: Proof of Proposition 1

If only (6) is binding we will have $\lambda_1 = 2$ and $\lambda_3 = 0$. Inserting into (8) and (7) yields

$$\lambda_2 = \frac{2G(\Delta a) - 1}{g(\Delta a)} \quad \text{and} \quad c''(e_1^*) = 2g(\Delta a)\frac{1 - c'(e_1^*)}{2G(\Delta a) - 1}. \tag{18}$$

Since the left-hand side of the last equality is positive, comparison with (2) immediately shows that $e_1^* < e^{FB}$.

If only (5) is binding (that is $\lambda_1 = 0$ and $\lambda_3 = 2$) conditions (7) and (8) together give

$$c''(e_3^*) = 2g(\Delta a). \tag{19}$$

(19) and (2) show that if $c''^{-1}(2g(\Delta a)) > c'^{-1}(1)$ we will have $e_3^* > e^{FB}$.

If both (6) and (5) are binding so that $\lambda_1, \lambda_3 > 0$ the two binding constraints together with (4) lead to (11). From equations (7) and (8) we obtain

$$\begin{aligned} \lambda_2 &= \frac{2[1 - G(\Delta a)] - c'(e_2^*)}{[1 - G(\Delta a)]c''(e_2^*) - c'(e_2^*)g(\Delta a)} \quad \text{and} \\ \lambda_1 &= \frac{c''(e_2^*) - 2g(\Delta a)}{[1 - G(\Delta a)]c''(e_2^*) - c'(e_2^*)g(\Delta a)}. \end{aligned} \tag{20}$$

Condition $\lambda_1 + \lambda_3 = 2$ leads to

$$\lambda_3 = \frac{2g(\Delta a)(1 - c'(e_2^*)) - (2G(\Delta a) - 1)c''(e_2^*)}{[1 - G(\Delta a)]c''(e_2^*) - c'(e_2^*)g(\Delta a)}. \tag{21}$$

If the denominator in (20) and (21) is positive, $\lambda_1, \lambda_3 > 0$ yields

$$c''(e_2^*) > 2g(\Delta a) \quad \text{and} \quad c''(e_2^*) < 2g(\Delta a)\frac{1 - c'(e_2^*)}{2G(\Delta a) - 1}.$$

Comparison with (19) and (18) shows that $e_2^* > e_3^*$ and $e_2^* < e_1^* < e^{FB}$. If the denominator in (20) and (21) is negative, $\lambda_1, \lambda_3 > 0$ implies

$$c''(e_2^*) < 2g(\Delta a) \quad \text{and} \quad c''(e_2^*) > 2g(\Delta a)\frac{1 - c'(e_2^*)}{2G(\Delta a) - 1}$$

and – together with (19) and (18) – $e_2^* < e_3^*$ and $e_2^* > e_1^*$.

Finally, we have to prove the claim on the cutoff-values $\bar{w}_1$ and $\bar{w}_2$ for $\bar{w}$ with $\bar{w}_1 > \bar{w}_2$. In the case of e_1^*, the non-binding limited-liability constraint (5) yields

$$\bar{w} > \frac{c'(e_1^*)}{2g(\Delta a)} 2[1 - G(\Delta a)] - c(e_1^*) =: \bar{w}_1$$

whereas in the case of e_3^* we obtain from the non-binding participation constraint (6)

$$\bar{w} < \frac{c'(e_3^*)}{2g(\Delta a)} 2[1 - G(\Delta a)] - c(e_3^*) =: \bar{w}_2.$$

Define

$$f(x) := \frac{c'(x)}{2g(\Delta a)} 2[1 - G(\Delta a)] - c(x).$$

To show that $\bar{w}_1 > \bar{w}_2 \Leftrightarrow f(e_1^*) > f(e_3^*)$ we must have that $e_1^* > e_3^* \Rightarrow f'(x) > 0, \forall x \in [e_3^*, e_1^*]$, but $e_1^* < e_3^* \Rightarrow f'(x) < 0, \forall x \in [e_1^*, e_3^*]$. Consider

$$f'(x) := \frac{c''(x)}{2g(\Delta a)} 2[1 - G(\Delta a)] - c'(x). \tag{22}$$

In case of $e_1^* > e_3^*$, the comparison of (18) and (19) implies $\frac{(1 - c'(e_1^*))}{2G(\Delta a) - 1} > 1 \Leftrightarrow 2[1 - G(\Delta a)] > c'(e_1^*)$ and hence $2[1 - G(\Delta a)] > c'(x), \forall x \in [e_3^*, e_1^*]$. Together with (19) we immediately obtain $f'(e_3^*) > 0$ from (22). Since $c''(\cdot)$ is monotonically increasing, we also must have that $f'(x) > 0, \forall x \in (e_3^*, e_1^*]$. If $e_1^* < e_3^*$ we will have $2[1 - G(\Delta a)] < c'(x), \forall x \in [e_1^*, e_3^*]$. Now, (19) leads to $f'(e_3^*) < 0$. Monotonicity of $c''(\cdot)$ yields $f'(x) < 0, \forall x \in [e_1^*, e_3^*)$, which completes the proof.

Appendix B: Derivation of (12) and (13)

First, we can calculate e_3^*. Because of the exponential cost function and the normal distribution, $c''(e_3^*) = 2g(\Delta a)$ can be written as

$$\begin{aligned} c^2 \exp\{ce_3^*\} &= \frac{2}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{\Delta a^2}{2\sigma^2}\right\} \quad \Leftrightarrow \\ e_3^* &= \frac{1}{c}\left[\ln\left(\frac{2}{c^2\sqrt{2\pi\sigma^2}}\right) - \frac{\Delta a^2}{2\sigma^2}\right] \end{aligned} .$$

Equation (2) for the first-best effort yields

$$e^{FB} = \frac{1}{c}\ln\left(\frac{1}{c}\right)$$

so that we obtain

$$e_3^* > e^{FB} \Leftrightarrow \ln\left(\frac{2}{c\sqrt{2\pi\sigma^2}}\right) > \frac{\Delta a^2}{2\sigma^2}.$$

Note that the right-hand side of the existence condition (3) boils down to c^2. The left-hand side of (3) (LHS(3)) can be written as

$$\max_x \Delta w\left(-\frac{(\Delta a + x)}{\sigma^3\sqrt{2\pi}} \exp\left\{-\frac{(\Delta a + x)^2}{2\sigma^2}\right\}\right).$$

Differentiating with respect to x and noting that the maximum satisfies $\Delta a + x < 0$, we obtain the solution $x = -(\sigma + \Delta a)$. Inserting into the LHS(3) gives

$$\Delta w\left(\frac{1}{\sigma^2\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\right\}\right).$$

Recall that $\Delta w = \frac{c'(e_3^*)}{g(\Delta a)}$ with

$$\begin{aligned} c'(e_3^*) &= c\exp\{ce_3^*\} = c\exp\left\{\ln\left(\frac{2}{c^2\sqrt{2\pi\sigma^2}}\right) - \frac{\Delta a^2}{2\sigma^2}\right\} \\ &= \frac{2}{c\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\Delta a^2}{2\sigma^2}\right) \end{aligned}$$

and

$$g(\Delta a) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{\Delta a^2}{2\sigma^2}\right\}$$

so that $\frac{c'(e_3^*)}{g(\Delta a)} = \frac{2}{c}$. Altogether, for the LHS(3) we have

$$\frac{2}{c\sigma^2\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\right\},$$

and for (3) the parameterized version (13).

Appendix C: Proof of Propositions 2 and 3

With individual prizes and a fixed prize sum, the employer maximizes $\tilde{\pi} = 2\tilde{e} - \Delta\tilde{w} - w_{2A} - w_{2B}$ subject to

$$w_{2A} + G(\Delta a)\Delta\tilde{w} - c(\tilde{e}) \geq 0 \tag{PC$_A$}$$

$$w_{2B} + (1 - G(\Delta a))\Delta\tilde{w} - c(\tilde{e}) \geq 0 \tag{PC$_B$}$$

$$\Delta\tilde{w}g(\Delta a) = c'(\tilde{e}) \tag{IC}$$

$$w_{2A} \geq -\bar{w} \tag{LL$_A$}$$

$$w_{2B} \geq -\bar{w}. \tag{LL$_B$}$$

The Lagrangian is given by

$$\begin{gathered} L = 2\tilde{e} - \Delta\tilde{w} - w_{2A} - w_{2B} + \lambda_1 (w_{2A} + G(\Delta a)\Delta\tilde{w} - c(\tilde{e})) \\ +\lambda_2 (w_{2B} + (1 - G(\Delta a))\Delta\tilde{w} - c(\tilde{e})) + \lambda_3 (\Delta\tilde{w}g(\Delta a) - c'(\tilde{e})) \\ +\lambda_4 (w_{2A} + \bar{w}) + \lambda_5 (w_{2B} + \bar{w}). \end{gathered}$$

Differentiating with respect to w_{2A} and w_{2B}, we obtain

$$\frac{\partial L}{\partial w_{2A}} = -1 + \lambda_1 + \lambda_4 \stackrel{!}{=} 0 \quad \text{and} \quad \frac{\partial L}{\partial w_{2B}} = -1 + \lambda_2 + \lambda_5 \stackrel{!}{=} 0.$$

This means that for each worker the participation constraint, the limited-liability constraint or both must bind. Assume $\lambda_1 = \lambda_2 = 1$, $\lambda_4 = \lambda_5 = 0$ in which case both participation constraints are binding. Moreover, differentiate the Lagrangian with respect to $\tilde{e}$ and $\Delta\tilde{w}$. We get

$$\frac{\partial L}{\partial \tilde{e}} = 2 - \lambda_1 c'(\tilde{e}) - \lambda_2 c'(\tilde{e}) - \lambda_3 c''(\tilde{e}) \overset{!}{=} 0$$

$$\frac{\partial L}{\partial \Delta\tilde{w}} = -1 + \lambda_1 G(\Delta a) + \lambda_2 (1 - G(\Delta a)) + \lambda_3 g(\Delta a) \overset{!}{=} 0.$$

Using $\lambda_1 = \lambda_2 = 1$, $\lambda_4 = \lambda_5 = 0$ it is straightforward to show that the conditions yield $c'(\tilde{e}) = 1$ so that $\tilde{e} = e^{FB}$.

It remains to check that the contract does not violate the limited-liability constraints. As both participation constraints are binding, worker A receives a lower loser prize than worker B. Hence, we must check that

$$w_{2A}^{*} = c\left(e^{FB}\right) - G(\Delta a)\frac{c'(e^{FB})}{g(\Delta a)} \geq -\bar{w}$$

This is equivalent to $\bar{w} \geq -c\left(e^{FB}\right) + \frac{G(\Delta a)}{g(\Delta a)}$ and coincides with the condition in Proposition 2. This completes the proof of this proposition.

To prove Proposition 3 as well, suppose now $\lambda_1 = \lambda_2 = 0$, $\lambda_4 = \lambda_5 = 1$ in which case both limited-liability constraints are binding. From the Kuhn-Tucker conditions derived before, it directly follows that $2 - \lambda_3 c''(\tilde{e}) = 0$ and $-1 + \lambda_3 g(\Delta a) = 0$ which yields $c''(\tilde{e}) = 2g(\Delta a)$. As both loser prizes are equally high, the winner prizes must be equally high, too. Finally, we have to show that the participation constraints are not violated. Since both workers receive the same wages, B's participation constraint is the relevant one. This constraint simplifies to

$$-\bar{w} + (1 - G(\Delta a))\frac{c'(\tilde{e}^{*})}{g(\Delta a)} - c(\tilde{e}^{*}) \geq 0$$

$$\Leftrightarrow \bar{w} \leq (1 - G(\Delta a))\frac{c'(\tilde{e}^{*})}{g(\Delta a)} - c(\tilde{e}^{*}),$$

which completes the proof of Proposition 3.

Appendix D: Proof of Corollary 1

Due to different cost functions, the workers' first-best efforts e_A^{FB} and e_B^{FB}, described by $1 = c_i'(e_i)$ $(i = A, B)$, will also differ. The equilibrium at stage 2 is characterized by the workers' first-order conditions

$$\Delta\tilde{w}g(e_A - e_B + \Delta a) = c_A'(e_A) \quad \text{and} \quad \Delta\tilde{w}g(e_A - e_B + \Delta a) = c_B'(e_B).$$

In addition, the workers' participation constraints are

$$\begin{aligned} w_{2A} + \Delta\tilde{w}G(e_A - e_B + \Delta a) - c_A(e_A) &\geq \bar{u}_A \quad \text{and} \\ w_{2B} + \Delta\tilde{w}\left[1 - G(e_A - e_B + \Delta a)\right] - c_B(e_B) &\geq \bar{u}_B. \end{aligned}$$

By choosing $\Delta\tilde{w} = 1/g\left(e_A^{FB} - e_B^{FB} + \Delta a\right)$, employer E can implement first-best efforts for both workers. If the workers are sufficiently wealthy, they are not protected by limited liability and E is indeed interested in efficiency[17]: the optimal tournament prizes

$$\begin{aligned} w_{2A} &= \bar{u}_A + c_A\left(e_A^{FB}\right) - \frac{G\left(e_A^{FB} - e_B^{FB} + \Delta a\right)}{g\left(e_A^{FB} - e_B^{FB} + \Delta a\right)} \\ w_{1A} &= \bar{u}_A + c_A\left(e_A^{FB}\right) + \frac{1 - G\left(e_A^{FB} - e_B^{FB} + \Delta a\right)}{g\left(e_A^{FB} - e_B^{FB} + \Delta a\right)} \\ w_{2B} &= \bar{u}_B + c_B\left(e_B^{FB}\right) - \frac{1 - G\left(e_A^{FB} - e_B^{FB} + \Delta a\right)}{g\left(e_A^{FB} - e_B^{FB} + \Delta a\right)} \\ w_{1B} &= \bar{u}_B + c_B\left(e_B^{FB}\right) + \frac{G\left(e_A^{FB} - e_B^{FB} + \Delta a\right)}{g\left(e_A^{FB} - e_B^{FB} + \Delta a\right)} \end{aligned}$$

extract all rents and, at the same time, fulfill the employer's self-commitment constraint (14).

[17] Of course, the sum of the workers' reservation values must not exceed the first-best surplus, as otherwise the employer would not want to hire the workers at all.

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