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Size and soft budget constraints*

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Abstract

There is much evidence against the so-called "too big to fail" hypothesis in the case of bailouts to sub-national governments. We look at a model where districts of different size provide local public goods with positive spillovers. Matching grants of a central government can induce socially-efficient provision, but districts can still exploit the intervening central government by inducing direct financing. We show that the ability of a district to induce a bailout from the central government and district size are *negatively* correlated.

Key Words: bailouts, soft-budget constraints, jurisdictional size, public goods, spillovers

JEL Codes: H4, H7, R1

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1 Introduction

There is much evidence supporting the conjecture that the occurrence of bailouts to sub-national governments in general contradicts the so-called "too big to fail" hypothesis[1]. A first example is that, based on the constitutional principle of uniformity of living conditions throughout the nation, the German federal court supported in 1992 the claims of the two smallest state governments in terms of population, Bremen and Saarland, in pursuing the federal government to support them in coping with their excessive public debt and unusual high ratio of interest payments to total expenditures, which together with the poor economic performance, could set the basic supply of local public services under risk (Seitz, 1999). At the beginning of the 1990s, the health system in Italy[2] faced in small and poor regions in the south of the country a deficit of about 15% and the central government stepped in and covered the deficits thus incurred to prevent health care in these regions break down (von Hagen et al, 2000). In Sweden, the central government was empowered by law during the period 1974-1992, to provide discretionary transfers to support municipalities in financial distress[3]. Econometric evidence for this period (Dahlberg and Pettersson, 2003), shows that population size and density have a significant negative association with realized bailouts and accumulation of municipal debt. On the other hand, the

[1]It is important to point out that we are not interested in episodes of generalized bailouts like, for example, the rescue operation implemented by the federal government in Mexico early after the financial crisis in December 1994 which included extraordinary transfers to *all* state governments. Another example is Brazil, where the federal government assumed all state and municipal debt in 1993 and 1997 (Dillinger and Webb, 1998).

[2]In 1992, ordinary regions spent 71% of their total resources on health services. Almost 96% of their revenues came from central government (matching) grants. (von Hagen et al, 2000).

[3]This relief program was not part of a regular intergovernmental transfer scheme (von Hagen et al, 2000).

recent fiscal crisis faced by the city of Philadelphia in the US in 1990 cannot be considered a case of bailout since the fiscal cost of the crisis was mainly internalized by its residents living with reduced public services, additional sales tax and city workers facing a wage freeze and a reduction in employee benefits (Inman 1995).

Also in Latin America, a number of recent experiences in Argentina, Colombia and Costa Rica contradict the "too big to fail" hypothesis. In Argentina for example, the central government has often used extraordinary resources to face fiscal and financial crises at provincial level since the return of democracy in 1983. In general, they took place in jurisdictions with the lowest level of GDP and which are among the smallest in terms of population[4] (Nicollini et al, 2002).

This paper investigates the ability and willingness of local governments to induce a central government to directly finance the provision of the local public goods, i.e. to induce bailouts[5]. Size differences among local jurisdictions play an important role in this paper. The paper's take is that, in a federation with lower-level governments of different size providing local public goods, the ability of a district to induce a bailout depends *negatively* on its size. This line of research is pioneered by Wildasin (1997), who develops a model where externalities in the provision of local public goods explain the allocation of bailouts among jurisdictions. In clear contrast to

[4]Bailout episodes during the 1990s include the provinces of Jujuy, La Rioja, Tucuman, Catamarca, Corrientes, Santiago del Estero and Rio Negro which are the smallest in terms of population if we exclude the extremely sparsely populated and oil producing provinces in Patagonia in the south of the country. Moreover, these provinces together represent less than 13% of the total population and less that 10% of national GDP.

[5]Although not discussing size effects, recent literature on soft budget constraints and bailouts also include: Qian and Roland (1998), Inman (2001), Goodspeed (2002) and Sanguinetti and Tommasi (2002). See also Kornai (1986), who introduces the discussion on soft budget constraints in the study of state-owned enterprises, and Maskin (1999) for a survey.

our results, he finds that the size of a subnational jurisdiction positively affects its likelihood of demand and obtaining a bailout.

We develop a two-tier hierarchy model with the central government at the top and several jurisdictions of different size at the bottom that provide local public goods[6]. We assume that there are economies of scale and (positive) externalities in the provision of public goods. As in for example Alesina and Spolaore (1997), *economies of scale* are modelled with a fixed cost associated with public goods' provision. Furthermore, the *spillover* effect is modelled in a similar way as Besley and Coate (2003), that is, public goods provided in a district do not only benefit individuals in this particular district, but also entail a positive externality for individuals in other districts.

The paper starts with looking at the non-cooperative outcome in Section 2. Individuals choose the optimal amount of public goods to be provided in their district. It follows that districts only provide local public goods when district size is large enough relatively to the economies of scale effect in local public good provision. A common finding in this form of decision making is, however, that the spillover effect is not taken into account and, therefore, underprovision of local public goods occur. Section 3 characterizes the optimal level of local public goods provision and defines a system of matching grants implemented by a central government that can be used to achieve an efficient outcome without completely centralizing decision making. Not only is the (strictly positive) amount of public goods provided in a non-cooperative

[6]Throughout this paper we write public goods though, strictly speaking, we mean publicly provided goods.

Nash equilibrium lower than the level in an efficient outcome, also in less cases than socially optimal, local public goods are provided when district size is small relative to the economies of scale in local public goods provision.

The paper then shifts attention to the issue of *soft budget constraints*. In Section 4, it analyzes whether and when the central government is willing to make an extraordinary transfer (*bailout*) to a district, which decides to underprovide local public goods. It turns out that the willingness of the central government to provide a bailout depends negatively on the size of the jurisdiction. In addition to that, since the amount of public goods provided under the bailout policy is lower than that in an efficient outcome, it is costly for individuals to induce a bailout - in case of a bailout there will be less local public goods in their districts than they are willing to pay for. For that reason, the central government's bailout policy does not fully characterize the occurrence of bailouts. Subsequently, the conditions under which local governments indeed choose to induce such a bailout are identified. In agreement with the empirical evidence, as jurisdiction size decreases, the bailout becomes in general more attractive for a jurisdiction, and the willingness of a local government to induce a bailout increases. In Section 5 we argue that these bailout policies can be the strategies in a subgame-perfect Nash equilibrium. Finally, Section 6 summarizes and concludes.

2 The non-cooperative equilibrium

Suppose that a country is divided in $\mathcal{N}$ districts of different size. The country has a population of N individuals and each district i has a population of n_i, where $n_i < N/2$ for all districts. Each individual has an endowment y and there are two types of goods in the economy, a private good x and a public good g. To simplify notation and to show that the results do not depend on heterogeneity among individuals, we assume that all individuals have identical preferences and endowments. We assume that an individual's payoff is quasilinear in the endowment and that the utility function is additively separable.

We assume that there are economies of scale in public good provision. This feature is modelled with a fixed cost F for providing public goods, regardless of the size of the region. There is also a variable cost that depends on the exact amount of public goods that individuals want to provide. A district i provides per capita an amount g_i of the local public goods and each individual in district i pays a lump-sum district tax t_i to finance public good provision in district i. If a district provides an amount g_i of the public good then individuals in this district will get a benefit $v(g_i)$ from these public goods. We assume that $v(.)$ is strictly concave, that $v'(.) > 0$ and that $v(0) = 0$. An individual, however, does not only get a payoff from the public good in his own district but also from the public goods in all other districts. The degree of this (positive) spillover effect is denoted by κ, $0 < \kappa < 1$, so that an individual in district i gets a benefit $\kappa v(g_j)$ of the public goods provided in district j, $(i \neq j)$ [7].

[7] The main argument behind the "too big to fail" hypothesis is that spillovers are important and increasing in district size. This is for example the case when the spillover effect is proportional to

To illustrate this consider two examples, health care and education. An individual in the first place benefits from vaccinations and basic literacy in his own district. There are, however, diminishing returns since an individual benefits less from say plastic surgery or some forms of university education. In the second place an individual also indirectly benefits from these goods provided in other districts. since an individual may sometimes interact with individuals from other districts, and the provision of public goods there make these interactions more beneficial. In these cases, the economies of scale in public good provision, represented by the fixed costs, is for example a single bureaucracy that is responsible for health care and education in each district. The variable costs then represents how much health care or education per individual is available.

Thus, the utility of an individual in district i is

$$v(g_i) + \sum_{j \neq i} \kappa v(g_j) + y - t_i \tag{1}$$

The costs of providing public goods differ per district and its variation is captured by the strictly positive parameter p_i. Since districts have balanced budgets, tax rates t_i are given by

$$t_i = \begin{cases} \frac{F}{n_i} + p_i g_i & \text{if } g_i > 0 \\ 0 & \text{if } g_i = 0 \end{cases} \tag{2}$$

We assume that all individuals in a district can choose the amount of public goods

district size, so that an individual in district i gets a benefit $\kappa n_j v(g_j)$ of the public goods provided in district j. In an Appendix we show that, when spillovers are indeed important, small districts are more likely to get a bailout. The main results of our paper are thus robust to this alternative specification.

provided in their district. Since the individuals within a district are identical, however, we only have to look at the preferences of a single individual as these preferences prevail for all individuals in the same district. The level of public goods provided in a district i is thus determined by the following maximization problem

$$\max_{g_i} \quad v(g_i) + \sum_{j \neq i} \kappa v(g_j) + y - t_i \tag{3}$$

where t_i is given by (2). The non-cooperative Nash equilibrium outcome is characterized by the following first-order condition of maximization problem (3):[8]

$$\begin{cases} v'(g_i) = p_i & \text{if } v(g_i) > \frac{F}{n_i} + p_i g_i \\ g_i = 0 & \text{otherwise} \end{cases} \tag{4}$$

From the first-order condition (4) it follows that districts only provide local public goods when district size is large enough compared to the economies of scale effect in the provision of local public goods.

3 Efficiency and grants

It is a common finding that in the form of decision making described in Section 2 the spillover effect is not taken into account and that, therefore, underprovision of local public goods occurs. A system of grants, however, can be used to achieve an efficient

[8] In this and in subsequent maximization problems the strict concavity of $v(.)$ implies that the first order conditions are sufficient. Moreover, the strict concavity implies that the solutions are unique.

Nash equilibrium without completely centralizing decision making. We assume that such a system is implemented by a central government and that to finance this system, individuals pay a national lump-sum tax T. In order to characterize such an equilibrium, we first characterize the optimal levels of local public good provision as a benchmark for normative evaluation of equilibrium outcomes. Then we characterize a system of matching grants that induces local governments to provide these optimal levels of local public goods.

Since in this model the payoffs are quasilinear in the endowment, for efficiency it suffices to focus on an outcome in which all individuals pay the same tax level. The objective is to maximize the equally weighed sum of all individual utilities. The maximization problem for determining g_i can therefore be written as

$$\max_{g_i} \; n_i v(g_i) + \textstyle\sum_{j\neq i} n_j \kappa v(g_i) + Ny - NT \tag{5}$$

and since the budget is balanced

$$T = \sum_{j|g_j>0} \frac{F + n_j p_j g_j}{N} \tag{6}$$

We define $\hat{g}_i$ to be the socially optimal or efficient per-capita amount of public goods if $\hat{g}_i$ satisfies the following first-order condition of (5):

$$\begin{cases} v'(\hat{g}_i) = \frac{n_i p_i}{n_i + (N-n_i)\kappa} & \text{if} \quad \begin{array}{l} n_i v(\hat{g}_i) + \sum_{j\neq i} n_j \kappa v(\hat{g}_i) > \\ F + n_i p_i \hat{g}_i \end{array} \\ \hat{g}_i = 0 & \text{otherwise} \end{cases} \tag{7}$$

From the first-order condition (7) it follows that it is only efficient to provide local public goods when district size is large enough compared to the economies of scale in local public good provision. A comparison of the first-order conditions (7) with (4) yields that there is indeed underprovision of public goods. Firstly, the strictly positive levels of public goods in a non-cooperative Nash equilibrium are lower than the level in an efficient outcome. Furthermore, the minimum district size for providing a positive amount of the public good is smaller than in the efficient outcome.

In the following we consider a system consisting of matching (or conditional) grants. The timing is now as follows.

1. The central government chooses a system of matching grants.
2. The local governments observe the system of matching grants and choose the amounts of local public goods that will be provided.

Let m_i denote the share of total spending the local government of district i can reimburse. This reimbursement is chosen such that the marginal incentives to provide local public goods are efficient. Again, districts have balanced budgets and therefore tax rates previously given by expression (2) are now given by

$$t_i = \begin{cases} \left(\frac{F}{n_i} + p_i g_i\right)(1 - m_i) & \text{if } g_i > 0 \\ 0 & \text{if } g_i = 0 \end{cases} \tag{8}$$

and the national tax rate is given by

$$T = \frac{\sum_{j|g_j>0} (F + p_j n_j g_j)\, m_j}{N} \tag{9}$$

The level of public goods provided in a district i is then implicitly given by maximization problem (10) with tax rates t_i and T given by expressions (8) and (9), respectively.

$$\max_{g_i} \quad v(g_i) + \sum_{j \neq i} \kappa v(g_j) + y - t_i - T \tag{10}$$

The first-order condition of this maximization problem is given by

$$\begin{cases} v'(g_i) = p_i(1-m_i) + \frac{n_i p_i m_i}{N} & \text{if} \quad \begin{array}{l} v(g_i) > \\ \left(\frac{F}{n_i} + p_i g_i\right)(1-m_i) + \frac{(F+n_i p_i g_i)m_i}{N} \end{array} \\ g_i = 0 & \text{otherwise} \end{cases} \tag{11}$$

From (7) and (11) it follows that the marginal incentives to provide local public goods is optimal with the following conditional transfers $\hat{m}_i$

$$\hat{m}_i = \frac{N\kappa}{n_i + (N - n_i)\,\kappa} \tag{12}$$

The marginal incentives to provide public goods are now efficient, moreover, a comparison of the conditions in (7) and (11) with $m_i = \hat{m}_i$ reveals that the decision whether to provide public goods is now also efficient, that is $g_i = \hat{g}_i$ for all i. Another feature of the transfer scheme characterized by (12) is that the transfers $\hat{m}_i$ do not depend on the exact values of the p_i's. Furthermore, when individuals choose g_i given matching grants $\hat{m}_i$ then $\hat{g}_i$ constitutes the unique Nash equilibrium outcome. Finally, note that the above transfer scheme would be the outcome that a benevolent, social-welfare maximizing, national government would choose. It is, however, also the scheme individuals would choose in case they would vote ex-ante, without knowing their district sizes, over a transfer scheme. Although majority voting, with knowledge of district sizes, might lead to a different transfer scheme, the result presented in the next section is stronger with the social-welfare maximizing transfer scheme: Even with such a scheme, the efficient outcome will not always be obtained when there is a soft budget constraint.

4 The soft budget constraint

In Section 3 we show that the first-best outcome can be reached with matching transfers. The motivation behind a system of matching grants is given by the benefits individuals outside a district get from the local public goods provided in this district. The principle that a central government makes transfers to increase efficiency creates, however, another possibility. When a district does not provide any local public goods at all, the central government could, with the same motivation as for the conditional transfers, make a transfer to this district so that at least some public goods are provided in this district so that people outside the district have the benefits from the spillovers. This motivation seems to be an essential feature of *bailouts* or *soft budget constraints*. The bailout policy is thus carried out in the interest of those individuals that are not located in the district needing a bailout. This leads to the same outcome as with majority voting, that is when individuals vote over pairwise comparisons of bailout levels.

We focus on the decision of the individuals in a single district i and in the analysis we assume that all other districts choose the positive levels given in Section 3. Even when the central government is willing to give a district a bailout, the choice of the individuals whether to induce such a bailout still depends on the increase in the central tax level necessary to finance the bailout, and the amount of local public goods provided in the district under a bailout. The decision on the bailout is taken after the decisions on the amount of local public goods are made by the districts. The timing is thus:

1. The central government chooses a system of matching grants.
2. The local governments observe the system of matching grants, choose the amounts of local public goods that will be provided and choose whether to induce a bailout.
3. The central government, observing the choices made by local governments, decides on bailouts induced by local governments.

In the following analysis we look at this game recursively, that is first at the central government's bailout policy and then at the decision over local public goods provision in district i. We assume that bailouts are costly, that is the central government has to put effort in finding out what the local cost parameter p_i is. The costs of this effort are denoted by c_{BO}. As already done for the system of matching grants, a majority voting argument is also given for the central and local governments' policies.

4.1 Central government bailout policy

In this section we look at the reaction of the central government when the individuals in a district choose a g_i and thus a t_i such that $g_i < \hat{g}_i$. Now the central government can intervene in district i's provision of local public goods g_i by making a lump-sum grant such that, per capita, an amount of local public goods in district i of $g_i + \underline{m_i}$ is provided. We do not drop the assumption that budgets are balanced, so to finance this transfer the central tax level is increased by $n_i p_i \underline{m_i}/N$. Finally, we assume that bailouts are costly, that is additionally the central tax rate increases by c_{BO}/N for each bailout.

Given these assumptions, the central government maximizes the payoff of an individual located outside the district that might get a bailout, and this optimization problem can be written as

$$\max_{\underline{m_i}} \quad \kappa v(g_i + \underline{m_i}) - T_{BO} \tag{13}$$

where T_{BO} denotes the raise in the central tax rate due to the bailout and is given by

$$T_{BO} = \begin{cases} \frac{F + n_i p_i g_i + c_{BO} + n_i p_i \underline{m_i} - t_i n_i}{N} & \text{if } \underline{m_i} > 0 \\ \frac{c_{BO}}{N} & \text{if } \underline{m_i} = 0 \end{cases} \tag{14}$$

The first-order condition of this maximization problem is given by

$$\begin{cases} \kappa v'(g_i + \underline{m_i}) = \frac{n_i p_i}{N} & \text{if } \begin{array}{l} \kappa v'(g_i) > \frac{n_i p_i}{N} \text{ and} \\ \kappa v(g_i + \underline{m_i}) > \frac{F + n_i p_i g_i + c_{BO} + n_i p_i \underline{m_i} - t_i n_i}{N} \end{array} \\ \underline{m_i} = 0 & \text{otherwise} \end{cases} \tag{15}$$

A comparison of conditions (15) and (7) reveals that the amount of public goods provided under the bailout policy is lower than the amount chosen by the individuals when there is a hard budget constraint. This implies that it is potentially costly for individuals to induce a bailout - in case of a bailout there will be less local public goods in their districts than they are willing to pay for. In the next subsection we look in more detail at the decision whether individuals will induce a bailout. In addition, from (15) it follows that when economies of scale in local public good provision become more important, the central government is less likely to provide a bailout.

Condition (15) makes it possible to characterize the central government's bailout policy.

Lemma 1 *There exist critical values $\overline{n_{i;C}}$, $\underline{t_{i;C}}$ and $\overline{g_{i;C}}$ such that:*

1. *if $n_i > \overline{n_{i;C}}$ the central government does not provide district i a bailout, even when district i chooses a zero level of own-contribution to local public good provision;*

2. *if $n_i < \overline{n_{i;C}}$ the central government provides a bailout to district i if and only if $t_i > \underline{t_{i;C}}$ and $g_i < \overline{g_{i;C}}$.*

Proof of Lemma 1:

(1): From condition (15) it follows that when $g_i = 0$ a necessary condition for $\underline{m_i} > 0$ is $\kappa v'(0) > \frac{n_i p_i}{N}$. Hence, for $n_i > \overline{n_{i;C}} = \frac{\kappa N v'(0)}{p_i}$ the central government never provides a bailout.

(2): Let $\overline{g_{i;C}}$ be so that $\kappa v'(\overline{g_{i;C}}) = n_i p_i / N$ and $\underline{t_{i;C}} = \frac{F + n_i p_i g_i + c_{BO} + n_i p_i \underline{m_i} - N \kappa v(g_i + \underline{m_i})}{n_i}$. Then for $g_i < \overline{g_{i;C}}$ it holds that $\kappa v'(\overline{g_{i;C}}) > n_i p_i / N$. If in addition $t_i > \underline{t_{i;C}}$ then from condition (15) it follows that the government will provide a bailout. $\square$

It follows from Lemma 1 that the willingness of the central government to provide bailouts and district size are negatively related. As argued above, the willingness of the central government to give a bailout is not sufficient for a bailout to take place. In the following section we therefore look at whether local governments indeed choose to induce such a bailout.

4.2 Local government bailout policy

The central government bailout policy, implicitly given by condition (15), does not fully characterize the occurrence of bailouts. The condition shows how and when a district can induce a bailout from the center. This does not, however, imply that such a bailout is attractive for a district. In other words, condition (15) is *necessary*, but not *sufficient*. Below we analyze the choice made by individuals in a district, given the soft-budget constraint.

First note that, for any g_i such that $\kappa v'(g_i) < n_i p_i / N$ the district will receive no bailout at all. In this case the optimal choice for the individuals in district i therefore is $\hat{g}_i$. Secondly, when g_i is such that $g_i < \hat{g}_i$ and as long as both conditions in the first line of (15) are met, it follows that the amount of local public goods provided under a bailout is not affected by the value of g_i. Individuals within the district that induces a bailout naturally are interested in making their own contribution to local public good provision as small as possible. An obvious way to do this is by choosing $g_i = t_i = 0$.

It may then be the case, however, that although $\kappa v'(0) > n_i p_i / N$ the second inequality of the first line of condition (15) does not hold and district i would therefore not get a bailout when $t_i = 0$. It can be, however, that although the second inequality of the first line of condition (15) does not hold for $t_i = 0$, it holds for $t_i = F(1-\hat{m}_i)/n_i$. In the latter case individuals in the district that gets a bailout minimize their own contribution by choosing $t_i = F(1-\hat{m}_i)/n_i$ and g_i positive but infinitesimally small. We assume, however, that individuals in district i can only induce bailouts with $t_i = 0$. Note that if district i gets a bailout with $t_i = 0$ then it would also get a bailout with any $t_i > 0$ and that we focus on the only type of bailouts one could observe when there are no economies of scale in local public good provision.

In the remaining of this section we focus on a particular class of the payoff functions $v(.)$, either $v(g) = ln(g+1)$ or $v(g) = g^{1-\alpha}/(1-\alpha)$ for $\frac{1}{2} \leq \alpha < 1$. This implies that $v(.)$ should be "concave enough".

Individuals within district i prefer to induce a bailout with $t_i = 0$ and T given by expression (6) over an optimal level of public good provision $g_i = \hat{g}_i$ when

$$v(\underline{m_i}) + \textstyle\sum_{j\neq i} \kappa v(\hat{g}_j) + y - \left(T + \frac{F+c_{BO}+n_i p_i \underline{m_i}}{N} - \frac{(F+n_i p_i \hat{g}_i)\hat{m}_i}{N}\right) >$$
$$v(\hat{g}_i) + \textstyle\sum_{j\neq i} \kappa v(\hat{g}_j) + y - \left(\frac{F}{n_i} + p_i \hat{g}_i\right)(1-\hat{m}_i) - T$$

which, using expressions (7), (12) and (15), can be rewritten as

$$v(\hat{g}_i) - v(\underline{m_i}) < v'(\hat{g}_i)\hat{g}_i + \frac{(N-n_i)(1-\kappa)F}{N\,(n_i + (N-n_i)\kappa)} - \kappa v'(\underline{m_i})\underline{m_i} - \frac{c_{BO}}{N} \qquad (16)$$

Condition (16) makes it possible to show how district size and the local government's bailout policy are related.

Lemma 2 *There exists critical values $\overline{n_{i;L}}$ such that if $n_i < \overline{n_{i;L}}$ and if the central government is willing to give a bailout to district i, then the local government of district i will induce a bailout.*

Proof of Lemma 2: First note that when the central government is not willing to give a bailout, the local government will not induce a bailout since the per-capita costs of inducing would be c_{BO}/N.

Secondly, look at the case with $F = 0$. The left-hand side of (16) then increases more when n_i increases than the right-hand side if

$$v'(\hat{g}_i)\frac{\partial \hat{g}_i}{\partial n_i} - v'(\underline{m_i})\frac{\partial \underline{\underline{m_i}}}{\partial n_i} > v''(\hat{g}_i)\frac{\partial \hat{g}_i}{\partial n_i}\hat{g}_i + v'(\hat{g}_i)\frac{\partial \hat{g}_i}{\partial n_i} - \kappa v''(\underline{m_i})\frac{\partial \underline{\underline{m_i}}}{\partial n_i}\underline{m_i} - \kappa v'(\underline{m_i})\frac{\partial \underline{\underline{m_i}}}{\partial n_i} \tag{17}$$

When $v(g) = log(g+1)$ then (17) can be rewritten as

$$\frac{1}{n_i} - \frac{N\kappa}{n_i\,(n_i + (N - n_i)\kappa)} > p_i\left(\frac{1}{N} - \frac{N\kappa}{(n_i + (N - n_i)\kappa)^2}\right)$$

and this inequality holds for $p_i \leq N/n_i$, and while from Lemma 1 we know that the central government only provides bailouts when $p_i \leq \kappa N v'(0)/n_i = \kappa N/n_i < N/n_i$, it follows that inequality (17) holds for $v(g) = log(g+1)$.

For $v(g) = g^{1-\alpha}/(1-\alpha)$ expression (17) can be rewritten as

$$\left(\frac{1}{\alpha} - \frac{\kappa}{\alpha} + \kappa\right) > \left(\frac{n_i + (N - n_i)\kappa}{N\kappa}\right)^{\frac{1}{\alpha}-2}$$

and this inequality holds when $\frac{1}{2} < \alpha < 1$ for possible value of κ, n_i and N, so inequality (17) holds for $v(g) = g^{1-\alpha}/(1-\alpha)$.

This leads to three possibilities when $F = 0$. Firstly, when (16) holds for all possible n_i then bailouts always take place, and this is the case when $\overline{n_{i;L}} = N/2$. Secondly, when (16) does not hold for any n_i then bailouts never take place and this is the case when $\overline{n_{i;L}} = 0$. Finally, when neither of these two does hold, then by the intermediate value theorem there exists an $\overline{n_{i;L}}$ such that condition (16) holds if and only if $n_i < \overline{n_{i;L}}$.

Finally, when $F \neq 0$, then the only difference is the term

$$\frac{(N - n_i)(1 - \kappa)F}{N\left(n_i + (N - n_i)\kappa\right)}$$

and it is straightforward to show that this term is decreasing in n_i. This implies that a similar reasoning holds for $F \neq 0$. □

It follows from Lemma 2 that individuals are more likely to induce a bailout when they are in a small district. Besides, from condition (16) it follows that an increase in F makes it more likely that local governments induce a bailout. From Section 4.1, however, it followed that the central government is less likely to give a bailout when the economies of scale in local public goods provision are more important, that is, when F is larger. It is therefore not clear how the economies of scale in local public good provision is related to the occurrence bailouts.

5 Bailouts in equilibrium

The analysis in Sections 4.1 and 4.2 specified the bailout policies of the central government and of the local government, respectively. In this section we argue that these bailouts can occur in a subgame-perfect Nash equilibrium.

For tractability reasons, we make further assumptions to do this. In the first place we look at a specific utility function $v(g) = g^{1-\alpha}/(1-\alpha)$ for $\alpha = 1/2$ and at cases where there are no economies of scale in local public good provision, so $F = 0$. Secondly, except district 1 with size n_1, districts are of equal size n. For the districts of size n, all cost parameters are equal to p_H while the possible values for the cost parameter p_1 of district 1 are p^L and p^H, with $p^L < p^H$, where $Prob[p_1 = p^L] = Prob[p_1 = p^H] = \frac{1}{2}$.

Now consider the following strategies:

> **Central government**: Give each district an earmarked lump-sum transfer equal to the amount of public goods that would be socially optimal in that district if $p_i = p^H$ and a matching grant given by expression (12). When a local government induces a bailout, provide one when the conditions of Lemma 1 hold.
>
> **Local government**: Induce a bailout when the conditions of Lemmas 1 and 2 are met, otherwise provide an amount of the public good that satisfies conditions (7).

As argued above, there are majority voting arguments behind the strategies of the central and local governments.

Conjecture 1 *The above-mentioned strategies are, under certain parameter restrictions, the unique subgame-perfect Nash equilibrium.*

The crucial requirement for the above outcome to be a subgame-perfect Nash equilibrium is that the central government does not have an incentive to change the system of matching grants to avoid bailouts. Recall that bailouts are costly from a social welfare point of view, since less public goods than the socially optimal amount are provided and since there are bailout costs c_{BO}. A social-welfare maximizing central government would therefore try to adjust the system of matching grants to avoid bailouts. Note that districts with higher costs of public good provision provide less of these goods. Bailouts can thus be avoided in all districts with $p_i = p^H$ by giving these district an earmarked lump-sum transfer equal to the amount of public goods that would be socially optimal in that district if $p_i = p^H$. These transfers are earmarked in the sense that they have to be spent on local public goods. In addition to that, districts get the matching grant given by expression (12). It is straightforward to show that, in the absence of bailouts, these grants lead to the socially optimal outcome.

Next to the matching grants there are now earmarked lump-sum grants. Lemma 1 and 2, however, are proven for the case in which there are no lump-sum grants. To make sure we can still use the results of these lemmas, we only look at cases where no lump-sum grant is given to district 1. We thus restrict our attention to those cases where it is not socially optimal to provide public goods in district 1 when $p_1 = p^H$, and therefore no earmarked lump-sum transfer is given to district 1. This is the case when district 1 is small enough. A more precise discussion of this can be found in an Appendix.

The second possible adjustment of the system of grants would be to change the matching grant $\hat{m}_1$. This changes the incentives of district 1 to induce a bailout, more specifically, an increase in the matching grant m_1 would decrease the incentives to induce a bailout. When district 1 is small enough, however, a social-welfare maximizing central government does not have an incentive to do this. A more precise argument can again be found in an Appendix.

It follows that the central government does not have an incentive to deviate from the system of matching grants $\hat{m}$ if district 1 is small enough. This is similar to the conditions lemmas 1 and 2 imply for bailouts to take place. For district 1 sufficiently small, the above strategies are therefore indeed the subgame-perfect Nash equilibrium outcome, and bailouts can take place.

Finally, note that it is important which equilibrium concept one uses. In a subgame-perfect Nash equilibrium players do not have the possibility to commit to strategies. A social-welfare maximizing government thus cannot commit not to provide bailouts, even though this would be welfare maximizing. The impossibility of such a commitment is, in our view, another typical characteristic of bailouts.

6 Concluding remarks

There is much evidence, in developing as well as developed countries, that relatively small subnational jurisdictions are more likely to be bailed out. Like Wildasin (1997), the paper focuses on the relationship between size and soft budget constraints in a model where positive externalities in the provision of local public goods motivates grants and bailouts from the central government to subnational jurisdictions. We extend the analysis by including economies of scale in local public good provision and we get results that differ from previous contributions, but that are in line with the evidence. From the model three broad conclusions emerge:

[1] The willingness of the central government to bail out a subnational jurisdiction depends (a) negatively on the size of the jurisdiction and (b) positively on the externalities associated with local public good provision.

[2] The willingness of a subnational jurisdiction to induce a bailout and the size of this jurisdiction are negatively related.

[3] Bailouts can occur in a subgame-perfect Nash equilibrium. As long as the subnational jurisdiction that might get a bailout is small enough, the prevention of a potential bailout is too costly.

7 Appendix: Spillovers

In this appendix we show that the main results of the paper are robust for a different specification of the spillover effect, namely that the spillover effect κ, is increasing in the size of the district where the public good is provided. An individual in district i

now gets a benefit $\kappa n_j v(g_j)$ of the public goods provided in district j, $(i \neq j)$. In a non-cooperative equilibrium the utility of an individual in district i thus is

$$v(g_i) + \sum_{j \neq i} \kappa n_j v(g_j) + y - t_i$$

where t_i is given by (2). It is straightforward to show that the non-cooperative equilibrium is again given by (4).

The socially optimal or efficient outcome is now determined by the following maximization problem

$$\max_{g_i} \; n_i v(g_i) + \textstyle\sum_{j \neq i} n_j \kappa n_i v(g_i) + Ny - NT$$

where T is defined by (6). Let $\hat{g}_i$ again denote the socially optimal or efficient outcome, where $\hat{g}_i$ satisfies the first-order condition of this maximization problem:

$$\begin{cases} v'(\hat{g}_i) = \frac{p_i}{1+(N-n_i)\kappa} & \text{if } \begin{array}{l} n_i v(\hat{g}_i) + \sum_{j \neq i} n_j \kappa n_i v(\hat{g}_i) > \\ F + n_i p_i \hat{g}_i \end{array} \\ \hat{g}_i = 0 & \text{otherwise} \end{cases} \tag{18}$$

A comparison of the first-order conditions (18) with (4) yields that there is again underprovision of public goods. As in Section 3, it is possible, however, to find a system of matching grants that induce the optimal outcome. Individuals choose to provide the social efficient when the central government chooses the following matching transfers $\hat{m}_i$

$$\hat{m}_i = \frac{N\kappa}{1 + (N - n_i)\,\kappa}$$

We now focus on soft budget constraints. As in Section 4.1, we first analyze the central government bailout policy. The central government maximizes the payoff of an individual located outside the district that might get a bailout, and this optimization problem can be written as

$$\max_{\underline{m_i}} \quad \kappa n_i v(g_i + \underline{m_i}) - T_{BO}$$

where T_{BO} is given by (14). The first-order condition of this maximization problem is given by

$$\begin{cases} \kappa v'(g_i + \underline{m_i}) = \frac{p_i}{N} & \text{if} \quad \begin{array}{l} \kappa v'(g_i) > \frac{p_i}{N} \text{ and} \\ \kappa n_i v(g_i + \underline{m_i}) > \frac{F + n_i p_i g_i + c_{BO} + n_i p_i \underline{m_i} - t_i n_i}{N} \end{array} \\ \underline{m_i} = 0 & \text{otherwise} \end{cases} \tag{19}$$

Condition (19) makes it possible to characterize the central government's bailout policy.

Lemma 3 *There exist critical values $\underline{\kappa_{i;C}}$, $\underline{t_{i;C}}$ and $\overline{g_{i;C}}$ such that:*

1. *if $\kappa < \overline{\kappa_{i;C}}$ the central government does not provide district i a bailout, even when district i chooses a zero level of own-contribution to local public good provision;*

2. *if* $\kappa > \overline{\kappa_{i;C}}$ *the central government provides a bailout to district* i *if and only if* $t_i > \underline{t_{i;C}}$ *and* $g_i < \overline{g_{i;C}}$.

Proof of Lemma 3: (1): From condition (15) it follows that when $g_i = 0$ a necessary condition for $\underline{m_i} > 0$ is $\kappa v'(0) > \frac{p_i}{N}$. Hence, for $\kappa < \underline{\kappa_{i;C}} = \frac{p_i}{Nv'(0)}$ the central government never provides a bailout.
(2): Let $\overline{g_{i;C}}$ be so that $\kappa v'(\overline{g_{i;C}}) = p_i/N$ and $t_{i;C} = \frac{F+n_i p_i g_i + c_{BO} + n_i p_i \underline{m_i} - N\kappa n_i v(g_i+\underline{m_i})}{n_i}$. Then for $g_i < \overline{g_{i;C}}$ it holds that $\kappa v'(\overline{g_{i;C}}) > p_i/N$. If in addition $t_i > t_{i;C}$ then the government will provide a bailout. □

As in Section 4.2, we now focus on the local government's bailout policy. Individuals within district i prefer to induce a bailout with $t_i = 0$ and T given by expression (6) over an optimal level of public good provision $g_i = \hat{g}_i$ when

$$v(\underline{m_i}) + \sum_{j\neq i} \kappa n_j v(\hat{g}_j) + y - \left(T + \frac{F+c_{BO}+n_i p_i \underline{m_i}}{N} - \frac{(F+n_i p_i \hat{g}_i)\hat{m}_i}{N}\right) > \\ v(\hat{g}_i) + \sum_{j\neq i} \kappa n_j v(\hat{g}_j) + y - \left(\frac{F}{n_i} + p_i \hat{g}_i\right)(1-\hat{m}_i) - T$$

which, using expressions (7), (12) and (15), can be rewritten as

$$v(\hat{g}_i) - v(\underline{m_i}) < v'(\hat{g}_i)\hat{g}_i + \frac{(N-n_i)(1-n_i\kappa)F}{Nn_i\left(1+(N-n_i)\kappa\right)} - \kappa n_i v'(\underline{m_i})\underline{m_i} - \frac{c_{BO}}{N} \qquad (20)$$

Condition (20) makes it possible to show how district size and the local government's bailout policy are related.

Lemma 4 *When* $\kappa > 1/n_1$ *there exists critical values* $\overline{n_{i;L}}$ *such that if* $n_i < \overline{n_{i;L}}$ *and if the central government is willing to give a bailout to district* i*, then the local government of district* i *will induce a bailout.*

Proof of Lemma 4: First note that when the central government is not willing to give a bailout, the local government will not induce a bailout since the per-capita costs of inducing would be c_{BO}/N.

Secondly, look at the case with $F = 0$. The left-hand side of (20) increases more when n_i increases than the right-hand side if

$$0 > v''(\hat{g}_i)\frac{\partial \hat{g}_i}{\partial n_i}\hat{g}_i - \kappa v'(\underline{m_i})\underline{m_i}$$

When $v(g) = log(g+1)$ then this inequality can be rewritten as

$$[1 + (N - n_i)\kappa](N - n_i)\kappa^2 N < p_i\{[1 + (N - n_i)\kappa]^2 - N\kappa\}$$

and since $p_i \leq [1 + (N - n_i)\kappa]$ for $g \geq 0$, a sufficient condition for this inequality to hold is that $\kappa > \frac{1}{n_i}$.

For $v(g) = g^{1-\alpha}/(1-\alpha)$ expression (17) can be rewritten as

$$[1 + (N - n_i)\kappa]^{1/\alpha - 2} < [\kappa N]^{1/\alpha - 1}$$

and this inequality holds when $\frac{1}{2} < \alpha < 1$ for values of $\kappa > \frac{1}{N}$.

This leads to three possibilities when $F = 0$. Firstly, when (20) holds for all possible n_i then bailouts always take place, and this is the case when $\overline{n_{i;L}} = N/2$. Secondly, when (20) does not hold for any n_i then bailouts never take place and this is the case when $\overline{n_{i;L}} = 0$. Finally, when neither of these two does hold, then by the intermediate value theorem there exists an $\overline{n_{i;L}}$ such that condition (20) holds if and only if $n_i < \overline{n_{i;L}}$.

Finally, when $F \neq 0$ the only difference is the term

$$\frac{(N - n_i)(1 - n_i\kappa)F}{Nn_i\,(1 + (N - n_i)\kappa)}$$

and since this is decreasing in n_i, a similar reasoning holds for $F \neq 0$. $\square$

8 Appendix: Bailouts in Equilibrium

First note that when $v(g) = g^{1-\alpha}/(1-\alpha)$ for $\alpha = 1/2$ and when it is efficient to provide a positive amount of local public goods in district 1 then this amount is given by

$$\hat{g}_1 = \left(\frac{n_1 + (N - n_1)\kappa}{n_1 p_1}\right)^2$$

and if a bailout is given to district 1 than the amount of public goods is given by

$$\underline{m_1} = \left(\frac{N\kappa}{n_1 p_1}\right)^2$$

No earmarked lump-sum grant It is socially optimal to provide no public goods in district 1 when $p_1 = p^H$ if

$$n_1 v(\hat{g}_1) + (N - n_1)\,\kappa v(\hat{g}_1) - n_1 p^H \hat{g}_1 < 0 \tag{21}$$

that is, when $p_1 = p^H$ if $p^H > 2/n_1$. Since a district should consist of at least one individual a sufficient condition is $p^H > 2$.

No change in the matching grant The central government does not have an incentive, from the social welfare point of view, to change the matching grant to district 1 when for any matching grant m the expected payoff is lower than for $\hat{m}_1$. There are two possible cases when the matching grant could be changed, one in which individuals in district 1 start providing public goods when $p_1 = p^H$, and the other in which this is not the case. For the first case the government does not have an incentive to change the matching grant if

$$\begin{gathered}\left\{Prob[p_1 = p^L]\right\}\left\{n_1 v(g(L,m)) + (N - n_1)\kappa v(g(L,m)) - n_i p^L g(L,m)\right\} + \\ \left\{Prob[p_1 = p^H]\right\}\left\{n_1 v(g(H,m)) + (N - n_1)\kappa v(g(H,m)) - n_i p^H g(H,m)\right\} < \\ \left\{Prob[p_1 = p^L]\right\}\left\{n_1 v(\underline{m_1}) + (N - n_1)\kappa v(\underline{m_1}) - n_1 p^L \underline{m_1} - c_{BO}\right\}\end{gathered}$$

and for the second case the government does not have an incentive to change the matching grants if

$$\begin{gathered}n_1 v(g(L,m)) + (N - n_1)\kappa v(g(L,m)) - n_1 p^L g(L,m) < \\ n_1 v(\underline{m_1}) + (N - n_1)\kappa v(\underline{m_1}) - n_1 p^L \underline{m_1} - c_{BO}\end{gathered} \tag{22}$$

where $g(L,m)$ denotes the amount of public goods individuals in district i provide when the matching grants are m and $p_1 = p^L$.

In case the matching grant differs from $\hat{m}_1$, an amount of public goods is provided in district 1 that differs from the efficient one, so the net aggregate payoff from providing public goods in district 1 decreases. From the discussion on earmarked grants it followed that n_1 and p^H are such that it is efficient to provide no public goods in district 1 when $p_1 = p^H$. With (21) this implies that for all $g(H, m)$ the following inequalities hold

$$n_1 v(g(H,m)) + (N - n_1)\kappa v(g(H,m)) - n_i p^H g(H,m) \leq$$
$$n_1 v(\hat{g}_1) + (N - n_1)\,\kappa v(\hat{g}_1) - n_1 p^H \hat{g}_1 < 0$$

From this it follows that it is sufficient to look at condition (22).

With a change in matching grants the central government tries to avoid a bailout. A bailout is less attractive for individuals in district 1 when they get a higher matching grant. On the other hand, however, the more the matching grant exceeds the optimal grant $\hat{m}_1$, the lower the net aggregate social welfare. That is, the left-hand side of (22) is decreasing in m. The central government therefore tries to find the matching grant m^* such that individuals in district 1 are indifferent between providing public goods and inducing a bailout. This m^* is implicitly given by

$$v\left(g(L,m^*)\right) - \frac{n_1 p^L g(L,m^*)}{n_1}(1 - m^*) - \frac{n_1 p^L g(L,m^*)}{N} m^* = v(\underline{m_1}) - \frac{n_1 p^L \underline{\underline{m_1}}}{N} - \frac{c_{BO}}{N}$$

It follows from Section 4.2 that bailouts are more attractive for individuals in smaller districts, so the smaller the district the bigger the m^* that makes the individuals indifferent between providing public goods and inducing a bailout. The left-hand side of inequality (22) is, however, decreasing in m while the right-hand side does not depend on m, so an increase in $m = m^*$ makes it more likely that this inequality is satisfied and that it is optimal not to change the system of matching grants.

Note that when $p_1 = p^L$ and $g(L, \hat{m_1}) = \hat{g}_1$, inequality (22) is not satisfied for any $c_{BO} > 0$. When $m = 1$, however, inequality (22) can be written as

$$c_{BO} < (1-\kappa)^2 \left(\frac{1}{2}N - n_1\right)$$

and is thus satisfied for some $c_{BO} > 0$ since, by assumption, $n_i < N/2$. As argued above, a decrease in n_1 increases m^*, so it follows from the intermediate value theorem that there exists an n^* such that when $n_1 < n^*$ then the central government does not have an incentive to change the matching grant to district 1.

References

[1] Alesina, A., Spolaore, E., 1997. On the Number and Size of Nations. The Quarterly Journal of Economics 112(4), 1027-1056.

[2] Besley, T., Coate, S., 2003. Central versus Local Provision of Public Goods: A Political Economy Analysis. Journal of Public Economics 87(12), 2611-2637.

[3] Bordignon, M., 2000. Problems of Soft Budget Constraints in Intergovernmental Relationships: The Case of Italy. Inter-American Development Bank, Research Network Working Papers R-398.

[4] Dahlberg, M., Petterson-Lidbom, P., 2003. An Empirical Approach for Evaluating Soft Budget Constraints. Working Paper 28, Uppsala University.

[5] Dillinger, W., Webb, S., 1998. Fiscal Management in Federal Democracies: Argentina and Brazil. World Bank Policy Working Paper 2121.

[6] Inman, R., 1995. How to Have a Fiscal Crisis: Lessons from Philadelphia. The American Economic Review 85 (2), 378-383.

[7] Inman, R., 2001. Transfers and Bailouts: Enforcing Local Fiscal Discipline with Lessons from US Federalism. In: Rodden,J., Eskelund, G., Litvack, J. (eds), Fiscal Descentralization and the Challenge of Hard Budget Constraints, MIT Press: Cambridge, Mass.

[8] Kornai, J., 1986. The Soft Budget Constraint. KYKLOS 29(1), 3-30.

[9] Maskin, E., 1999. Recent Theoretical Work on the Soft Budget Constraint. The American Economic Review 89 (2), 421-425.

[10] Nicollini, J. P., Posadas, J., Sanguinetti, J., Sanguinetti, P., Tommasi, M., 2002. Decentralization, Fiscal Discipline in Sub-National Governments and the Bailout Problem: The Case of Argentina. Inter-American Development Bank, Research Network Working Papers R-467.

[11] Qian, Y., Roland, G., 1998. Federalism and the soft Budget Constraint. The American Economic Review 88 (5), 1141-1162.

[12] Rodden, J., 2003. Soft Budget Constraints and German Federalism. In: Rodden, J. (ed), Fiscal Decentralization and the Challenge of Hard Budget Constraints, MIT Press: Cambridge, Mass.

[13] Sanguinetti, P., Tommasi, M., 2004. Intergovernmental Transfers and Fiscal Behavior: Insurance versus Aggregate Discipline. Journal of International Economics 62(1), 149-170.

[14] Seitz, H., 1999. Subnational Government Bailouts in Germany. ZEI Working Paper B20.

[15] von Hagen, J., Bordignon, M., Dahlberg, M., Grewal, B., Petterson, P., Seitz, H., 2000. Subnational Government Bailouts in OECD Countries: Four Case Studies. Inter-American Development Bank, Research Network Working Paper R-399.

[16] Wildasin, D.E., 1997. Externalities and Bailouts: Hard and Soft Budget Constraints in Intergovernmental Fiscal Relations, mimeo. Vanderbilt University.