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Performance measure congruity in linear agency models with interactive tasks*

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Abstract

This note demonstrates how performance measure congruity and noise determine an agency's total surplus within an linear agency framework with multiple tasks. It provides a decomposition of agency costs, leading back to a congruity index previously proposed in the literature. In addition, it generalizes this index to a more general cost function, thereby highlighting the context specificity of the original criterion. Finally, it suggests a redefinition of tasks under which the criterion prevails.

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1 Introduction

Over the last 15 years, the linear principal-agent framework has become an increasingly popular device for studying a variety of questions concerning the provision of incentives. Its main attraction is its analytical tractability, which allows for closed form solutions even in situations for which results are hardly derived in the standard principal agent model. One such situation is the now familiar multi-task agency model, in which the agent provides a diversity of actions. By the analysis of such situations, the aspect of performance measure congruity—which is absent in the single-action standard agency framework—has become an important subject in the performance measurement literature. Starting with the work of Feltham and Xie (1994), several papers (Baker (2000, 2002); Datar et al. (2001)) have used the setting, thereby deriving metrics of (in-)congruity. All of these metrics try to capture the welfare effects due to the misalignment of performance measures with the principal's objective, some of them in absolute and some in relative terms. All of them, however, build on a specific cost function in which actions are independent and marginal costs are equal among tasks. In particular, they are capable of identifying the lowest misallocation of effort only for this class of cost functions (Schnedler (2003)).

We take up this deficiency and derive a more general measure of congruity that adjusts and generalizes the measure previously proposed by Baker (2000, 2002). We show that this measure naturally arises from a decomposition of the agency's net surplus. We then return to the initial objection and show that under an alternative representation of the agency problem, the shortcoming no longer endures. The main feature of this adaptation is a redefinition of task, restoring the separability of the cost function.

The remainder of this note is organized as follows. Section 2 recapitulates the linear agency framework to derive an adjusted version of Baker's measure of congruity. Section 3 generalizes this model to incorporate interactive actions, thereby deriving a generalized measure of congruity. Section 4 finally provides a redefinition of the agent's tasks to restore the initial measure of congruity. Section 5 concludes.

2 Independent tasks and an intuitive metric of congruity

Consider a situation in which an agent influences the net present value of his principal by exerting a multidimensional $\mathbf{a} \in \mathbb{R}^n$ which cannot be legally enforced. Neither the principal's net present value $V(\mathbf{a}) = \mathbf{d}'\mathbf{a} = \sum_{i=1}^{n} d_i a_i$ of this activity nor the cost $C(\mathbf{a})$ accruing to the agent is verifiable. In this section, we assume that the agent's cost of taking action $\mathbf{a}$ is given by:

$$C(\mathbf{a}) = \frac{1}{2}\mathbf{a}'\mathbf{a} = \frac{1}{2}\sum_{i=1}^{n} a_i^2. \tag{1}$$

This cost function is used in most linear agency models, merely for mathematical convenience. In the next section, we present a more general cost function. It then becomes apparent that the metrics of congruity proposed in the literature build on the specific cost function of (1).

To motivate the agent for the activity, the principal has to rely on a performance measure $P(\mathbf{a}) = \mathbf{y}'\mathbf{a} + \epsilon = \sum_{i=1}^{n} y_i a_i + \epsilon$, where $y_i \in \mathbb{R}$ denotes the performance measure's sensitivity with respect to action a_i, and $\epsilon \sim N(0, \sigma^2)$ is a normal error term reflecting the uncertainty related to measure P.

The agent is effort- and risk-averse, which is reflected by an exponential utility function $U(S, \mathbf{a}) = -\exp\left(-r(S - C(\mathbf{a})\right)$, where S is any transfer received and r

is the Arrow–Pratt measure of absolute risk aversion. Given this specification, the agent's preferences can equally be represented by his certainty equivalent $CE(S, \mathbf{a}) = S - C(\mathbf{a}) - \frac{r}{2}\sigma^2$. The principal remunerates the agent using a linear compensation scheme $S = s_0 + sP$. The reservation level of the agent's certainty equivalent is CE^R.

The principal's contracting problem in this model is a special case of that analyzed by Feltham and Xie (1994), who allow for multiple performance measures. By choosing s, the principal maximizes the expected total surplus $V(\mathbf{a}) - C(\mathbf{a}) - \frac{r}{2}\sigma^2$, subject to the the incentive compatibility constraint $\mathbf{a} = s\mathbf{y}$. The optimal contract parameter is $s = \frac{\mathbf{d}'\mathbf{y}}{\mathbf{y}'\mathbf{y}+r\sigma^2}$, from which the agency's net total surplus

$$\Pi_{RA}^{SB} = \frac{1}{2}\frac{(\mathbf{d}'\mathbf{y})^2}{\mathbf{y}'\mathbf{y} + r\sigma^2} \tag{2}$$

can be derived (Feltham and Xie 1994, p. 433).

Using this simple framework, measures of congruity and risk can easily be derived by ceteris paribus comparisons. In detail, we compare:

1. The net total surplus of the agency under first-best to that under second-best with a risk-neutral agent. This comparison provides a measure of congruity.

2. The net total surplus under second-best with a risk-neutral agent to that under second-best with a risk-averse agent. From this we derive an index of the risk incorporated in the performance measure.

Since both measures are defined as ratios of total surplus numbers, the second-best surplus equals the first-best surplus, multiplied by the respective measures of congruity and risk. The measures are computed as follows:

1. Congruity: The total surplus under first-best conditions,

$$\Pi^{FB} = \frac{(\mathbf{d}'\mathbf{d})}{2}, \tag{3}$$

results from (2) with $\mathbf{y} = \mathbf{d}$ and $r = 0$. The total surplus

$$\Pi^{SB}_{RN} = \frac{(\mathbf{d}'\mathbf{y})^2}{2\mathbf{y}'\mathbf{y}} \tag{4}$$

under second-best and risk neutrality results from $r = 0$ and arbitrary sensitivities $\mathbf{y}$. Relating (4) to (3), a measure

$$\phi(\mathbf{d}, \mathbf{y}) = \frac{\Pi^{SB}_{RN}}{\Pi^{FB}} = \frac{(\mathbf{d}'\mathbf{y})^2}{(\mathbf{y}'\mathbf{y})(\mathbf{d}'\mathbf{d})} = (\cos(\beta))^2 \tag{5}$$

of congruity can be defined, where β is the angle between the vectors $\mathbf{d}$ and $\mathbf{y}$. The cosine of β has already been promoted by Baker (2000, 2002) as a measure of congruity. By reference to the *squared* cosine, the present measure is scaled to the unit interval and monotonically relates congruity to surplus numbers.

2. Risk: Relating the total surplus (4) under second-best and risk neutrality to the second-best total surplus (2) under risk aversion, a measure ψ of risk can be defined by

$$\psi(\mathbf{y}, \sigma^2) = \frac{\Pi^{SB}_{RA}}{\Pi^{SB}_{RN}} = \frac{(\mathbf{y}'\mathbf{y})}{(\mathbf{y}'\mathbf{y}) + r\sigma^2} = \frac{1}{1 + r\frac{\sigma^2}{\mathbf{y}'\mathbf{y}}}.$$

Using the term $\frac{\sigma^2}{\mathbf{y}'\mathbf{y}}$ in the denominator of ψ, the signal's variance is normalized with respect to the marginal products $\mathbf{y}$ of the performance measure. Referring to Banker and Datar (1989), it can be denoted as the signal's intensity with respect to a. Borrowing from the engineering terminology, its reciprocal value is also referred to as the signal-to-noise ratio of $\mathbf{y}$ (e.g. Baker 2002, p. 732).

Applying the above definitions, the total surplus of an agency in the linear model with a separable quadratic cost function can be decomposed in the following manner:

$$\Pi_{RA}^{SB} = \Pi^{FB}\phi(\mathbf{d},\mathbf{y})\psi(\mathbf{y},\sigma^2).$$

The metrics $\phi(\mathbf{d},\mathbf{y})$ and $\psi(\mathbf{y},\sigma^2)$ quantify the performance measure's relative effectiveness with respect to congruity and precision.

3 Interactive tasks

The cost function in (1) assumes that the agent's actions are completely independent. To capture interaction between tasks, a generalized quadratic cost function

$$\hat{C}(\mathbf{a}) = \mathbf{a}'\mathbf{K}\mathbf{a} \tag{6}$$

can be used, where $\mathbf{K}$ denotes a positive definite and symmetric $(n \times n)$-matrix. $\hat{C}(\mathbf{a})$ is identical to $C(\mathbf{a})$ for $\mathbf{K} = \mathbf{I}/2$. Moreover, the quadratic form in (6) is capable of covering almost any degree of complementarity and substitutability. With regard to marginal cost, it provides a linear approximation of any convex cost function, as considered by Holmström and Milgrom (1991). From the generalized cost function, the agent's action choice becomes $\mathbf{a}(s,\mathbf{K},\mathbf{y}) = \frac{s}{2}\mathbf{K}^{-1}\mathbf{y}$. Substitution in the principal's optimization problem yields a net total surplus of

$$\Pi_{RA}^{SB} = \frac{1}{4}\frac{(\mathbf{d}'\mathbf{K}^{-1}\mathbf{y})^2}{\mathbf{y}'\mathbf{K}^{-1}\mathbf{y} + 2r\sigma^2}. \tag{7}$$

Applying the above measure of risk to the modified model, a paradox seemingly emerges: under the generalized cost function, an unbiased performance measure in general does *not* maximize total surplus in a class of performance measures with identical signal-to-noise ratio (Schnedler 2003). The paradox is exemplified by the following example:

Example 1 *Let $\mathbf{d} = (1,0)'$ determine the result of the agent's two-dimensional effort. We compare two performance measures, $P^1 = \mathbf{d}'\mathbf{a} + \epsilon_1$ and $P^2 = \mathbf{y}'\mathbf{a} + \epsilon_2$, with $\mathbf{y} = (1,1)'$, $\epsilon_1 \sim N(0,1)$ and $\epsilon_2 \sim N(0,2)$. Thus, (i) P^1 is perfectly congruent with the principal's objective, whereas P^2 is distorted; and (ii) both measures exhibit the same signal-to-noise ratio $\mathbf{d}'\mathbf{d}/\sigma_1^2 = \mathbf{y}'\mathbf{y}/\sigma_2^2 = 1$. According to the above measures ϕ and ψ, P^1 should be preferred to P^2.*

Now consider the agent's cost of effort (6) with

$$\mathbf{K} = \begin{bmatrix} 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{bmatrix}.$$

The two tasks are complements: The marginal cost of one action decreases the level of the other. Consequently, the actions under both performance measures tend to balance the two tasks. Optimization yields $\mathbf{a}(s, \mathbf{K}, \mathbf{d}) = s\left(\frac{2}{3}, \frac{1}{3}\right)'$ and $\mathbf{a}(s, \mathbf{K}, \mathbf{y}) = s\,(1,1)'$. The modified cost function alleviates the consequences of the incongruity of P^2. Nevertheless, the action under P^2 is still distorted, whereas under P^1 the first-best action $\left(\frac{2}{3}, \frac{1}{3}\right)'$ can be induced.

Inspection of the second-best solutions

$$\Pi_{RA}^{SB,P^1} = \frac{1}{4} \cdot \frac{\left(\frac{4}{3}\right)^2}{\frac{4}{3} + 2r} \qquad \text{and} \qquad \Pi_{RA}^{SB,P^2} = \frac{1}{4} \cdot \frac{2^2}{4 + 4r},$$

however, reveals that for a sufficiently risk-averse agent, the principal prefers

P^2 to P^1: for $r > 2$ the principal's second-best profit is strictly less under the unbiased performance measure P^1.

The example seems to conflict with the proposed decomposition of congruity and risk effects: if risk effects can be expressed as a function of the signal-to-noise ratio, then signals of identical signal-to-noise relation should be comparable with respect to the above measure of congruity. The example contradicts this intuition.

The discrepancy arises from the definition of risk associated with a performance measure. This becomes clear when we take a similar approach to that in the preceding section in order to separate the effects of congruity and risk. Working along the lines of Section 2, we obtain:

1. Congruity: The net total surplus under first-best now equals

$$\Pi^{FB} = \frac{(\mathbf{d}'\mathbf{K}^{-1}\mathbf{d})}{4}.$$

   Relating this to the net total surplus under second-best and risk neutrality,

$$\Pi^{SB}_{RN} = \frac{(\mathbf{d}'\mathbf{K}^{-1}\mathbf{y})^2}{4\mathbf{y}'\mathbf{K}^{-1}\mathbf{y}}, \tag{8}$$

   a modified measure of congruity can be defined as

$$\hat{\phi}(\mathbf{d}, \mathbf{K}, \mathbf{y}) = \frac{\Pi^{SB}_{RN}}{\Pi^{FB}} = \frac{(\mathbf{d}'\mathbf{K}^{-1}\mathbf{y})^2}{(\mathbf{y}'\mathbf{K}^{-1}\mathbf{y})(\mathbf{d}'\mathbf{K}\mathbf{d})}. \tag{9}$$

   Comparing $\hat{\phi}(\mathbf{d}, \mathbf{K}, \mathbf{y})$ to $\phi(\mathbf{d}, \mathbf{y})$, it emerges that the cosine interpretation is no longer apparent under the generalized cost function. Congruity is now a function not only of marginal products $\mathbf{d}$ and $\mathbf{y}$, but also of the agent's cost function determined by $\mathbf{K}$. The reason for this change is quite obvious:

what really matters is not a performance measure's degree of alignment with the principal's objective *per se*, but the alignment of the resulting *efforts* with the first-best action choice. If all actions are independent and equally costly [as in the cost function (1)], the two comparisons yields identical results because $\mathbf{a}^{FB} = \mathbf{d}$ and $\mathbf{a}^{SB} = s\mathbf{y}$. Under the general cost function (6), however, the first-best action $\mathbf{a}^{FB} = \frac{1}{2}\mathbf{K}^{-1}\mathbf{d}$ as well as the second best action $\mathbf{a}^{SB} = \frac{s}{2}\mathbf{K}^{-1}\mathbf{y}$ depends on $\mathbf{K}$. A substitution of $\mathbf{a}^{FB}$ and $\mathbf{a}^{SB}$ for $\mathbf{d}$ and $\mathbf{y}$ in the cosine formula (5) yields the generalized measure (9), which therefore describes the squared cosine of the angle γ between the first-best and second-best *effort* vectors.

This effect is illustrated in Figure 1, which takes up the data of example 1.

[Figure 1 about here.]

Here, the original cosine criterion highly underestimates the alignment of Performance measure P^2 with the principal's interests. While Baker's congruity index $\phi(\mathbf{d}, \mathbf{y}) = \frac{1}{2}$ refers to the angle β between the vectors $\mathbf{d}$ and $\mathbf{y}$, the modified measure $\hat{\phi} = \frac{3}{4}$ refers to the smaller angle γ between the first-best and the second-best action. Obviously, γ indicates a much greater congruity than β.

Note that despite the modification of ϕ, an unbiased performance measure with $\mathbf{y} = \mathbf{d}$ still leads to the maximal congruity of 1. This is worth mentioning because of the paradox described in the example. Since in the present decomposition the principal's profit monotonically increases in the congruity measure $\hat{\phi}$, the conflict must arise from the definition of risk.

2. Risk: Relating the net total surplus in (8) under second-best and risk neutrality to the respective net total surplus (7) under risk aversion, the measure of risk becomes

$$\hat{\psi}(\mathbf{y}, \mathbf{K}, \sigma^2) = \frac{\Pi_{RN}^{SB}}{\Pi^{FB}} = \frac{(\mathbf{y}'\mathbf{K}\mathbf{y})}{(\mathbf{y}'\mathbf{K}\mathbf{y}) + 2r\sigma^2} = \frac{1}{1 + \frac{2\sigma^2}{\mathbf{y}'\mathbf{K}\mathbf{y}}}, \tag{10}$$

which obviously is not in line with the signal-to-noise ratio described above. Different to the case of a separable quadratic cost function of equally costly tasks, the signal's variance is normalized with respect to the marginal product $\mathbf{y}$ of the performance measure, related to the respective marginal cost of effort. Therefore, performance measures of equal signal-to-noise ratio are no longer equally risky in the modified notion of risk. A justification of the risk measure in (10) is straightforward: if the marginal cost of a particular action is high, the agent will spend only low effort on this task. Consequently, a performance measure's sensitivity with respect to this task should have only a minor impact on a risk metric associated with that task. Similar to congruity, what matters is not a performance measure's risk *per se*, but the risk resulting from the agent's consequential *action*. This is exactly accounted for by rescaling the variance σ^2 by $\mathbf{y}'\mathbf{K}^{-1}\mathbf{y}$ instead of $\mathbf{y}'\mathbf{y}$.

In the example, the original signal-to-noise ratio overestimates the risk associated with performance measure P^2, compared to that of P^1. Applying the risk metric (10), the modified signal-to-noise ratio $\mathbf{y}'\mathbf{K}^{-1}\mathbf{y}/\sigma_2^2 = 4$ of P^2 is higher than that of P^1, which amounts to $\mathbf{d}'\mathbf{K}^{-1}\mathbf{d}/\sigma_1^2 = 8/3$. Thus, P^1 and P^2 do not belong to the same risk class under the modified cost function. Since P^2 is now "less risky" than P^1, for a sufficiently risk-averse agent the higher precision outweighs its lower incongruity, and the principal is better off using the distorted measure.

4 Redefinition of tasks and independent actions

Since the difference in the congruity indexes in (5) and (9) is caused by the matrix $\mathbf{K}$ determining the agent's cost of effort, it is worthwhile inspecting how $\mathbf{K}$ relates the cost functions (1) and (6). If $\mathbf{K}$ is a scalar matrix (proportionate to the identity matrix $\mathbf{I}$), all actions are independent and equally costly. The initial cost function (1) can then be derived from the generalized cost function (6) by simple rescaling. If $\mathbf{K}$ is diagonal, the different actions are independent, but of different marginal cost. In this case, the initial cost function is obtained by rescaling the different actions by their respective marginal cost. These two cases are obvious and have already been treated in the literature (Schnedler 2003). If the matrix $\mathbf{K}$ is not diagonal, however, the marginal cost of one action in principle depends on the chosen level of another, and the cost function C cannot be obtained by simply rescaling.

It can be restored, however, by a proper *redefinition* of tasks: Since K is positive definite and symmetric, it is *diagonalizable*, i.e., there exists an orthogonal $(n \times n)$-matrix $\mathbf{U}$ such that:

$$\mathbf{K} = \mathbf{U}\mathbf{Q}\mathbf{U}^{-1}, \tag{11}$$

where $\mathbf{Q}$ is a diagonal matrix, the elements of which are the eigenvalues of $\mathbf{K}$ (Sydsaeter et al. 1999, p. 137). This can be used to give an alternative presentation of the principal's optimization problem with independent actions. To this end, we substitute the diagonalized matrix K in the general cost function (6),

$$\hat{C}(\mathbf{a}) = \mathbf{a}'\mathbf{U}\mathbf{Q}\mathbf{U}^{-\mathbf{1}}\mathbf{a} = \mathbf{a}'\mathbf{U}\mathbf{Q}\mathbf{U}'\mathbf{a},$$

where the equality follows from the property $\mathbf{U}\mathbf{U}' = \mathbf{I}$ of orthogonal matrices (Sydsaeter et al. 1999, p. 141). By defining actions $\tilde{\mathbf{a}} = \mathbf{U}'\mathbf{a}$, we obtain a separable cost function

$$\hat{C}(\tilde{\mathbf{a}}) = \tilde{\mathbf{a}}'\mathbf{Q}\tilde{\mathbf{a}}.$$

In a second step, actions can be rescaled to $\hat{\mathbf{a}} = \mathbf{Q}^{\frac{1}{2}}\tilde{\mathbf{a}}$ in order to generate a cost function of the form given in (1):

$$\hat{C}(\hat{\mathbf{a}}) = \hat{\mathbf{a}}'\hat{\mathbf{a}}.$$

Redefining tasks, however, requires rewriting of the principal's gross benefit and the agent's performance measure. From the two steps of the redefinition of tasks, we have $\hat{\mathbf{a}} = \mathbf{Q}^{\frac{1}{2}}\mathbf{U}'\mathbf{a}$ or $\mathbf{a} = \mathbf{U}\mathbf{Q}^{-\frac{1}{2}}\hat{\mathbf{a}}$. Substitution yields

$$V(\hat{\mathbf{a}}) = \mathbf{d}'\mathbf{a} = \mathbf{d}'\mathbf{U}\mathbf{Q}^{-\frac{1}{2}}\hat{\mathbf{a}} = \hat{\mathbf{d}}'\hat{\mathbf{a}},$$

where $\hat{\mathbf{d}} = \mathbf{Q}^{-\frac{1}{2}}\mathbf{U}'\mathbf{d}$ denotes the marginal products of the redefined tasks. Similarly, $\hat{\mathbf{y}} = \mathbf{Q}^{-\frac{1}{2}}\mathbf{U}'\mathbf{y}$ can be defined as the sensitivities of the performance measures with respect to these tasks, yielding a performance measure $P(\hat{\mathbf{a}}) = \hat{\mathbf{y}}'\hat{\mathbf{a}} + \epsilon$.

Application of this procedure to the example renders

$$\mathbf{Q} = \begin{bmatrix} \frac{3}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \quad \text{and} \quad \mathbf{U} = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}.$$

The redefined tasks are $\hat{a}_1 = \sqrt{3}\,(a_2 - a_1)\,/2$ and $\hat{a}_2 = (a_1 + a_2)\,/2$. After redefining marginal products $\hat{\mathbf{d}} = \mathbf{Q}^{-\frac{1}{2}}\mathbf{U}'\mathbf{d} = \left(1/\sqrt{3}, 1\right)'$ and sensitivities $\hat{\mathbf{y}} = \mathbf{Q}^{-\frac{1}{2}}\mathbf{U}'\mathbf{y} = (0,2)'$, the original cosine criterion in fact is in line with the above definition of congruity:

$$\phi(\hat{\mathbf{d}}, \hat{\mathbf{y}}) = \frac{(\hat{\mathbf{d}}'\hat{\mathbf{y}})^2}{(\hat{\mathbf{y}}'\hat{\mathbf{y}})(\hat{\mathbf{d}}'\hat{\mathbf{d}})} = \frac{(0+2)^2}{(0+4)\left(\frac{1}{3}+1\right)} = \frac{3}{4}.$$

On inspection of the example, a shortcoming of the redefinition of tasks becomes obvious: the redefined action $\hat{a}_1 = \sqrt{3}\,(a_2 - a_1)\,/2$ is not naturally interpretable as a *combination of different tasks* because it includes negative levels of action a_1. This fact holds for any redefinition as described above: since $\{\hat{a}_1, \ldots, \hat{a}_n\}$ form an *orthonormal* basis for the action space $\mathbb{R}^n$, any such basis different from the natural basis [which is given for the separable cost function (1)] will comprise negative entries in its base vectors.[1] Consequently, at least one action cannot be interpreted as 'doing parts of the original tasks'.

5 Conclusion

This note has revisited the subject of performance measure congruity in a linear agency setting. Building on the previous work of Baker (2000, 2002), a geometric interpretation of congruity under a separable cost function could be based on the squared cosine of marginal product vectors. For more general cost functions, the measure had to be generalized or the agent's action space had to be refined. The latter, however, resulted in tasks that are not easily explained in terms of elementary tasks because they include negative levels of the initial actions. Therefore, while the cosine measure can be formally restored as a metric of congruity, even

[1]This fact is illustrated best by a geometrical argument: the redefinition described is simply a *rotation* of the natural basis.

under effort interaction, an economic interpretation seems to require the more general metric of congruity proposed in this paper.

Probably the most important fact revealed by considering task interdependencies is that performance measure congruity, as measured by the metrics proposed in the literature, is not the primary goal in performance measure selection, even in the absence of risk sharing issues. It is only useful by the extent to which it supports the alignment of the agent's action with the principal's objectives.

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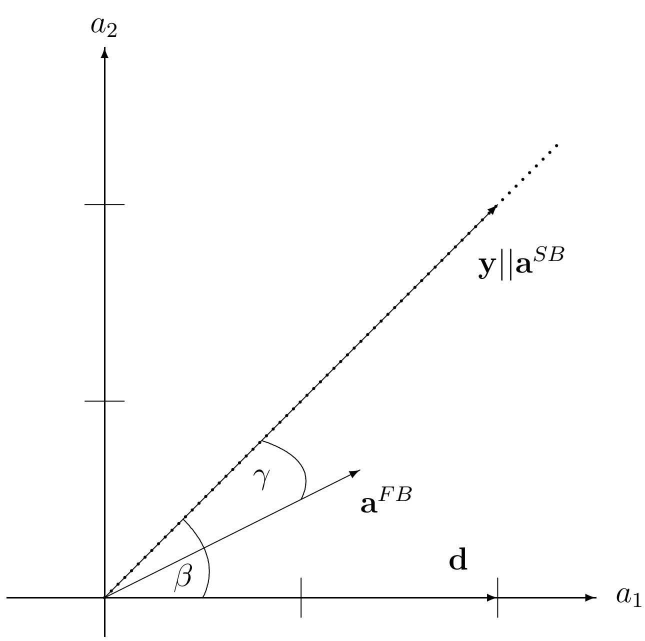

Figure 1: Congruity of measures and actions