GOVERNANCE AND THE EFFICIENCY OF ECONOMIC SYSTEMS
GESY

Discussion Paper No. 138

Labour market screening with intermediaries

Paul Schweinzer*

June 2006

*Paul Schweinzer, Department of Economics, University of Bonn Lennéstraße 37, 53113 Bonn, Germany
paul.schweinzer@uni-bonn.de

Financial support from the Deutsche Forschungsgemeinschaft through SFB/TR 15 is gratefully acknowledged.

Sonderforschungsbereich/Transregio 15 · www.gesy.uni-mannheim.de
Universität Mannheim · Freie Universität Berlin · Humboldt-Universität zu Berlin · Ludwig-Maximilians-Universität München
Rheinische Friedrich-Wilhelms-Universität Bonn · Zentrum für Europäische Wirtschaftsforschung Mannheim

Speaker: Prof. Konrad Stahl, Ph.D. · Department of Economics · University of Mannheim · D-68131 Mannheim,
Phone: +49(0621)1812786 · Fax: +49(0621)1812785

Labour market screening with intermediaries*

Paul Schweinzer
Department of Economics, University of Bonn
Lennéstraße 37, 53113 Bonn, Germany
Paul.Schweinzer@uni-bonn.de

Abstract

We consider a Rothschild-Stiglitz-Spence labour market screening model and employ a centralised mechanism to coordinate the efficient matching of workers to firms. This mechanism can be thought of as operated by a recruitment agency, an employment office or head hunter. In a centralised descending-bid, multi-item procurement auction, workers submit wage-bids for each job and are assigned stable jobs as equilibrium outcome. We compare this outcome to independent, sequential hiring by firms and conclude that, in general, a stable assignment can only be implemented if firms coordinate to some extent. (JEL *C78, D44, E24, J41.* Keywords: *Matching, Multi-item auctions, Sequential auctions, Screening.*)

Introduction

"The entire recruitment market is estimated at more than $300 billion a year. The market for executive search is approximately $10 billion annually with middle-management recruitment estimated at more than $30 billion a year on a global basis. The fragmented market includes many facets from MBA and college recruitment to career management, human resource outsourcing, candidate tracking and company job postings." Korn/Ferry (2000) Apart from private recruitment, there is the government employment office, industry matching programs such as the US National Resident Matching Program (and its international counterparts), central recruiting divisions in large corporations and the public

*Thanks for helpful discussions to Benny Moldovanu, Heidrun Hoppe, Christian Groh and Alex Gershkov. Financial support from the German Science Foundation through SFB/TR 15 is gratefully acknowledged.

sector. “Many large companies today spend in excess of $30 million on search fees per annum and this is a growing phenomenon for sure.” (Scott A. Scanlon, CEO Hunt-Scanlon Avdisors)

Why do firms use recruiters? We study this question as the problem of matching a number of heterogenous workers to a number of non-identical jobs in the well-known screening framework of Rothschild-Stiglitz-Spence.[1] If firms recruit through a single centralised intermediary, this matching problem is similar to that of auctioning multiple items of non-identical goods to a number of heterogenous buyers demanding only a single good each. Demange, Gale, and Sotomayor (1986)—referred to as DGS below—solve this problem by devising an incentive compatible ascending price mechanism for general preferences. We adopt this mechanism to our special setting where workers face some privately known cost of doing some job which is offset by the wage that job pays. We let workers bid the wages for which they are willing to do a particular job in a descending version of the DGS mechanism. Hence the idea is that competition between workers drives down the wages paid by firms.

We contrast this centralised mechanism with a sequence of independent second-price sealed-bid wage auctions, each conducted separately by a single firm.[2] We show that only a very particular sequence of independent second-price auctions is able to implement the stable and efficient DGS assignment. For this sequence to prevail, firms require some coordination and information sharing among themselves which we ascribe to an intermediary.

Shapley and Shubik (1972) initiate the analysis of the assignment problem. Its application to the problem of assigning jobs to workers is studied by Crawford and Knoer (1981), Kelso and Crawford (1982), and Leonard (1983). Sequential auctioning of two jobs with single-crossing preferences is discussed by Elmaghraby (2003) and in a general fashion by Kittsteiner, Nikutta, and Winter (2004).

[1] This model is described in detail, for example, in Mas-Colell, Whinston, and Green (1995, 13.D).
[2] By the revelation principle, the precise type of auction mechanism used is unimportant.

1 The model

There is a set $\mathcal{N}$ of N workers competing for a set of vacant jobs $\mathcal{M}$ containing M jobs. Each worker can accept at most one job. Since we are interested in the effects of competition among workers for jobs we assume that $N \geq M \geq 2$. Each worker—typically indexed by $i \in \mathcal{N}$—has a private skill type θ_i which is ex-ante drawn from some known distribution over a subset of $\mathbb{R}_+$. Jobs—typically indexed by $j \in \mathcal{M}$—are ordered in increasing complexity ('task level') $T^j \in \mathbb{R}_+$ as $T^1, T^2, \ldots, T^M$. These jobs are exogenously given and commonly known.

Workers are equipped with quasi-linear preferences over wages w and private cost of effort $c(\theta, T)$ summarised by $u_i(\theta_i, w^j, T^j) = w^j - c(\theta_i, T^j)$ pinned down by the costless, increasing outside option $w^o(\theta_i)$ at $c(\theta, T = 0) = 0$. The vector of all workers' utilities is $\mathbf{u}$. We denote the set of all players' outside options by $\mathcal{O}$. Outside options are not in $\mathcal{M}$—elements of which we call 'inside options'—and workers matched to their outside option are called unmatched. The cost function is common knowledge and the same for all workers. Crucially, for all T and θ, workers' preferences satisfy the Spence-Mirrlees ('single-crossing') condition on the non-negative reals $c_T(\theta, T) > 0, c_{TT}(\theta, T) > 0, c_\theta(\theta, T) < 0, c_{T,\theta}(\theta, T) < 0$. Thus worker i has an 'indifference' wage $\tilde{w}_i^j$ for a job j such that $\tilde{w}_i^j - c(\theta_i, T^j) = w^o(\theta_i)$ and worker i is indifferent between her outside option and job j. Thus $\tilde{w}_i^j$ is the lowest wage at which worker i is prepared to do job j. We denote the rent i obtains from job j at wage w^j by $r_i^j = w^j - \tilde{w}_i^j$. Worker i's demand for jobs at wages $\mathbf{w}$ and the set of workers demanding a particular job $j \in \mathcal{M}$ at $\mathbf{w}$ are written as

$$\begin{aligned} D^j(\mathbf{w}) &= \{i \in \mathcal{N} | r_i^j = w^j - c(\theta_i, T^j) = \max_{l \in \mathcal{N}} \{w^j - c(\theta_l, T^j))\}\}, \\ D_i(\mathbf{w}) &= \{j \in \mathcal{M} \cup \mathcal{O} | r_i^j = w^j - c(\theta_i, T^j) = \max_{k \in \mathcal{M} \cup \mathcal{O}} \{w^k - c(\theta_i, T^k))\}\}. \end{aligned} \tag{1}$$

There are M firms offering a single job $j \in \mathcal{M}$ each. When the job is filled with *any* capable worker, the job generates a constant exogenous revenue of $\bar{w}^j$.[3] Firms' preferences are given by $v^j(\bar{w}^j, w^j) = \bar{w}^j - w^j$ and thus $\bar{w}^j$ is the highest wage a firm is willing to pay to a

[3] As firms are non-strategic about the type of worker they hire (as long as she can fulfill her job), alternative interpretations of our model are the market for delegation, outsourcing with centralised negotiation and centralised (inter-department) procurement.

worker accepting job j. Again the vector of all firms' utilities is $\mathbf{v}$. We assume the vector of reserve wages $\bar{\mathbf{w}}$ to be publicly known.

We adopt the following mechanism devised by DGS to descending wage-prices:

t=1 The auctioneer (intermediary) announces the starting wage vector $\mathbf{w}(1) = (\bar{w}^1, \bar{w}^2, \ldots, \bar{w}^M)$. Each worker bids by revealing her demand $D_i(\mathbf{w}(1))$ at wage $\mathbf{w}(1)$. $D_i(\mathbf{w})$ is nonempty because if a worker accepts no job in $\mathcal{M}$ and is thus unmatched, she still demands her outside option.

t+1 After bids are announced, if it is possible to assign each worker i to a job in her demand set $D_i(\mathbf{w}(t))$ at price $\mathbf{w}(t)$, the procedure stops. If no such assignment exists, there must be some *overdemanded* set, that is, a set of jobs such that the number of workers demanding only jobs in this set is greater than the number of jobs in this set. The intermediary chooses a *minimal* overdemanded set, that is, an overdemanded set S such that no strict subset of S is an overdemanded set and *decreases* the offered wage for each job in this set by one unit. All other wages remain at the level $\mathbf{w}(t)$. This defines $\mathbf{w}(t+1)$.[4]

Finally we follow DGS in assuming that all wages and workers' rents take integer values. It can be relaxed if a more involved mechanism were to be employed. The precise information structure of the sequence of independent second-price auctions is not important since we do not attempt to derive an equilibrium of this sequential auction. All we require is that firms are unable to coordinate on a particular sequence and for that it suffices to assume that firms draw their position in the sequence of auctions randomly. The following section discusses and illustrates our results. All proofs and an example are presented in the appendix.

[4] This implicitly defines a bidding decrease $\varepsilon = 1$ which we use in the results section. Ties are the measure zero events of more than one worker having the same type. We break these with equal probability in either way.

2 Results

We follow DGS and their textbook treatment Roth and Sotomayor (1990, p209ff) in making the below definitions:

1. A wage-vector $\mathbf{w}$ is called *feasible* if for all $j \in \mathcal{M}$ it is true that $w^j \leq \bar{w}^j$.
2. A feasible wage-vector $\mathbf{w}$ is called *competitive* if there is an assignment $\mu : \mathcal{N} \to \mathcal{M}$ such that if $\mu(i) = j$, then $j \in D_i(\mathbf{w})$ for all $i \in \mathcal{N}$. In this case μ is said to be *compatible* with $\mathbf{w}$.
3. A pair $(\mathbf{w}, \mu)$ is called a *competitive equilibrium* if (i) $\mathbf{w}$ is competitive, (ii) μ is compatible with $\mathbf{w}$, and (iii) $w^j = \bar{w}^j \; \forall j \notin \mu(\mathcal{N})$.
4. An outcome $(\mathbf{u}, \mathbf{v})$ with assignment $\mu(i) = j$ compatible with $\mathbf{w}$ is called (pairwise) *stable* if, for all $h \neq j \in \mathcal{M}$, no other worker $l \neq i \in \mathcal{N}$ demands $D_l(w) = j$.[5]

The proofs of the first two propositions differ only slightly from the original arguments developed by DGS in that we use a decreasing-price version of their mechanism. Our restriction to single-crossing preferences compared to DGS plays a role only in the arguments which follow the first three propositions—particularly for the sequencing of independent auctions—they can be relaxed at no cost in the DGS mechanism.

Proposition 1. *Let* $\mathbf{w}$ *be the wage-vector obtained from the DGS mechanism. Then* $\mathbf{w}$ *is the maximum competitive wage.*

Proposition 2. *If* $\mathbf{w}$ *is the maximum competitive wage obtained from the DGS mechanism, then there is an assignment* μ *such that* $(\mathbf{w}, \mu)$ *is an equilibrium and* $\mathbf{w}$ *is a competitive equilibrium wage-vector.*

Following Demange (1982), Roth and Sotomayor (1990, p212f) identify the DGS mechanism as a generalised Vickrey-Clarke-Groves (VCG) mechanism. Holmström (1979) shows that the VCG mechanism is the unique direct reporting mechanism with dominant strategies, efficient outcomes, and zero payments by losing bidders. Thus with independent bidder types, any other (centralised) auction design leading to efficiency must involve the same wage-payments as the DGS mechanism.

Proposition 3. *The outcome* $(\mathbf{u}, \mathbf{v})$ *under the competitive equilibrium* $(\mathbf{w}, \mu)$ *is stable.*

[5] Notice that stability as defined implies efficiency.

Our restriction to specific preferences allow us to ensure additional structure on the equilibrium assignment resulting from propositions 1–3.

Lemma 1. *Competitive equilibrium wages* $\mathbf{w}$ *in the DGS assignment are increasing in* T*.*

Lemma 2. *The skill types* θ *of workers matched to jobs in the competitive equilibrium assignment are increasing in* T*.*

We now introduce the concept of an *autonomous* match as a job whose wage is determined by an unmatched outsider. We proceed to identify such a job in the DGS assignment. Since the equilibrium wage assigned to an autonomous job by the DGS mechanism can be implemented using a single second-price sealed-bid auction, we know that the *final* auction in a sequence of second-price auctions can implement the autonomous DGS job settlement.

Definition. A job match (j, w^j) is called *autonomous* if its wage component w^j either equals firm j's reservation wage $\bar{w}^j$, or the indifference wage $\tilde{w}_i^j - \varepsilon$ of an unmatched worker i.

Proposition 4. *Every DGS competitive equilibrium assignment* $(\mathbf{w}, \mu)$ *contains an autonomous match.*

Proposition 5. *In order to implement the DGS equilibrium with a sequential mechanism, it is necessary that an autonomous job is auctioned last in a sequence of independent, non-autonomous job auctions.*

There can be assignments containing more than one autonomous match; indeed there are assignments containing only autonomous matches. Hence there are preference-profiles where many sequences of one-shot auctions may lead to a stable assignment. In general, however, a random sequence of one-shot spsb auctions cannot ensure that an autonomous match is auctioned last. Therefore, some amount of cooperation among the firms—perhaps in the form of an intermediary—is required to implement a stable competitive equilibrium assignment.

To appreciate the strategic difference between the DGS and the sequential mechanism it suffices to recall that winners of previous rounds do not participate in the sequential mechanism while they do participate in the DGS mechanism. In the DGS mechanism, workers tentatively assigned to different jobs bid down to their indifference wage plus the rent obtained from their tentative inside option. Hence there are (weakly) more potential players at each round of the DGS mechanism than in the sequential version. These additional players cannot make a difference to the wage outcome if they are not better qualified than the players also present in the sequential mechanism. But if they are more efficient than the competitors in the sequential mechanism, their presence must lower the negotiated wages. The example in the appendix illustrates this effect for the case of three workers competing for two jobs.

Conclusion

We show that the stable and efficient assignment obtained from a centralised labour market cannot be in general implemented through a sequence of independent auctions. This result is due to a centralised recruiter's informational advantage over the individual firms. We arrive at our results using the strong assumption that preferences satisfy the Spence-Mirrlees single-crossing condition. This assumption, however, is widely employed in labour-market, signalling and screening contexts alike.

Appendix

Proof of prop 1. This adjusts theorem 1 of Demange, Gale, and Sotomayor (1986) to fit our descending version of their mechanism. We argue by contradiction and suppose that there is a competitive wage-vector $\mathbf{y}$ such that $\mathbf{y} > \mathbf{w}$ (in at least one component wage). Hence all workers prefer $\mathbf{y}$ to $\mathbf{w}$. We set the initial (step 1) wages to equal the known reserve wages of the firms $\bar{\mathbf{w}}$, so $w^j(1) = \bar{w}^j, \ \forall j \in \mathcal{M}$ and thus $\mathbf{w}(1) \geq \mathbf{y}$. Call t the last step at which this inequality still holds. For $j \in D_i(t)$, we know that $u_i(p(t)) = w^j - c(\theta_i, T^j) \leq y^j - c(\theta_i, T^j) = u_i(y)$ (and the same is true at all previous steps).

Let S be the minimal overdemanded set of jobs whose wages are decreased at $t+1$ defined as $S = \{j | w^j(t+1) < w^j(t)\}$. Similarly, define the set of jobs containing all minimal overdemanded sets including and after $t+1$ as $S_1 = \{j | w^j(t+1) < y^j\}$. Clearly $S_1 \subset S$ and $y^j = w^j \ \forall j \in S_1$ from our integer assumptions.

We want to show that $S - S_1$ nonempty and overdemanded, contradicting that S is a minimal overdemanded set of jobs contrary to the rules of the DGSM. To do this, define the set of workers demanding jobs in S as $R = \{i | D_i(p(t)) \subset S\}$. Since S is overdemanded, we know that $\#R > \#S$. Similarly, define the nonempty $R_1 = \{i \in R | D_i(p(t)) \subset S_1\}$. We claim that $D_i(y) \subset S_1 \; \forall i \in R_1$ and choose $j \in \{S_1 \cap D_i(p(t))\}$. Either

1. $k \notin S$, then i prefers j to k at wage $\mathbf{y}$, or

2. $k \in S - S_1$, then i likes j at least as well as k at $p(t)$ but $p^k(t) > p^k(t+1) \geq y^k$ (and $p^j(t) = y^j$).

So i prefers j to k at wage $\mathbf{y}$. Since $\mathbf{y}$ is competitive, there are no overdemanded sets at $\mathbf{y}$. Hence $\#R_1 \leq \#S_1$. Thus $\#(R - R_1) > \#(S - S_1)$ and $R - R_1 \neq \emptyset$ and $R - R_1 = \{i \in R | D_i(p(t)) \subset S - S_1\}$. Hence $S - S_1 \neq \emptyset$ and $S - S_1$ is overdemanded giving the required contradiction. □

Proof of prop 2. This adjusts theorem 2 of Demange, Gale, and Sotomayor (1986) to fit our descending version of their mechanism. Let μ be an assignment compatible with $\mathbf{w}$. Call a job j *underpaid* if it is unassigned by μ but $w^j < \bar{w}^j$. If $(\mathbf{w}, \mu)$ is not an equilibrium, there must be at least one underpaid job. We define a procedure to eliminate all underpaid jobs. We construct a directed graph whose vertices are $\mathcal{N} \cup \mathcal{M}$. There are two types of arcs: (1) if $\mu(i) = j$, there is an arc from i to j, and (2) if $j \in D_i(\mathbf{w})$, there is an arc from j to i.
Let k be an underpaid job. Then $k \in D_i(\mathbf{w})$ for some i because otherwise we could increase w^k and still have a competitive wage contradicting maximality of $\mathbf{w}$. Let $\bar{\mathcal{N}} \cup \bar{\mathcal{M}}$ be all vertices which can be reached by a directed graph starting from k.

1. $\bar{\mathcal{N}}$ contains an unmatched bidder i. Let $(k, i_1, j_2, i_2, j_3, i_3, \ldots, j_l, i)$ be a path from k to i. Then we may change μ by matching i_1 to k, i_2 to j_2,…, i to j_l. The match is still competitive and k is no longer underpaid and the number of underpaid jobs has been reduced.

2. All i in $\bar{\mathcal{N}}$ are matched. Then we claim that there must be some $j \in \bar{\mathcal{M}}$ such that $w^j = \bar{w}^j$. Suppose that this is not the case. By definition of $\bar{\mathcal{N}} \cup \bar{\mathcal{M}}$ we know that if $i \notin \bar{\mathcal{N}}$, then i does not demand any job in $\bar{\mathcal{M}}$. Hence we can increase the wage paid to any job in $\bar{\mathcal{M}}$ by some $\delta > 0$ and still have competitive $\mathbf{w}$ contradicting maximality of $\mathbf{w}$.
So choose a $j \in \bar{\mathcal{M}}$ such that $w^j = \bar{w}^j$ and let $(k, i_1, j_2, \ldots, j_l, i_l, j)$. Again change μ by matching i_1 to k, i_2 to j_2, …, i_l to j_l leaving k unmatched and again the number of underpaid jobs has been reduced. □

Proof of prop 3. Since $\mathbf{w}$ is competitive, if $\mu(i) = j$, then $j \in D_i(\mathbf{w})$. Hence in equilibrium every worker gets the job which gives her the highest rent at $\mathbf{w}$ and nobody envies somebody else's job not in her demand set. Moreover, since our mechanism only terminates when there are no overdemanded jobs remaining, there is no worker $h \neq i$ who prefers to do job $j = \mu(i)$ for $\hat{w}^j < w^j$ over any job in $D_h(\mathbf{w})$. □

Proof of lemma 1. Suppose the opposite is true and consider any two matched jobs $1, 2 \in \mathcal{M}$ with $T^1 < T^2$ and $\tilde{w}^1 \geq w^2$. Then 1 must be overdemanded from single-crossing of preferences contradicting the rules of the DGS mechanism. $\square$

Proof of lemma 2. Suppose the opposite is true and consider any two jobs $1, 2 \in \mathcal{M}$ with $T^1 < T^2$ matched to two workers $L, H \in \mathcal{N}$ with $\theta_L < \theta_H$ such that H is assigned to job 1 and L is assigned to job 2. From the previous lemma we know that $w^1 < w^2$. But then, from single-crossing, player L must prefer job 1 to his equilibrium assignment and is willing to do it for less than w^1. Hence job 1 is overdemanded contradicting the rules of the DGS mechanism. $\square$

Proof of prop 4. Since the DGS mechanism's equilibrium assignment is stable, we have each matched worker $i = D^j$'s indifference curve passing through his assigned job $j = \mu(i)$ at the point (T^j, w^j). On the one hand, if a matched job j was never overdemanded, then its matched wage is $\bar{w}^j$ because $i = D^j$ is the only worker who ever demanded j. If, on the other hand, a job was overdemanded at some stage prior to the equilibrium assignment, then there is an intersection of the indifference curves of the equilibrium match for j and the worker $h \neq i$ who switched his demand from j at $w^j + \varepsilon$. If worker h is unmatched by μ, then j is autonomous and $w^j + \varepsilon = \tilde{w}^j_h$. If there is a match $k = \mu(h)$, then j is non-autonomous. Thus each non-autonomous job j is connected through the unique indifference curve of worker D^k passing through $(T^j, w^j + \varepsilon)$ to another assigned job k. The demand withdrawal of worker D^k from j to k decides the wage w^j.

From single-crossing we know that each pair of workers' indifference curves can intersect only once. Since a non-autonomous job takes a crossing between 2 matched workers' indifference curves, we need at least one unmatched worker deciding the equilibrium wage on some job if $N \geq M$. Hence there must be at least one autonomous job in any DGS assignment. $\square$

Proof of prop 5. In a single spsb auction, it is well known to be a weakly dominant strategy to bid one's true valuation; Vickrey (1961). Thus in order to duplicate the assignment of the DGS mechanism, it is necessary to auction an autonomous job last in any sequence of spsb auctions. $\square$

Example: Consider the following example where a single worker L is best suited for more than one job: There are two jobs $1, 2$ with $T^1 < T^2$ and three workers L, M, H with $\theta_L < \theta_M < \theta_H$ with preferences and reserve wages as drawn in fig. 1.

(1) The DGS mechanism results in the stable allocation of L to 1, M to 2 and the wage profile $w^* = (\tilde{w}^1_M + r_M - 2\varepsilon, \tilde{w}^2_H - \varepsilon)$. To see this, let the initial wage vector be $w_0 = (\bar{w}^1, \bar{w}^2)$ resulting in the demand $D_L(w_0) = D_M(w_0) = D_H(w_0) = \{1\}$. Hence 1 is overdemanded and the going wage w^1 is decreased to some $w_1 = (w^1_1, \bar{w}^2)$ at which H no longer demands 1 but prefers 2. Since, at that wage, 1 is still overdemanded by L and M, w^1 is further decreased to some $w_2 = (w^1_2, \bar{w}^2)$ at which $D_L(w_2) = \{1\}, D_M(w_2) = \{1, 2\}$ and

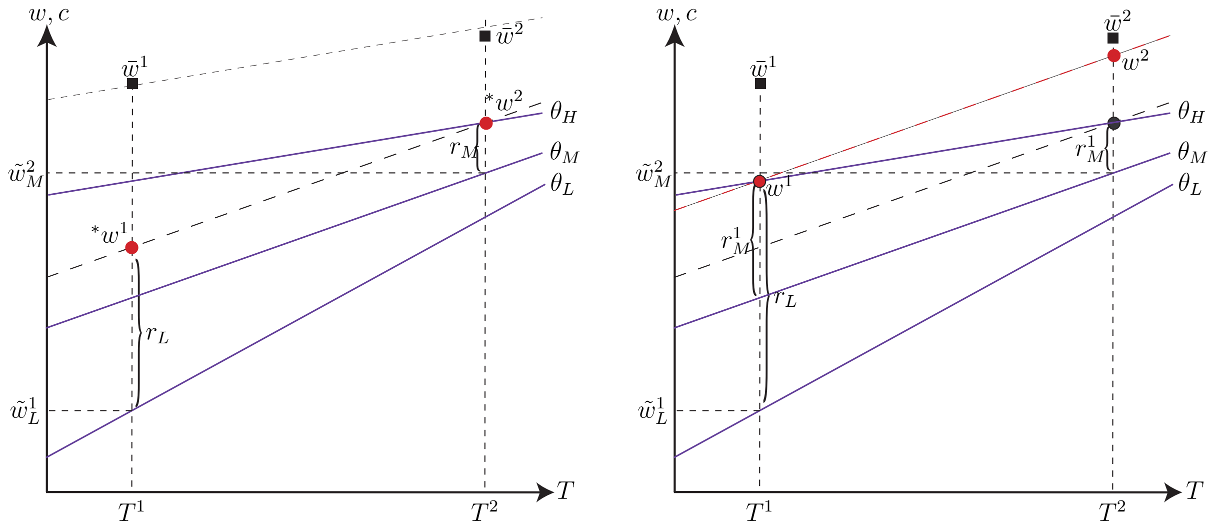

Figure 1: The stable DGS outcome (left). Job 2 is autonomous and thus auctioning job 2 first using independent sequential auctions is either unstable because $r^1_M > r^2_M$ or more expensive (right).

$D_H(w_2) = \{2\}$. Hence both 1 and 2 are overdemanded at w_2 and both w^1, w^2 are reduced simultaneously until $w^2 < \tilde{w}^2_H$ at which point H switches demand to her outside option $\mathcal{O}$ and $D_L(w_3) = \{1\}, D_M(w_3) = \{1, 2\}$ and $D_H(w_3) = \{\mathcal{O}\}$. Hence, at w^3, 1 is overdemanded and a final ε-reduction in w^1 results in each worker demanding a unique job (or outside option) at w^*. In this example, job 2 is autonomous since its wage is determined by H.

(2) Let us verify that the same outcome is reached when 1 is auctioned before the autonomous 2 using two independent second-price auctions: Start at some initial wage vector $w_0 = (\bar{w}^1, \mathbb{E}[w^2|N^2 = 1, \theta] = \bar{w}^2)$ resulting in the demand $D_L(w_0) = D_M(w_0) = D_H(w_0) = \{1\}$. As before, 1 is overdemanded and the going wage w^1—which is the only wage which can be adjusted sequentially—is decreased to some $w_1 = (w^1_1, \mathbb{E}[w^2|N^2 = 1, \theta] = \bar{w}^2)$ at which H no longer demands 1 but quits the auction for job 1. If H does not misrepresent her preferences, this reveals her true type to her opponents who conclude that if they quit next, the should expect wages of $\mathbb{E}[w^2|N^2 = 2, L] \leq \tilde{w}^2_H$ and $\mathbb{E}[w^2|N^2 = 2, M] \leq \tilde{w}^2_H$ for the next-auctioned job 2. At the same time, 1 is still overdemanded by L and M. Further reducing w^1 to some $w_2 = (w^1_2, \mathbb{E}[w^2|N^2 = 2])$ must result at some point in $D_L(w_2) = \{1\}, D_M(w_2) = D_H(w_2) = \{\mathcal{O}\}$ which clears the market for 1 at the same wage $^*w^1 = \tilde{w}^1_M + r_M - 2\varepsilon$ as above. The single subsequent second-price auction at 2 terminates at $^*w^2 = \tilde{w}^2_M - \varepsilon$. Since this is the VCG outcome, no worker has incentives to misrepresent her true preferences.

(3) The same is not true if the autonomous job 2 is independently auctioned before 1: Since 1 is auctioned last and we want to implement a stable allocation, we know that M must be assigned to 2 leaving only L and H in the contest for 1. Thus a second-price auction must result in $w^1 = \tilde{w}^1_H - \varepsilon >^* w^1$. At this wage, however, M prefers 1 to 2, thus 1 is overdemanded, and in order to reach a stable outcome, the sequential mechanism must

offer a wage $w^2 >^* w^2$ to M. Hence in this example, decentrally negotiated stable wages exceed the wages negotiated through the recruiter. $\triangleleft$

References

CRAWFORD, V. P., AND E. M. KNOER (1981): "Job matching with heterogeneous firms and workers," *Econometrica*, 49, 437–50.

DEMANGE, G. (1982): "Strategyproofness in the assignment market game," *Laboratoire d'Économétrie de l'École Polytechnique, Paris*, Mimeo.

DEMANGE, G., D. GALE, AND M. SOTOMAYOR (1986): "Multi-item auctions," *Journal of Political Economy*, 94(4), 863–72.

ELMAGHRABY, W. (2003): "The Importance of Ordering in Sequential Auctions," *Management Science*, 49(5), 673–82.

HOLMSTRÖM, B. (1979): "Groves Schemes on Restricted Domains," *Econometrica*, 47, 1137–44.

KELSO, A. S., AND V. P. CRAWFORD (1982): "Job matching, coalition formation, and gross substitutes," *Econometrica*, 50, 1483–504.

KITTSTEINER, T., J. NIKUTTA, AND E. WINTER (2004): "Declining valuations in sequential auctions," *International Journal of Game Theory*, 33, 89–106.

KORN/FERRY (2000): "Annual Report 2000," *Korn/Ferry International*, `http://www.kornferry.com`.

LEONARD, H. B. (1983): "Elicitation of honest preferences for the assignment of individuals to positions," *Journal of Political Economy*, 91, 461–79.

MAS-COLELL, A., M. WHINSTON, AND J. GREEN (1995): *Microeconomic Theory*. Oxford University Press, Oxford.

ROTH, A. E., AND M. A. O. SOTOMAYOR (1990): *Two-sided matching: A study in game-theoretic modeling and analysis*, vol. 18 of *Econometric Society Monographs*. Cambridge University Press, Cambridge.

SHAPLEY, L., AND M. SHUBIK (1972): "The Assignment Game I: The Core," *International Journal of Game Theory*, 1(2), 111–30.

VICKREY, W. (1961): "Counterspeculation, Auctions and competitive sealed tenders," *Journal of Finance*, 16, 8–37.