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Speculative Attacks with Multiple Sources of Public Information

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Speculative Attacks with Multiple Sources of Public Information

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Abstract:

We propose a speculative attack model in which agents receive multiple public signals. It is characterised by its focus on an informational structure which sets free from the strict separation between public information and private information. Diverse pieces of public information can be taken into account differently by players and are likely to lead to different appreciations *ex post*. This process defines players' private value. The main result is to show that equilibrium uniqueness depends on two conditions: (i) signals are sufficiently dispersed (ii) private beliefs about the relative precision of these signals sufficiently differ. We derive economic policy implications of such a result.

Keywords:

Speculative attack – Private value game – Multiple equilibria – Public and private information.

JEL Classification:

F31 – D82.

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1 – Introduction

Speculative attacks are based on information which is in parts publicly available or provided by media and agencies that are recognised by all major traders on foreign exchange markets. Public information is not just helpful in predicting the future course of an economy, but also induces higher order beliefs that allow for crises occurring out of self-fulfilling beliefs. In this paper, we analyze the question whether multiple sources of public information prevent self-fulfilling prophecies.

Second generation speculative attack models in the tradition of Obstfeld (1986, 1996) can be modelled as coordination games with multiple equilibria. Whether a central bank devaluates a currency depends on market pressures that arise from traders' beliefs about the probability of devaluation. If traders believe in devaluation and speculate against a currency, market pressure may force a central bank to abandon a peg that it would have kept without the additional pressure generated by speculators. Applying the global-game approach, Morris and Shin (1998) have shown that this kind of coordination games has a unique equilibrium, if traders' information is private instead of public.[1] Morris and Shin (2003) and Hellwig (2002) show that equilibrium uniqueness requires that agents attach a sufficiently large weight to private information when both, private and public signals are available. Bayesian rationality requires that weights are positively related to the precision of information, which is the inverse of the variance of the respective signals. Thus, uniqueness relies on private signals being sufficiently precise in comparison to public signals. The most precise information, however, is provided publicly by transparent central banks and well informed agencies. This raises concerns of whether economic transparency may lead the inclination to self-fulfilling prophecies. One counterargument is that agents deal the same information differently and posterior beliefs are private information even if all information about economic fundamentals is publicly available.

While many figures about an economy are provided publicly and become common knowledge (at least in theory), the precision of these figures is usually not public information. A rare exception is the reports by the Bank of England that publishes "fan charts" in addition

[1] A signal is private information if it is received by a single agent and public information if it is received by all agents, all agents know that all agents received the same signal, and so on.

to inflation forecasts.[2] With multiple public signals, beliefs about the relative precision of these signals may differ between agents and lead to different posterior beliefs about the state of the world.

In this paper, we introduce multiple sources of public information in the currency-attack model by Morris and Shin (1998, 2003). Agents receive noisy public signals and have different opinions about the relative precision of these signals. We analyze conditions for uniqueness of the equilibrium.

The model has a unique equilibrium, if there are sufficiently many or strong announcements that hint at states at which either attacking or not-attacking are dominant strategies. In addition, there must be a sufficient mass of agents who attribute enough weight to these signals, so that attacking or non-attacking, respectively, are dominant strategies given their posterior beliefs.

Because, an algebraic solution is intractable for a general number of public signals, we restrict our formal analysis to three cases distinguished by the number of public signals. Each of the three cases gives an additional insight for the intuition that carries over to more general cases. If there are just two signals and agents' beliefs about the relative precision of these signals has a uniform distribution, there is a unique equilibrium if and only if at least one of the signals indicates a state at which neither attacking nor non-attacking is a dominant strategy. With more than two signals or with a uni-modal distribution of beliefs about their precision, uniqueness may require that signals are sufficiently different and agents put a sufficiently strong weight on the most extreme signals. When the number of agents approaches infinity, the distribution of posterior beliefs becomes common knowledge. This turns the private-information game into a private-value game, for which we know that it has a unique equilibrium, provided that there is a sufficient mass of agents for whom either action is a dominant strategy (Dönges and Heinemann, 2001). In our model, this requires that the average precision of public signals is sufficiently low.

In terms of economic policy, we conclude that the central bank should benefit from at least two tools: if used appropriately, number and precision of public announcements can be effective at stabilising the economy in situations where it might be prone to self-fulfilling crises otherwise. The provision of different specialized data about the fundamentals of an economy reduces the inclination to self-fulfilling prophecies in comparison to the provision of

[2] Fan charts indicate estimated probabilities for future inflation rates. These probabilities account for estimated forecast errors, but not for possible errors in the model underlying these estimates.

just one compound announcement. With a sufficiently large number of public signals, the probability that an economy is hit by a crisis due to self-fulfilling beliefs can be reduced to almost zero, provided that these signals are not too precise.

Section 2 introduces the formal model. In Section 3 we give conditions for equilibrium uniqueness. We solve the model for three special cases and develop the intuition that yields robust insights in the interaction between the distribution of signals and the distribution of private beliefs for the determinacy of equilibrium behaviour. In section 4 we draw conclusions for the optimal modes of information dissemination. Section 5 concludes the paper by summing up the main results.

2 – Model

The model builds on the reduced form of a currency-attack game introduced by Morris and Shin (1998, 2003). It deals with an open economy in which the central bank has anchored its exchange rate on a fixed parity. Our main innovation is the introduction of multiple public signals.

2.1. Reduced form game

There is a continuum of agents $i \in [0,1]$ who decide simultaneously whether or not to attack the currency peg by short selling one unit of domestic currency. An attack is associated with transaction costs $t > 0$ that are linked to the differential of interest rates between domestic and foreign currency. The fundamentals of the economy are summarized by an aggregate state variable θ. If the proportion of agents who decide for attacking the currency exceeds θ, the central bank devaluates the currency and attacking agents earn an amount $R > t$. If the proportion of attacking agents is less than or equal to θ, the central bank keeps the peg and attacking agents just loose transaction costs. A high (low) value of θ represents a good (respectively bad) fundamental state. If $\theta \geq 1$, the economy is in a sound condition where the central bank can always defend the currency against an attack. If $\theta < 0$, the currency must be devaluated even without the additional market pressure from speculating traders. The aggregate state θ may be interpreted as a measure of the additional market pressure from speculation that is needed to enforce devaluation.

If a speculator knows that $\theta < 0$, attacking is a dominant strategy, because it leads to a positive payoff independent of the other traders' actions. If a trader knows that $\theta \geq 1$, it is a dominant strategy not to attack, because an attack cannot be successful. If it is common knowledge amongst traders that $0 \leq \theta < 1$ there are two equilibria in pure strategies: either all traders believe in devaluation and attack the currency. In this case, the central bank gives in and beliefs turn out to be correct. Or, traders do not believe in devaluation and abstain from an attack. In this case, the central bank keeps the peg and beliefs are also fulfilled.

2.2. Different informational assumptions

Morris and Shin (2003) distinguish private and public information by assuming that each agent receives two signals, x^i and *y*, that differ from θ by independent noise terms with normal distributions. Signal x^i is private information of the respective agent *i*, while the public signal *y* is commonly observed by all agents. If the variance of $x^i - \theta$ is sufficiently small in comparison to the variance of $y - \theta$, the model has a unique equilibrium with a threshold function $x^*(y)$, such that for a given public signal *y* all agents with signals $x^i < x^*(y)$ attack the currency, while agents with higher signals do not attack.

Metz (2002) and Bannier and Heinemann (2005) analyze the comparative statics of the equilibrium with respect to signals' precision, provided that the condition for uniqueness holds. Heinemann and Illing (2002) suggest that public information should be intermediated by private agencies to prevent agents from exactly inferring which information other agents possess.

If public information is relatively more precise, so that

$$Var(y - \theta) < \sqrt{\frac{Var(x^i - \theta)}{2\pi}},$$

then the model has multiple equilibria and the thresholds to attack are not uniquely determined for all *y*. Here, an attack can be triggered by events that are unrelated to economic fundamentals (sunspots), because traders' beliefs are self-fulfilling. In this light, transparency can have destabilizing effects: if central banks provide accurate information about their foreign currency reserves and publish their statistics and predictions about the future course of the economy, the high precision of this information raises the danger of sudden currency crises triggered by unpredicted shifts of beliefs.

On the other hand, public information is not homogeneous. There is a plurality of channels via which information is provided to the public and even central banks publish different kinds of information that may be more or less relevant for predicting future exchange rates. These bits and peaces of information differ in their relevance for predicting the aggregate state summarized by θ. Furthermore, there is no reason to believe that traders agree on the relative importance of various kinds of information for predicting θ. In the absence of a commonly agreed model (or a common prior) agents may even be aware about their different evaluations and agree to disagree. Consequently, agents may hold different posterior beliefs even if they all receive the same signals about the state of the economy. This raises the question whether multiple sources of public information with unknown precisions are sufficient to guarantee a unique equilibrium.

As so often, the answer is that "it depends." In the following pages we analyze conditions under which multiple sources of public information lead to a unique equilibrium.

2.3. Multiple Public Signals

We extend the model by introducing $K > 1$ public signals received by speculators. Each signal y_k differs from the fundamental state θ by a noise term with a normal distribution, i.e. $y_k = \theta + \varepsilon_k$, with $\varepsilon_k \sim N(0, \tau_k^2)$. Noise terms ε_j and ε_k are pair-wise independent for all $j \neq k$. Denote the vector of public signals by $Y = (y_1, y_2, ..., y_K)$ and assume (without loss of generality) that $y_1 \leq y_2 \leq \cdots \leq y_K$. We interpret each k as one source of public information. Each agent takes into account the whole vector of K commonly observable signals. But, they do not know the true variances and attribute subjective weights to each of these signals.

The posterior associated with a vector of normally distributed signals Y is a weighted average of these signals, $E(\theta \mid Y) = \sum_{k=1}^{K} q_k y_k$, where the weights are given by the relative precision (inverse variance) of these signals, $q_k = \frac{\frac{1}{\tau_k^2}}{\sum_{k=1}^{K} \frac{1}{\tau_k^2}}$. The conditional variance is given by $V(\theta|Y) = \frac{1}{\sum_{k=1}^{K} \frac{1}{\tau_k^2}}$.

We assume that agents know the aggregate level of uncertainty in the economy, so that $V(\theta|Y)$ is common knowledge. But, they do not know the objective weights q_k for each of the public signals that they should use to form their expectations. Instead, each agent has a private belief about these weights that we denote by $q^i = (q_1^i, \ldots, q_K^i)$. Of course, these weights must sum up to one, so that they are contained in a *K*-dimensional simplex,

$$q^i \in \Delta^K = \left\{ q \in \Re^K \,\middle|\, 0 \le q_k \le 1 \quad \wedge \quad \sum_k q_k = 1 \right\}.$$

An agent who believes that relative precisions are given by q^i has a posterior subjective belief about θ that is described by a normal distribution with $E^i(\theta | Y) = \sum_k q_k^i y_k$ and $V^i(\theta | Y) = V(\theta|Y)$.

2.4. Equilibrium Strategies

A strategy is a function a^i, such that: $a^i(Y) \in \{0,1\}$, where $a^i(Y) = 0$ means that agent *i* will not attack and $a^i(Y) = 1$ that she will attack. Denote a strategy profile by $a = (a^i)_{i \in [0,1]}$.

For a given vector of public signals *Y*, the proportion of attacking speculators is $\int_0^1 a^i(Y)\, di$.

The central bank devaluates the currency if this proportion exceeds θ. Thus, for any vector of public announcements *Y* and for any strategy profile *a*, the currency will be devaluated if and only if

$$\theta < \theta^*(Y) = \int_0^1 a^i(Y) di .$$

Thereby, the decision problem of a single agent boils down to attack if and only if the subjective probability for the state being worse than some threshold θ^* is sufficiently large.

The expected payoff from an attack for agent *i*, given the strategy combination *a*, the vector of public signals *Y*, and the agent's subjective beliefs q^i, is

$$R \Pr\left(\theta < \theta^*(Y) \middle| Y, q^i\right) - t ,$$

where $\Pr(\cdots|Y,q^i)$ denotes the subjective probability that an agent with subjective beliefs q^i attributes to the event of devaluation. Given the normality of subjective conditional distributions, we can express the expected payoff using the cumulative standard normal distribution Φ. Agent *i* attacks the currency if

$$R\,\Phi\left(\frac{\theta^*(Y)-E^i(\theta|Y)}{\sqrt{V^i(\theta|Y)}}\right)-t>0$$

$$\Leftrightarrow \quad E^i(\theta|Y)<\theta^*(Y)-\sqrt{V(\theta|Y)}\,\Phi^{-1}\left(\frac{t}{R}\right). \qquad (1)$$

Recall that conditional variances are the same for all agents. Equation (1) shows that an agent attacks if her posterior expectation is below some threshold, at which the expected reward from an attack equals its costs. The proportion of attackers is the proportion of all agents with a subjective expectation below this threshold. For an equilibrium strategy combination, this proportion must equal the critical proportion at the marginal state θ^*, i.e.

$$\theta^*(Y)=\frac{\left|\left\{q^i\in\Delta^K \,\middle|\, E^i(\theta|Y)<\theta^*(Y)-\sqrt{V^i(\theta|Y)}\,\Phi^{-1}\left(\frac{t}{R}\right)\right\}\right|}{\left|\Delta^K\right|}, \qquad (2)$$

and the associated equilibrium strategy is $a^{*i}(Y)=1$ if and only if inequality (1) holds for *i*'s subjective weights q^i.

Equation (2) is equivalent to

$$\theta^* = \frac{1}{\left|\Delta^K\right|}\cdot\left|\left\{q^i\in\Delta^K \,\middle|\, \sum_{k=1}^K q_k^i y_k<\theta^*-\frac{\Phi^{-1}\left(\frac{t}{R}\right)}{\sqrt{\sum_{k=1}^K\frac{1}{\tau_k^2}}}\right\}\right|. \qquad (3)$$

Any solution to this equation $\theta^*(Y)$ characterizes an equilibrium. To proceed the analysis, let us first define

$$\underline{\theta}=-\frac{\Phi^{-1}\left(\frac{t}{R}\right)}{\sqrt{\sum_{k=1}^K\frac{1}{\tau_k^2}}} \qquad \text{and} \qquad \overline{\theta}=1+\underline{\theta}.$$

Private beliefs about the relative precision of multiple public signals are in general not sufficient for equilibrium uniqueness. Consider, for example, the case where all signals hint at some intermediate state of the economy, in particular $\underline{\theta} < y_k < \overline{\theta}$ for all *k*. All agents update their information by forming posterior beliefs that are a weighted average of these signals, so that $\underline{\theta} < E^i(\theta|Y) < \overline{\theta}$ for all *i*. For these posteriors, an attack has a positive expected payoff if all agents attack and a negative expected payoff if almost nobody attacks. Agents agree to disagree in their posterior expectations, but it is common knowledge that everybody believes an attack to be rewarding if everybody attacks, and to fail if almost nobody attacks. The game has two pure-strategy equilibria as in the standard model for intermediate states. There are at least three solutions to equation (3), $\theta^* = 0$, $\theta^* = 1$, and at least one equilibrium with $0 < \theta^* < 1$ where agents with expectations below some interior threshold attack.

It is a necessary condition for a unique equilibrium that at least one of the signals is outside the intermediate region $(\underline{\theta}, \overline{\theta})$. Whether there is a unique equilibrium or multiple equilibria depends on the vector of public announcements *Y* and on the distribution of private beliefs q^i. A general algebraic characterization is intractable. To get an intuition for uniqueness conditions, we characterise them for a particular distribution of private weights q^i and for three special cases for the number of signals, $K = 2$, $K = 3$ and $K \to \infty$. Then, we explain the rationale that carries over to general settings.

3 – Equilibrium uniqueness

In this section, we derive some conditions for equilibrium uniqueness for different numbers of public signals. We show that public information, if interpreted or dealt differently by agents, can lead to a unique equilibrium, even in some cases where the objective posterior hints at a state at which an attack may occur out of self-fulfilling prophecies, if this posterior is common knowledge. While the two-dimensional case is useful to illustrate the consequences of private information about variances, some results are not robust with respect to the number of signals. We solve the case for three signals which is more complicated, but yields robust insights in the interaction between the particular signals and the distribution of private beliefs for the determinacy of equilibrium behaviour. Finally, we solve the case for an infinite number of public signals under more general conditions. This case shows how the accuracy of public announcements affects the existence of multiple equilibria.

For our formal analysis we assume that subjective weights q^i have a uniform distribution on the K-dimensional unit simplex. The corresponding density function is $h(q^i) = 1/S$, $\forall q^i \in \Delta^K$, where $S = |\Delta^K|$ is the size of the K-dimensional unit simplex.

3.1. Two public signals (K = 2)

Suppose there are just two public announcements, y_1 and y_2. We know already that there are multiple equilibria, if both signals are in the interval $(\underline{\theta}, \overline{\theta})$. Now assume, instead, that one signal hints at a particular bad state at which an attack is a dominant strategy, e.g. $y_1 < \underline{\theta}$. Then, there is a positive mass of agents, who believe that attacking is a dominant strategy. Since the distribution is common knowledge, other agents know that there are some fellows, who believe strongly in the worst news and will attack. Thus, they expect a critical mass of attacking capital that raises their own threshold up to which an attack is a dominant strategy. Agents with higher posteriors will attack, because they know that a certain fraction of agents attacks anyway. Since other agents know this as well, some traders with even higher posteriors attack. Higher order beliefs, expressed by the iterative elimination of dominated strategies, lead agents to attack up to some threshold that may represent a unique equilibrium. But, uniqueness requires that at least one signal is outside the multiplicity region and that the distribution is sufficiently thick (in particular at the edges), so that enough mass is attracted in each step of the elimination procedure.

With one signal in the “attack” region, $y_1 < \underline{\theta}$, and the other in the multiplicity region, $\underline{\theta} < y_2 < \overline{\theta}$, there exists one equilibrium, in which all agents attack. Here, the elimination procedure may eliminate any other equilibrium. Vice versa, if there is one signal in the “not attack” region, $y_2 > \overline{\theta}$, and the other is in the multiplicity area: there exists one equilibrium, in which no agent attacks and it may be the only one. Whether the elimination process stops before the threshold reaches the other signal and there are multiple equilibria or not, depends on the distribution of private weights q^i.

If there is one signal in each of the two extreme regions, the elimination procedure reduces the multiplicity region from both sides and may lead to a unique equilibrium with an intermediate threshold, such that all agents with pessimistic beliefs (below the threshold) attack, while agents with more optimistic beliefs refrain from attacking. Whether the elimination from both sides stops at the same point and yields a unique equilibrium or not, depends once more on the distribution of private weights.

For a uniform distribution of weights q^i on the simplex Δ^2, we can show that there are multiple equilibria if and only if *both* signals are inside the multiplicity region. For just two signals, the equilibrium condition (3) can be simplified to

$$\theta^* = \left| \left\{ q \in [0,1] \middle| \, q y_2 + (1-q) y_1 < \theta^* + \underline{\theta} \right\} \right| = \left| \left\{ q \in [0,1] \middle| \, q < \frac{\theta^* + \underline{\theta} - y_1}{y_2 - y_1} \right\} \right|.$$

An equilibrium with $\theta^* = 0$ exists, whenever $y_1 \geq \underline{\theta}$. An equilibrium with $\theta^* = 1$ exists, whenever $y_2 \leq \overline{\theta}$. For a uniform distribution of weights, an intermediate equilibrium is given by

$$\theta^* = \frac{\theta^* + \underline{\theta} - y_1}{y_2 - y_1} \quad \Leftrightarrow \quad \theta^* = \frac{y_1 - \underline{\theta}}{1 + y_1 - y_2}.$$

An intermediate equilibrium exists if and only if $0 < \frac{y_1 - \underline{\theta}}{1 + y_1 - y_2} < 1$. For $y_2 - y_1 < 1$ this condition is equivalent to $\underline{\theta} < y_1 < y_2 < \overline{\theta}$. For these public signals there are three equilibria. For $y_2 - y_1 > 1$, an equilibrium with $0 < \theta^* < 1$ exists, if and only if $y_1 < \underline{\theta} < \overline{\theta} < y_2$, i.e. if the lower signal hints at a state, at which attacking is a dominant strategy and the high signal hints at a state where not-attacking is a dominant strategy. Since this condition rules out equilibria in which all or no agent attack, the game has a unique equilibrium with an interior threshold that arises from an iterative elimination procedure as illustrated in Figure 1.

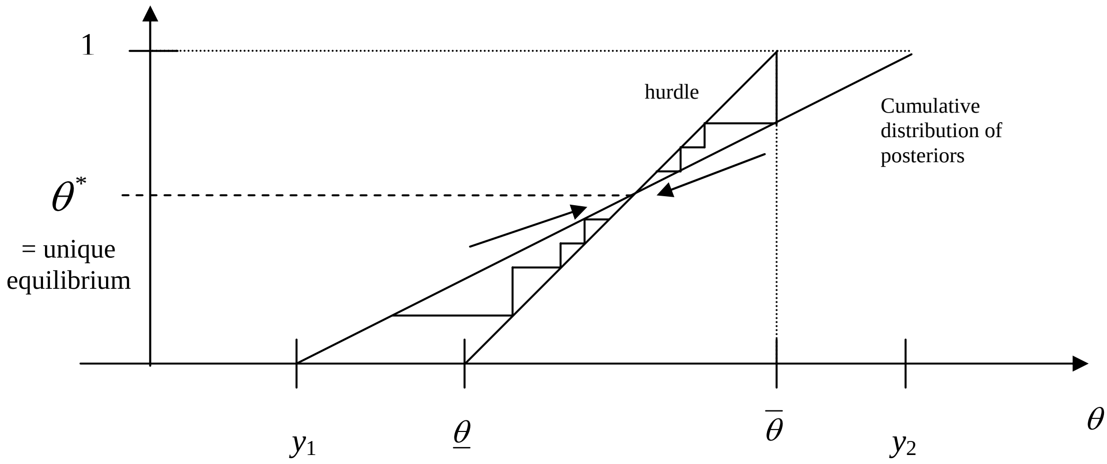

Figure 1. Iterative elimination of dominated strategies.

If $y_1 < \underline{\theta}$ and $\underline{\theta} < y_2 < \overline{\theta}$, there is only one equilibrium, in which all agents attack. If $y_2 > \overline{\theta}$ and $\underline{\theta} < y_1 < \overline{\theta}$, there is a unique equilibrium, in which no agent attacks. Combining these results, multiple equilibria exist if and only if all signals are in the intermediate region.

Proposition 1: For a uniform distribution of subjective weights, the game with two public signals has multiple equilibria if and only if $\underline{\theta} < y_k < \overline{\theta}$ for both k.

This result shows that it makes a crucial difference, whether agents know the variances of public signals or not. For known variances, agents agree on the posterior and multiple equilibria exist, whenever this posterior is in $(\underline{\theta}, \overline{\theta})$. For unknown variances, multiplicity may require that all signals are in this region.

The simplicity of this result is due to the assumptions that the weights q are uniformly distributed and $K = 2$. However, it is not a general condition for multiplicity that all signals must be contained in the intermediate region. This can be seen by either assuming another distribution of weights or by considering more than two signals. For the case with $K = 2$, suppose that the distribution of subjective weights q_2 is uni-modal around 0.5. If the center of the interval $[y_1, y_2]$ is in the multiplicity region, there are less agents with posterior expectations in the dominance regions than for a uniform distribution. The cumulative distribution of posterior beliefs may intersect the hurdle to success up to three times, which may give us multiple equilibria even if $y_1 < \underline{\theta} < \overline{\theta} < y_2$. An example is shown in Figure 2.

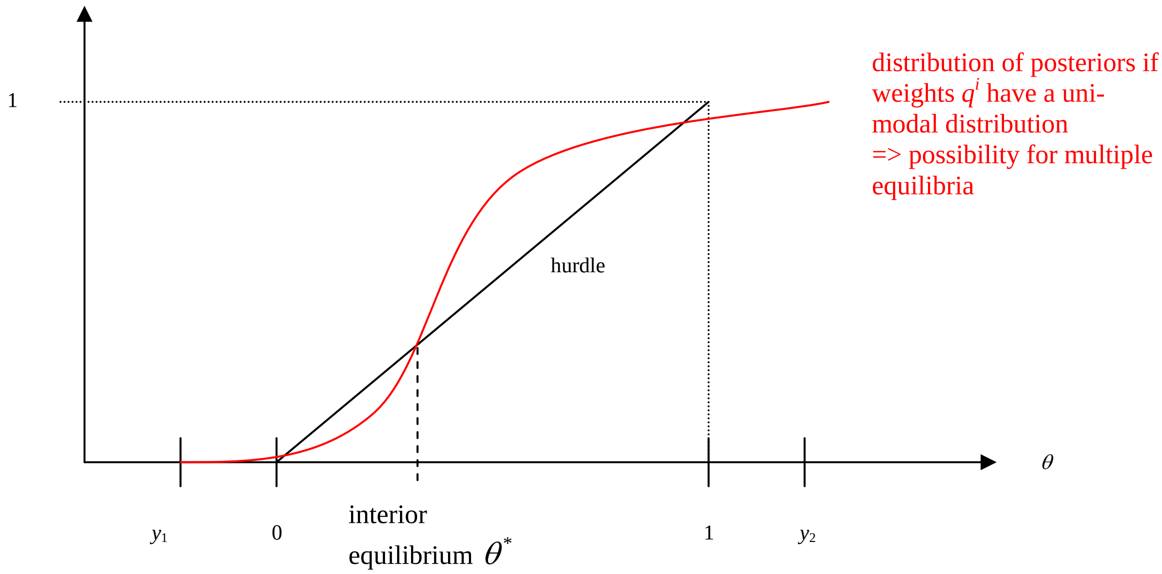

Figure 2. Multiple equilibria for a uni-modal distribution of posteriors.

Indeed, with K=3, we get a similar shape for the cumulative distribution of posterior beliefs even with a uniform distribution of q on the unit simplex.

3.2. Equilibrium in the case of three public signals

To analyze uniqueness conditions with more than two signals, we treat both sides of equation (3) as functions of θ^*. Both sides are continuous and increasing in θ^*, and the right hand side is restricted between zero and one. At the lowest and at the highest equilibrium, the derivative of the right hand side with respect to θ^* stays below 1. Multiplicity requires that there is an intermediate equilibrium, at which the cumulative distribution of posteriors rises faster than the hurdle. That is

$$\frac{d\left|\left\{ q \in \Delta^K \,\middle|\, \sum_k q_k^i y_k < \theta^* + \underline{\theta} \right\}\right|}{d\theta^*} > \left|\Delta^K\right|. \qquad (4)$$

For K = 3, the equilibrium condition is written as follows:

$$\theta^* = \frac{2}{\sqrt{3}} \left| \left\{ q \in \Delta^3 \,\middle|\, q_1 y_1 + q_2 y_2 + q_3 y_3 < \theta^* + \underline{\theta} \right\} \right|.$$

If $\frac{d|\{q\}|}{d\theta^*} < \frac{\sqrt{3}}{2}$, the game has a unique equilibrium.

Proposition 2: For a uniform distribution of subjective weights, the game with three public signals has a unique equilibrium if $y_3 - y_1 > 2$.

Proof: See appendix.

For more than two public signals, uniqueness or multiplicity depends on the precise interaction between the distribution of signals and subjective weights. If all signals are close to each other and cover the intermediate region $(\underline{\theta}, \overline{\theta})$, then there are multiple equilibria, even for a uniform distribution of weights. However, if there is sufficient dispersion between the highest and the lowest signal, then uniqueness of the equilibrium is guaranteed independent from the range that is covered by these signals.

In particular, for $y_3 - y_1 > 2$, the slope of the cumulative distribution of posteriors is smaller than 1. Therefore, it can intersect the hurdle function between $\underline{\theta}$ and $\overline{\theta}$ only once, so that there is a unique equilibrium. If the distribution of weights is more concentrated on the center of the simplex, then the extreme signals need to be even further away from each other to guarantee uniqueness.

The intuition behind this result is the following. If at least one signal is outside the region $(\underline{\theta}, \overline{\theta})$, the equilibrium may be unique and it can be derived by iterative elimination of dominated strategies. The iteration process starts with agents whose posteriors are such that either attacking or not-attacking is a dominated strategy. For the remaining agents with posteriors close to the edges of $(\underline{\theta}, \overline{\theta})$, either action looses its appeal, if they know that the proportion of attacking agents is bounded away from zero or one, respectively. This leads to a smaller region for which neither action is dominant strategy. The size of these steps of elimination depends on the mass of agents for whom either action can be predicted from their extreme beliefs. If the number of agents with extreme beliefs is small, then the iteration procedures stop early and the interval for which posterior beliefs are self-fulfilling is reduced only slightly. However, if a sufficiently large mass of agents is in the respective dominance region, the iteration steps are large and converge to a single threshold.

3.3. Equilibrium in the case of an infinite number of public signals

We determine the analytical solution for equilibrium uniqueness in the case where the number of public signals tends to infinity and give some intuition for a large but finite number of signals.

To ease the exposition, we assume $\tau_k^2 = \tau^2$ for all *k*. That is, all signals have the same precision. However, we keep the assumption that agents have private beliefs about these precisions. While the objective weights are $q_k = 1/K$ for all *k*, individuals attach private weights to the signals. When all signals have the same precision, the conditional variance of θ is $Var(\theta|Y) = \tau^2 / K$. The aggregate uncertainty after realization of signals becomes smaller with an increasing number of signals. With an infinite number of signals, $K \to \infty$, the uncertainty vanishes and agents are almost sure that their private posterior coincides with the true state θ. However, since agents differ in their evaluation of the various signals, they still disagree in their posterior beliefs. With $Var(\theta|Y) \to 0$, the range of posteriors for which there is no dominant strategy converges to the unit interval, $(\underline{\theta}, \overline{\theta}) \to (0,1)$.

Due to the law of large numbers, the distribution of realized signals is almost certainly identical to the prior distribution of signals, $y_k \sim N(\theta, \tau^2)$. However, the distribution of posterior beliefs, $E^i(\theta) = \sum_{k=1}^{\infty} q_k^i \, y_k$, depends also on the distribution of private weights $q^i \in \Delta^\infty$. Any distribution of weights induces a distribution of posteriors with probability one. Denote the cumulative density function of the distribution of posterior beliefs by *F*. The equilibrium condition (3) is then equivalent to $\theta^* = F(\theta^*)$.

Multiplicity of equilibria requires that there is a solution to this equation, at which $f(\theta^*) > 1$, where *f* is the non-cumulative density of posteriors. For a uniform distribution of weights on the simplex and for any single peaked symmetric distribution on the simplex, the induced density of posteriors *f* has its maximum at the true state θ. This maximum decreases to zero with an increase in $\tau^2 \to \infty$. Hence, there is a critical level for the variance of public signals, such that for a higher variance there is a unique equilibrium for all realizations of θ. For lower variances, there may be multiple equilibria for some realizations of $\theta \in (0,1)$. An example is illustrated in Figure 3.

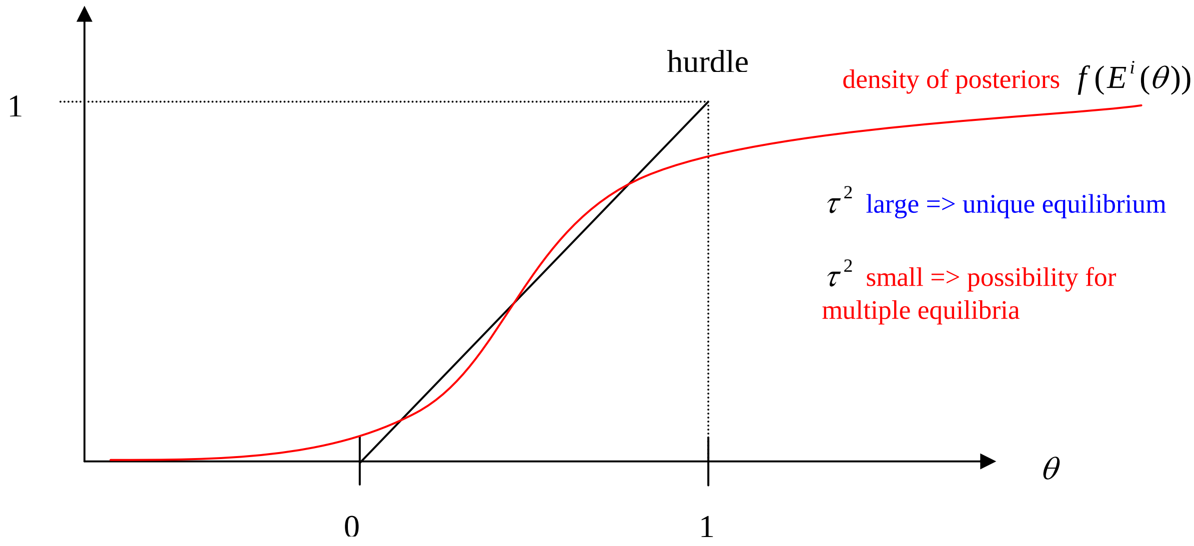

Figure 3. Multiple equilibria may exist for some $\theta \in (0,1)$ *,if* τ^2 *is sufficiently small.*

The higher τ^2, the flatter is the distribution of posteriors.

Proposition 3: For any single-peaked symmetric distribution of weights on the simplex Δ^{∞}*, multiplicity of equilibria requires that* τ^2 *is sufficiently small.*

3.4. Intuition for *K* finite larger than 3

If public signals are rather precise, then most signals are close to the true state θ. Thereby, most agents' posteriors are close to the true state, even though they differ in their opinion about the relative precision of signals. If the true state happens to be in the interior of the region $(\underline{\theta},\overline{\theta})$, then there are multiple equilibria for a sufficiently high precision of public signals. This occurs with some positive probability. The lower the precision of public signals, the wider the dispersion of posterior beliefs and the smaller is the region of states, for which multiple equilibria exist. Thereby, the probability that the economy is endangered by self-fulfilling beliefs gets smaller. If the precision of public signals is sufficiently small, then there is a unique equilibrium for all states θ.

This shows that the precision of public signals is related to the prior probability of an economy being endangered by crises out of self-fulfilling beliefs. In this sense, our results lead to a similar conclusion as the global-game approach by Morris and Shin (2003): uniqueness requires that public information is not too precise. However, our results differ from those by Morris and Shin, because we did not rely on the existence of rather precise private information. Instead, all that we need for uniqueness is a sufficient dispersion of public signals *and* private beliefs about their relative importance. These private beliefs are common knowledge in our model: all agents know the distribution of these weights, and in an economy with a finite number of agents, our results would still hold, if agents know the actual weights of all other agents. Therefore, posterior beliefs are common knowledge, while uniqueness in the global-game approach requires that posterior beliefs are not only different, but private information as well.

4 – The effect of public announcements: some implications for economic policy

Our analysis of the currency-attack model with multiple public signals has some consequences in terms of economic and informational policies. The model contributes to shed light on the current debate on the effects of reinforced transparency. Indeed, the fact that central banks and newspapers release information publicly raises concerns of whether economic transparency may be destabilising, by rendering the economy prone to self-fulfilling crashes. This question is rather important with the adhesion by the IMF to programmes like the SDDS (Special Data Dissemination Standard). Nevertheless, our model suggests a counterargument to the traditional view: agents deal the same information differently and posterior beliefs may differ even if all information is publicly disclosed, as soon as there are multiple public signals.

We have shown that there may be a unique equilibrium if there is at least one public signal that hints at a state at which either attacking or not-attacking is a dominant strategy. With multiple public signals, multiplicity of equilibria requires that (i) signals are not too dispersed and (ii) private beliefs about the relative precision of these signals do not differ too much. These two conditions interact: if signals are dispersed over a wide range, there may still be multiple equilibria if most agents attach the same weights to these signals and vice versa.

With a large number of public signals, the probability of the economy being prone to self-fulfilling beliefs depends on the average precision of signals. If signals get very precise, we approach the case with perfect information. The lower the precision of public signals, the smaller gets the set of states with multiple equilibria and the smaller is the prior probability that the true state falls in this region. For a sufficiently low precision, the equilibrium is always unique.

The economic policy implications of these results require to distinguish three dimensions of transparency: transparent central banks provide more information, i.e. a larger number of public signals. Another dimension of transparency is the precision of the information provided to markets. A third dimension concerns information about the precision of statements, for example reliable figures on expected forecast errors. A larger number of signals (or more frequent provision of information) helps to avoid overreactions to any single announcement. A higher precision is useful for markets in determining the consequences of actions. But, it also raises the probability that most signals are in the multiplicity region. Finally, if agents agree on the precision of signals, their posteriors coincide, which leads to the same effects as providing a summary statistic as a single public signal. It induces high common weights to the announcements that may lead to crises out of self-fulfilling beliefs if the common posterior indicates a critical situation.

We have shown that agents do not always over-react to public information. Indeed, when there are multiple public signals whose precisions are not common knowledge- agents do not always have self-fulfilling beliefs. In the case where K=1, the result is completely different from K>1. There is a place for equilibrium uniqueness under certain conditions (whereas this is impossible when agents receive only one public signal because of common knowledge). As a consequence, the economy should be relatively more stable with $K > 1$. This gives a role to the precision of signals: apart from its degree, uncertainty on it can represent an effective tool for the central bank to control for the beliefs of the agents. The number of signals is also essential; especially having two (appropriate) public signals instead of one on the market can prevent from self-fulfilling equilibria by avoiding common posterior beliefs.

However, when K>1, then increasing the number of public signals K beyond two might not be helpful (in terms of stabilisation) insofar as equilibrium uniqueness requires (from K>2) a sufficient mass on the "extreme" values (i.e. external to the interval $\left[\underline{\theta},\overline{\theta}\right]$). For example, if the new disclosed signals accumulate in the intermediate region, it can be worse for the central bank to give more announcements even if they are more precise. The contents of announcements is also very important. Suppose agents receive two public signals. If some additional announcements (say two) cross either border, the equilibria switch to another regime, as represented on the next figure.

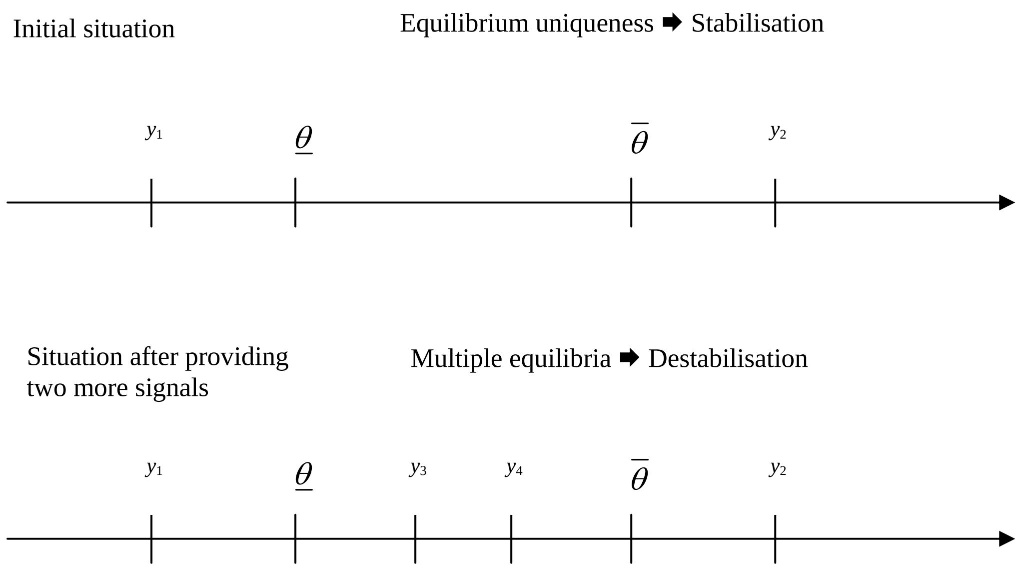

Figure 4. Destabilization by Public Announcements.

This effect can be reinforced if signals are of high precision: increasing the number of too precise public signals can lead to a situation equivalent to common knowledge and damage the stability of the economy.

Finally, the number of signals is also ambiguous. Two intertwined effects go in opposite directions: with a large number of signals, there is a higher chance of getting signals in extreme areas while with a higher dimension, there might be some amplifying effects due to higher order beliefs.

We thus make the case that public information is not per se (automatically) destabilising. Our model is less deterministic than second generation models that always give multiple equilibria in the intermediate zone and private information models that always find some conditions for uniqueness (as soon as private information is sufficiently precise). Providing multiple public signals does not exclude multiple equilibria, but reduces the likelihood that conditions for multiplicity are met.

5 – Conclusion

This paper sheds light on the difficulties linked with the dichotomy between public information on the one hand and private information on the other. How increasing public information without increasing private information, and vice versa? Those two notions should be linked although theory clearly distinguishes them. In the literature, there typically lacks a model that could show how diverse sources of information or differences in the treatment of information could avoid common posterior beliefs, thus creating sufficient differences in the evaluation of publicly available information to prevent self-fulfilling beliefs equilibria. Here, we try to fill in the theoretical gap between public and private information, by proposing a private value game applied to the traditional speculative-attack model.

It is well known that common knowledge is difficult to establish in practice. However, financial markets are very transparent and many informational signals are disclosed by the central bank, or any other institution. On the exchange rate market, there is a plurality of channels (*media*) which disclose more or less precise (but "objectively mistaken") public information. Hence, any information is observed by all the agents; as agents are rational, they are aware of that. Common knowledge of *posterior* beliefs does not only require that all agents share the same information, it also requires that agents share the beliefs about the conditional distribution of the revealed information, given the fundamentals. As a consequence, even if all agents share the same information, agents may differ in their evaluation of these signals, and thus in their posterior beliefs. This does not require private information. Agents may agree to disagree. By creating disparities between agents' posterior beliefs, multiple sources of public information can avoid self-fulfilling beliefs equilibria. Such a model can help to explain why and how attacks are determined, even when the most relevant information about fundamentals is publicly disclosed.

6 – References

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7 – Appendix: PROOF of *Proposition 2*

First, note that $\left|\Delta^3\right| = \sqrt{3}/2$. For $K = 3$ the equilibrium condition (3) is given by

$$\theta^* = \frac{2}{\sqrt{3}} \left| \left\{ q \in \Delta^3 \,\middle|\, q_1 y_1 + q_2 y_2 + q_3 y_3 \leq \theta^* + \underline{\theta} \right\} \right|. \tag{4}$$

With a uniform distribution of weights on the simplex, there is a positive mass of agents, who believe strongly in the worst signal and a positive mass of agents who believe strongly in the best signal. Hence, an equilibrium in which all agents attack and $\theta^* = 1$ exists, if and only if $y_k \leq \overline{\theta}$ for all *k*. An equilibrium in which no agent attacks and $\theta^* = 0$ exists if and only if $y_k \geq \underline{\theta}$ for all *k*.

Multiple equilibria require the existence of at least one interior equilibrium, $0 < \theta^* < 1$, at which the derivative of the right hand side of (4) with respect to θ^* exceeds 1.

For any interior equilibrium,

$$\theta^* = \frac{2}{\sqrt{3}} \left| \left\{ q \in \Delta^3 \,\middle|\, q_1 y_1 + q_2 y_2 + q_3 y_3 = \theta^* + \underline{\theta} \right\} \right|. \tag{5}$$

Hence, an interior equilibrium requires that $y_1 < \theta^* + \underline{\theta} < y_3$. So, there exists a linear combination of y_1 and y_3 with $(1 - q_A)y_1 + q_A y_3 = \theta^* + \underline{\theta}$, which is equivalent to

$$q_A = \frac{\theta^* + \underline{\theta} - y_1}{y_3 - y_1}.$$

In Figures 3a and 3b this point is given by A. Now we distinguish two cases: If $y_2 \geq \theta^* + \underline{\theta}$, then there also exists a linear combination of y_1 and y_2 that equals $\theta^* + \underline{\theta}$. This is indicated by point B in Figure 6a. Any combination of weights on the straight line between A and B is associated with the same expected state. In an equilibrium of this type, the area on the simplex below the line AB divided by the total size of the simplex equals θ^*.

If $y_2 < \theta^* + \underline{\theta}$, then there exists a linear combination of y_2 and y_3 that equals $\theta^* + \underline{\theta}$. This is indicated by point B in Figure 6b. Again, any combination of weights on the straight line between A and B is associated with the same expected state. In an equilibrium of this type, θ^* equals the area on the simplex below the line AB divided by the total size of the simplex.

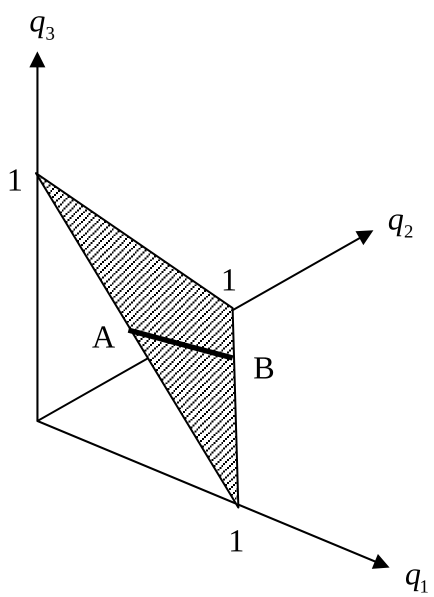

Figure 6a.

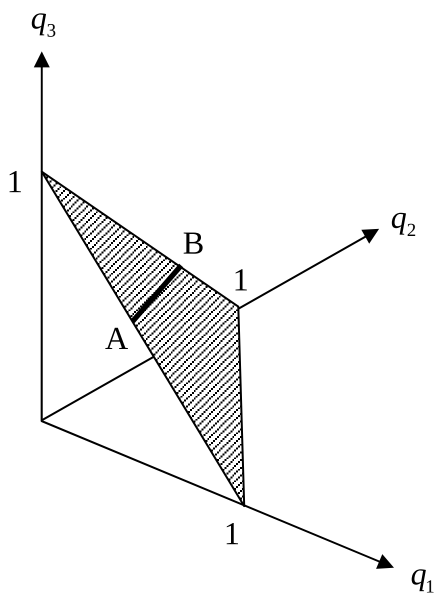

Figure 6b.

In both figures the shaded area is the unit simplex. Points on the simplex below AB *are associated with posterior expectations* $\sum_{k=1}^{3} q_k y_k \leq \theta^* + \underline{\theta}$.

If $y_2 \geq \theta^* + \underline{\theta}$, the coordinates of B are $\left(\frac{y_2 - \theta^* - \underline{\theta}}{y_2 - y_1}, \frac{\theta^* + \underline{\theta} - y_1}{y_2 - y_1}, 0 \right)$. Basic rules of trigonometry enable us to calculate that the area below AB; the latter has the size

$\frac{\sqrt{3}}{2} \cdot \frac{(\theta^* + \underline{\theta} - y_1)^2}{(y_3 - y_1)(y_2 - y_1)}$. Thus, the condition for an interior equilibrium (5) is equivalent to $\theta^* = \frac{(\theta^* + \underline{\theta} - y_1)^2}{(y_3 - y_1)(y_2 - y_1)}$. The right-hand side is increasing and concave in θ^*. So, the derivative of the right-hand side is maximal at the highest θ^* at which the condition $y_2 \geq \theta^* + \underline{\theta}$ applies, i.e. at $\theta^* = y_2 - \underline{\theta}$. Here the derivative is $\frac{2}{(y_3 - y_1)}$.

If $y_2 < \theta^* + \underline{\theta}$, the coordinates of B are $\left(0, \frac{y_3 - \theta^* - \underline{\theta}}{y_3 - y_2}, \frac{\theta^* + \underline{\theta} - y_2}{y_3 - y_2}\right)$ and the area below AB has the size $\frac{\sqrt{3}}{2} \cdot \left[1 - \frac{(y_3 - \theta^* - \underline{\theta})^2}{(y_3 - y_2)(y_3 - y_1)}\right]$. Thus, the condition for an interior equilibrium (5) is equivalent to $\theta^* = 1 - \frac{(y_3 - \theta^* - \underline{\theta})^2}{(y_3 - y_2)(y_3 - y_1)}$. The right-hand side is increasing and convex in θ^*. So, the derivative of the right-hand side is maximal at the lowest θ^* at which the condition $y_2 < \theta^* + \underline{\theta}$ applies, i.e. at $\theta^* = y_2 - \underline{\theta}$. Here the derivative is $\frac{2}{(y_3 - y_1)}$.

Combining the two cases, we see that for any interior equilibrium the derivative of the right-hand side of (4) is lower than 1 if $y_3 - y_1 > 2$.

Thus, we conclude that $y_3 - y_1 > 2$ is a sufficient condition for a unique equilibrium.

QED