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Allocative and Informational Externalities in Auctions and Related Mechanisms*

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Abstract

We study the effects of allocative and informational externalities in (multi-object) auctions and related mechanisms. Such externalities naturally arise in models that embed auctions in larger economic contexts. In particular, they appear when there is downstream interaction among bidders after the auction has closed. The endogeneity of valuations is the main driving force behind many new, specific phenomena with allocative externalities: even in complete information settings, traditional auction formats need not be efficient, and they may give rise to multiple equilibria and strategic non-participation. But, in the absence of informational externalities, welfare maximization can be achieved by Vickrey-Clarke- Groves mechanisms. Welfare-maximizing Bayes-Nash implementation is, however, impossible in multi-object settings with informational externalities, unless the allocation problem is separable across objects (e.g. there are no allocative externalities nor complementarities) or signals are one-dimensional. Moreover, implementation of any choice function via ex-post equilibrium is generically impossible with informational externalities and multidimensional types. A theory of information constraints with multidimensional signals is rather complex, but indispensable for our study.

1 Introduction

General equilibrium analysis has identified several forms of externalities as obstacles on the road towards economic efficiency. It is well known that the First Welfare Theorem may fail to hold in the presence of allocative externalities, i.e., when agents care about the physical consumption bundles of

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others. Akerlof's (1970) famous analysis demonstrated that the First Welfare Theorem may also fail in the presence of informational externalities, i.e., when agents care about the information held by others.

In contrast to general equilibrium analysis, auction theory is based on the premise of individual strategic behavior. This theory offers explicit models of price formation and allocative distribution that can be applied also to small markets. The belief that auctions yield competitive outcomes even if information is dispersed is behind the practical appeal of auctions, and behind their recent popularity.

Since, as mentioned above, Walrasian equilibria need not be efficient in the presence of various forms of externalities, it is of interest to understand what are the parallel consequences of external effects in auctions, and in other related mechanisms. This is the main purpose of the research summarized in the present paper.

Traditionally, the focus of auction theory has been on models that view auctions as isolated events. In practice, however, auctions are often part of larger transactions: for example, in privatization exercises such as license allocation schemes (see Jehiel and Moldovanu, 2003, and the surveys in Janssen, 2004), auctions or other mechanisms shape the size and composition of future markets. Thus the auction typically affects the nature of the post-auction interaction among bidders. On the other hand, anticipated scenarios about the future interaction influence bidding behavior: already at the bidding stage agents need to care about who gets what, and about the information revealed to, or possessed by others, since these features will be reflected in the equilibrium of the post-auction interaction. Thus, we want to stress here that allocative and informational externalities naturally arise in models that embed auctions in larger economic contexts. This, in our view, constitutes the main motivation for the present study.

In Section 2 we present a social choice model with a finite number of alternatives that includes, as a special case, a general multi-object auction model where the alternatives are partitions of objects among agents. The model can incorporate allocative and informational externalities, as well as complementarities. We also sketch a typical application to license auctions.

In Section 3 we focus on the effects created by allocative externalities. The induced endogeneity of valuations is the main driving force behind a wealth of new, specific phenomena. Generally speaking, traditional auction formats need not be efficient. They may give rise to the possibilities of strategic non-participation and multiple equilibria with varied outcomes, thereby suggesting that the outcomes of auctions are hard to predict. We also note that the presence of allocative externalities may be responsible for the emptiness of the core, thereby suggesting another channel through which allocative externalities are an important source of (coalitional) instability. We also discuss the use of optimal threats in revenue maximization, and the severe conflicts that may arise among various designer's goals, such as welfare maximization and revenue maximization. We note that more flexible auction formats need not be preferable as they may allow bidders to achieve more concentrated market structures.

We also show that in environments with limited commitment abilities, resale markets ensure that in the long run the welfare performance is unaffected by the initial allocation of property rights (if agents are patient enough). But the final outcome induced by the resale markets need not be efficient, thereby suggesting that a desirable initial allocation coupled with restrictions on the resale markets may be preferable. Finally, we briefly survey a variety of recent applications.

In Section 4 we add private information about values, and we study multi-object auction models without informational externalities (these are so called *private values* models). The analysis focuses on dominant strategy mechanisms that achieve desirable goals such as value maximization for buyers, or revenue maximization for the seller. In particular, welfare-maximization is achieved by the celebrated Vickrey-Clarke-Groves mechanisms even in the presence of allocative externalities.

We also discuss how other dominant strategy mechanisms, not necessarily welfare maximizing, can be characterized in such frameworks. An application is made to revenue boosting via mixed bundling in auctions for several heterogenous objects. Note that the revenue-maximizing auction in this setting is still unknown.

In Section 5 the emphasis is on informational externalities, and on several impossibility results in such frameworks. We start with technical observations that are needed for the proofs of our general impossibility results. In order to consistently and generally model the preferences of bidders, private signals must be vectors rather than scalars. This feature distinguishes our framework from most auction models for a single object (that were the traditional domain of much of auction theory) that generally assumed one-dimensional private information[1]. Analyzing incentive constraints with multidimensional signals is technically complex, but indispensable. For the general social choice model developed in Section 2 we characterize Bayes-Nash incentive compatible mechanisms under the assumption that signals are independent. A main new requirement is that equilibrium utility (as a function

[1]For one-object auction models that allow for several informational dimensions (e.g., on a private value component and common value component), see, among others, Maskin (1992), Pesendorfer and Swinkels (1998), Compte and Jehiel (2002a), and Jackson (2003).

of type) is a convex potential. Immediate corollaries are general payoff and revenue equivalence theorems.

We next proceed to show that Bayes-Nash implementation of the welfare-maximizing choice function is impossible in generic settings with multidimensional signals, and, moreover, that robust implementation of any non-trivial choice function is generically impossible. In particular, welfare-maximizing multi-object auctions do not exist, unless the allocation problem is separable across objects (e.g., there are neither allocative externalities nor complementarities) or signals are one-dimensional. Here (as in Section 4) robust implementation refers to implementation via mechanisms that do not finely depend on the beliefs of the agents or designer. In contexts with informational externalities, the relevant concept is the *ex-post equilibrium*: It is shown that no social choice rule that makes use of the private information can be ex post implemented, as soon as two agents have at least two dimensions of private information. These results are in sharp contrast with the finding obtained in the private values setup (Section 4), and illustrate the complex effects of the combination of allocative and informational externalities on mechanisms. We also identify non-generic settings where ex-post implementation and ex-post welfare-maximization are possible, and we mention several recent applications. Section 6 concludes.

We want to emphasize here that the present paper is not meant to be a survey of auction theory and mechanism design: there is a wealth of interesting and relevant issues that will not be addressed here. Interested readers can consult, for example, Klemperer (1999).

2 A General Multi-Object Auction Model

We start with a general social choice model with $N+1$ agents, indexed by $i = 0, 1, 2, ..N$ and K social alternatives, indexed by $k = 1, 2, ..K$. Each agent gets a private signal about the state of the world $\theta^i \in \Theta^i \subseteq \Re^m$. We denote $\theta = (\theta^0, \theta^1, ...\theta^N)$, $\Theta = \times_{i=0}^N \Theta^i$, $\theta^{-i} = (\theta^0, ..., \theta^{i-1}, \theta^{i+1}, ..., \theta^N)$ and so on...

Agents have quasi-linear utility functions that depend on the chosen alternative, on private signals, and on monetary payments: if alternative k is chosen, and if agent i obtains a monetary transfer[2] t^i, then her utility is given by

[2]More generally, agent i' s utility may depend also on monetary transfers made to other players. These are situations with "financial externalities". We do not include them in our present analysis. The reader interested in the effect of such externalities on auctions should consult Dasgupta and Tsui (2004), Ettinger (2002) and Goeree et al. (2004).

$$u^i(k,\theta,t^i) = v^i_k(\theta^0,\theta^1,...\theta^N) + t^i$$

where v^i_k may a priori be any function of $(\theta^0,\theta^1,...\theta^N)$.

The special case of auctions is included as follows: A set of M objects (possibly heterogenous) is allocated among a seller (who will be called agent zero) and N potential buyers. Here a social alternative is a partition of the goods among the agents: $P = (P_0, P_1, ..P_N)$, where P_i represents the bundle allocated to agent i. Let $\mathcal{P}$ denote the set of all partitions.

For each partition P , agent i obtains a signal θ^i_P that influences values for that partition. Thus $\theta^i = (\theta^i_P)_{P\in\mathcal{P}}$, and we write $v^i_P(\theta) = v^i_P(\theta^0_P, .., \theta^N_P)$. This specification includes a large variety of auction / mechanism design models studied in the literature. Here are a few prominent examples:

1. For any partitions P and P' such that $P_i = P'_i$ assume that $\theta^i_P = \theta^i_{P'} \equiv \theta^i_{P_i}$, and $v^i_P(\theta) = v^i_{P'}(\theta) \equiv v^i_{P_i}(\theta^i_{P_i})$. This is a "pure private values" model where agent i only cares about the bundle allocated to her in each partition, and about a signal pertaining to that bundle. There are neither allocative nor informational externalities.

2. For any θ and θ' such that $\theta^i_P = \theta'^i_P$, assume that $v^i_P(\theta) = v^i_P(\theta') \equiv v^i_P(\theta^i_P)$. This is a model where agent i only cares about his own signal about the partition, (i.e., there are no informational externalities), but i may care about the entire partition of objects (i.e., there are allocative externalities)

3. For any partitions P and P' such that $P_i = P'_i$ assume that $v^i_P(\theta) = v^i_{P'}(\theta)$. It follows that θ^j can be re-parameterized as $\theta^j = (\theta^j_X)_{X\in 2^M}$ where X is a subset of the M objects, and $v^i_P(\theta)$ can be re-written as $v^i_{P_i}(\theta^0_{P_i}, ..., \theta^N_{P_i})$. This is a model where agent i only cares about the bundle allocated to him in each partition (i.e., there are no allocative externalities), but i does care about the information about that bundle available to other agents (i.e., there are informational externalities).

4. $v^i_P(\theta)$ depends in a general way on the entire partition P and on the entire profile of signals θ. This is the most general model that admits both allocative and informational externalities.

5. Assume that there are no allocative externalities. If $v^i_{P_i\cup P'_i}(\cdot) > (<) v^i_{P_i}(\cdot) + v^i_{P'_i}(\cdot)$ for some bundles of object P_i, P'_i, then this is a model that exhibits complementarities (substitutabilities).

2.1 A Typical Application

There are many applications in which both allocative and informational externalities naturally arise in auction contexts. Allocative externalities often arise because bidders care about the ensuing market structure that is affected by the auction's outcome (or who gets what). Informational externalities arise because private information on the cost structure typically affects the competitors' profits. Information is naturally multi-dimensional because there are various aspects to the cost structure and different objects for sale.

As an illustration, consider the recent European process of allocating UMTS licenses to telecom firms (for surveys see, among others, Jehiel and Moldovanu, 2003 and Klemperer, 2002). The allocation proceeded via a sequence of national auctions and beauty contests. This was a complicated process with a variety of aspects not directly connected to auctions. We focus below only on the basic features that parallel those in the model sketched above.

1. The auctioned objects were licenses to operate a third-generation mobile telephony network in a certain country. The licenses differed in a multitude of dimensions such as the size and distribution of the population, spectrum capacity, duration, required investment size and deployment speed, and many other physical or regulatory constraints. Some of the participating firms were small and had only national interests, while most of the bigger firms competed continent-wide. Thus, both auctioned objects and bidders were heterogenous, and bidders had to aggregate various dimensions of information.

2. To a large extent, national licenses were substitutes (and firms were generally not allowed to buy more than one license in each country), while licenses in various countries were complements (since they allow, among other things, integrated and seamless service, roaming, billing, etc... within one large network, and they offer more bargaining power versus equipment producers and regulators)

3. The value of a bundle of licenses for a given firm equals, basically, the expected revenue in the future telecom market from holding that bundle, minus the required infrastructure and operation costs. In any reasonable scenario for the future telecom market, the expected profits crucially depend on the number of licenses (or competitors) within one country, on their identity (e.g., small local firms or large supranational ones), on the license holdings of competitors in other countries (that determine what integrated services they can offer), and so on... Thus,

a major feature of the license allocation example is the presence of allocative externalities among bidders[3].

4. Private information is multidimensional since many types of licenses were sold (see also point 1 above), and since the bidding firms had heterogenous assessments about technical network requirements, present and future regulatory frameworks, future demand characteristics, future operation costs, future technological developments, etc... All these factors influence future revenue and costs, and thus valuations.

5. In any reasonable oligopoly scenario, competitors' cost parameters[4] affect one own's profit. Since competitors have some information about their operation method, future technological development and future demand, the auctions also involved informational externalities.

3 Allocative Externalities

In this section we review design issues that are related to the presence of allocative externalities. In order to abstract from other effects, we first assume that there is complete information, and we come back later to the additional phenomena due to asymmetric information.

We illustrate a number of insights through the following situation appearing in Industrial Organization models: There are N potentially active firms in the market. An innovation protected by a patent is auctioned among the firms. The acquiring firm is able to produce at a lower cost, but the magnitude of the cost reduction may depend on the identity of the acquiring firm. Let v_i^i denote the change in profit of firm i if i acquires the innovation; v_i^i is referred to as i's valuation. Let v_i^j denote the change of profit of firm j when firm i acquires the innovation. This change of profit is caused by the modified oligopolistic competition after the innovation is introduced (which is typically less favorable for j). We refer to v_i^j as the *externality* exerted by i on j. This specification fits into the general model presented above by noting that, in the case of one single object, the partition P can be simply described by the firm i who acquires the innovation.

[3]The sellers (i..e, the national governments) had also preferences involving allocative externalities since they were concerned with consumer welfare. Thus they cared about how many licenses are awarded, how many of them are bought by new entrants, who are the wining firms (local or foreign, etc...).

[4]This is just an example; information about demand parameters is another.

3.1 Endogenous Valuations

Assume that the innovation is auctioned using a second-price sealed-bid auction[5]. That is, each firm i submits a bid b_i, and the firm with highest bid wins the auction and pays the second highest bid. Ties are resolved as usual.

In the traditional setup without externalities, it is a (weakly) dominant strategy for firms to bid their values for the auctioned object (see Vickrey, 1961). Here, the mere notion of *value* is not well defined. Indeed, how much i is willing to pay in order to win very much depends on her expectation about who is going to win if she does not. For example, if i expects j to win her net value of winning (compared to the loss scenario) is $v_i^i - v_j^i$. Similarly, if i expects k to win her value is $v_i^i - v_k^i$. These two values need not coincide, and thus it is impossible to say how much i values the innovation, independently of her expectations over alternative market scenarios. Of course, expectations must be consistent with equilibrium play. The observation that valuations depend here on expectations translates into the possibility of multiple equilibria with quite different outcomes:

Example 1 *(Jehiel and Moldovanu, 1996) Let* $N = 3$,*and let* $v_i^i = v$ *for all* i. *Let the externality terms be:* $v_1^2 = v_2^1 = -\alpha$, $v_1^3 = v_2^3 = -\gamma$, *and* $v_3^1 = v_3^2 = -\beta$ *where* $\alpha > \gamma > \beta > 0$. *It is readily verified that, in one equilibrium, firms 1 and 2 compete with each other (since they are very afraid of each other). The resulting outcome is that either firm 1 or 2 wins the auction and pays* $v + \alpha$. *In another equilibrium, 1 and 3 are in competition. The resulting outcome is that 3 wins the auction (because 3 is more afraid of 1 than 1 is afraid of 3, e.g.* $\gamma > \beta$*) and pays* $v + \beta$ *- a much lower price than in the previous equilibrium. Firm 2 is not willing to outbid 3 because 2 is not that afraid of 3.*

3.2 Strategic Non-Participation

In auctions without externalities, not participating in the auction is equivalent to participating and making an irrelevant bid[6]. In the presence of externalities, this is no longer the case. By staying out, a bidder may induce an outcome that turns out to be more favorable to her than the outcome that would have arisen if she had participated (see Jehiel and Moldovanu, 1996). We illustrate it through the following example:

Example 2 *(Hoppe, Jehiel and Moldovanu, 2005) Let* $N = 3$. *Firms* $1, 2$ *are incumbents, while firm* 3 *is a potential entrant. The incumbents do not*

[5]The same insights apply to all other standard formats.

[6]Participation costs are assumed to be nul.

value the object (innovation, license, etc...) per se: $v_1^1 = v_2^2 = 0$. *Moreover,* $v_1^2 = v_2^1 = 0$. *The entrant has value* $v_3^3 = v$ *, and it creates an externality* $v_3^1 = v_3^2 = -\alpha$ *on incumbents. We assume that* $v < \alpha$. *We now check why there is no equilibrium where all three firms participate in the auction: If all three firms participate, there are, essentially, three possible bidding equilibria: two in pure strategy, one in mixed strategy. In a pure strategy equilibrium, one of the incumbents, say firm 1, wins and pays* v. *In this equilibrium firm 2 bids zero since there is no point winning: that outcome is equivalent to the outcome when she lets 1 win. In the mixed strategy equilibrium, firms 1 and 2 mix between a bid of zero (say), and a bid slightly above* v, *and the entrant sometimes wins. The strategic interaction between the incumbent firms resembles a war of attrition: each incumbent is willing to deter entry (since* α *is large), but prefers that the other incumbent pays the price* v *of entry deterrence. Assume then that all firms participate, and that one of the above bidding equilibria is played. At least one incumbent, say 1, wins at price* v *with positive probability. But, 1 would be strictly better off by not participating in the auction. Indeed, in that case, the auction is among bidders 2 and 3, and 2 wins because* $\alpha > v$. *Clearly, firm 1 benefits from such a deviation: entry is deterred and firm 1 pays nothing for it. Thus, in any equilibrium, at least one of the firms will choose not to participate with positive probability*[7].

In Hoppe, Jehiel and Moldovanu (2005), we build on Example 2 to show that increasing the number of objects may change the nature of the interaction between the incumbents. When two objects are auctioned, the two incumbents have an easy way to collude within the auction: each buys one object at price v, thereby deterring entry. If allowing two entries is also an equilibrium, the two-object auction resembles now a coordination game between the two incumbents, rather than a war of attrition (as in the one-object auction). This is somewhat reminiscent of the finding that, in multi-object auctions, collusion may mean sharing the items for sale (see Wilson 1979). But, the channel through which this occurs here is specific to the presence of allocative externalities.

3.3 Participation Decisions and Optimal Mechanisms

We considered above standard auction formats. We wish now to analyze how the auction designer can exploit the bidders' participation decisions in

[7]The argument assumes that the set of participants is public information. One way to avoid strategic non-participation is to keep the set of bidders secret. But, such policies may have other drawbacks (see Compte and Jehiel, 2002b and DasVarma, 2002).

order to increase her revenue. The key observation is that, by augmenting the auction design by appropriate threats, the designer is able to extract payments also from bidders who do not win. This subsection summarizes insights from Jehiel, Moldovanu and Stacchetti (1996) (see also Kamien, Oren and Tauman 1992 for an early analysis of a setting where non-acquirers make payments to the auctioneer).

Assume that all externalities are negative, i.e., $v_i^j \leq 0$ for all $i, j \neq i$. If firm i stays out, the worst scenario for firm i would be that the winner is firm $j(i)$ where $j(i) \in \arg\min_j v_j^i$. We let $\underline{v}^i = \min_{j \neq i} v_j^i$, and augment the mechanism by the specification that if agent i does not participate, the winner is firm $j(i)$. In line with the mechanism design literature we assume first that the designer has the commitment power to implement such threats.

If firm i refuses to participate, it will get a minimal payoff. In equilibrium[8], all firms participate, and the outcome is chosen so as to maximize welfare. This is so because the designer can internalize social welfare by asking every firm i to pay the difference between i's payoff in the welfare maximizing outcome and $\underline{v}^i$.

Proposition 3 *The outcome of the revenue maximizing mechanism also maximizes welfare for the agents. The extracted revenue is:*

$$R = -\sum_i \underline{v}^i + \max\left\{0, \max_i \sum_j v_i^j\right\}$$

The above argument can be viewed as an expression of the celebrated Coase theorem in our setup[9]. Observe that, in the presence of allocative externalities, as soon as there are at least three bidders, welfare is usually not maximized by standard auctions.

To illustrate Proposition 3, consider example 2 again. Given that $v < \alpha$, the outcome that maximizes welfare (among agents) is that the object is sold to either incumbent (or, equivalently, that the seller keeps the object). The threat to either incumbent is that, if either of them refuses to participate, the object is sold to the entrant. The entrant is not threatened. Each incumbent is willing to pay α to avoid entry, and the revenue to the designer is:[10] 2α.

[8]There could, a priori, be other equilibria where several firms decide not to participate. But, by suitably defining what the mechanism does when several firms do not participate, one can guarantee that participation is a weakly dominant strategy. (see Jehiel et al., 1996).

[9]This holds despite the fact that the participation constraints are endogenous, unlike those in Coase's original analysis. The point is that reservation values can be set independently of the chosen outcome.

[10]If we restrict attention to mechanisms where only the winner can make payments, the revenue falls down to α.

More generally, if the designer also cares about consumers' surplus and not only about revenue, then total welfare will be maximized in equilibrium, and the payoff of the designer will:

$$W = -\sum_i \underline{v}^i + \max \left\{ 0, \max_i \left(\sum_j v_i^j + CS_i \right) \right\}$$

where CS_i is the change in consumers' surplus resulting from a sale to firm i.

3.4 Credibility and Resale

The mechanism design approach assumes a high commitment power on the designer's part. In the above application, the designer can commit to personalized and fine-tuned threats. What happens if the commitment power is limited? In Jehiel and Moldovanu (1999) we consider[11] a model where firms can sell and further resell the object before a (sufficiently far away) deadline T , at which time the current owner of the good must use it. We assume that agents are unable to commit to actions at future stages (in particular, they are unable to commit not to resell, or to sell to a specific agent if some pre-specified event occurs). At each stage, the current owner makes an offer to a set of agents. The offer may include a sale in exchange for payments. If an approached agent refuses the deal, one period of time elapses, and the owner makes a new offer. The main finding of Jehiel and Moldovanu (1999) is:

Proposition 4 *: The identity of the initial owner does not affect the identity of the agent who consumes at date T. The consumer at date T need not maximize total welfare unless all agents have veto power over all transactions.*

The following example illustrates the result:

Example 5 *Let $N = 4$, and let $T \geq 2$. The values are: $v_1^1 = 6.5$, $v_2^2 = 10.1$, $v_3^3 = 9$, $v_4^4 = 7$; $v_1^2 = v_1^3 = v_1^4 = 0$; $v_2^1 = v_2^3 = -1$, $v_2^4 = -2$; $v_3^1 = -2$, $v_3^2 = 0$, $v_3^4 = -1$; $v_4^1 = v_4^2 = 0$, $v_4^3 = -1$. The welfare maximizing firm is 1. But, no matter who the initial owner is, the object will be consumed by firm 2 at stage T. At stage $T-1$, firm 1 sells to 2 without extracting any payment from 3 and 4 because these prefer that 1 is the final owner rather than 2. Firm 2 sells to 3 while extracting $v_3^4 - v_2^4 = 1$ from 4. Firm 3 sells to 2 while*

[11] See also Brocas (2003) for a study of an auction with negative externalities where the seller has no credible threats.

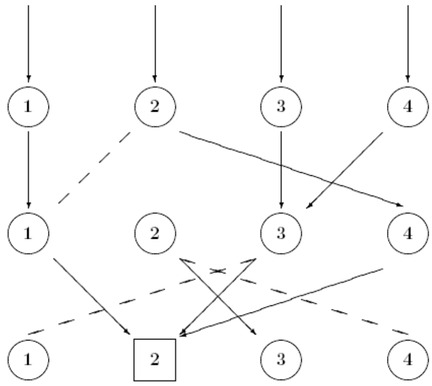

Figure 1:

extracting $v_2^1 - v_3^1 = 1$ *from 1.Firm 4 sells to 2 without extra payments from 1 and 3. At stage* $T-2$*, firms 1 and 3 do not sell. Firm 2 sells to 4 while extracting* $v_2^1 - v_3^1 = 1$ *from firm 1. Firm 4 sells to 3 without extra payment from firms 1 and 2. At stage* $T-3$ *and any earlier stages, no matter who is the current owner, the innovation ends up in the hands of firm 2 at stage* T*.*

A main feature of the above equilibrium is that, at some stages, firms that oppose the deal are simply excluded from the agreement. It can be showed that if one introduces veto power (i.e., all agents must agree to a change of ownership) the outcome maximizes welfare.

When resale cannot be forbidden, the above result suggests that it is welfare irrelevant how the initial property rights are assigned: eventually the same final physical outcome results. But, it is erroneous to conclude that mechanism design is irrelevant. Indeed, the final outcome need not be efficient. Thus, it may be a good idea to try to allocate the object efficiently in the first place, and then control the resale market to some extent[12].

There are several limitations to the above model: 1) The result relies on the existence of a deadline[13]; 2) There is only good for sale; 3) Only the current owner is able to make proposals. Gomes and Jehiel (2005) were able to generalize the main result without these assumptions. They also show that efficiency must occur in the long run if the efficient allocation of goods

[12]An extreme option is to forbid resales, as was done in the case of spectrum license auctions.

[13]Chien (2004) analyzes the same model with an infinite horizon and focuses on the resulting differences.

is such that no group of agents can force a move to another allocation that hurts i without her consent.

3.5 Core and Externalities

We make now a brief detour to show that, from the viewpoint of coalitional deviations, the presence of allocative externalities is a source of instability. The following result is a corollary of Proposition 6 in Jehiel and Moldovanu (1996).

Proposition 6 *Suppose that welfare is maximized by letting firm i^* buy the object . If there exists a subset T of buyers, $i^* \notin T$, and a buyer $i \in T$ such that*[14]

$$\sum_{j \in T \cup \{i^*\}} v_{i^*}^j < \sum_{j \in T \cup \{i^*\}} v_i^j \tag{1}$$

then the core of the associated market with externalities is empty[15]*.*

The intuition for Proposition 1 is as follows: stability against coalitional deviations requires welfare maximization, so that in any core outcome the object is sold to i^*. Buyers $j \neq i^*$ can always refuse to make any payment, and buyers $j \neq i$ are thus assured to get at least $\sum_{j \neq i^*} v_{i^*}^j$ collectively. Buyer j cannot get strictly more than $v_{i^*}^j$ as otherwise the coalition of the seller and firm i^* could do better by just ignoring j. Thus, j must get exactly $v_{i^*}^j$. Thus, if condition (1) holds, the coalition of the seller and buyers in $T \cup \{i^*\}$ can improve on the candidate core allocation by re-allocating the object to i rather than i^*. It follows that the core must be empty when condition (1) holds.

Ranger (2005) generalizes the Ausubel-Milgrom (2002) ascending proxy auction for several objects to a framework with allocative externalities (see also Lamy, 2005), and shows that the outcome must be in the "core" of the market. This looks puzzling, since, as we just showed, the core is often empty

[14] When condition (1) holds, the welfare maximizing state i^* is not *negative externality-free* (see Gomes and Jehiel 2005) since the coalition of the seller and buyers in T can allocate the innovation to firm i , hurting agents outside $T \cup \{i^*\}$. The spoliation is the key reason for the inexistence of a stable outcome.

[15] With externalities , the core notion depends on assumptions about reactions to coalitional deviations by agents in the complement. Our result holds for the most permissive definition, the α-core, where the complement is assumed to choose the worst course of action from the point of view of the deviators. Any core is empty if the $\alpha-$core is.

when there are (negative) externalities. The point is that Ranger's notion of core restricts what coalitions can do. For example, side-payments (e.g., compensation) among bidders are not allowed in his model.

3.6 The Conflict between Welfare and Revenue

In this subsection, we briefly consider an auction where the number of auctioned objects is endogenously determined by the bidders' behavior, and where externalities are created by the effects on downstream payoffs[16].

Conventional wisdom suggests that disaggregating spectrum, say, into small capacity blocks, and letting the bidders aggregate the blocks to form licenses of whatever capacity they need is a good idea. After all, the designer is usually not knowledgeable about how much the firms value the licenses, or about how valuable extra capacity is. But, this argument ignores the possibility that the auction's flexibility may be used by firms to induce concentrated market structures.

Example 7 *There are four identical blocks for sale, and let* $N = 5$*. In order to operate a bidder needs at least one block (small license), and bidders may buy up to two blocks (large license). Each bidder* i *submits a schedule* $b_i = (b_i(1), b_i(2))$ *where* $b_i(m)$ *is the bid for* m *blocks,* $m = 1, 2$*. Blocks are allocated and payments are made according to a uniform price auction. For any partition* P *of the four blocks in which firm* i *receives* m *blocks, and a total of* n *firms get at least one block, define* $v_P^i = \pi(m, n)$*. If* $\pi(2, 2) - \pi(1, 3) > \pi(1, 3)$*, the auction outcome is a duopoly because the extra profit gained by switching from a small license in a triopoly to a large license in a duopoly is larger than the profit with a small license in triopoly. That is, two firms buy two blocks each, the equilibrium price for a block is* $\pi(1, 3)$*, and a winner pays* $2\pi(1, 3)$*. In contrast, a less flexible format could, for example, mandate that three licenses (two small, one big) are sold. The assessment of the two formats depends, basically, on the partial derivatives of* π*. If the derivative with respect to capacity is larger (presumably it is positive), the flexible format is likely to be preferable. If, however, the derivative with respect to the size of the market (presumably negative) is larger, then the less flexible format is likely to be preferable because of the increased consumer surplus in a less concentrated market.*

In a related vein to the above, it is sometimes argued that welfare maxi-

[16]This is inspired by the German UMTS license auction that took place in 2001. The treatment follows Jehiel and Moldovanu (2003).

mization for bidders and revenue go hand in hand in auctions[17]. But, again this view ignores that "value", and hence "revenue" may be driven by the desire to squeeze consumers' surplus. Since the value of a monopoly position is larger than the combined values of oligopolists, it is intuitive that an auction for monopoly (which is a form of bundling -Section 4 below) will yield more revenue than an auction that creates several winners that compete against each other (at an extreme such an auction yields no revenue at all if firms expect a "Bertrand" type of interaction). But welfare, including consumer surplus, will be small under monopoly. This simple argument shows that revenue and efficiency may be quite unrelated to each other if there are allocative externalities (see also Janssen and Moldovanu, 2004).

3.7 Private information

So far, we have completely ignored informational issues. In the next Sections, we will develop several general insights about incentive constraints in models with informational asymmetries and with externalities (allocative or informational). Here, we want only to briefly review a number of insights for simple auction formats with allocative externalities and informational asymmetries.

Note that private information on allocative externality terms can yield *private value* models without informational externalities if i knows the externality terms v_j^i caused to her, or it can yield *interdependent value* models if i has private information on the externality terms v_i^j she causes to others (of course, mixtures are also possible).

Jehiel and Moldovanu (2000) analyze a sealed-bid second-price auctions with entry fees and reserve prices in a model where bidders have one dimensional private information that influences both their valuations, and the externality terms. Specifically, in a two bidder model, $\theta^i = v_i^i$ is private information, and v_j^i is a function of both θ^i and θ^j, i.e. $v_j^i(\theta^i, \theta^j)$. This model captures, for example, situations where each firm has private information about the reduction on marginal cost induced by an innovation, and where the cost structure is public information after the auction, and before the market interaction.

This model displays both private and interdependent value components: when i compares the alternative where he wins to the one where the seller keeps the object, his net value is θ^i. When i compares the alternative where he wins with the one where the other bidder wins, his net value is $\theta^i -$

[17]There are many caveats to this claim even without allocative externalities. For example, revenue maximization requires the use of reserve prices, or handicaps in asymmetric contexts (Myerson 1981) , or quantity discounts in multi-object auctions (see Palfrey 1983, Jehiel-Moldovanu 2001 , and Jehiel, Meyer-ter-Vehn and Moldovanu , 2005).

$v_j^i(\theta^i, \theta^j)$, which depends both on i and j's private information. Only the second comparison is relevant if there is no reserve price, and then the analysis is analogous to the one in Milgrom and Weber' (1982) framework: in a symmetric two-bidder setting, i.e., $v_j^i(x, y) \equiv v_i^j(x, y) = e(x, y)$, where the function $x \to x - e(x, x)$ is increasing, bidder i with type θ^i bids[18]

$$b(\theta^i) = \theta^i - e(\theta^i, \theta^i) \tag{2}$$

The auction with a reserve price R is more interesting. A first observation is that, when externalities are negative, there will be no bids in the neighborhood of R.[19] For the sake of illustration, assume that the externality function is given by a constant $-e$, where $e \geq 0$, and consider R inside the support of valuations[20]. The equilibrium is such that bidders with valuation below R bid below R, (say zero), and bidders with valuations above R bid $\theta^i + e$. Thus, there is no relevant bid between R and $R + e$. The reason for this discontinuity is as follows: when the marginal type $v_i^i = R$ considers whether to make a relevant bid or not, the benchmark is that the seller keeps the object. As soon as a bidder makes a relevant bid, the effect of marginally decreasing the bid (say) is now that the other bidder sometimes wins. Thus, in the relevant bid area, the bid function does satisfy (2).

Another interesting observation is that the seller's optimal reserve price may be below the seller's valuation, a situation that never occurs without externalities[21]. The reason is that, selling more often frightens bidders when there are negative externalities, and therefore they bid higher in order to win.

In the case of positive externalities, the equilibrium bidding function still has two distinct parts, one where the benchmark is that the seller keeps the object, and another where the benchmark is that the competitor wins. But now consistently combining the two parts requires that a positive measure of valuations θ^i bids R [22]. Moreover, with positive externalities, entry fees and reserve prices do not lead to equivalent revenues, and one can typically achieve higher revenues with reserve prices than with entry fees.

Moldovanu and Sela (2003) study a patent auction where the post-auction interaction is à la Bertrand. There, the analog of the function $\theta^i - v_j^i(\theta^i, \theta^j)$ is decreasing and pooling occurs even with negative externalities. Goeree

[18] With more than two bidders and with asymmetries, the features mentioned in the above subsections (multiplicity of equilibria, strategic non-participation, etc...) appear also here.

[19] A similar feature arises in the affiliated model of Milgrom and Weber (1982).

[20] We also assume that the valuations are identically and independently distributed.

[21] The optimal reserve price also depends on the number of bidders.

[22] Pooling also appears in the resale auctions studied by Haile (2000). The reason is that the possibility of resale translates into a reduced form positive externality.

(2000), Das Varma (2003) and Molnar and Virag (2004) study a variant of the above model where the cost structure is not made public after the auction. In this case, the winning bid has the extra feature of conveying some information about the cost structure of the winning bidder, and there is an extra signaling motive appearing in the bidding strategy.

We now briefly consider revenue-maximizing auctions. There are two difficulties linked to the presence of allocative externalities: 1) Information is typically multidimensional; 2) Participation constraints are typically determined by the mechanism itself and are type-dependent.

Jehiel, Moldovanu and Stacchetti (1999) consider a symmetric, private values setup with multidimensional private information and negative externalities: bidder i knows her valuation v_i^i and the externalities v_j^i caused to her. For example, the unique symmetric equilibrium of a second-price auction among N bidders is given by:

$$b(v_i^i, v_{-i}^i) = v_i^i - \frac{1}{N-1} \sum v_j^i$$

Thus, the equilibrium bids are set at the valuation minus the average externality.

As in the complete information case (see subsection 3.3), the optimal auction will include some threat in case a bidder does not participate. But, in this private value model, there is no way to fine-tune the threat for i by using information revealed by others. It turns out that it is enough to care about the participation constraint of the type whose valuation is smallest and whose externalities are closest to zero. The threat (taking the form of a fixed allocation rule in case i does not participate) can be designed so that the participation constraints of all other types of bidder i are automatically satisfied as soon as i's incentive constraints are satisfied. Thus, even though participation constraints are *a priori* type-dependent, the fact that they are endogenously determined by the mechanism (through the choice of threats) allows us to avoid some of the complications inherent to exogenous type-dependent participation constraints[23].

Jehiel, Moldovanu and Stacchetti (1999) show that a second-price auction with an appropriately defined entry fee is the revenue-maximizing mechanism in a class of mechanisms where the object is always sold, and where agents are allowed to make one-dimensional bids. They also characterize the equilibrium of the second price auction with reserve prices, and show that it is never optimal to set a small reserve price. Thus, either there should be no reserve

[23]See Jullien (2000) for an analysis of such constraints in one-dimensional principal-agent setting.

price or the reserve price should be such that the object is not sold with a significant probability. This result cannot arise in a one-dimensional setting. It should also be contrasted with the finding in multidimensional monopoly problems (where there is no competition among consumers). There, it is always optimal to exclude some set of consumers no matter where the support of consumers' valuations lies (see Armstrong, 1996). By contrast, in the auction setup, it may be revenue-enhancing for the seller to always sell the good[24]

Jehiel, Moldovanu and Stacchetti (1996) construct the revenue maximizing auction in a setting where bidder i knows her valuation v_i^i and the negative externality $v_i^j \leq 0$ she exerts on other bidders j. This is now a model with interdependent values or informational externalities. In line with the idea of targeted threats outlined above, if i does not participate, the auctioneer optimally decides to sell the good to the agent $j(i) = \arg\min_j \widetilde{v}_j^i$ where $\widetilde{v}_j^i$ is agent j's report in a direct mechanism.

Figueroa and Skreta (2004) generalize the analysis of optimal mechanisms to multi-object settings with externalities, but assume that agents have one-dimensional signals[25].

Das Varma (2002) builds on the model of Jehiel, Moldovanu and Stacchetti (1999) and he observes that the ascending English auction may sometimes generate more revenue than sealed-bid auctions. This happens because bidders have incentives to stay longer in the auction in cases where their worst enemy also stays in the auction. The insight is particularly interesting in light of the revenue equivalence theorem in auctions (see subsection 6.1 below). It illustrates that, even in a symmetric, independent, private values context with risk neutral bidders, the ascending and the sealed-bid auctions need not be revenue-equivalent if there are allocative externalities.

Caillaud and Jehiel (1998) examine the possibility of collusion in the presence of negative externalities among agents when private information bears on the sole valuation. They show that the information sharing among the ring members need not be efficient even if side-payments are allowed, which is in sharp contrast with the finding in externality-free auction setups (see Graham and Marshall (1986), McAfee and McMillan (1992), and Mailath and Zemsky (1991).

[24] Having one bidder participate more often increases the others' incentive to bid more aggressively, which in turn may dominate the supply restriction effect.

[25] See also Caillaud and Jehiel (1998) and Brocas (2003).

3.8 Applications

Auction models with allocative externalities offer an unified framework for all situations where competing firms or agents buy important inputs that affect the nature of downstream interaction.

Some effects of externalities on bidding behavior have been previously identified in the literature on the "persistence of monopoly" (see, for example, the classical paper by Gilbert and Newbery, 1982 , and also Krishna, 1993 and Rodriguez, 2002) and in the literature on patent licensing (see the classical contributions of Katz and Shapiro (1986), and Kamien and Tauman, 1986)[26]. Similar effects appear in the auctions of capacity studied by McAfee (1998) and Ranger (2004). Roughly speaking, the general framework presented here extends "the persistence of monopoly" approach beyond monopoly/duopoly market structures, and it extends the literature on patent licensing by allowing for asymmetries between agents[27], and by considering allocation mechanisms that go beyond standard auctions.

Norbaeck and Persson (2003), Molnar (2003) and Inderst and Wey (2004) analyze large, auction-like deals (such as privatizations, mergers and takeovers) that change the nature of an industry by affecting the number and the identity of the operating firms. For example, it is an immediate consequence of the theory of auctions with negative externalities that a takeover premium must be observed in horizontal mergers where each firm is negatively affected if the target is acquired by another competitor. Molnar derives several related testable hypothesis and looks at the empirical data.

Dana and Spier (1994), Auriol and Laffont (1992), McGuire and Riordan (1995) and Jehiel and Moldovanu (2004) study the efficient design of private industries in frameworks where the designer (the state) has preferences that combine measures of consumer surplus and revenue.

Alboth, Lerner and Shalev (2001), Perez-Castrillo and Wettstein (2002), and Waehrer (2003, 2004) study the properties of bidding games and other mechanisms for allocating public goods or bads, such as contributions to joint ventures, or siting of hazardous facilities.

Sorana (2000), and Anton, Van der Weide and Wettas (2002) focus on bidding and pricing behavior in markets where universal service regulation applies. The externalities occur across sectors of the market: large urban areas are often profitable, rural areas are not.

Maeda (2003) and Burguet et Sempere (2005) analyze trading models for the emission of noxious gases. Using some insights developed for resale markets with externalities (see Jehiel and Moldovanu, 1999), Burguet et

[26]See also the more recent papers of Moldovanu and Sela (2002), and Sarbar (2005).

[27]Rockett (1990) considers asymmetries, but does not offer a general analysis.

Sempere emphasize both the inadequacy of bilateral trading mechanisms for internalizing all existing externalities, and the need for multilateral schemes. A related point is made by Bagwell, Mavroidis and Staiger (2004) who propose an innovative auction of retaliation rights for dispute settlements within the World Trade Organization.

We have emphasized above the effects of allocative externalities on bidding behavior/mechanism design. A small recent literature studies the effect of financial externalities, i.e., agents directly care about the monetary payments made by others. Dasgupta and Tsui (2004) and Ettinger (2000) study bidding behavior in auctions where buyers and sellers are intertwined through cross-shareholding. Their finding about the revenue comparison of first-price and second-price sealed-bid auctions with vertical toeholds has antecedents in Engelbrecht-Wiggans (1994) and Bulow et al. (1999). Morgan, Steiglitz, and Reiss (2003) describe the effects of "spite" in auctions. Their agents enjoy when other pay much (this can be driven by pure spite, but also by some post-auction interaction where it is better to have financial weaker competitors). In a related framework, Goeree, Maasland, Onderstal and Turner (2004) look at charity auctions, and emphasize the effects of the endogenous financial externalities.

4 Private Values

In this Section we study private values models: there are no informational externalities. Note, however, that allocative externalities are allowed. The focus is on the possibility of implementing the welfare-maximizing social choice rule via Vickrey-Clarke-Groves mechanisms and on the form of more general social choice rules that can be implemented in dominant strategy (Roberts, 1979).

4.1 Welfare-Maximization

We briefly review the celebrated Vickrey-Clarke-Groves analysis, and several extensions relevant for our purposes. We assume that $v_P^i(\theta) \equiv v_P^i(\theta^i)$. Thus, agent i cares only about his own private signal, but may care about the entire partition of objects.

In private values settings, the VCG analysis allows for multidimensional signals, allocative externalities and complementarities without any further theoretical difficulties.

For any profile of signals θ , define

$$P^*(\theta) = \arg\max_{P \in \mathcal{P}} \sum_j v_P^j(\theta^j) \tag{3}$$

Partition $P^*(\theta)$ maximizes the economic value for the agents[28]. Let P_{-i} be a partition of objects where agent i is allocated the empty set, and let $\mathcal{P}_{-i}$ denote the set of such partitions. For any profile θ^{-i} define

$$P^*_{-i}(\theta^{-i}) = \arg\max_{P_{-i} \in \mathcal{P}_{-i}} \sum_{j \neq i} v^j_{P_{-i}}(\theta^j) \tag{4}$$

Partition $P^*_{-i}(\theta^{-i})$ represents the welfare-maximizing partition of objects in the absence of i.

Definition 8 (Vickrey-Clarke-Groves) *The VCG mechanism is the direct revelation mechanism where, for each reported profile of signals θ, the designer implements partition $P^*(\theta) = \arg\max_{P \in \mathcal{P}} \sum_j v_P^j(\theta^j)$, and makes a transfer to i given by*

$$t^i(\theta^{-i}) = \sum_{j \neq i} v^j_{P^*(\theta)}(\theta^j) - \sum_{j \neq i} v^j_{P^*_{-i}(\theta^{-i})}(\theta^j) \tag{5}$$

It is well-known that truth-telling is a dominant strategy for each player in the VCG mechanism. Moreover, if all agents reveal their information truthfully, the welfare maximizing partition of objects is obviously chosen (see Vickrey, 1961, Clarke, 1971, and Groves, 1973). It is important to notice that, except in degenerate cases, agents in a VCG mechanism have strictly positive interim incentives to report truthfully because of the uncertainty (still to be resolved at the time of the reporting) about the chosen alternative. All these aspects certainly justify Paul Milgrom's claim (see Milgrom, 2004) that " The VCG analysis has become an important standard. It is the work by which nearly all other mechanism design work is judged and in terms of which its contribution is assessed."

4.2 Dominant-Strategy Implementation without Welfare-Maximization

In some applications, one may be interested in goals other than welfare. An appealing property of the VCG mechanism is that the truthful equilibrium

[28] If the argmax is not unique, any selection will do.

is in dominant strategies. This subsection reports a characterization, due to Roberts (1979), of those social choice rules that can be implemented in dominant strategy. Such a characterization can be used, for example, in an analysis of how revenue can be maximized in dominant strategy mechanisms.

Roberts' model assumes private values, and requires that each agent i gets a one-dimensional signal for each social alternative[29]. Applied to auctions, this means $\theta^i = (\theta^i_P)_{P\in\mathcal{P}}$, where $\theta^i_P \in \Re$, and $v^i_P(\theta) \equiv v^i_P(\theta^i_P)$.

Crucial assumptions for Roberts' characterization are:

1. There are at least 3 relevant partitions.
2. For any $x \in \Re^{|\mathcal{P}|}$ there exists $\theta^i = (\theta^i_P)_{P\in\mathcal{P}} \in \Theta^i$ such that $\{v^i_P(\theta^i_P)\}_{P\in\mathcal{P}} = x$.

With these assumptions, Roberts shows that a deterministic social choice function Ψ is dominant strategy implementable only if there is a set of real weights $\{\alpha_i\}_{i=1,2,..N}$, not all equal to zero, and a set of real weights $\{\lambda_P\}_{P\in\mathcal{P}}$ such that

$$P^\lambda(\theta) = \arg\max_{P\in\mathcal{P}}[\sum_j \alpha^j v^j_P(\theta^j_P) + \lambda_P]$$

Thus, dominant strategy implementable social choice functions must maximize a weighted average of the agents values, augmented by a partition-specific weight. Jehiel et al. (2004) call these functions *affine maximizers.* The dominant-strategy implementability[30] of affine maximizers is an easy consequence of the Vickrey-Clarke-Groves analysis (just think about the λ_P's as the designer's preferences on partitions). The key contribution of Roberts (1979) is that under conditions 1 and 2 above, nothing but affine maximizers can be implemented in dominant strategy.

Besides the unbounded range assumption, the crucial condition 2 above implies that agents must care about the entire partition of goods. Thus, there must be allocative externalities for Roberts' characterization to hold !

Bickchandani et al. (2004), Gui et al. (2004) and Lavi et al. (2004) characterize dominant-strategy implementability on various restricted domains where condition 2 need not hold[31]. Their results are in terms of monotonicity conditions that need to be satisfied by an implementable social choice function.

[29]This last assumption can be relaxed. See Jehiel et. al. (2004).

[30]Not every affine maximizer is implementable! Problems arise if some weight α^i is zero. But, the definition of an affine maximizer can be slightly adapted so that all affine maximizers are implementable.

[31]The characterization for situations with only two social alternatives is due to Laffont and Maskin (1982).

For example, Bickchandani et al. (2004) study a model where there are L identical units for sale, and where signals are drawn from a compact hypercube: here $\theta^i = (\theta^i_1, .., \theta^i_L)$ and $v^i_P(\theta^i) = v^i_{P_i}(\theta^i) = \sum_{l=1}^{|P_i|} \theta^i_l$. Thus, θ^i_l represents i's marginal utility from the l -th unit, and there are no allocative externalities. A social choice function is implementable in dominant strategy if it is *non-decreasing in marginal utilities* (NDMU). That is, if i is allocated more units when he reports $\widetilde{\theta}^i$ than when he reports θ^i, then it must be the case that his valuation at $\widetilde{\theta}^i$ for the additional units is at least as large as his valuation at θ^i.[32] Affine maximizers satisfy the monotonicity condition, but, on the present restricted domain, there are other social choice functions that satisfy NDMU, and thus are dominant-strategy implementable.

Gui et al. (2004) offer characterizations on several other restricted domains (e.g., auctions of heterogeneous objects without allocative externalities - these are also studied by Lavi et al., 2004) via a generalization of NDMU that is formulated in terms of preference cycles on the type space. Generally speaking, Roberts's result does not hold on restricted domains unless one imposes additional conditions on the social choice functions[33].

Holzman and Monderer (2004), and Holzman et al. (2004) focus on ex-post equilibria of the VCG mechanisms for combinatorial auctions without allocative and informational externalities. Holzman et.al. (2004) study specific strategies of the following type: a certain family of bundles is specified in advance, and is common to all bidders. These only report valuations for this family[34]. If there are at least three players, such strategies form a symmetric ex-post equilibrium if and only if the designated family is a quasi-field (i.e., it is closed under complements and disjoint unions). Holzman and Monderer (2004) show that all ex-post equilibria in the VCG mechanism must revolve around a quasi-field of bundles, as explained above. Again, due to the restricted domain of preferences, the implemented social choice functions need not be affine maximizers.

[32]Formally, let $\Psi_i(\theta)$ be the number of units allocated to agent i. Ψ is *non-decreasing in marginal utilities* if if for every $i, \theta^{-i}, \theta^i, \widetilde{\theta}^i$:

$$\text{If } \ \Psi_i(\widetilde{\theta}^i, \theta^{-i}) > \Psi_i(\theta^i, \theta^{-i}) \text{ then } \sum_{l=\Psi_i(\theta^i,\theta^{-i})+1}^{\Psi_i(\widetilde{\theta}^i,\theta^{-i})} \widetilde{\theta}^i_l \geq \sum_{l=\Psi_i(\theta^i,\theta^{-i})+1}^{\Psi_i(\widetilde{\theta}^i,\theta^{-i})} \theta^i_l$$

[33]For example, Lavi et. al. (2004) introduce an independence of irrelevant alternative assumption, while Meyer-ter-Vehn and Moldovanu (2003) introduce player decisiveness.

[34]Valuations for other bundles are mechanically calculated based on the declarations for bundles in the designated family.

4.3 Applications: Revenue Maximization and Mixed Bundling

Myerson (1981) and Riley and Samuelson (1981) have described the revenue-maximizing auction for the one-object case without allocative or informational externalities. If bidders are ex-ante symmetric, the optimal auction is a second-price auction with a reserve price, and hence it is implementable in dominant strategies. In that mechanism, the object is allocated efficiently whenever it is allocated at all to one of the buyers. Note that a reserve price is nothing else than a weight on the alternative where the seller keeps the object. Thus, for ex-ante symmetric bidders, the one-object revenue maximizing auction is a very simple affine maximizer, as defined above.

The analog revenue-maximizing multi-object auction is not yet known, and we doubt that the problem is analytically tractable (the problem is how to deal with complex integrability constraint due to multidimensional signals in the Bayes-Nash framework - see next Section for details).

Given the above difficulty, the main idea in Jehiel et al. (2005) is to study the revenue properties of the restricted class of affine maximizers. This class well captures the idea of *bundling* and *mixed-bundling* that are known to be revenue-enhancing in monopolistic non-linear pricing models.

Jehiel et al. (2005) consider the simplest model with heterogeneous objects: there are no allocative or informational externalities, nor complementarities and substitutabilities[35]: Each buyer i obtains a private signal $\theta^i_m \in \Re$ about each object m, and $v^i_P(\theta) = v^i_{P_i}(\theta^i) = v^i_{P_i}(\theta^i_{P_i}) = \sum_{m\in P_i} \theta^i_m$. For the seller, we assume here $v^0_P \equiv 0$. $\mathcal{P}_{bun} \subset \mathcal{P}$ is the subset of pure bundling allocations, i.e. $P \in \mathcal{P}_{bun} \Rightarrow \exists i$ such that $P_j = \emptyset$ for all $j \neq i$.

Two simple auction rules with equilibria in dominant strategies immediately come to mind: separate Vickrey auctions (one for each object), and a Vickrey auction for the entire bundle of all the objects. In the separate auctions, every object goes to the bidder who values it most, and each winning bidder pays the highest rejected bid on each object she receives. In the pure bundling auction, the set of all objects goes to the bidder who values highest the entire lot and the winner pays the highest rejected bid on the entire lot.

Jehiel et al. (2005) define a large class of auctions, called λ-auctions, that encompass many well-known formats, including the two described above[36]:

[35] The extension that allows for complementarities or substitutabilities between objects is straightforward, but we focus on additive valuations in order to single out the competitive effect of bundling rather than the "technological" one.

[36] Note that, however large, the class of choice rules implemented by λ-auctions is a strict subclass of dominant-strategy implementable choice rules since Roberts' domain restrictions do not hold here.

To each partition $P \in \mathcal{P}$ we attach a real-valued parameter $\lambda_P \in \mathbb{R}$ that enhances the probability that the allocation P will be chosen by the auction, and we then add transfers (defined in the spirit of the VCG mechanism) ensuring that truth-telling is a dominant strategy:

Definition 9 *The $(\lambda_P)_{P \in \mathcal{P}}$-auction (which we shall also call λ-auction) is defined by the allocation rule*

$$P^{\lambda}(\theta) = \arg\max_{P \in \mathcal{P}} \left\{ \left(\sum_{j=1}^{N} v_P^j(\theta^j) \right) + \lambda_P \right\} \tag{6}$$

and by the transfer rule

$$t^i(\theta) = -\left(\left(\sum_{j \neq i} v^j_{P^{\lambda}(\theta)}(\theta^j) \right) + \lambda_{P^{\lambda}(\theta)} \right) + \tau^i\left(\theta^{-i}\right) \tag{7}$$

where $\tau^i\left(\theta^{-i}\right) = \max_{P \in \mathcal{P}} \left(\sum_{j \neq i} v^j_{P_j}(\theta^j) + \underline{v}^i_{P_i} + \lambda_P \right)$ and $\underline{v}^i_{P_i}$ is i's minimum valuation for P_i. [37]

Just to give an example, separate second-price auctions for the M objects with reserve price r_m for object m are replicated by the λ-auction where , for each partition P, $\lambda_P = \sum_{m \in P_0} r_m$, i.e. the weight on an allocation is the sum of the reservation prices over all the objects that are not sold.

Observe that revenue maximization in the above defined class reduces to a problem of maximization over a finite collection of real numbers $\{\lambda_P\}_{P \in \mathcal{P}}$ - this problem can be solved by numerical simulation with a small computer.

Given truthfully reported valuations in an λ-auction, denote by $R(\lambda)$ and $S(\lambda)$ the expected revenue and expected social surplus, respectively. Jehiel et al. (2005) show that the effect of changing λ on expected revenue can be decomposed in an effect on social surplus and an effect on bidder surplus:

$$\frac{\partial R}{\partial \lambda_P}(\lambda) = \frac{\partial S}{\partial \lambda_P}(\lambda) + \sum_{i=1}^{N} [\Pr\left(P^{\lambda}(\theta) \neq P\right) - \Pr\left(P^{\lambda}_{-i}(\theta^{-i}) \neq P\right)]$$

By definition, the social surplus $S(\lambda)$ is maximized at the efficient auction where $\lambda \equiv 0$, and therefore $\nabla S(0) = 0$. Thus, a necessary condition for the efficient auction to maximize revenue in the class of $(\lambda_P)_{P \in \mathcal{P}}$-auctions is :

$$\forall P \in \mathcal{P}, \ \sum_{i=1}^{N} [-\Pr\left(P^{\lambda}(\theta) = P\right) + \Pr\left(P^{\lambda}_{-i}(\theta^{-i}) = P\right)] = 0 \tag{8}$$

[37] τ^i is set so that the type with minimum valuations receive a zero payoff.

This is a non-generic condition on the density function governing the distribution of signals. In other words, by introducing a small λ_P for some P one can almost always improve upon the efficient auction. Since $\lambda \neq 0$ creates an inefficiency, this shows, in particular, that the revenue-maximizing auction need not allocate the objects to those buyers who value them most, in marked contrast to the one-object case[38], and to various claims in the literature (see for example, Armstrong 2000).

In Jehiel et al. (2005), we also show that pure bundling auctions[39] can always be improved upon in terms of revenues. Thus, the best λ-auction necessarily involve some form of mixed bundling in which sometimes the goods are sold in bundles, and sometimes they are not.

5 Informational Externalities

In this Section we focus on the role of informational externalities, i.e., the information θ^j held by agent j affects agent i's utility in some alternative k. The theoretical interest in settings with informational externalities is not new. The classical contributions of Wilson (1969) and Milgrom and Weber (1981) , and the large literature following them analyzed one-object auctions with symmetric bidders. In contrast, the focus of the more recent literature is on multi-object auctions, and on the effects of asymmetries (see Maskin, 1992) on welfare-maximization.

The analysis will be divided in three parts. In Subsection 5.1 we first report some general insights about incentive compatibility and payoff equivalence in contexts with multidimensional signals. In Subsection 5.2 we next inquire whether the Vickrey-Clarke-Groves idea for private values can be extended to settings with informational externalities. The main result is that, generically, if at least one agent holds a signal with at least two dimensions, no Bayes-Nash mechanism exists that implements the welfare maximizing allocation if signals are independently distributed across agents. A fortiori, for any distribution of signals (not necessarily independent across agents), no mechanism exists that implements the welfare-maximizing allocation in

[38]See also Jehiel and Moldovanu (2001b) who connect Pafrey's results to the payoff equivalence theorem.

[39]Pure bundling auctions has been first studied by Palfrey (1983), who showed that the pure bundling auction is revenue superior to separate, efficient auctions if there are only two bidders. Under some technical assumptions on the distribution of types, Chakraborty (1999) showed that there is a critical number of bidders, below which pure bundling outperforms separate auctions, and above which separate auctions outperform the pure bundling auction.

ex-post equilibrium[40], if at least one agent holds a signal with at least two dimensions. If the signals of all agents are one-dimensional, welfare-maximizing implementation is possible under a *single-crossing* condition. This and other possibility scenarios are discussed in subsection 5.3.

In Subsection 5.4, we inquire what social choice functions (not necessarily welfare-maximizing) can be implemented in a robust way, independently of the distributions of signals (i.e., in an ex-post equilibrium). The main result is that, generically, if at least two agents hold a signal with at least two dimensions, no choice rule that conditions on the signals can be robustly implemented.

Thus, the two main insights obtained for private value settings with allocative externalities in Section 4 cannot be extended to frameworks with informational externalities, unless valuations are not generic or signals are one-dimensional.

5.1 Incentive Compatibility and Payoff Equivalence

Recall the social choice framework described in Section 2. Agents have quasi-linear utility functions that depend on the chosen alternative k, on private signals $\theta = (\theta^0, \theta^1, ...\theta^N)$, and on a monetary payment t^i :

$$u^i(k, \theta, t^i) = v^i_k(\theta) + t^i$$

The revelation principle asserts that, for any Bayes-Nash equilibrium of any given mechanism, one can construct an equivalent equilibrium of a direct revelation mechanism where all agents truthfully report their private information. Thus, for the characterization results, we focus below on truth-telling equilibria in direct revelation mechanisms.

Denote by $v^i(\theta)$ the vector $(v^i_k(\theta))_{k\in K}$, and Let $\Delta^{|K|-1}$ denote the simplex of probability distributions over the finite set of alternatives. A *direct revelation mechanism* is given by a pair (Ψ, t) where $\Psi : \Theta = \times_{i=0}^{N}\Theta^i \to \Delta^{|K|-1}$ is the *allocation rule*, and $t : \Theta \to \Re^{N+1}$ is the *payment rule*. Given a mechanism (Ψ, t), define

$$\overline{v}^i(\tau^i, \theta^i) = E_{\theta^{-i}}[\Psi(\tau^i, \theta^{-i}) \cdot v^i(\theta)] \tag{9}$$

and

$$\overline{t}^i(\tau^i) = E_{\theta^{-i}}[t^i(\tau^i, \theta^{-i})] \tag{10}$$

[40]In frameworks with interdependent values, this concept forms the natural generalization of dominant strategy equilibrium.

Then

$$U^i(\tau^i, \theta^i) = \overline{v}^i(\tau^i, \theta^i) + \overline{t}^i(\tau^i) \tag{11}$$

represents agent i's expected utility when all other agents report truthfully, and when i has true type θ^i but reports type τ^i.

A mechanism (Ψ, t) is *incentive compatible* if:

$$\forall i, \theta^i, \; V^i(\theta^i) \equiv U^i(\theta^i, \theta^i) = \sup_{\tau^i \in \Theta^i} U^i(\tau^i, \theta^i) \tag{12}$$

In other words, $V^i(\theta^i)$ represents agent i's expected utility in a truthful Bayes-Nash equilibrium.

There are several versions of characterization results in the literature. The one below is due to Krishna and Maenner (2001):

Theorem 10 *Assume that, for each i, the type set Θ^i is convex and that $u_i(k, \theta^{-i}, \cdot, t^i)$ is a convex function of θ^i. Then, in any incentive compatible mechanism (Ψ, t), the expected equilibrium utility function $V^i(\theta^i)$ is convex, and is determined by the allocation rule Ψ up to an additive constant. For any $\theta^i, \tau^i \in \Theta^i$, and for any smooth path γ joining θ^i to τ^i in Θ^i, it holds that*

$$V^i(\theta^i) = V^i(\tau^i) + \int q^i \cdot d\gamma \tag{13}$$

where q^i is a subgradient[41] *of V^i at θ^i.*

The main complication added by the presence of multidimensional signals is the requirement that q^i , which is fully determined by Ψ, be a (sub)gradient of a convex function V^i. Whereas this requirement reduces to a standard monotonicity condition in the one-dimensional case, it involves both a monotonicity condition, and a differential condition on the cross derivatives of q^i (yielding the path independence condition) if signals are multidimensional - see Subsection 5.2 for further illustration). These consequences of multidimensional constraints in the context of auctions have been first pointed out by Jehiel, Moldovanu, and Stacchetti (1996, 1999).[42]

To get some geometric intuition about the added complexity, consider the following example:

[41] Consider a convex function $F : C \to \Re$ where $C \subset \Re^d$. A vector $x^* \in \Re^d$ is a *subgradient* of F at $x \in C$ if for all $y \in C$ it holds: $F(y) \geq x^* \cdot (y - x)$. Whenever F is differentiable (a.e.) the subgradient is unique and coincides with the usual gradient. Note that convex functions are twice continuously differentiable almost everywhere.

[42] Analogous results appeared earlier in the monopolistic screening literature (see Rochet, 1985).

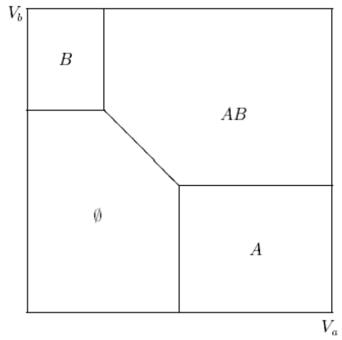

Figure 2:

Example 11 *There are two objects, A and B , and one bidder. The bidder obtains a two-dimensional private signal (v_A, v_B) about the values of the two goods. The bundle is worth $v_A + v_B$. Any deterministic, incentive compatible mechanism divides the two-dimensional set of types into areas of constant allocation. The gradient of the equilibrium utility is given here by the probabilities with which the bidder gets the objects. Thus, the vector q is either $(0,0)$, $(0,1), (1,0)$ or $(1,1)$. Convexity of V (or, equivalently, monotonicity of q) implies that, as we increase valuations, q is only allowed to "jump" from $(0,0)$ to either $(0,1)$, $(1,0)$ or $(1,1)$, and from $(0,1)$, $(1,0)$ to $(1,1)$. This is similar to the insight obtained in one-dimensional models. But, there is an additional twist here: if a jump occurs, the integral of q will not depend on the path of integration (i.e., the vector field q is conservative) if and only if "the jump vector" is perpendicular to the boundary between the areas where each alternative is chosen. For example, the boundary between areas where the buyer gets both objects or none must have a slope of $45°$ (since the jump is $(1,1) - (0,0) = (1,1)$), while the boundaries between areas where the buyer gets one object or two are either horizontal, or vertical lines. While in the present example with one bidder these observations are simple (and could be derived from the well-known Taxation Principle of monopolistic screening), the analog conditions become very involved when there are several interacting agents, as is generally the case in auctions. The requirement of conservativeness of the vector field q yields partial differential equations that determine the boundaries, and these need not be straight lines (for these insights and an application, see Jehiel, Moldovanu and Stacchetti, 1999).*

An important corollary of Theorem 10 is the so called payoff and revenue equivalence result. Consider two incentive compatible mechanisms (Ψ, t)

and (Ψ, s) that implement the same allocation rule Ψ, and thus yield the same expressions for $\overline{v}^i$ and q^i. Choose an arbitrary type $\widetilde{\theta}^i$, and let $V_t^i(\widetilde{\theta}^i)$ and $V_s(\widetilde{\theta}^i)$ denote the expected equilibrium utility in the truth-telling equilibria of (Ψ, t) and (Ψ, s), respectively. Payoff and revenue equivalence follows immediately from the above Theorem by noting that :

$$\begin{aligned}
\forall i, \theta^i, \ \overline{t}^i(\theta^i) &= V_t^i(\widetilde{\theta}^i) - \overline{v}^i(\theta^i, \theta^i) + \int q^i \cdot d\gamma \\
\forall i, \theta^i, \ \overline{s}^i(\theta^i) &= V_s^i(\widetilde{\theta}^i) - \overline{v}^i(\theta^i, \theta^i) + \int q^i \cdot d\gamma
\end{aligned}$$

The above equations yield together:

$$\forall i, \theta^i, \ \overline{t}^i(\theta^i) - \overline{s}^i(\theta^i) = V_t^i(\widetilde{\theta}^i) - V_s^i(\widetilde{\theta}^i) = const$$

In other words, expected transfers in the two mechanisms are, up to a constant, the same.

Analogous payoff equivalence results for dominant-strategy or ex-post implementation can be proved along similar lines, but these no longer require that the distribution of signals be independent across agents. Moreover, they deliver equivalence up to a constant of actual transfers rather than expected transfers (see, for example, Chung and Ely, 2001).

5.2 The Impossibility of Welfare Maximization

To illustrate this impossibility result, we consider a linear specification of preferences, and we assume that the designer has no private information θ^0. Let

$$v_k^i(\theta^1, ..., \theta^N) = \sum_{j=1}^{N} a_{ki}^j \theta_{ki}^j,$$

and assume that $a_{ki}^i \geq 0$ for all k, i. The signal θ^i of agent i is drawn from a space $\Theta^i \subseteq 211d^{K\times N}$ according to a continuous density $f_i(\theta^i) > 0$, independently of other agents ' signals θ^{-i}.

Consider a direct revelation mechanism (DRM) (Ψ, t) where $\Psi_k(\tau^1, ...\tau^N)$ is the probability that alternative k is chosen given the report profile $\tau = (\tau^1, ...\tau^N)$, and $t^i(\tau^1, ...\tau^N)$ is the transfer received by agent i given the report profile τ.

We now use the methodology developed for Theorem 10 in subsection 5.1. Recall that the function

$$V^i(\theta^i) = U^i(\theta^i, \theta^i) = \sup_{\tau^i} U^i(\tau^i, \theta^i) \tag{14}$$

represents $i's$ expected utility in a truth-telling equilibrium of a direct revelation mechanism.

The function V^i is convex, and hence twice differentiable almost everywhere. Assuming that V^i is differentiable at θ^i, we obtain by the Envelope Theorem that:

$$\begin{aligned} \frac{\partial V^i}{\partial \theta^i_{ki}}(\theta^i) &= a^i_{ki} q^i_k(\theta^i) \\ \frac{\partial V^i}{\partial \theta^i_{kj}}(\theta^i) &= 0 \text{ for } j \neq i \end{aligned}$$

where

$$q^i_k(\tau^i) = \int_{\Theta^{-i}} \Psi_k(\tau^i, \theta^{-i}) f_{-i}(\theta^{-i}) d\theta^{-i}$$

is $i's$ equilibrium interim expected probability that alternative k is chosen conditional on i's report τ^i. Assuming that V^i is twice continuously differentiable at θ^i, we obtain by Schwarz's Theorem that the cross derivatives of V^i at θ^i must be equal. This implies for all $k, k', i, j \neq i$:

$$a^i_{ki} \frac{\partial q^i_k(\theta^i)}{\partial \theta^i_{k'i}} = a^i_{k'i} \frac{\partial q^i_{k'}(\theta^i)}{\partial \theta^i_{ki}} \tag{15}$$

$$a^i_{ki} \frac{\partial q^i_k(\theta^i)}{\partial \theta^i_{k'j}} = 0 \tag{16}$$

Consider now a welfare-maximizing allocation rule where $\Psi_{k^*}(\theta^1, ...\theta^N) = 0$ if $k^* \notin \arg\max_k \sum_{i=1}^{N} v^i_k(\theta^1, ..., \theta^N)$. For such rules one can directly compute the induced interim expected probabilities that each alternative is chosen. The impossibility result is obtained by showing that these functions cannot generically satisfy (15) and (16):

Theorem 12 *(Jehiel-Moldovanu 2001a) Let* (Ψ, t) *be a welfare maximizing DRM, and assume that the following conditions are satisfied. (1) There exist* i*,* j*,* k *such that* $i \neq j$*,* $a^i_{ki} \neq 0$ *and* $a^i_{kj} \neq 0$*. (2) There exists an open set of* θ^i *such that, depending on the realization of* θ^{-i}*, the welfare maximizing alternative is either* k *or* k'*. Then* (Ψ, t) *cannot be incentive compatible.*

Proof. Conditions 1 and 2 ensure that, for an open set of signals θ^i, the interim expected probabilities (that each alternative is chosen) induced by any value maximizing DRM must satisfy $\partial q^i_k(\theta^i)/\partial\theta^i_{kj} \neq 0$. This is incompatible with (16). ■

To prove Theorem 12 we have only used identities (16). This is enough because the signal θ^i of agent i has dimension $N \times K$, which, in particular, is larger than the dimension of the alternatives K. This result is not too surprising, as it seems impossible to elicit an information that has more dimensions than the number of alternatives the agent cares about. Maskin (1992) provided an early example with this flavor, but he used the stronger ex-post equilibrium concept.

It is important then to understand what happens to the impossibility result if the dimension of the signals is no larger than K, the number of payoff-relevant alternatives, i.e., when the simple insight obtained above does not necessarily hold. Consider the same setting as above, except that agent i receives a K-dimensional signal[43] $\theta^i = (\theta^i_k)$ and

$$v^i_k(\theta^1, ..., \theta^N) = \sum_{j=1}^{N} a^j_{ki}\theta^j_k.$$

The analogues of identities (15) are now: for all i and $k, k' \neq k$,

$$\text{For all } i, k, k' \neq k,\ a^i_{ki}\frac{\partial q^i_k(\theta^i)}{\partial\theta^i_{k'}} = a^i_{k'i}\frac{\partial q^i_{k'}(\theta^i)}{\partial\theta^i_k}. \tag{17}$$

Theorem 13 *(Jehiel-Moldovanu 2001a) Let* (Ψ, t) *be a welfare maximizing DRM, and assume that the following conditions are satisfied: (1) There exist* i*,* k *and* k' *such that* $a^i_{k'i} \neq 0$*. (2) There exists an open set of* θ^i *such that, depending on the realization of* θ^{-i}*, the welfare maximizing alternative is either* k *or* k'*. Then, if* (Ψ, t) *is Bayes-Nash incentive compatible, it must be the case that:*

$$\frac{a^i_{ki}}{a^i_{k'i}} = \frac{\sum_{j=1}^{N} a^i_{kj}}{\sum_{j=1}^{N} a^i_{k'j}}. \tag{18}$$

[43] That is, from the viewpoint of the above more general model, we assume that $\theta^j_{ik} = \theta^j_k$.

Proof. The result follows from the observation that a welfare-maximizing DRM must satisfy

$$\sum_{j=1}^{N} a^i_{kj} \frac{\partial q^i_k(\theta^i)}{\partial \theta^i_{k'}} = \sum_{j=1}^{N} a^i_{k'j} \frac{\partial q^i_{k'}(\theta^i)}{\partial \theta^i_k}. \tag{19}$$

To see this, consider a setting where all agents' preferences coincide with total welfare. If a DRM chooses the welfare maximizing alternative (without any transfer), every agent has obviously an incentive to report truthfully. Thus, the analog of conditions (17) must be satisfied for such preferences, yielding conditions (19). In other words, condition (19) must hold for the interim expected probabilities generated by any mechanism that chooses the welfare-maximizing alternative! But, the incentive constraints (with respect to the original preferences) imply conditions (17). Combining (17) and (19) we get (18). ■

Since the above formulation is relatively abstract, it is helpful to consider a simple auction example:

Example 14 *There are two objects A and B, and two bidders $i = 1, 2$. Each bidder i receives a two-dimensional signal $\theta^i = (\theta^i_A, \theta^i_B)$. There are no allocative externalities*[44]*. Bidder i's preference (which depends only on i's bundle) is assumed to take the following linear form:*

$$v^i_X(\theta) = \theta^i_X + a^{-i}_{Xi}\theta^{-i}_X \ , \quad X = A, B$$

$$v^i_{AB}(\theta) = v^i_A(\theta) + v^i_B(\theta) + s^i_{AB}$$

We refer to s^i_{AB} as i's synergy term, and assume here that these terms are common knowledge.

Consider a direct truthful mechanism with associated interim expected probabilities $q^i_P(\theta^i)$, where the partition $P = P_{ij}$, denotes the allocation where object A is allocated to agent i and object B to agent j (thus P_{ii} denotes the allocation where i gets both objects, and so on..). Let $V^i(\theta^i)$ denote the corresponding equilibrium utility of bidder i. Assuming that V^i is differentiable at θ^i, we obtain by the Envelope Theorem that:

$$\begin{aligned} \frac{\partial V^i}{\partial \theta^i_A}(\theta^i) &= q^i_{P_{ii}}(\theta^i) + q^i_{P_{ij}}(\theta^i) \\ \frac{\partial V^i}{\partial \theta^i_B}(\theta^i) &= q^i_{P_{ii}}(\theta^i) + q^i_{P_{ji}}(\theta^i) \end{aligned}$$

[44]Note that every bidder receives a signal of lower dimension than the the number of partitions, or even than the number of bundles he cares about.

Assuming that V^i is twice continuously differentiable at θ^i, we obtain that:

$$\frac{\partial}{\partial \theta^i_B}[q^i_{P_{ii}}(\theta^i) + q^i_{P_{ij}}(\theta^i)] = \frac{\partial}{\partial \theta^i_A}[q^i_{P_{ii}}(\theta^i) + q^i_{P_{ji}}(\theta^i)] \tag{20}$$

Consider now the welfare-maximizing allocation rule, and let $\widetilde{q}^i_P(\theta^i)$ denote the expected probability that the partition P is chosen conditional on i's signal θ^i. Observing that such an allocation can be implemented whenever agents' preferences coincide with total welfare, we obtain that $\widetilde{q}$ must satisfy:

$$\begin{aligned} & \frac{\partial}{\partial \theta^i_B}[\widetilde{q}^i_{P_{ii}} + \widetilde{q}^i_{P_{ij}} + a^i_{Aj}\widetilde{q}^i_{P_{ji}} + a^i_{Aj}\widetilde{q}^i_{P_{jj}}](\theta^i) & (21) \\ = \; & \frac{\partial}{\partial \theta^i_A}[\widetilde{q}^i_{P_{ii}} + \widetilde{q}^i_{P_{ji}} + a^i_{Bj}\widetilde{q}^i_{P_{ij}} + a^i_{Bj}\widetilde{q}^i_{P_{jj}}](\theta^i) & (22) \end{aligned}$$

In general, equations (20) and (21) are not compatible unless $a^i_{Aj} = a^i_{Bj} = 0$, i.e., unless there are no informational externalities! To see that, assume for concreteness that the partition P_{jj} is never welfare-maximizing (say, because s^j_{AB} is negative with a large absolute value), and assume that $a^i_{Bj} = 0$ while $a^i_{Aj} > 0$. Then $\widetilde{q}^i_{P_{jj}} = 0$, and it is readily verified that (20) and (21) are incompatible because $\frac{\partial}{\partial \theta^i_B}\widetilde{q}^i_{P_{ji}} \neq 0$ whenever[45] $\frac{\partial}{\partial \theta^i_B}[\widetilde{q}^i_{P_{ii}} + \widetilde{q}^i_{P_{ij}} + a^i_{Aj}\widetilde{q}^i_{P_{ji}}] = \frac{\partial}{\partial \theta^i_A}[\widetilde{q}^i_{P_{ii}} + \widetilde{q}^i_{P_{ji}}]$.

In some special (yet non-generic) cases, the welfare-maximizing allocation rule can be implemented even when there are informational externalities. This is, for example, the case when $s^i_{AB} = 0$, $i = 1, 2$ and when $a^{-i}_{Xi} < 1$, $X = A, B, i = 1, 2$. Then the welfare-maximizing rule can be implemented using separate, one dimensional second price auctions (see Maskin, 1992 who studies the one-dimensional one-object case and who emphasizes the role of a single crossing condition, amounting here to $a^{-i}_{Xi} < 1$).

It is worth understanding why a welfare-maximizing allocation rule simultaneously satisfies (20) and (21) when $s^i_{AB} = 0$, $i = 1, 2$. Given the separability of the problems in this case, the efficient allocation takes the form:

$$\begin{aligned} q^i_{P_{ii}}(\theta^i) &= q^i_A(\theta^i_A)q^i_B(\theta^i_B) \\ q^i_{P_{ij}}(\theta^i) &= q^i_A(\theta^i_A)[1 - q^i_B(\theta^i_B)] \\ q^i_{P_{ji}}(\theta^i) &= [1 - q^i_A(\theta^i_A)]q^i_B(\theta^i_B) \\ q^i_{P_{jj}}(\theta^i) &= [1 - q^i_A(\theta^i_A)][1 - q^i_B(\theta^i_B)] \end{aligned}$$

[45] For example, if the synergy term s^i_{AB} is non-positive, and if partition P_{ji} is sometimes welfare maximizing, one gets $\frac{\partial}{\partial \theta^i_B}\widetilde{q}^i_{P_{ji}} > 0$.

where $q_X^i(\theta_X^i)$ denotes the interim expected probability that good X is allocated to i conditional on θ_X^i (it solely depends on θ_X^i because of the separability property). It is then an elementary exercise to check that conditions (20) and (**??**) are always met for such separable allocation functions, no matter what a_{Xj}^i are[46].

To sum up, the welfare-maximizing allocation rule cannot be implemented in auction setups with interdependent values and multidimensional signals except in cases where the allocation problem can be divided in separate one-object, one-dimensional auctions[47]. Such a separation is usually impossible if there are complementarities/substitutabilities or allocative externalities.

5.3 Possibility Results and Applications

5.3.1 One-dimensional signals

For a two-bidder, one-object auction with interdependent values Maskin (1992) has shown that welfare maximization can be achieved if both bidders have a one-dimensional signal, and if a single crossing condition holds. This observation has been extended to more general, one-dimensional settings by Dasgupta and Maskin (2000), Ausubel (1997), Jehiel and Moldovanu (2001a), Perry and Reny (2002), Bergemann and Valimaki (2002) and Krishna (2003).

Within the above described linear model, we provide now a condition ensuring that welfare maximization can be obtained when each agent i receives a one dimensional signal $\theta^i \in \Re$, and i's value of alternative k is given by:

$$v_k^i(\theta^1, ..., \theta^N) = \sum_{j=1}^{N} a_{ki}^j \theta^j.$$

Theorem 15 *(Jehiel and Moldovanu 2001a) Suppose that for all i, k, k'*

$$\frac{a_{ki}^i}{a_{k'i}^i} > 1 \Rightarrow \frac{\sum_{j=1}^{N} a_{kj}^i}{\sum_{j=1}^{N} a_{k'j}^i} > 1.$$

[46]The requirement that $a_{Xj}^i < 1$ is derived from the standard one-dimensional incentive compatibility condition that an agent with a higher signal should receive the good with a higher probability.

[47]Jehiel, Moldovanu and Stacchetti (1996) contains an early impossibility result in a one-object auction with allocative externalities, interdependent values and multidimensional signals.

Then there exists a welfare-maximizing, Bayesian incentive compatible mechanism.

The set of parameters for which welfare maximization can be achieved is now an open set, and it has positive measure. The fundamental difference is due to the incentive constraints that now reduce to a simple monotonicity condition (without the complex integrability requirement). The relevant mechanism for the above result follows the Vickrey-Clarke-Groves logic by completely dampening the influence of one's own signal on one's own transfer.

Several papers work within an one-dimensional setup in order to analyze conditions under which specific bidding formats can lead to welfare maximization, or in order to study the role of additional requirements such as participation constraints or budget balancedness. Building on Maskin (1992)'s result , Krishna (2003) derives conditions (stronger than single-crossing) ensuring the efficiency of the English ascending auction even if there are more bidders. Kirchkamp and Moldovanu (2004) experimentally compare the performance of the English and second-price sealed-bid auctions with interdependent valuations: as predicted by theory, the English auction is superior from an efficiency point of view. Dasgupta and Maskin (2000) and Perry and Reny (2002) present welfare-maximizing bidding schemes for multi-object auctions where agents have one-dimensional signals.

Gresik (1991), Fieseler, Kittsteiner and Moldovanu (2003), Kittsteiner (2003), Jehiel and Pauzner (2004), Ornelas and Tuner (2004) and Brusco, Lopomo, and Wiswanathan (2004) analyze properties of trading models with interdependent values and one-dimensional signals in a variety of settings that combine features of the classical models due to Akerlof (1970), Myerson-Satterthwaite (19831) and Cramton-Gibbons-Klemperer (1987). For example, Kittsteiner et al. (2003) show that negative (positive) informational externalities make it easier (harder) to construct welfare-maximizing, budget balanced and individual-rational mechanisms than in analogous cases with private values. The sign of the informational externalities plays also a crucial role for over/under - investment in the information acquisition model with interdependent values studied by Bergemann and Valimaki (2002). Jehiel and Pauzner (2005) show, in contrast to Cramton et al. (1987)'s private value result, that extreme ownership structures may dominate mixed ownerships in partnership dissolution setups with interdependent values and one-sided private information.

Hain and Mitra (2004) and Kittsteiner and Moldovanu (2005) analyze scheduling and queueing problems, respectively, where agents have private information about processing times: this naturally yields a model with informational externalities since waiting costs depend on information available to others. Gruener and Kiel (2004) focus on collective decisions with interdependent values in the absence of monetary transfers[48].

The impossibility results established above has consequences for the design of private industries. If the competing firms hold multi-dimensional private information about their cost structure (say on fixed and marginal cost), then it is impossible to induce a welfare maximizing market structure even if there are no costs to public funds. This is illustrated in Jehiel and Moldovanu (2004), and it contrasts with the analysis performed in Dana and Spier (1994), Auriol and Laffont (1992), and McGuire and Riordan (1995) in which private information is one-dimensional and the sole economic friction comes from the shadow cost of public funds.

5.3.2 Multi-dimensional signals

The correlated case: Our main impossibility results (Theorems 12 and 13) assume that the signals are independently distributed across agents. If there are several agents and if the signals held by the various agents are correlated, it is possible to design subtle transfer devices whereby the belief held by agent i on the signal of agent j is elicited for free by the designer. When j's belief about i's signal completely determines the signal held by i, a welfare-maximizing allocation can be (approximately) implemented[49], using the logarithmic scoring rule used in Johnson et al. (1990) and Johnson et al. (2003). It is important to note that the approach developed by Cremer-McLean (1985, 1988) is not useful[50] here since it relies on the existence of a welfare-maximizing mechanism, but as shown above, this assumption is not fulfilled if there are informational externalities and multidimensional signals.

To illustrate Johnson et al. 's idea, suppose that agent j has the belief that i's signal is distributed according to the density $\beta(\theta^i)$, and that agent j is asked to report her belief to a designer who observes the realization of θ^i (in equilibrium this will be reported by i). Suppose further that (up to a constant) agent j receives $\ln \widetilde{\beta}(\theta^i)$ conditional on θ^i , where $\widetilde{\beta}(\cdot)$ is the report made by j about her belief on i's signal.[51] It is readily verified that agent j

[48] They give the example of decision making within the European Central Bank.

[49] In two important papers, Neeman (2004) and Heifetz and Neeman (2005) challenge the view that, generically, there is a one-to one mapping from j's belief onto i's type in the correlated case.

[50] McLean and Postelwaite (2004) use this approach, but only for situations where agents are *informationally small.*

[51] We assume (for now) that the report made by j is not used to implement a social alternative.

will choose optimally to report her true belief. This is because

$$\begin{aligned}\int \beta(\theta^i) \ln \widetilde{\beta}(\theta^i) d\theta^i &\leq \int \beta(\theta^i) \ln \beta(\theta^i) d\theta^i \Leftrightarrow \\ \int \beta(\theta^i) \ln \frac{\widetilde{\beta}(\theta^i)}{\beta(\theta^i)} d\theta^i &\leq 1\end{aligned}$$

and because

$$\int \beta(\theta^i) \ln \frac{\widetilde{\beta}(\theta^i)}{\beta(\theta^i)} d\theta^i \leq \ln(\int \beta(\theta^i) \frac{\widetilde{\beta}(\theta^i)}{\beta(\theta^i)} d\theta^i) = 0$$

by the concavity of the logarithm, and by the property that $\int \widetilde{\beta}(\theta^i) d\theta^i = 1$ which is met by any probability measure $\widetilde{\beta}$.

It is not hard to see how the welfare-maximizing allocation can be (approximately) implemented whenever there is a one-to-one mapping from j's belief onto i's signal: by having a transfer that puts a sufficiently large weight on $\ln \widetilde{\beta}(\theta^i)$, agent j will have the right incentive to report her true belief on i's signal (the belief being isomorphic to her type). This result does not rely at all on the form of the preferences, and it is solely driven by the facts that different types correspond to different beliefs, and that beliefs can be elicited for free when there is no restriction on the size of the transfers.

Arbitrarily large transfers (via the weight on $\ln \widetilde{\beta}(\theta^i)$) are required whenever distributions of types are nearly independent across agents. If bounds on transfers are imposed (because, say, of limited liability constraints) our impossibility result persists whenever the distribution of signals is not too far from the independent case. This is a continuity result in the vein of Robert (1990).[52]

Conditioning on extra information: Mezzetti (2004) considers a different informational environment: after the social alternative has been chosen, and before monetary payments have been concluded, the agents observe their payoff from the chosen alternative. Thus, agents receive information in addition to their initial signals, and mechanisms should depend on this extra information.

Mezzetti observes that an efficient mechanism can be implemented in two stages: In stage 1, agents report their signals; based on the reports, the value maximizing alternative is implemented. In stage 2, transfers are implemented

[52] Adding the possibility of an ex-post veto is also likely to invalidate such a possibility result. See Compte and Jehiel (2004) for an exploration of such constraints in Myerson-Satterthwaite's model with correlated types.

according to a Vickrey-Clarke-Groves mechanism based on reports about the observed payoffs in the alternative chosen at stage 1. Assuming that agents report honestly in stage 2, agents also have the right incentive to report honestly in stage 1 because, given the VCG transfers at stage 2, their objective coincides with the social welfare criterion

Mezzetti's insight[53] is valuable. But, the specific mechanism proposed by Mezzetti suffers from a number of serious drawbacks. First, at stage 2, agents are completely indifferent about their announcement, since it only serves to compute the transfers received by others (the social alternative has been already chosen at stage 1. This is in sharp contrast with the agents' strictly positive incentive (due to the unresolved uncertainty about the chosen alternative) to report their true types in the VCG mechanism in the private values case. Second, in many applications, a significant amount of time may elapse before the payoff attached to a chosen alternative is revealed to an agent. Then, allowing for transfers that are contingent on information that becomes available in a distant future seems impractical[54].

5.4 Ex-post Implementation without Welfare Maximization

Bayesian mechanism design has been criticized on the ground that both the designer and the agents need a lot of information about the distribution of the private signals in order to choose their best course of action[55]. *Robust implementation* seeks for a stronger notion of implementation where the agents and the designer do not need such precise information. In private value contexts, this leads to consider *dominant strategy implementation*, whereas the analogous concept for settings with informational externalities is *ex-post implementation*[56]: Agents should find it optimal to report their true signals even after learning the signals received by others. Such a notion is necessary for robustness since implementation should be possible for any belief, in particular for the degenerate belief that other agents' signals are given

[53] Hansen (1985) is an early paper on auction with contingent payments in settings where additional information becomes available.

[54] This issue pertains also to the celebrated lemons market in Akerlof (1970). Mezzetti's analysis suggests that there is no problem for the buyer and the seller to agree on the price after the buyer observes quality (say after one year of driving a used car). But, our opinion is that such mechanisms are fragile due to the noisy and subjective assessments of quality, moral hazard, verifiability, etc....

[55] This is sometimes referred to as *Wilson's critique.*

[56] This notion corresponds to the *uniform equilibrium* first defined by d'Aspremont and Gerard Varet (1979), and to *uniform incentive compatibility* as defined by Holmstrom and Myerson (1983). The term *ex-post equilibrium* is due to Cremer and McLean (1985).

by a specific realization. It is also sufficient for robustness in quasi-linear environments since ex-post implementation basically means that, whatever their beliefs about others' types, agents find it optimal to report their true types whenever other agents are expected to report truthfully. These ideas have been recently formalized by Bergemann and Morris (2005) [57]

An immediate corollary of the main result in the previous section is that, generically, the welfare-maximizing allocation cannot be robustly implemented if there are multidimensional signals and informational externalities (if it were, the welfare-maximizing Bayes-Nash implementation would be possible for any prior, including the independent case; but this was shown to be false!)

In this Subsection we considerably strengthen the above insight by showing that it extends to any deterministic social rule that makes use of the agents' reports. In our view, the resulting impossibility result casts a serious doubt about the role of ex-post implementation as a form of robust implementation. The analysis follows Jehiel, Meyer-ter-Vehn, Moldovanu, and Zame (2005).

We focus on choice functions $\psi : S \to K$, with the property that there are transfers functions $t^i : S \to \mathbb{R}$, such that truth-telling is an *ex-post equilibrium* in the incomplete information game that is induced by the direct revelation mechanism $\left(\psi, (t^i)_{i\in N}\right)$, i.e.

$$v^i_{\psi(\theta)}(\theta) + t^i(\theta) \geq v^i_{\psi\left(\widetilde{\theta}^i, \theta^{-i}\right)}(\theta) + t^i\left(\widetilde{\theta}^i, \theta^{-i}\right) \tag{23}$$

for all $\theta^i, \widetilde{\theta}^i \in \Theta^i$ and $\theta^{-i} \in \Theta^{-i}$. Such a choice rule ψ is said to be *ex post implementable*. We call a choice function ψ *trivial* if it is constant on the interior Θ of the type space.

By requiring optimality of i's truth-telling for every realization of other agents types θ^{-i}, inequality (23) treats θ^{-i} as if it was known to agent i. Her incentive constraint is thus equivalent to a monopolistic screening problem for every θ^{-i}. Thus, the designer can post personalized prices $p^i_k(s^{-i})$ for the various alternatives, and let the individuals choose among them. In equilibrium all agents must agree on a most favorable alternative:

Lemma 16 (Ex-Post Taxation Principle) *A choice function ψ is ex post implementable, if and only if for all $i \in \mathcal{N}$, $k \in \mathcal{K}$ and $s^{-i} \in S^{-i}$, there are transfers $\left(p^i_k\left(\theta^{-i}\right)\right)_k \in (\mathbb{R} \cup \{\infty\})^N \setminus (\infty, ..., \infty)$ such that:*

$$\psi(\theta) \in \arg\max_{k\in\mathcal{K}} \left\{v^i_k(\theta) - p^i_k\left(\theta^{-i}\right)\right\}. \tag{24}$$

[57] See also Dasgupta, Hammond, Maskin, (1979) and Ledyard (1978) for early, related arguments in private value frameworks.

The difficulty of finding an ex-post implementable rule is that (24) should be simultaneously satisfied for all agents i.

For illustration, we now reduce the problem by assuming that there are only two agents, $i = 1, 2$ and two alternatives k, l. Because agents' incentives are only responsive to differences in payoffs, it is convenient to focus on *relative* valuations[58] μ^i and *relative* prices δ^i :

$$\begin{aligned}
\mu^i(\theta) &= v_k^i(\theta) - v_l^i(\theta) \\
\delta^i\left(\theta^{-i}\right) &= p_k^i\left(\theta^{-i}\right) - p_l^i\left(\theta^{-i}\right)
\end{aligned}$$

Assuming, that relative prices δ are continuous, the taxation principle implies that at a signal θ such that agent i is indifferent between the two alternatives, agent j should also be indifferent (since they agree on the preferred alternative). That is,

$$\mu^i(\theta) - \delta^i\left(\theta^{-i}\right) = 0 \Leftrightarrow \mu^j(\theta) - \delta^j\left(\theta^{-j}\right) = 0 \tag{25}$$

Assuming further that relative prices δ are differentiable, condition (25) implies that the gradients of agents's payoff functions must be parallel on the *indifference* set (i.e., the set of signal profile where agents are indifferent between the two alternatives). That is,

$$\begin{pmatrix} \nabla_{\theta^i}\mu^i(\theta) \\ \nabla_{\theta^{-i}}\mu^i(\theta) - \nabla_{\theta^{-i}}\delta^i\left(\theta^{-i}\right) \end{pmatrix} \text{ and } \begin{pmatrix} \nabla_{\theta^i}\mu^{-i}(\theta) - \nabla_{\theta^i}\delta^{-i}\left(\theta^i\right) \\ \nabla_{\theta^{-i}}\mu^{-i}(\theta) \end{pmatrix}$$
$$\text{are parallel on the indifference set.} \tag{26}$$

For differentiable relative price functions, this implies:

Proposition 17 *Let (ψ, t) be a non-trivial ex-post incentive compatible mechanism. If the relative transfers δ^i are differentiable for all $i \in \{1, 2\}$ then, there exist an indifference signal profile $\widehat{\theta}$, and a vector y (with the dimensionality of θ^i) such that $\nabla_{\theta^i}\mu^i(\theta)$ and $(\nabla_{\theta^i}\mu^{-i}(\theta) - y)$ are parallel for every indifference signal profile $\theta = (\widehat{\theta}^i, \theta^{-i})$.*

Jehiel et al. (2005) extend the above intuition to the case where the relative price function is neither differentiable, nor continuous, and they show that the above geometric condition cannot be generically[59] satisfied. This yields:

[58]For technical simplicity, we assume that relative valuations satisfy the mild requirement $\nabla_{\theta^i}\mu^i(\theta) \neq 0$ for all $\theta \in \Theta$.

[59]See Jehiel et al (2005) for precise notions of genericity in the infinite dimensional space of preferences. The result holds for both topological and measure-theoretic notions.

Theorem 18 *(Jehiel et al. 2005) Assume that the dimension of the signal θ^i is at least two for each agent $i = 1, 2$. Then, for generic preferences, only trivial social choice rules are ex-post implementable.*

For illustration, consider now a setting with bilinear valuations and two-dimensional signals $\theta^i = \left(\theta^i_k, \theta^i_l\right) \in [0,1]^2$.

Example 19 *Define valuations v by:*

$$\begin{aligned} v^i_k(\theta) &= a^i_k\theta^i_k + b^i_k\theta^i_k\theta^{-i}_k = \theta^i_k\left(a^i_k + b^i_k\theta^{-i}_k\right) \\ v^i_l(\theta) &= a^i_l\theta^i_l + b^i_l\theta^i_l\theta^{-i}_l = \theta^i_l\left(a^i_l + b^i_l\theta^{-i}_l\right) \end{aligned}$$

where $a^i_k, b^i_k, a^i_l, b^i_l \neq 0$. Thus,

$$\mu^i(\theta) = a^i_k\theta^i_k - a^i_l\theta^i_l + b^i_k\theta^i_k\theta^{-i}_k - b^i_l\theta^i_l\theta^{-i}_l.$$

For a vector $y = \begin{pmatrix} y_k \\ y_l \end{pmatrix}$, we have

$$\begin{aligned} \nabla_{\theta^i}\mu^i(\theta) &= \begin{pmatrix} a^i_k + b^i_k\theta^{-i}_k \\ -a^i_l - b^i_l\theta^{-i}_l \end{pmatrix} \\ \left(\nabla_{\theta^i}\mu^{-i}(\theta) - y\right) &= \begin{pmatrix} b^{-i}_k\theta^{-i}_k - y_k \\ -b^{-i}_l\theta^{-i}_l - y_l \end{pmatrix} \end{aligned}$$

It is readily verified that $b^1_l b^2_k - b^1_k b^2_l = 0$ is necessary for such vectors to remain parallel when we vary θ^{-i}_k and θ^{-i}_l. It follows from Proposition 17 that a non-trivial choice function ψ is implementable only if

$$b^1_l b^2_k - b^1_k b^2_l = 0. \tag{27}$$

The above condition is obviously non-generic: the set of parameters where it is satisfied has zero Lebesgue-measure in the 8-dimensional space of coefficients that parameterize the bi-linear valuations in this example.

Possibility results We now review several situations that allow for non-trivial ex-post implementation. All of them result from settings where the geometric condition displayed in Proposition 17 above is less restrictive.

1) Theorem 18 heavily relies on the impossibility of simultaneously satisfying the incentive constraints of several agents. If there is only one strategic agent, non-trivial ex post implementation is possible. This should be contrasted with the impossibility of efficient Nash-Bayes implementation, which holds as soon as at least one agent has multidimensional private information.

2) If all agents have a one-dimensional signal[60], or if only one agent has a multi-dimensional signal then non-trivial ex post implementation is possible for some open set of preferences (see Jehiel et al. 2005).

3) For some non-generic, yet interesting, preferences non-trivial ex post implementation is possible.

3a) Consider preferences $v_k^i(\theta)$ that are additively separable, i.e. $v_k^i(\theta) = f_k^i(\theta^i) + h_k^i(\theta^{-i})$. It is readily verified by the standard VCG analysis that $\arg\max_k \sum_i f_k^i(\theta^i)$ can be ex-post implemented. Under some technical conditions, only such affine maximizers can be implemented (see Jehiel, Meyer-ter-Vehn and Moldovanu, 2004). This is the counterpart to Roberts' (1979) result in the private values case.

3b) Another non generic but interesting class is studied by Bikchandani (2004). He considers a one-object auction without allocative externalities and observes that, by not selling the object for a sufficiently large subset of signals, a non-trivial (yet very inefficient) choice rule can be ex-post implemented.

6 Conclusion

We have studied the effects of allocative and informational externalities in auctions and related mechanisms. Because values become endogenous, standard auctions cease to be welfare maximizing in the presence of allocative externalities, and they give rise to a wealth of new phenomena. But the traditional Vickrey-Clarke-Groves mechanisms achieve welfare maximization in such frameworks. Moreover, robust (not necessarily welfare maximizing) implementation is generically possible only for a class of weighted Vickrey-Clarke-Groves mechanisms. Informational externalities can be satisfactorily dealt with only in settings where signals are one-dimensional (and where a single-crossing property holds). For example, an English ascending auction is welfare maximizing in one-object symmetric settings, and generalized Vickrey-Clarke-Groves mechanisms are welfare- maximizing even in asymmetric settings as long as signals are scalars. The situation drastically changes when information is multidimensional (as required by general multi-object applications): robust welfare maximization is generically impossible.

We hope that the above survey has shown that externalities naturally arising in many applications have a significant effect on the outcome of auctions and other mechanisms. From an empirical viewpoint, it is now time to investigate the role (magnitude and effect on bidding strategies) of alloca-

[60]In this case, the geometric condition merely requires that some scalars (rather than vectors) are multiples of each other

tive externalities in auctions - this could parallel the exciting recent work on common value auctions (see Athey and Haile, 2005).

From a theoretical viewpoint, the above analysis leaves open a number of important questions. In particular, since informational externalities make it impossible to implement the welfare-maximizing allocation, how does the second-best allocation look like in such contexts? Since ex-post implementation is generally impossible with informational externalities, what is a good way to achieve robust implementation? What are the pros and the cons of resale markets, in contexts with externalities ? Finally, the difficulties posed by multidimensional signals hampered advances also in contexts without externalities: how does the revenue-maximizing auction for multiple, heterogenous objects looks like ?

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