



GOVERNANCE AND THE EFFICIENCY
OF ECONOMIC SYSTEMS
GESY

Discussion Paper No. 123

**Fragmented property rights and
R&D competition**

Derek J. Clark*
Kai A. Konrad**

June 2006

*Derek J. Clark, Department of Economics and Management, University of Tromsø, N-9037 Tromsø, Norway.
Derek.Clark@nfh.uit.no

**Kai A. Konrad, WZB, Reichpietschufer 50, D-10785 Berlin, Germany, and Free University of Berlin.
kkonrad@wz-berlin.de

Financial support from the Deutsche Forschungsgemeinschaft through SFB/TR 15 is gratefully acknowledged.

Sonderforschungsbereich/Transregio 15 · www.gesy.uni-mannheim.de
Universität Mannheim · Freie Universität Berlin · Humboldt-Universität zu Berlin · Ludwig-Maximilians-Universität München
Rheinische Friedrich-Wilhelms-Universität Bonn · Zentrum für Europäische Wirtschaftsforschung Mannheim

Speaker: Prof. Konrad Stahl, Ph.D. · Department of Economics · University of Mannheim · D-68131 Mannheim,
Phone: +49(0621)1812786 · Fax: +49(0621)1812785

Fragmented property rights and R&D competition*

Derek J. Clark[†] and Kai A. Konrad[‡]

June 14, 2006

Abstract

Where product innovation requires several complementary patents, fragmented property rights can be a factor that limits firms' willingness to invest in the development and commercialization of new products. This paper studies multiple simultaneous R&D contests for complementary patents and how they interact with patent portfolios that firms may have acquired already. We also consider how this interaction and the intensity of the contests depends on the type of patent trade regimes and the product market equilibria that result from these regimes. We solve for the contest equilibria and show that the multiple patent product involves an important hold-up problem that considerably reduces the overall contest effort.

Keywords: fragmented property rights, patents, contests, hold-up, R&D, patent pools, licensing

JEL classification numbers: D44

*We should like to thank participants at the Norwegian Economics Conference in Bergen, 2006 for helpful comments. Errors are our own. This paper is part of the project "The knowledge-based society" sponsored by the Research Council of Norway (Project 172603/V10). Konrad acknowledges support by the German Science Foundation (DFG grant SFB-TR-15).

[†]Department of Economics and Management, University of Tromsø, N-9037 Tromsø, Norway. E-mail: Derek.Clark@nfh.uit.no.

[‡]WZB, Reichpietschufer 50, D-10785 Berlin, Germany, and Free University of Berlin. E-mail: kkonrad@wz-berlin.de.

1 Introduction

Many modern goods are produced using multiple, complementary technology components, which are often protected under a number of patents. This complementarity in production and in intellectual property generates several challenges. Heller and Eisenberg (1998), for instance, discuss that such fragmented property rights defined around gene fragments in biotechnology may reduce firms' incentives to invest and commercialize products:

Foreseeable commercial products, such as therapeutic proteins or genetic diagnostic tests, are more likely to require the use of multiple fragments. A proliferation of patents on individual fragments held by different owners seems inevitably to require costly future transactions to bundle licenses together before a firm can have an effective right to develop these products. (Heller and Eisenberg 1998, p.699.)

Other problems with fragmented property rights have been discussed. Firms that invest in a new product face the risk that, after sinking considerable resources in this, another firm may essentially blackmail this firm by claiming that this new product infringes on some patent held by this firm. Ziedonis (2004, p.806) reports the case of Intel in 1998: "After developing the architecture and tailoring its fabrication facilities to produce the new chip, Intel was sued by a small communications company, S3, for allegedly infringing patents S3 had purchased from a failed start-up company."

These examples illustrate that fragmentation of intellectual property rights is a widespread phenomenon and involves important strategic issues both for firms and policy makers. Ziedonis (2004) discusses whether firms may choose to acquire large sets of patents as 'bargaining chips' to deal with this problem. Lerner and Tirole (2004) discuss the welfare properties of patent pools. They briefly discuss the implications of patent pools for innovation, but mostly concentrate on a situation with a given set of patents. Shapiro (2001) also acknowledges the problems that emerge if complementary patents are allocated between several firms. He discusses different arrangements of trading to circumvent hold-up problems and problems that are similar in type as the problem of double marginalization in the context of a downstream monopoly that purchases an input from an upstream monopoly and is restricted to linear pricing and discusses several institutional arrangements by which firms

essentially trade their patent rights. He also acknowledges the existence of dynamic efficiency problems of the different trading arrangements in terms of their implications for the incentives on R&D. However, similar to Lerner and Tirole (2004), he focusses on static allocation efficiency aspects, i.e., on the problem of inefficient use of an existing set of patents.

The dynamic aspect of the role of market structure for R&D has received much attention in the literature as well, starting with Arrow (1962). In this context much of the focus is on single patents that may be attained in an R&D contest that may have one or multiple stages. The interaction of multiple patents and their complementarity has received less attention. Some work in this context focuses on a sequential structure of innovation, the incentives to expend R&D effort in a first or in a second stage of a cumulative innovation, and its policy implications for the allocation rules for the trading of patents (e.g., Scotchmer 1996, Green and Scotchmer 1995, Denicolo 2002).

We focus on the innovation incentives in a situation similar to the one discussed by Heller and Eisenberg (1998): firms can make simultaneous investments in patents for several technology components, all of which are needed to innovate and produce a new consumer product. We consider symmetric firms who start this process without having acquired a stock of patents, and the competition between firms in which some firm acquired some patents already, but who competes with another firm about a set of further patents.

Dynamic commitment issues which emerge in sequential R&D, or uncertainty about a possible infringement when commercializing a new product, are absent from this consideration. We assume that the R&D processes for the different technology components are random and independent of each other. For this reason it will often be the case that one firm wins some patents and a competitor wins the complementary set of patents. In this case, none of the firms can produce without further arrangements or a reallocation of the rights to use the information that is protected by the patents. The competition between two firms, A and B , that takes place for n complementary patents has essentially three outcomes: A owns all patents and receives all rent, B owns all patents and receives all rent, or both firms own some patents and share the rent. In the first two cases the firm that owns all patents will typically produce and earn a monopoly profit. However, the outcome in the third case depends on the rules and arrangements according to which patent rights can be traded between firms.

We analyse a benchmark case in which firms can freely trade exclusive rights to use single patents and three other, more restricted regimes that

can be related to the different types of transactions among patent holders that are discussed in Shapiro (2001). We concentrate on the incentives of firms to innovate if they expend resources simultaneously in a number of parallel patent contests. We then compare this outcome with different patent trading regimes and note that different trading regimes induce rather similar incentives in patent contests. A generalization of this analysis looks at a situation that also extends some of the analysis on cumulative, sequential innovation. Here we consider the case in which patenting occurs in two consecutive periods. We allow for k patents to be awarded in the first period and n patents in the second period. It turns out that some, but not all of the specific properties of the simultaneous multi-patent contest carry over to this partially sequential problem.

2 The analytical problem

Consider two firms A and B that compete with each other in the following three stage game.

In stage 1 the two firms spend efforts on R&D. They already hold $k = k_A + k_B$ patents, of which firm A holds $k_A \geq 0$ and firm B holds $k_B \geq 0$. They need to innovate and patent n further essential components of a new good. In this stage each firm chooses one level of R&D expenditure for each component. These effort choices are made simultaneously and can be seen as n independent, parallel patent contests. To save on notation, we define $\mathbf{x} \equiv (x_1, \dots, x_n)$ and $\mathbf{y} \equiv (y_1, \dots, y_n)$.

A firm's effort in an R&D contest typically has two effects that relate to two different sources of uncertainty, as described by Loury (1979). First, R&D is genuinely a risky activity, as it is uncertain whether and when own research effort will yield the desired information. This type of uncertainty may be called 'technological uncertainty'. Second, as other firms search for the same information, there is some uncertainty about who innovates first, and, hence, receives the patent. In line with Loury (1979) we call this the 'market uncertainty'. Both types of uncertainty are important.

We concentrate on market uncertainty. i.e., the information how to produce component i is eventually revealed, for any levels of effort. However, the probability that firm A gets hold of the relevant information about component i prior to firm B and wins the patent, is a function of the efforts in

the respective single patent contest as follows:

$$p_i = \frac{x_i}{x_i + y_i} \text{ if } \max\{x_i, y_i\} > 0, \text{ and } p_i = 1/2 \text{ otherwise.} \quad (1)$$

This description of a firm's market uncertainty in the R&D contest between two firms can be justified using an important equivalence result that has been developed by Baye and Hoppe (2003). They show that many types of innovation contest and patent race in which the process of innovation follows a stochastic process can be represented equivalently as a simple lottery contest in which the contestants' win probabilities equal their shares in aggregate expenditure, and in which the value of winning the lottery prize is a function of the aggregate contest efforts that depends more specifically on the properties of the stochastic process. For some stochastic processes the lottery prize is a constant with respect to aggregate efforts. We concentrate on this case which corresponds to considering only market uncertainty.¹

Once the efforts are chosen, the random process that is governed by win probabilities (1) allocates the patents on the components $i = 1, \dots, n$ to the two firms. We assume that the random processes that determine the win probabilities in the different components are stochastically independent of each other.

At the beginning of stage 2 all components are innovated and patented. We define the vector $\mathbf{z} = (z_1, \dots, z_k, z_{k+1}, \dots, z_{k+n})$ with $z_i = a$ if firm A holds patent i , and $z_i = b$ if firm B holds patent i . Firms negotiate with each other about who is allowed to use which set of patents. We generally assume that a firm needs to be in control of all patents, $k + n$ to be able to produce and market the good in question. To be in control means here that the firm can use the respective technology covered by this patent without any risk of being sued by the actual holder of the patent, either because the firm itself is the holder of the patent, or because the outcome of trading and negotiating between firms leads to this security. For the different types of negotiation regimes, we draw on Shapiro (2001), and we explain these further below.

At the beginning of stage 3, these negotiations are completed. Patent rights are finally allocated between firms. Any firm that is in control of all $k + n$ patents can produce and market the good, and any firm that is not in control of all patents cannot enter the market. We do not model and

¹For applicability of this function for the limit of a low discount rate see also Nti (1997). The function also has been axiomatized for contests more generally by Skaperdas (1996).

solve an explicit market game. Instead, we make the following assumptions that are compatible with a large number of explicit market games, including Cournot and Bertrand competition. If none of the firms is in control of all rights, none of the firms can produce and both firms earn zero profits in the product market. If only one firm is in control of all patents, this firm is a monopolist and will earn the monopoly rent which is denoted as F_M . Consumer rent under monopoly will be denoted C_M . By symmetry, these values do not depend on which firm is the monopolist. If each firm is in control of all patents, the firms will compete and each firm will earn the equilibrium profit in duopoly, denoted by F_D . Aggregate consumer rent in the duopoly is denoted as C_D . We will assume that the firm's monopoly rent exceeds the sum of the firms' profits in a duopoly, and that consumer rents in the monopoly are smaller than in the duopoly:

$$\begin{aligned} F_M &> 2F_D, C_M < C_D, \text{ and} \\ F_M + C_M &< 2F_D + C_D. \end{aligned} \tag{2}$$

Consider stage 2. Suppose the allocation of patent ownership is described by some \mathbf{z} at the beginning of stage 2. Shapiro (2001) discusses several transaction modes by which firms may settle the conflict resulting from a set of complementary patents that are allocated between several firms and yield control of all patents to exactly one firm. For instance, firm A may acquire firm B , including firm B 's intellectual property rights. Similarly, the two firms may write a contract that allows A to use the technology components that are protected by B 's patents, with B promising to not sue A for patent right infringement, which will put A in the position of a monopoly. Third, the two firms may form a patent pool and auction the exclusive right to produce the good that needs all these technology components to a third firm, C which then earns the monopoly rent.²

In the absence of informational asymmetries or contractual constraints between firms, the two firms will end up with an allocation of their patents that maximizes the sum of the joint profits they can obtain in the market competition in stage 3, and this joint profit is the monopoly profit. If $\mathbf{z} = (a, a, \dots a)$ or if $\mathbf{z} = (b, b, \dots b)$, then it follows from subgame perfection that any bargaining outcome will yield the monopoly profit to the firm that

²Implicitly we rule out that consumers take part in bargaining. While this is plausible, given their much higher transaction cost, it is not crucial for the qualitative findings we have on the R&D contest.

owns all patents at the beginning of stage 2. If each firm holds at least one patent, what needs to be determined is the function by which the monopoly rent is shared between the two firms if both firms own at least one patent. Many bargaining concepts that could be used to determine this distribution will yield the same outcome, given the assumption about risk neutrality of firms and symmetry. For simplicity, we assume symmetric Nash bargaining among risk-neutral firms. Let firms have zero profits as their outside options. Then the surplus from efficient negotiations is the monopoly profit F_M and they share this evenly in the Nash bargaining solution. Hence, if $\mathbf{z} \notin \{(a, a, \dots, a), (b, b, \dots, b)\}$, then the firms' payoffs are

$$\pi_A = \frac{F_M}{2} - \sum_i x_i \text{ and } \pi_B = \frac{F_M}{2} - \sum_i y_i. \quad (3)$$

We now turn to the multi-patent R&D contest in stage 1. We can state the properties of the subgame perfect equilibrium for three different cases, depending on whether some firm has acquired some complementary patents in a previous stage.

Proposition 1 *If $k = 0$, and $n \in \{1, 2, \dots, 6\}$, a subgame perfect equilibrium of the multi-patent contest with unconstrained bargaining between two firms exists and is described by monopoly in the product market, efficient bargaining that allocates patent rights such that only one firm can produce, and an interior symmetric equilibrium in the simultaneous contests for n patents with efforts*

$$x_i^* = y_i^* = \frac{F_M}{2^{n+1}} \text{ for all } i = 1, \dots, n. \quad (4)$$

and equilibrium payoffs

$$\pi_A^* = \pi_B^* = \frac{F_M}{2} - n \frac{F_M}{2^{n+1}} > 0. \quad (5)$$

Proof. We only consider stage 1 and use the properties of the equilibrium in the continuation game at stage 2 that have been discussed already. When $n = 1$, this is a standard Tullock (1980) contest, and each contestant will dissipate $\frac{1}{4}$ of the monopoly rent F_M , and this equilibrium is unique (Szidarovszky and Okuguchi 1997).

Consider A 's objective function for $n > 1$. With an efficient bargaining arrangement after the patent contest, the expected profit of firm A is a

function of the two firms' contest efforts. Using (3), the payoff is given by:

$$\pi_A = \frac{F_M}{2} + \frac{\prod_{i=1}^{i=n} x_i}{\prod_{i=1}^{i=n} (x_i + y_i)} \frac{F_M}{2} - \frac{\prod_{i=1}^{i=n} y_i}{\prod_{i=1}^{i=n} (x_i + y_i)} \frac{F_M}{2} - \sum_i x_i. \quad (6)$$

Firm A receives the monopoly rent if it wins all patents, which explains the additional payoff described by the second term in (6), and firm A does not receive any rent if firm B wins all patents, which explains the third, negative term on the right hand side. Finally, the firm has to pay the sum of its efforts in the n parallel single patent contests, and this constitutes the fourth term on the right-hand side. Equation (6) uses that the payoff is the same for all outcomes in which both firms win at least one patent, no matter how asymmetric is the distribution of patents in this case.³

For $\mathbf{y} = (y, y, \dots, y)$, we use Lemma 1 in the Appendix that shows a property of optimal replies $\mathbf{x}(\mathbf{y})$ and $\mathbf{y}(\mathbf{x})$: if B chooses $\mathbf{y} = (y_1, y_2, \dots, y_n)$ with $y_1 = y_2 = \dots = y_n \equiv y$, then the optimal reply by firm A is some $\mathbf{x}(\mathbf{y})$ with $x_1(y) = x_2(y) = \dots = x_n(y) \equiv x(y)$, and, analogously for B 's optimal reply $\mathbf{y}(\mathbf{x})$. In words, if one firm spends the same effort along all patent contests, then the other firm's optimal reply is characterized by uniform effort along all patent contests. Using this result in (6) the first-order condition for a uniform reply to the uniform vector \mathbf{y} is the vector $\mathbf{x}(\mathbf{y}) = (x, x, \dots, x)$ that maximizes

$$\pi_A(x) = \frac{F_M}{2} + \frac{x^n - y^n}{(x + y)^n} \frac{F_M}{2} - nx. \quad (7)$$

Note that

$$\frac{d\pi_A(x)}{dx} = n \frac{x^{n-1}y + y^n}{(x + y)^{n+1}} \frac{F_M}{2} - n. \quad (8)$$

This term is zero for $y = F_M/2^{n+1}$ at $x = F_M/2^{n+1}$. Hence, for the candidate equilibrium efforts the first-order conditions for an optimal reply by firms A and B are fulfilled. Note also that this is the only symmetric solution of $\frac{d\pi_A(x)}{dx} = 0$. For x^* and y^* as in (4) the two firms' payoffs are strictly positive. This also shows that \mathbf{x}^* is superior to a choice of $\mathbf{x} = \mathbf{0}$. Moreover, the strictly positive payoff for \mathbf{x}^* implies that $x < F_M/n$ must hold in the global

³An interesting generalization of our approach addresses a situation in which the patents are not essential, in which case production cost may simply be a function of the number of patents which a firm is allowed to use. However, the linearly-limitational case we consider yields stark results and is of particular interest.

maximum, as $\pi_A(x) \leq 0$ for $x \geq F_M/n$. In the Appendix, we show that \mathbf{x}^* also characterizes a global maximum on the interval $x \in (0, F_M/n)$. ■

The case in Proposition 1 describes a symmetric contest, in which two prizes are at stake. By winning up to $n - 1$ patents, a firm gains veto-power. If both firms are veto-players, they share the prize. If only one firm has all patents, and, hence is the only veto-player, the firm receives the whole prize F_M . The results for this case are intuitive: as any patent is like any other, each one being similarly contested for, and both firms expend the same effort on the contest for each patent. The two firms' total effort in all patent contests sums up to

$$nx^* + ny^* = \frac{nF_M}{2^n}. \quad (9)$$

This sum has its maximum for $n = 1$ and is strictly decreasing in n . Intuitively, if many complementary patents are needed to produce a particular good, even if one firm, say A , expends much effort on each of the single patent contests, it becomes very likely that both firms fail to obtain all patents. In each of these cases firm A receives $F_M/2$, independently of whether it holds 1, 2, or even $n - 1$ patents. This makes it less worthwhile to expend much effort in the simultaneous contests.

Note that, for $n \geq 7$, the problem is not 'well-behaved', and (4) does not characterize a symmetric equilibrium in pure strategies. This is also illustrated numerically in Figure 2 in the Appendix. It shows that (4) does not represent mutually optimal replies, as, for instance, A can do better than choosing (4) if B chooses (4). We now turn to the situation in which one of the firms holds some patents already at the stage where the two firms enter into the simultaneous contests for n remaining patents. Without loss of generality, we assume that this firm is firm A , and the next proposition characterizes an equilibrium for all possible n .

Proposition 2 *Suppose $k_A > 0$ and $k_B = 0$.*

If $n \in \{1, 2\}$ then there is a pure strategy symmetric equilibrium in which

$$x_i^* = y_i^* = \frac{F_M}{2^{n+2}}. \quad (10)$$

If $n > 2$, then a mixed strategy equilibrium exists and is characterized by firm B choosing

$$y_i^* = \left(\frac{n-1}{n}\right)^n \frac{F_M}{2n} \frac{1}{n-1}$$

and firm A choosing $\mathbf{x}^* = [\frac{y_i^*}{p^*}, \dots, \frac{y_i^*}{p^*}]$ with probability $p^* = 1/(n-1)$ and $\mathbf{x}^* = \mathbf{0}$ with probability $(1-p^*)$. Firm A 's expected payoff is $\pi_A^* = F_M/2$ and firm B 's expected payoff is $\pi_B = \frac{F_M}{2}(1 - \frac{2(n-1)^{n-1}}{n^n})$ in this equilibrium.

Proof. Suppose that firm A holds a positive number of patents. We call A the leader and B the follower in this case. Consider the contest over the remaining n patents. When the ex post allocation of patents is resolved using efficient bargaining (or licensing with exclusive exploitation of intellectual property), the expected payoffs to the firms are:

$$\pi_A = \frac{F_M}{2} + \prod_{i=1}^n \frac{x_i}{x_i + y_i} \frac{F_M}{2} - \sum_i x_i \quad \text{and} \quad (11)$$

$$\pi_B = (1 - \prod_{i=1}^n \frac{x_i}{x_i + y_i}) \frac{F_M}{2} - \sum_i y_i. \quad (12)$$

Since B cannot become the sole owner of all $k+n$ patents, the only way to share in the surplus is if firm B wins at least one of the n patents. A straightforward solution of the system of equations that consist of the first-order conditions for A maximizing (11) and B maximizing (12) yields

$$x_i^* = y_i^* = \frac{F_M}{2^{n+2}} \quad (13)$$

and total expenditure per firm

$$nx_i^* = \frac{nF_M}{2^{n+2}}. \quad (14)$$

This interior solution yields $\pi_A^* > F_M/2$ and $\pi_B^* > 0$ if $n = 1$ and $\pi_A^* = F_M/2$ and $\pi_B^* > 0$ if $n = 2$. Hence, x_i^* as in (13) also dominates $x_i = 0$ for $n = 1$ and weakly dominates $x_i = 0$ for $n = 2$.

If $n > 2$, then $x_i = y_i = \frac{F_M}{2^{n+2}}$ yields $\pi_A \leq F_M/2$, and, as $\mathbf{x} = (0, 0, \dots, 0)$ always yields at least $\pi_A \geq F_M/2$, (13) is no longer an equilibrium. Consider the candidate equilibrium in Proposition 2. For this to be an equilibrium, $\mathbf{y}^* = (y^*, \dots, y^*)$ must maximize

$$\pi_B = \left[(1 - \prod_{i=1}^n \frac{x^*}{x^* + y_i}) \frac{F_M}{2} - \sum_{i=1}^n y_i \right] p^* + (1 - p^*) \left[\frac{F_M}{2} - \sum_{i=1}^n y_i \right], \quad (15)$$

Further, $\mathbf{x}^* = (x^*, \dots, x^*)$ must maximize

$$\pi_A = \frac{F_M}{2} + \prod_{i=1}^n \frac{x_i}{x_i + y^*} \frac{F_M}{2} - \sum_i x_i \quad (16)$$

for all \mathbf{x} with $x_i > 0$. Further, for A to be indifferent between \mathbf{x}^* and $\mathbf{x} = \mathbf{0}$,

$$\pi_A(\mathbf{x} = \mathbf{0}) = \pi_A(\mathbf{x} = \mathbf{x}^*)$$

must hold. For a solution the following three conditions must hold simultaneously: the first-order condition for x_j :

$$\frac{y^*}{(x^* + y^*)^2} \left(\frac{x^*}{x^* + y^*} \right)^{n-1} \frac{F_M}{2} = 1, \quad (17)$$

the first-order condition for y_j :

$$p^* \frac{x^*}{(x^* + y^*)^2} \left(\frac{x^*}{x^* + y^*} \right)^{n-1} \frac{F_M}{2} = 1 \quad (18)$$

and the condition that makes firm A indifferent between $\mathbf{x} = \mathbf{0}$ and $\mathbf{x} = \mathbf{x}^*$:

$$\left(\frac{x^*}{x^* + y^*} \right)^n \frac{F_M}{2} - nx^* = 0. \quad (19)$$

Note that we have again made use of Lemma 1 in the Appendix: for a given candidate equilibrium effort vector \mathbf{y}^* with uniform components, the optimal reply \mathbf{x}^* must also have $x_1^* = x_2^* = \dots = x_n^* \equiv x^*$.

The indifference condition (19) can be transformed into $\left(\frac{x^*}{x^* + y^*} \right)^{n-1} \frac{F_M}{2} = n(x^* + y^*)$, and this can be substituted in (17) and (18) to obtain $\frac{y^*}{x^* + y^*} n = 1$ and $\frac{x^*}{x^* + y^*} np^* = 1$. This yields

$$y^* = x^* p^*. \quad (20)$$

We now use (20) in the indifference condition (19) and obtain

$$x^* = \left(\frac{1}{1 + p^*} \right)^n \frac{F_M}{2n} \text{ and } y^* = p^* \left(\frac{1}{1 + p^*} \right)^n \frac{F_M}{2n}. \quad (21)$$

Inserting (21) into (17) and simplifying leads to

$$p^* = \frac{1}{n - 1}. \quad (22)$$

Hence, $1 + p^* = \frac{n}{n-1}$. Accordingly,

$$x^* = \left(\frac{1}{1+p^*}\right)^n \frac{F_M}{2n} = \left(\frac{n-1}{n}\right)^n \frac{F_M}{2n} \quad (23)$$

and

$$y^* = \left(\frac{n-1}{n}\right)^n \frac{F_M}{2n} \frac{1}{n-1}. \quad (24)$$

Given $\mathbf{y}^* = (y^*, y^*, \dots, y^*)$, among all uniform vectors \mathbf{x} , the vector \mathbf{x}^* is optimal but yields just a payoff of $\pi_A = F_M/2$ for firm A . Accordingly, A is indifferent between $\mathbf{x} = \mathbf{0}$ and $\mathbf{x} = \mathbf{x}^*$. A may therefore randomize between these two optimal choices and may choose $\mathbf{x} = \mathbf{x}^*$ with probability p^* as in (22). Moreover, given this choice by A , firm B maximizes the objective function (15) with x^* and p^* given by (22) and (23).

The payoff of A in this equilibrium is obtained using one of the actions from A 's equilibrium support, e.g., $\mathbf{x} = \mathbf{0}$, together with y^* ; hence, the payoff is $\pi_A^* = F_M/2$. Firm B 's payoff is obtained by inserting the equilibrium values in (15). Note that B 's expected payoff is below $F_M/2$.

It remains to confirm that $x = 0$ and $x = x^*$ and $y = y^*$ indeed globally maximize π_A and π_B , respectively. Using the homogeneity in F_M we simplify the notation and consider $F_M = 2$. It remains to be shown that x^* is a global maximum of

$$\pi_A(x) = 1 + \frac{x^n}{(x + y^*)^n} - nx \quad (25)$$

for $x \in (0, \frac{1}{n})$ for given $y^* = \frac{(n-1)^{n-1}}{n^{n+1}}$ and

$$\pi_B = p^* \left(1 - \frac{(x^*)^n}{(x^* + y)^n}\right) - ny \quad (26)$$

takes a global maximum for y^* for $y \in (0, \frac{1}{p^*n})$ for given $x^* = \frac{(n-1)^n}{n^{n+1}}$ and $p^* = \frac{1}{n-1}$.

Consider first (26). Taking the second derivative with respect to y yields

$$\frac{d^2(\pi_B)}{(dy)^2} = -\frac{x^n p^*}{(x + y^*)^{n+2}} (n+1)n < 0$$

for all $x > 0$.

Consider next (25). Taking the second derivative with respect to x yields

$$\frac{d^2(\pi_A)}{(dx)^2} = y^* (ny^* - 2x - y^*) \frac{x^{n-2}n}{(x + y^*)^{n+2}}.$$

This second derivative is negative if and only if

$$x > \frac{(n-1)}{2}y^*.$$

This shows that $\pi_A(x; y^*)$ starts out convex at the corner extremum for $x = 0$, has a turning point at $x^0 = \frac{(n-1)}{2}y^*$, is concave on the whole range for $x > x^0$, and reaches its only further extremum at $x^* = 2x^0 = (n-1)y^*$. Therefore, the only possible candidates for a global maximum are $x = 0$ and x^* , but $\pi_A(x = x^*; y^*) = \pi_A(x = 0; y^*)$. This completes the proof. ■

Figure 1 illustrates the mixed strategy equilibrium for $n > 2$. It maps $\pi_A(x)$ as a function of n and x .

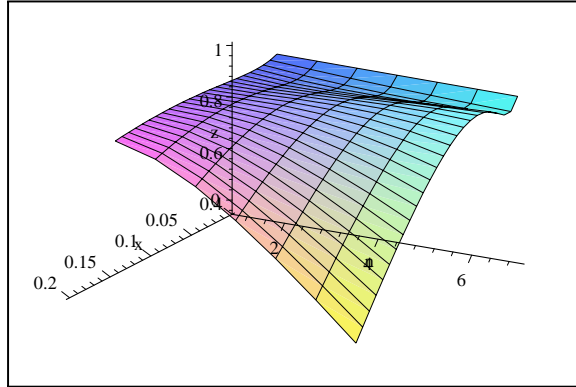


Figure 1: The payoff π_A as a function of $x \in [0.001, 0.2]$ and $n \in \{1, 2, \dots, 7\}$ for given y^* for the case with $k_A > 0$ and $k_B = 0$.

Proposition 2 describes an asymmetric situation. One firm already holds one or several patents when the two firms start fighting about the allocation of the remaining patents. We call this firm the leading firm. The leading firm has already secured half of the monopoly rent at the outset, and so the contest over the remaining patents is about the other half of the monopoly rent. The leading firm wins this second half only if it wins all remaining patents, whereas the other firm needs to win only one of the patents to

secure this other half. The competition for this other half of the monopoly rent is, therefore, asymmetric. The leading firm is at a disadvantage. The disadvantage can be so large that the leading firm may actually give up and not attempt to prevent the rival from also attaining a veto right. Indeed, for large n , π_B as in Proposition 2 converges towards $F_M/2$, i.e., in the equilibrium B receives half of the monopoly rent without expending any significant effort.

The equilibrium in Proposition 2 is of some interest, particularly as the leading firm does not really gain from actively participating in this contest. Unless the leading firm wins in all n of the parallel contests that take place in stage 1, it will lose the prize that is awarded in this stage to the other firm. To win all these n patents is rather costly, and the cost increases in the number of patents that have to be won in this stage. To match the effort that is expended by the other firm along all dimensions becomes more and more costly, because the probability that this effort succeeds along all contests is drops rapidly as the number n of contests increases. The expected benefit of the leading firm in the stage 1 contest is zero, and, to stay out of the contest in this stage is an alternative, optimal option for the leading firm. Of course, zero bids by the leading firm cannot constitute an equilibrium either, as the other firm's optimal reply to zero would be very low bids, so low that the leading firm would like to make even higher positive bids, even if n is large. The leading firm will therefore play a mixed strategy in the equilibrium. It will bid zero in all patent contests in stage 1 with some positive probability, and it will bid uniformly on all patents with an amount of effort that is just optimal and also yields zero payoff with the remaining probability. Given this bid strategy by the leading firm, the other firm is in a situation in which any effort is valuable only in the states in which the leading firm chooses the strictly positive effort. The larger the probability by which the leading firm makes a zero bid, the smaller becomes the marginal increase in expected payoff that the other firm can gain from an increase in its expenditure. As this other firm's payoff is concave in y , a larger p^* reduces the optimal y of firm B . Notice that when the leader makes a positive bid it is larger than that of the rival, and that this difference increases the more patents that must be won at stage 1; as n increases, the firm that holds no patents initially knows that its chance of gaining a veto right are large by virtue of the fact that the probability of the leader winning all of the patent contests is reduced for a given expenditure.

Proposition 3 *If $k_A > 0$ and $k_B > 0$, then the equilibrium efforts in the different patent contests are $x_i^* = y_i^* = 0$ and the equilibrium payoffs are $\pi_A^* = \pi_B^* = F_M/2$.*

Proof. If $k_A > 0$ and $k_B > 0$, then, whatever is the allocation of the remaining n patents, efficient negotiations will lead to monopoly profit which is shared equally between the two firms. Accordingly, the optimal amounts of effort are zero. This case highlights the importance of the assumption that the contest effort is not valuable per se, but only reduces market uncertainty.

■

The case covered in Proposition 3 is straightforward. If both firms already own some patents, they are already veto-players as regards the bargaining and production stages, and the further allocation of the remaining n patents is not payoff relevant for the subgames that may emerge in stages 2 and 3. Therefore, the firms are unwilling to expend effort in stage 1 on acquiring these patents.

3 Alternative patent trade regimes

We now discuss deviations from the benchmark case in the previous section and consider regimes in which patent trade is restricted in different ways.

No patent trade Consider first the most severe patent trade restrictions one could think of: suppose patents cannot be traded at all. The only case in which production can take place in stage 3 is when one firm wins all patents.

Consider first the case $k = 0$. The payoff of firm A becomes

$$\pi_A = \frac{\prod_{i=1}^{i=n} x_i}{\prod_{i=1}^{i=n} (x_i + y_i)} F_M - \sum_i x_i, \quad (27)$$

and equivalently for firm B . A firm wins only if it wins all patents, and if the different single patent contests are mutually independent, this is the product of the win probabilities for all patents $i = 1, \dots, n$.

Maximization of this payoff with respect to x_j yields n identical first-order conditions of the type

$$\frac{\partial \pi_A}{\partial x_j} = \frac{\prod_{i \neq j} x_i \prod_{i=1}^{i=n} (x_i + y_i) - \prod_{i \neq j} (x_i + y_i) \prod_{i=1}^{i=n} x_i}{\left(\prod_{i=1}^{i=n} (x_i + y_i) \right)^2} F_M - 1 = 0 \quad (28)$$

Making use of the symmetry assumption, one obtains

$$y_j^* = x_j^* = \frac{F_M}{2^{n+1}}. \quad (29)$$

Efforts and aggregate effort in this equilibrium is exactly the same as in the equilibrium in Proposition 1 without any restrictions on patent trade:

Proposition 4 *If all patent contests take place simultaneously, prohibition of trade of exclusive patent rights between the firms does not change the contest effort in a symmetric interior equilibrium of the multi-patent contest.*

Despite this similarity with respect to the contest stage, the equilibrium payoffs differ from those in the benchmark case. Aggregate profit in the interior equilibrium with efforts as in (29):

$$\pi_A = \pi_B = \frac{F_M}{2^n} - \frac{nF_M}{2^{n+1}} = \frac{F_M(2-n)}{2^{n+1}}. \quad (30)$$

For $n = 2$, this payoff is smaller than if patent trade is unconstrained. Firms expend the same amount of R&D, but each firm wins the monopoly rent only with probability 1/4, if patents cannot be traded. Firms and consumers are worse off ex-ante for $n = 2$, if patents cannot be traded. For $n > 2$, the profit (30) becomes negative, and symmetric effort as in (29) does not constitute an equilibrium in this case.

For $k > 0$, if $k_A > 0$ and $k_B > 0$, then, under this regime, no profits accrue, because no production will take place, and no contest effort is expended in this case. Accordingly, the contest effort in this case is also the same as in the case with unrestricted bargaining as in Proposition 3, but for somewhat different reasons.

For $k_A > 0$ and $k_B = 0$, it does not make sense for firm B to expend positive effort. Even if firm B wins all n patents that are awarded in the stage 1, under the no-trade regime the industry profit that accrues in stage 3 will still be zero. Accordingly, the effort choices differ in the no-trade regime from the effort choices in Proposition 2.

Selective patent licensing agreements Suppose now that firms can agree on sharing patents but cannot establish exclusive user rights on patents

that have been traded, or restrict each firm's use of the patent in other ways.⁴ Institutionally, this type of property rights regime may be the outcome of cross-licensing of two firms, and may be the outcome if the trade regimes that yield a monopoly in stage 3, such as a merger of the two firms, are not feasible, for instance due to competition policy.

Perhaps surprisingly, this regime leads to equilibrium R&D efforts and payoffs that are the same as in the unconstrained benchmark regime. If one firm wins all $k + n$ patents, this firm will become a monopolist, like in the framework with free trade. If, instead, each firm wins at least 1 patent, efficient negotiations will take place as follows. For each single patent, firms can negotiate the right to jointly use a patent for each single patent. An agreement that maximizes their joint surplus that is compatible with the restriction on patent sharing is as follows. Suppose firm A wins patents $1 \dots m$ and firm B wins patents $m + 1, \dots (n + k)$. In this case one and only one of the firms may offer the other firm the right to join in the use of all its patents. For instance, A may sell the right to make use of the technology components $1, \dots m$, and ask for a fee equal to $\frac{F_M}{2}$ in this case, whereas firm B sticks to its exclusive rights to use patents $m + 1, \dots (n + k)$. As a result, B will be the only firm that is able to produce the good, and will earn the monopoly profit. This outcome is an equilibrium of any subgame in which both firms hold at least one patent. It maximizes the total payoff of the two firms, as it leads to the monopoly profit, and, as in the benchmark case, it shares this profit between the two firms. Accordingly, the objective functions of firms at the contest stage are the same as in the benchmark case. We summarize this as

Proposition 5 *Trade restrictions that allow only for selective patent licensing agreements between the firms do not change the contest effort in a symmetric interior equilibrium of the multi-patent contest.*

Joint use of patent pools Consider a different regime that is more restrictive with respect to which contractual arrangements are feasible. We again rule out contracts that transfer exclusive user rights for a patent from one

⁴One such restriction could be a limit on the number of units of the good which the licensee is allowed to produce using a particular technology component. Such licensing arrangements are not uncommon and are seemingly anti-competitive. Note, however, that such arrangements are not needed here to obtain the monopoly outcome in the equilibrium.

firm to the other. We also rule out patent specific contracts as regards making use of a particular patent. Instead, firms may either keep the patents they have and use them exclusively, or they may pool all information and patent rights they have, like in a grand patent pool, and use them non-exclusively among them.

If one firm exclusively owns the patent rights for all $n + k$ components, sequential rationality in stages 2 and 3 will imply that the firms do not form a patent pool, but the firm that owns all patents will produce as a monopolist in stage 3 and earn the monopoly profit F_M .

If each firm owns at least one patent, if they do not form a patent pool each firm will earn zero profits in the market game. If, instead, the firms form a patent pool, both firms can produce in the market game. The outcome in the market game is described by competition between duopolists. Each firm earns F_D .

Turning to stage 1, let us assume that $k = 0$. Of course, $n > 1$, as otherwise there is nothing to pool. For reasons of symmetry, we can consider the payoff for one firm. Firm A 's objective function is

$$\pi_A = F_D + \frac{\prod_{i=1}^{i=n} x_i}{\prod_{i=1}^{i=n} (x_i + y_i)} (F_M - F_D) - \frac{\prod_{i=1}^{i=n} y_i}{\prod_{i=1}^{i=n} (x_i + y_i)} F_D - \sum_i x_i. \quad (31)$$

Firm A receives the duopoly profit in all patent allocations that emerge from the contest stage, except if firm A wins all patents or if firm A wins none of the patents. The first exception yields an additional payoff for firm A equal to $(F_M - F_D)$ and the exception occurs with the probability that is described by the ratio term in the second term on the right hand side of (31). The second exception yields firm A a payoff of zero, as the other firm has all patents. This is described by the third term in (31). Finally, and as in previous cases, the firm has to pay for the efforts in the n parallel R&D contests, and this constitutes the fourth term on the right-hand side.

Maximization of this payoff with respect to x_j for $j = 1, \dots, n$ yields n identical first-order conditions

$$\begin{aligned} & \frac{\prod_{i \neq j} x_i \prod_{i=1}^{i=n} (x_i + y_i) - \prod_{i \neq j} (x_i + y_i) \prod_{i=1}^{i=n} x_i}{(\prod_{i=1}^{i=n} (x_i + y_i))^2} (F_M - F_D) \\ & + \frac{\prod_{i \neq j} (x_i + y_i) \prod_{i=1}^{i=n} y_i}{(\prod_{i=1}^{i=n} (x_i + y_i))^2} F_D - 1 = 0 \end{aligned}$$

Using symmetry, this reduces to

$$x_j^* = y_j^* = \frac{F_M}{2^{n+1}}. \quad (32)$$

Again, the effort choice in a symmetric interior equilibrium is the same as in the regime in which exclusive patent rights can be traded freely, whereas the equilibrium payoffs differ. The payoff is equal to $F_D + \frac{1}{2^n}(F_M - F_D) - \frac{1}{2^n}F_D - \frac{nF_M}{2^{n+1}}$ which is equal to

$$\pi_A = \pi_B = \frac{2F_D(2^{n-1} - 1)}{2^n} + \frac{F_M(2 - n)}{2^{n+1}}.$$

This payoff is non-negative for $n = 2$. Whether it is positive or negative for larger values of n depends upon the relative size of F_D and F_M . For $n > 2$ it is positive for

$$\frac{F_D}{F_M} > \frac{n - 2}{2^{n+1} - 4}. \quad (33)$$

The ratio on the right hand side peaks at $n = 3$ (it is about 0.0833), and decreases (rapidly) in n thereafter, and whether this condition is fulfilled depends on the type of competition in a duopoly. Consumers gain from this regime compared to unconstrained trade of exclusive patent rights if the symmetric interior equilibrium exists, as the product is always supplied by at least one firm in both regimes, but the market is monopolized if exclusive patent rights can be traded without any restrictions, and a jointly used patent pool leads to a duopoly with a considerable probability which becomes ever larger if the number of patents is higher. This is summarized as a proposition:

Proposition 6 *Let $k = 0$ and $n > 1$. If firms cannot trade single patent rights, but can form a grand patent pool, condition (33) characterizes a necessary condition for the symmetric interior R&D contest equilibrium to exist. In this equilibrium they spend the same contest effort as with free patent trade. Firm payoffs are lower and consumer rents are higher than in the case with free patent trade.*

To get an intuition whether condition (33) is likely to hold, we consider Bertrand and Cournot competition with perfect substitutes. With Bertrand competition between identical firms, $F_D = 0$, and the condition fails, i.e., no interior symmetric equilibrium at the contest stage for $n > 2$ exists. With Cournot competition with homogenous products, constant marginal cost and

linear demand, $F_D/F_M = 4/9$, and participants make positive payoffs for all n .

We now turn to cases in which firms may have acquired some patents in stages prior to stage 1.

Proposition 7 *Suppose $k_A > 0$ and $k_B = 0$. If $\frac{F_M}{F_D} \geq n \geq 1$ then there is a pure strategy equilibrium in which*

$$x_i^* = \left(\frac{F_M - F_D}{F_M}\right)^n \frac{F_D(F_M - F_D)}{F_M} \quad (34)$$

$$y_i^* = \left(\frac{F_M - F_D}{F_M}\right)^n \frac{F_D F_D}{F_M} \quad (35)$$

The corresponding expected payoffs are:

$$\begin{aligned} \pi_A &= F_D + \left(\frac{F_M - F_D}{F_M}\right)^n (F_M - F_D) \left(\frac{F_M - nF_D}{F_M}\right) \\ \pi_B &= F_D - \left(\frac{F_M - F_D}{F_M}\right)^n F_D \left(\frac{F_M + nF_D}{F_M}\right). \end{aligned}$$

For $n > \frac{F_M}{F_D}$ a mixed strategy equilibrium exists that is characterized by firm B choosing

$$y_i^* = \left(\frac{n-1}{n}\right)^n \frac{(F_M - F_D)}{n(n-1)}, i = 1, \dots, n$$

and firm A choosing $\mathbf{x}^* = \left[\frac{y_i^*(F_M - F_D)}{p^* F_D}, \dots, \frac{y_i^*(F_M - F_D)}{p^* F_D}\right]$ with probability $p^* = \frac{F_M - F_D}{F_D(n-1)}$ and $\mathbf{x}^* = \mathbf{0}$ with probability $(1 - p^*)$. Firm A's expected payoff is $\pi_A^* = F_D$ and firm B's expected payoff is $\pi_B = F_D - \frac{2(F_M - F_D)}{n-1} \left(\frac{n-1}{n}\right)^n$ in this equilibrium.

Proof. Analogously to the proof of Proposition 2, the expected payoffs to the firms are:

$$\pi_A = F_D + \prod_{i=1}^n \frac{x_i}{x_i + y_i} (F_M - F_D) - \sum_i x_i \quad \text{and} \quad (36)$$

$$\pi_B = \left(1 - \prod_{i=1}^n \frac{x_i}{x_i + y_i}\right) F_D - \sum_i y_i. \quad (37)$$

A straightforward solution of the system of equations that consist of the first-order conditions for A maximizing (36) and B maximizing (37) yields the result in (34). Furthermore, it is straightforward to verify $\pi_A \geq F_D, \pi_B \geq 0$ for $\frac{F_M}{F_D} \geq n \geq 1$.

If $n > \frac{F_M}{F_D}$, then the pure strategy equilibrium breaks down since the interior solution is dominated by zero. There is a mixed strategy equilibrium, however. Consider the candidate equilibrium in Proposition 7. For this to be an equilibrium, $\mathbf{y}^* = (y^*, \dots, y^*)$ must maximize

$$\pi_B = \left[\left(1 - \prod_{i=1}^n \frac{x^*}{x^* + y_i}\right) F_D - \sum_{i=1}^n y_i \right] p^* + (1 - p^*) \left[F_D - \sum_{i=1}^n y_i \right], \quad (38)$$

Further, $\mathbf{x}^* = (x^*, \dots, x^*)$ must maximize

$$\pi_A = F_D + \prod_{i=1}^n \frac{x_i}{x_i + y^*} (F_M - F_D) - \sum_i x_i \quad (39)$$

for all \mathbf{x} with $x_i > 0$. Further, for A to be indifferent between \mathbf{x}^* and $\mathbf{x} = \mathbf{0}$,

$$\pi_A(\mathbf{x} = \mathbf{0}) = \pi_A(\mathbf{x} = \mathbf{x}^*)$$

must hold. Proceeding as in the proof to Proposition 2 yields the stated values for \mathbf{x}^* , \mathbf{y}^* , and p^* . Substitution yields the expected payoffs. For p^* to be bounded below 1 requires $nF_D > F_M$. This condition guarantees that $\pi_B \geq 0$ in equilibrium. ■

Qualitatively, there is a similarity between the equilibrium in Proposition 7 and that in Proposition 2. For sufficiently low n there is a pure strategy Nash equilibrium, but as the number of patents grows a mixed strategy equilibrium will emerge. The intuition behind this is analogous to Proposition 2. The larger is the relative gain to monopoly power ($\frac{F_M - F_D}{F_D}$), the higher is the effort by the leading firm for the remaining n patents, and the larger is the probability that this positive effort is made.

The case $k_A > 0$ and $k_B > 0$ is more straightforward and the result is the same as in Proposition 3. As both firms already hold at least one of the complementary patents, any allocation of the n remaining patents in stage 1 will not change the industry profits and their division in the stages 2 and 3; each firm will receive the duopoly profit, independent of their contest efforts in stage 1. Hence, their equilibrium efforts will be zero, just like in Proposition 3.

4 Conclusions

The number of patents that are used for single products is considerable, for many standard products. In this paper we have considered R&D contests if firms may, but need not have acquired some patents, but need n additional complementary patent rights for components from which they can produce a consumer good. We show that this complementarity generally weakens the incentives to invest in R&D effort. If firms can freely trade the rights to use each single of these n patents, the sum of all R&D effort in all single patent contests is decreasing in the number of complementary patents. Intuitively, if many complementary patents are needed to produce a particular good, even if one firm spends much effort on each of the single patent contests, it becomes very likely each firm fails to obtain all patents. But if each of the firms obtains at least one patent, this patent yields veto power, and a firm's payoff is therefore the same whether it holds 1, 2, or even $n - 1$ patents. This makes it less worthwhile to spend much effort in the n simultaneous contests.

Considering the R&D competition for fully complementary patents between firms who already hold one or several such patents and firms who do not, we find that holding such patents yields some secure payoff to such leading firms. However, it also yields a disadvantage for the leading firms in the ongoing patent contest for further patents. With a large number of further patents, the leading firms cannot gain from participating in these further contests. However, the mixed strategy equilibrium requires that they participate with a certain equilibrium probability nevertheless.

We also show that this result is robust with respect to alternative trading regimes among patent holders. Particularly, if patent rights cannot be traded at all, or if exclusive rights in patents cannot be traded, but patents can only be licensed, or if firms can only enter into a general patent sharing arrangement (a 'patent pool') and use all their patents jointly and non-exclusively, the marginal incentives to spend effort in the simultaneous single patent contests remain unchanged. The profits for firms are lower in some of these regimes, however, and these marginal incentives need not characterize an interior equilibrium if the number n of components becomes large.

From the buyers' point of view, among the regimes we study, the patent pool yields the highest consumer rents, provided that n is small enough such that the interior symmetric contest equilibrium is sustained under this regime.

5 Appendix

Optimality of uniform effort In this appendix we first prove the following lemma:

Lemma 1 *Let*

$$\pi(r) = \prod_{i=1}^n \frac{r_i}{r_i + s} - \sum_i r_i \quad (40)$$

or

$$\pi(r) = 1 - \prod_{i=1}^n \frac{s}{s + r_i} - \sum_i r_i \quad (41)$$

for some given $s > 0$. Then for any $\sum_i r_i > 0$ with $r_i \geq 0$ the vector $\mathbf{r} = (r_1, r_2, \dots, r_n)$ that maximizes (40) has $r_1 = r_2 = \dots = r_n$.

Proof. The proof is by contradiction. Consider (40). Let $\mathbf{r} = (r_1, r_2, r_3, r_4, \dots, r_n)$ be a vector that maximizes (40). Denote $\sum_{i=1}^n r_i \equiv \rho$. We first note that any vector \mathbf{r} that maximizes (40) is either $\mathbf{r} = \mathbf{0}$, or has components that are all strictly positive, as \mathbf{r} is otherwise strictly dominated by $(0, 0, \dots, 0)$. If $\mathbf{r} = \mathbf{0}$, then the claim in the lemma is true, as all components r_i are identically zero.

Consider now the case $r_j \neq 0$. The payoff π in (40) can be written as

$$\pi(r) = \frac{r_1}{r_1 + s} \frac{r_2}{r_2 + s} \prod_{i=3}^n \frac{r_i}{r_i + s} - \sum_i r_i \equiv \frac{r_1}{r_1 + s} \frac{r_2}{r_2 + s} \Psi - \rho.$$

Assume that $r_1 < r_2$. Then a marginal increase in r_1 by $dr_1 > 0$ and a simultaneous change in r_2 by dr_2 such that $dr_1 + dr_2 = 0$ changes profit by

$$\begin{aligned} \frac{d\pi}{dr_1} &= \left[\frac{s}{(r_1 + s)^2} \frac{r_2}{r_2 + s} - \frac{r_1}{r_1 + s} \frac{s}{(r_2 + s)^2} \right] \Psi \\ &= \left[\frac{r_2}{r_1 + s} - \frac{r_1}{r_2 + s} \right] \left[\frac{s}{(r_1 + s)(r_2 + s)} \right] \Psi > 0. \end{aligned}$$

The strict inequality holds because the last two terms on the left-hand side are always strictly positive, whereas $\left[\frac{r_2}{r_1 + s} - \frac{r_1}{r_2 + s} \right] > 0$ if $r_2 > r_1$. But this shows that \mathbf{r} with $r_1 \neq r_2$ is not optimal and this constitutes the contradiction.

The proof for (41) follows analogous lines. ■

Global optimality of \mathbf{x}^* in Proposition 1 In this section of the Appendix, we show that \mathbf{x}^* is a global maximum for $n \leq 6$. For this purpose, we first consider the range $n \leq 4$ analytically, and turn to the range $n \in \{5, 6\}$ numerically.

Lemma 2 *The function (7) is concave in x in the relevant range $x \in [0, F_M/n]$ for $n \leq 4$.*

Proof. Note first that the second derivative of π_A in (7) with respect to x is

$$\frac{d^2\pi_A}{(dx)^2} = \frac{nF_M}{2} \frac{(x^{n-2} - y^{n-2})y^2(n-1) - 2x^{n-1}y - 2y^n}{(x+y)^{n+2}}. \quad (42)$$

This expression (42) is negative for all $x \leq y + \delta$ for some strictly positive δ . Accordingly, $\pi_A(x)$ is strictly concave in the whole range $x \in (0, y + \delta)$.

Consider now concavity for the remaining range $x \in (\frac{F_M}{2^{n+1}}, \frac{F_M}{n})$. Note that the optimization problem is homogenous in the size of F_M and simplify notation by assuming that $F_M = 2$, such that this interval can be denoted $x \in (1/2^n, 2/n)$. The right hand side of (42) is negative if $(x^{n-2} - y^{n-2})y^2(n-1) - 2x^{n-1}y - 2y^n < 0$, or after inserting $y = 1/2^n$, if

$$(x^{n-2} - \frac{1}{2^{n(n-2)}}) \frac{1}{2^{2n}}(n-1) - 2x^{n-1} \frac{1}{2^n} - 2 \frac{1}{2^{nn}} < 0.$$

Multiplying with 2^n and regrouping yields

$$(x^{n-2} - \frac{1}{2^{n(n-2)}}) \frac{1}{2^n}(n-1) - 2x^{n-1} - 2 \frac{1}{2^{n(n-1)}} < 0. \quad (43)$$

One sufficient condition for this term to be negative is therefore $x^{n-2} \frac{1}{2^n}(n-1) - 2x^{n-1} < 0$, or equivalently

$$\frac{n-1}{2^{n+1}} < x.$$

It remains to be shown that (43) holds also for the range $x \in [1/2^n, (n-1)/2^{n+1}]$. Consider again the general condition (43). Divide by x^{n-2} and transform to $\frac{1}{2^n}(n-1) - 2x < \frac{1}{2^n}(n-1) \frac{1}{2^{n(n-2)} x^{n-2}} + 2 \frac{1}{2^{n(n-1)} x^{n-2}}$, or equivalently

$$\frac{1}{2^n}(n-1) - 2x < (n+1) \frac{1}{2^{n(n-1)}} \frac{1}{x^{n-2}}$$

Multiply by 2^n

$$(n-1) - 2^{n+1}x < \frac{(n+1)}{(2^n x)^{n-2}} \quad (44)$$

Define now $z \equiv 2^n x$, and rewrite (44) as $(n-1) - 2z < \frac{(n+1)}{z^{n-2}}$ or equivalently

$$z^{n-2}(n-1-2z) < n+1. \quad (45)$$

A sufficient condition for (45) to hold is that the bracketed expression on the left is negative; this is least likely to be the case when z is small, i.e. $x = \frac{1}{2^n}$. Inserting this into $(n-1-2z) = n-3$, so that the inequality holds for $n=2$ and $n=3$. For $n > 3$,

$$\text{sign} \frac{\partial(z^{n-2}(n-1-2z))}{\partial z} = \text{sign}((n-1)(n-2-2z)).$$

On $z \in [1, \frac{(n-1)}{2}]$, $(n-2-2z)$ takes a maximum for $z=1$, and this shows that concavity can be obtained analytically for $n \leq 4$. The payoff π_A as in (7) is concave for $x \in (0, F_M/n)$ for $n \leq 4$ and this establishes that \mathbf{x}^* describes a global maximum.

For larger n , the value x^* as in Proposition 1 is a local extremum, but the function (7) is not globally concave. However, we can establish that x^* is an optimal reply to y^* for $n=5$ and $n=6$, and its suboptimality of x^* for $n > 6$ numerically, and also show that x^* is not an optimal reply to y^* for $n \geq 7$.

Consider $\pi_A(x^*; y^*) - \pi_A(x; y^*)$ for $F_M = 2$. Note that the interval of values of x that could possibly yield higher payoff than x^* is the interval $x \in [\frac{1}{2^n}, \frac{n-1}{2} \frac{1}{2^n}]$. The payoff difference can be parametrized in this interval by parameters n and a , such that

$$\begin{aligned} & \pi_A(x^*; y^*) - \pi_A(x; y^*) \\ = & \left(1 - n \left(\frac{1}{2^n}\right)\right) - \left(1 + \frac{((1-a)\frac{1}{2^n} + a\frac{n-1}{2^{n+1}})^n - (\frac{1}{2^n})^n}{(((1-a)\frac{1}{2^n} + a\frac{n-1}{2^{n+1}}) + (\frac{1}{2^n}))^n} - n \left(\frac{1-a}{2^n} + a\frac{n-1}{2^{n+1}}\right)\right) \end{aligned}$$

with $a \in [0, 1]$, where $a=0$ corresponds to $x = 1/2^n$ and $a=1$ corresponds to $x = \frac{n-1}{2} \frac{1}{2^n}$. Calculating this difference for a grid for the number of patents that equals the natural numbers, and for a much finer grid for a yields the following value profile, where only positive values of $\pi_A(x^*; y^*) - \pi_A(x; y^*)$ are depicted, and the surface breaks off at the front part where it hits negative values. This illustrates that x^* is an optimal reply to y^* only for $n \leq 6$. ■

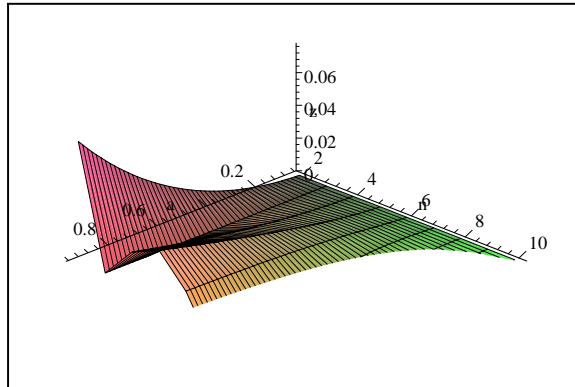


Figure 2: The surface describes the positive values of $\pi_A(x^*; y^*) - \pi_A(x; y^*)$ for $a \in [0.05, 0.95]$ for values of $n \in \{2, \dots, 10\}$. The south-east frontier of this surface denotes combinations of a and n for which $\pi_A(x^*; y^*) - \pi_A(x; y^*)$ becomes negative. Hence, for $n > 6$, x^* is not an optimal reply to y^* .

6 References

Arrow, K.J., 1962, Economic welfare and the allocation of resources for innovation, in: R. Nelson, *The Rate and Direction of Inventive Activity*, Princeton, Princeton University Press, 609-626.

Baye, M.R., and H.C. Hoppe, 2003, The strategic equivalence of rent-seeking, innovation, and patent-race games, *Games and Economic Behavior*, 44(2), pp. 217-226.

Denicolo, V., 2002, Sequential innovation and the patent-antitrust conflict, *Oxford Economic Papers*, 54(4), 649-668.

Green, J.R., and S. Scotchmer, 1995, On the division of profit in sequential innovation, *Rand Journal of Economics*, 26(1), 20-33.

Heller, M.A. and R.S. Eisenberg, 1998, Can patents deter innovation? The anticommons in biomedical research, *Science*, 280, 698-701.

Lerner, J., and J. Tirole, 2004, Efficient patent pools, *American Economic Review*, 94(4), 691-711.

Loury, G.C., 1979, Market structure and innovation, *Quarterly Journal of Economics*, 93(3), 395-410.

Nti, Kofi O., 1997, Comparative statics of contests an rent-seeking games, *International Economic Review*, 38(1), 43-59.

Scotchmer, S., 1996, Protecting early innovators: should second-generation products be patentable? *Rand Journal of Economics*, 27(2), 322-331.

Shapiro, C., 2001, Navigating the patent thicket: cross licenses, patent pools, and standard setting, in: A. Jaffe, J. Lerner and S. Stern, eds., *Innovation Policy and the Economy*, Vol. 1 NBER, Cambridge, MA., 119-150.

Skaperdas, Stergios, 1996, Contest success functions, *Economic Theory* 7, 283-290.

Szidarovszky, Ferenc, and Koji Okuguchi, 1997, On the existence and uniqueness of pure Nash equilibrium in rent-seeking games, *Games and Economic Behavior*, 18, 135-140.

Tullock, G., 1980, Efficient rent seeking, in J.M. Buchanan, R. D. Tollison, and G. Tullock, eds., *Toward a Theory of Rent Seeking Society*, Texas A&M Univeristy Press, College Station, 97-112.

Ziedonis, R.H., 2004, Don't fence me in: fragmented markets for technology and the patent acquisition strategies of firms, *Management Science*, 50(6), 804-820.