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Contests with Investment

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Abstract

Perfectly discriminating contests (or all pay auction) are widely used as a model of situations where individuals devote resources to win some prize. In reality such contests are often preceded by investments of the contestants into their ability to fight in the contest. This paper studies a two stage game where in the first stage, players can invest to lower their bid cost in a perfectly discriminating contest, which is played in the second stage. Different assumptions on the timing of investment are studied. With simultaneous investments, equilibria in which players play a pure strategy in the investment stage are asymmetric, exhibit incomplete rent dissipation, and expected effort is reduced relative to the game without investment. There also are symmetric mixed strategy equilibria with complete rent dissipation. With sequential investment, the first mover always invests enough to deter the second mover from investing, and enjoys a first mover advantage. I also look at unobservable investments and endogenous timing of investments.

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1 Introduction

Several recent papers have studied investment incentives ahead of different market institutions, such as standard first- and second-price auctions.[1] However, some market institutions are best described as contests where individuals devote nonrecoverable resources in order to win a prize, and little is known about investment incentives ahead of contests. Research tournaments are a case in point.[2] For example, Lichtenberg (1988) points out the importance of 'design and technical competitions' for public procurement, and argues that these competitions are best understood as contests. Officially, these contests begin when a federal agency issues a formal Request for Proposals. But even before the official start of a contest, potential contractors are very well aware of the areas of interest of the relevant federal agencies, and monitor the agencies' interests closely (Danhof 1968). Firms react on new developments by acquiring skills through selective hiring, or even purchasing another firm that already has the necessary experience. In fact, Danhof (1968, p. 237) argues that these pre-contest investments are essential: "The firm that first becomes aware of an agency's interest in an area through the receipt of a Request for Proposal will normally find itself severely if not impossibly handicapped should it wish to submit a proposal."

Investments ahead of contests also play an increasingly important role in markets with intense promotional competition and advertising. The interaction in these markets clearly are contests, since the market shares depend on nonrecoverable advertising expenditures (see, for example, Schmalensee 1976). According to The Economist, it is getting increasingly harder to reach consumers, as they are using "technologies that are getting better at enabling them to avoid ads, such as "pop-up" blockers in web browsers and personal video recorders that let viewers easily assemble their own TV schedules and skip commercial breaks" (The Economist, 2004a). Thus, "target practice" -

[1] In particular Tan (1992), Piccione and Tan (1996), Bag (1997), Arozamena and Cantillon (2004), Krähmer and Strausz (2006). See also Erbenová and Vagstad (1999) on investment incentives when a government cannot commit to future policies.

[2] Windham (1999) and Maurer and Scotchmer (2004) list many examples of research tournaments. Important papers on research tournaments include Lichtenberg (1988), Taylor (1995), Fullerton and McAfee (1999), Che and Gale (2003).

market research in order to find out how to reach a large audience of people with similar interests, ahead of actually starting an advertising campaign - is increasingly important (The Economist, 2005; see also The Economist, 2004b). Such market research activities can be viewed as investments that enhance the ability of firms to compete in a contest for market shares.

The objective of this paper is to model investments ahead of contests in detail. It studies a contest between symmetric players. In a first stage, contestants invest in their abilities. In the second stage, they compete in a perfectly discriminating contest, where the abilities or bid costs of the contestants depend on their investments in the first stage. The paper also studies the case where the investments increase the value of winning the contest, which turns out to be quite similar to the case of bid-cost reducing investment.

Perfectly discriminating contests or all pay auctions have been used to model several situations where individuals devote resources in order to win a prize: R&D races and research tournaments (Dasgupta 1986), election campaigns (Che and Gale 1998), rent-seeking and lobbying (Hillman and Riley 1989, Ellingsen 1991, Baye et al. 1993), conflicts in hierarchies (Konrad 2004), rivalry between exporting firms in international trade (Konrad 2000b), and even sport tournaments (Groh et al. 2003) and wars (Bester and Konrad 2005). The strategic interaction is modelled as a one shot game, where the players simultaneously choose their efforts or "bids", the player who chooses the highest bid wins a prize, and all players have to pay their bids. This paper argues that, in all these applications, the contestants can make an investment which enhances their ability to fight in the upcoming contest. Ahead of wars, or sport contests, it is obvious that players put a lot of effort into preparing for the contest. Before trying to influence some politicians on some specific issue, lobbyists often build up a relationship with the politicians. Having a better relationship helps to "dine and wine" and have an influence on politicians. In election races, candidates build up support prior to engaging in the actual election campaigns. As argued above, in R&D races and research tournaments, firms build up staff and research capabilities prior to entering the contest.

One main question of the analysis is how the possibility of investments affects expected efforts. Surprisingly, expected efforts are not always higher than in a benchmark case where the investments are exogenously fixed at zero. A second main question concerns rent-dissipation - that is, the expected total costs of the contestants (investment and bid cost) in relation to the rent that is at stake or prize to be won. In a completely discriminating contest between identical players, rent dissipation is complete. Several reasons for lower rent dissipation have been noted in the literature, including exogenous differences between players (Baye et al. 1996), imperfectly discriminating contests (see Nitzan (1994) for a survey), risk aversion (Hillman and Katz 1984), and population uncertainty (Myerson and Wärneryd 2006). This paper shows that investments in bid costs reduction can give an additional reason to expect incomplete rent-dissipation - even in a perfectly discriminating contest, with complete information, between ex ante symmetrical contestants, who are risk neutral.

Whether the investments are observable by the other contestants is important. If they are not, then, although the decisions are taken at different points in real time, we have a simultaneous move game. For each bid in the contest, there is one associated level of investment that minimizes the total cost of that bid (investment cost plus bid cost). Thus, the game collapses to a game in only one decision variable. As in the standard all pay auction, rent dissipation is complete. Surprisingly, expected bids can be smaller than in a world where no investment is possible.

If the investments are observable, there is an additional strategic effect: having low bid cost discourages the rival and makes him fight less hard in the ensuing contest. I study three different scenarios for the timing of the investment decisions. First, simultaneous timing. Here we have a two stage game, where in stage one contestants simultaneously decide upon their investment, and in stage two, after observing the investments of their rivals, they compete in the contest. This model seems appropriate when the contestants are ex ante symmetric. There is a symmetric equilibrium with complete rent dissipation. Under some conditions, there are also asymmetric equilibria where only one of the contestants invests, and rent dissipation is incomplete.

Second, sequential timing of the investments. This version of the model is appropriate when there is one incumbent who decides first about his investment level. I show that he will always choose to invest an amount sufficiently high to deter the followers from investing. Third, endogenous timing of the investment decisions. If the training decisions are long term decisions, and players do not necessarily start investing at the same point in time, a game with an endogenous order of moves might be most appropriate. Here I show that there exists an equilibrium where ex ante symmetric players will invest different amounts. This introduces an endogenous asymmetry into the contest stage, which lowers expected effort and rent dissipation. However, a symmetric equilibrium with complete rent dissipation exists as well.

In all three versions, I find that the possibility of lowering one's bid cost by investing leads to an endogenous asymmetry between the contestants even when they are symmetric ex ante. If they use pure strategies in the investment stage, the equilibrium is always asymmetric. On the other hand, any symmetric equilibrium involves mixing in the investment stage and leads to an asymmetric situation in the contest stage with positive probability.

The paper is related to Konrad, Peters and Wärneryd (2004) and Kräkel (2002, 2004) on strategic delegation in contests. The common topic is that interactions ahead of a contest tends to introduce an endogenous asymmetry even when the game is symmetric ex ante. Thus, Konrad, Peters and Wärneryd (2004) find that delegation contracts are asymmetric. Kräkel (2004) is perhaps most closely related to the present paper, since it studies investment activities that enhance the abilities of the contestants, embedded in a model of strategic delegation. However, Kräkel (2004) assumes an imperfectly discriminating contest in the tradition of Tullock (1980), whereas I look at a perfectly discriminating contest, and the results differ: in Kräkel (2004), if players are symmetric in the investment stage, their investments are also symmetric; in the case of a perfectly discriminating contest, the investment decisions introduce an endogenous asymmetry.[3]

[3] A second, albeit minor, difference is that in my paper, the investment reduces the bid cost, whereas in Kräkel (2004) it enhances the impact of a bid. Formally, the model of Kräkel (2004) is more closely related to contests with two activities as studied in Hefeker and Epstein (2003) and Arbatskaya and Mialon (2005).

Dixit (1987), Baik and Shogren (1992) and Leininger (1993) compare different timing structures in contest games. Dixit (1987) considers contests with an exogenously given sequential timing of moves. He shows that, when the underdog (the player who is less likely to win in a simultaneous move contest) moves first, he undercommits effort relative to his effort in the Cournot-Nash equilibrium of a contest with simultaneous moves. On the other hand, if the favorite moves first, he overcommits. When the underdog moves first, total expenditures are lower than with simultaneous moves; they are higher if the favorite moves first. Baik and Shogren (1992) and Leininger (1993) extend Dixit's analysis by adding an announcement stage, where the players simultaneously announce whether they want to move early or late. The announcements become common knowledge, and then the players play either a simultaneous move contest or one of the two possible sequential move contests. The main result in these papers is that the underdog will move first, and the favorite second. This endogenous timing of moves leads to lower total expenditures than simultaneous play.[4] The main difference between this literature and the present paper is that, in these papers, the timing of the decision about contest effort is discussed. In my paper, contest efforts are always simultaneous, and different assumptions on the timing of the investment decisions are discussed. However, there is a common underlying structure: a contest that is ahead induces a hold-up problem for expenditures in an earlier stage, and thereby potentially reduces total expenditures. Such a hold-up problem has first been observed by Ellingsen (1991), who studies consumer opposition against the winner of a contest for a monopoly position. Similarly, Konrad (2004) studies contests between groups where the winning group engages in an internal contest later on, which also induces

[4] Yildirim (2005) considers a different way to model endogenous timing of moves in contests. In his model, players simultaneously choose effort in a first stage, and have the opportunity to add to their previous efforts in a second stage, after observing the first-stage effort of the rivals. The probability of winning depends on the sum of effort over the two stages. Yildirim finds that there are multiple subgame perfect equilibria, both the Cournot-Nash simultaneous move contest outcome and the sequential outcome where the favorite moves first can emerge. However, the sequential outcome where the underdog moves first is not an equilibrium.

a hold-up problem.

Finally, the paper is related to Sahuguet and Persico (2006) who study campaign spending in a model of redistributive politics which has a close connection with all-pay auctions.

Section 2 sets out the basic model. Section 3 studies the case of unobservable investments, and Section 4 the case of observable investments. As a robustness check, Section 5 looks at extensions to more than two players and a different investment technology. Section 6 concludes.

2 The model

There are two players $1, 2$ (see Section 5 for an extension to $n > 2$ players). In the investment stage of the game, players choose their investments e_i. Investments are measured in monetary units. After the investment stage there is a contest stage. Here players choose their "bids" or efforts x_1 and x_2. Player i's bid costs are $c(e_i)\, x_i$, where $c : R \to R$ is a twice differentiable function, $c'(e) < 0$ and $c''(e) > 0$ for all $e \geq 0$. That is, the higher the investment, the lower the bid cost in the contest stage, and the investment has diminishing marginal returns. To check robustness, in Section 5 I comment on the case $c'' \leq 0$.

Player i wins a prize V in stage two if he submits the highest bid, that is, if $x_i > x_j$. On the other hand, if $x_j > x_i$, player i loses and gets nothing. Ties are broken randomly. Thus, the probability that i wins the contest equals

$$p_i = \begin{cases} 1, & \text{if } x_i > x_j, \\ 0, & \text{if } x_i < x_j, \\ \frac{1}{2}, & \text{if } x_i = x_j. \end{cases}$$

Contestants are risk neutral and thus the objective function of contestant i is

$$u_i = p_i V - c(e_i)\, x_i - e_i. \tag{1}$$

Closely related is a situation where players can invest in order to increase the value of winning. That is, $V = V(e_i)$ with $V'(e) > 0$ and $V''(e) \leq 0$ and

$u_i = p_i V(e_i) - x_i - e_i$. Here there is a clear sense in which one investment is socially optimal - it is given by $V'(e_{opt}) = 1$. Sometimes I will refer to this version of the model to talk about the optimal amount of investment.

3 Unobservable investments

If the investment is not observable, then, from a game theoretic point of view, the investments and the contest are simultaneous. Here, contestants influence only their own bid cost through their investment decision, but not the behavior of the rival. Contestant i chooses (e_i, x_i) in order to maximize (1).

Since $c''(e) > 0$, for any given bid x there is a unique optimal investment level $e^*(x)$ which minimizes the total cost of that bid:

$$e^*(x) = \arg\min_e (c(e)x + e). \tag{2}$$

Any combination (x, e) where $e \neq e^*(x)$ is strictly dominated by $(x, e^*(x))$. Thus, the two dimensional problem actually collapses to one dimension. Let $k(x)$ denote the minimized total cost of a bid x, i.e.

$$k(x) = c(e^*(x))x + e^*(x). \tag{3}$$

Then the problem of contestant i is to maximize $p_i V - k(x_i)$. Proposition 1 describes the equilibrium.

Proposition 1 *When investments are not observable, then there is a unique equilibrium which is in mixed strategies. Contestants mix over pairs (e_i, x_i) along the path $e_i = e^*(x_i)$, where $e^*(\cdot)$ is given in (2). Bids are continuously distributed according to the probability density function*

$$f(x) = \frac{c(e^*(x))}{V}$$

with support $[0, \bar{x}]$, where $\bar{x}$ is defined by $k(\bar{x}) = V$. Rent dissipation is complete. Expected bids in the contest stage can be higher or lower than *without investment.*

Proof. See Appendix A.1. ■

4 Observable investments

When investments are observable, there is an additional strategic effect: having low bid cost makes the rival fight less hard in the contest. I consider three different timings of the investment decisions: simultaneous investments, sequential investments, and finally endogenous timing of investments. In the latter two versions of the model, the investment stage consists of two periods of time, $t = 1,2$. With sequential timing, one exogenously specified player (say 1) invests in $t = 1$; and in $t = 2$, the other player 2 observes the investment of the first mover and chooses his own investment. With endogenous timing, in $t = 1$ a player can either invest or wait. In $t = 2$, players observe all actions taken in $t = 1$, and then those players who waited in $t = 1$ can invest.[5]

4.1 The contest stage

To solve the model by backward induction, consider the contest in stage two. It is an all pay auction with identical valuations of the price but different bidding costs. This game is identical with the usual all pay auction with complete information (see Baye et al. 1996, p. 292). To see this, note that we can divide the payoff of player i by $c(e_i)$ and add $e_i/c(e_i)$. Since e_i is given in stage two of the game, this is just an affine transformation and doesn't change behavior. Denoting the transformed utility function by $\tilde{u}_i$ we have

$$\tilde{u}_i := \frac{u_i}{c(e_i)} + \frac{e_i}{c(e_i)} = \begin{cases} \frac{V}{c(e_i)} - x_i, & \text{if } x_i > x_j, \\ -x_i, & \text{if } x_i < x_j, \\ \frac{1}{2c(e_i)}V - x_i, & \text{if } x_i = x_j \ . \end{cases}$$

[5]In the IO literature, this endogenous timing structure has been studied by Hamilton and Slutsky (1990), see their "game of action commitment".

This is the payoff in a standard all pay auction with complete information where the valuation of player i equals $V/c(e_i)$.

It is well known that there is a unique equilibrium which is in mixed strategies.[6] In the equilibrium, player i has an expected payoff (in units of the transformed utility function) of

$$E(\tilde{u}_i) = max\left\{\frac{V}{c(e_i)} - \frac{V}{c(e_j)}, 0\right\}.$$

Note that $\frac{V}{c(e_i)} > \frac{V}{c(e_j)}$ iff $e_i > e_j$. Transforming backwards we get

$$E(u_i) = c(e_i)E(\tilde{u}_i) - e_i = \begin{cases} V - \frac{c(e_i)}{c(e_j)}V - e_i, & \text{if } e_i > e_j, \\ -e_i, & \text{if } e_i \leq e_j. \end{cases} \tag{4}$$

4.2 The investment stage

Consider now the investment decision in stage one. The investment stage resembles a perfectly discriminating contest in one respect: if i invests less than j does, i 's payoff is minus the cost of his investment. However, if $e_i > e_j$, the "prize" $\left(V - \frac{c(e_i)}{c(e_j)}V\right)$ that i gets depends both on e_i and e_j. This is different in a standard all-pay auction where the prize does not depend on the bids. It is also different from the all-pay auctions with variable rewards studied by Kaplan et al. (2002, 2003) where the value of winning depend's on one's own bid, but not on the rival's bid.

To rule out the rather uninteresting case where there is no investment at all in equilibrium, for the rest of the paper I will assume that[7]

$$-\frac{c'(0)}{c(0)}V > 1. \tag{5}$$

[6] See Hillman and Riley (1989); Baye et al. (1996); and the textbook of Hirshleifer and Riley (1992), Chapter 10.

[7] Line (5) is violated if the prize is not very valuable and/or the investment not very effective. It is straightforward to show that, in this case, in there exists a unique subgame perfect equilibrium where $e_1 = e_2 = 0$.

4.2.1 Simultaneous timing of investment

Suppose for a moment that in equilibrium the contestants do not randomize in the investment stage (we will consider existence of this type of equilibrium below). Then the equilibrium has to be asymmetric: only one player invests a positive amount, whereas the other player chooses zero investment. To see this, suppose to the contrary that there is an equilibrium in which both players invest positive amounts $e_1^*, e_2^* > 0$. Without loss of generality assume $e_1^* \geq e_2^* > 0$. Then player 2 has an expected payoff of $-e_2^* < 0$. This cannot be an equilibrium since by playing $e_2 = 0$ player 2 can guarantee himself a payoff of zero. On the other hand, given (5) there is no equilibrium in which no player invests.

Thus the two players, who are identical ex ante, have different bid costs in the contest stage. Relative to the case where there is no investment, this leads to reduced expected bids in the contest. Suppose player 2 chooses does not invest. Given (5), the optimal response of player 1, is to invest

$$e^*(0) := c'^{-1}\left(-\frac{c(0)}{V}\right) > 0.$$

The expected bids in the contest stage are

$$E(x_1) = \frac{V}{2c(0)}, \; E(x_2) = \frac{c(e^*(0))}{c(0)} \frac{V}{2c(0)}$$

(see Baye et al. 1996). Thus

$$E(x_1 + x_2) = \frac{V}{c(0)} \frac{c(0) + c(e^*(0))}{2c(0)}.$$

If the two players were forbidden to invest, their expected bids would equal

$$\frac{V}{c(0)} > \frac{V}{c(0)} \frac{c(0) + c(e^*(0))}{2c(0)}.$$

Furthermore, expected rent dissipation is reduced. To see this, note that the expected bid costs for each player are

$$c(e_1^*) E(x_1) = c(0) E(x_2) = \frac{c(e^*(0))}{c(0)} \frac{V}{2}.$$

The expected total expenditures (investment plus bid cost) equal

$$e_1^* + c(e_1^*) E(x_1) + c(0) E(x_2) = e^*(0) + \frac{c(e^*(0))}{c(0)} V.$$

By (5) and the assumption that $e_1^* = e^*(0) > 0$ is on the equilibrium path, we have

$$V - \frac{c(e^*(0))}{c(0)} V - e^*(0) > 0,$$

and therefore rent dissipation is not complete.

Depending on the value of the prize V and the shape of the cost function c, there may or may not be such an equilibrium where players do not randomize in the investment stage. Proposition 2 makes this precise and sums up the discussion.

Proposition 2 *Consider the case of simultaneous timing of investment. If*

$$\max_{e \geq e^*(0)} \left\{ V - \frac{c(e)}{c(e^*(0))} V - e \right\} \leq 0, \tag{6}$$

there are two asymmetric subgame perfect equilibria where players do not randomize in the investment stage. Expected bids and rent dissipation are lower than in a situation where the investments are exogenously fixed at zero. On the other hand, if (6) doesn't hold, then there are no subgame perfect equilibria where players use pure strategies in the investment stage.

Proof. Suppose that player $i = 1, 2$ invests $e_i = e^*(0)$. If player $j \neq i$ invests $e_j = 0$, his payoff is zero. If j invests $e_j \in (0, e^*(0)]$, j 's bid cost in the contest stage are higher than i 's, thus j 's overall payoff is $-e_j < 0$. Moreover, if (6) holds, j has no incentive to invest more than $e^*(0)$. Thus, for j, investing zero is a best reply. Now consider i : given $e_j = 0$, investing $e_i = e^*(0)$ is by definition optimal. Thus, $(e_i, e_j) = (e^*(0), 0)$ is an equilibrium of the first stage of the game; the properties of this equilibrium follow from the discussion above. On the other hand, if (6) does not hold, then player j can gain a strictly higher utility by investing more than $e^*(0)$. ■

Proposition 2 shows that there can be equilibria in which both players use pure strategies in the investment stage. This contrasts to a usual all pay auction, where there are no pure strategy equilibria. In an all pay auction, if one player bids zero, then the other player optimally bids a marginal positive amount ε. But then the first player has an incentive to outbid the second by bidding (say) 2ε. This gives him a payoff of $V - 2\varepsilon$ which is positive for small ε. This kind of argument does not apply to the first stage of the present model. If one player invests only marginally more than the other player, then his expected utility gain from the contest in stage two is marginally small. In order to get a strictly positive payoff from the contest, a player has to invest strictly more than the other does. This is why $e^*(0)$ will usually be strictly positive, and it can be the case that it doesn't pay to invest more than your opponent, given he invests $e^*(0)$.

The logic underlying Proposition 2 is related to the logic of entry into oligipolistic competition when competition is strong. For example, consider the following two stage game. On the first stage, firms choose whether to enter a market. Each firm that enters has to pay a setup cost. In the second stage, the firms that have entered compete in a Betrand game with constant marginal cost. In equilibrium, only one firm will enter, and thus there also arises an endogenous asymmetry even though firms are symmetric ex ante.[8]

An empirical example of endogenous asymmetry arising from investments ahead of contests is the sharp asymmetry among teams in professional sports leagues.[9]

[8]See, for example, Mas-Colell et al. (1995), Chapter 12E. The relationship to games of entry is even more obvious in the case where $c'' \leq 0$ discussed in Section 4.2 below: there the only undominated investment alternatives are zero investment and a certain positive investment. In the all-pay auction literature, not much has been done on entry. The analysis of minimum outlays for participation in Hillman and Samet (1987) differs because Hillman and Samet (1987) look at a one-shot simultaneous move game, where the set of pure strategies is restricted by a minimum expenditure requirement.

[9]I thank an anonymous referee for pointing this out. See Szymanski (2003) for a survey of the literature on sporting contests.

However, Proposition 2 also shows that equilibria where players play pure strategies in the investment stage do not always exist. Further, if they exist, they are asymmetric, and each player would prefer to be the only one who invests. So there might be a coordination failure. This suggests that we should take a look at mixed strategy equilibria. The next proposition describes a symmetric mixed strategy equilibrium.

Proposition 3 *In the game with simultaneous investment there is a symmetric equilibrium in mixed strategies. In this equilibrium player i does not invest with probability*

$$r = -\frac{c(0)}{Vc'(0)},$$

and with probability $1-r$ player i mixes according to the probability density function

$$f(e_i) = \frac{c''(e_i)}{(c'(e_i))^2} \frac{c(e_i)}{\left(1+\frac{c(0)}{Vc'(0)}\right)V}$$

over $[0, \bar{e}]$, where $\bar{e}$ is the unique solution to

$$-\frac{c(\bar{e})}{c'(\bar{e})} + \bar{e} = V. \tag{7}$$

Rent dissipation is complete.

Proof. See Appendix A.2. ■

Both players put the probability mass $r \geq 0$ on zero investment. This mass is zero only if $\lim_{e \to 0} \frac{c(e)}{c'(e)} = 0$. The remaining probability mass is distributed over $[0, \bar{e}]$. The support of the mixed strategy is connected due to the assumption that $c''(e) > 0$; we will see below that the support is not connected if $c''(e) < 0$.[10]

Expected bids in the contest stage can be greater or smaller than $V/c(0)$. First of all, if $c(0) = \infty$, then without investment the expected bids are zero. If furthermore $\lim_{e \to 0} \frac{c(e)}{c'(e)} = 0$, in the mixed strategy equilibrium both players

[10]The difference between this result and the corresponding Remark 1 in Kaplan et al. (2003) is due to the fact (already noted above) that, in the present paper, the payoff from investing more than one's competitor does depend on the investment of the competitor.

invest with probability one, and hence expected bids are positive. Therefore, expected bids can be greater than $V/c(0)$. On the other hand, one can also construct examples where expected bids are lower (see Example 1 below).

If the investment increases the value of winning instead of lowering bid cost, then we have the socially optimal investment in the asymmetric equilibria discussed in Proposition 2, but inefficient investment with probability one in the symmetric equilibria in Proposition 3.

4.2.2 Sequential timing of investment

In this section I consider the game with sequential investment. In this version of the model, player 1 invests first. Player 2 then observes e_1 and chooses e_2. Afterwards the first mover observes e_2, and the contest starts.

If inequality (6) holds, then the optimal investment of 1 is $e^*(0)$. Player 2 will react with $e^*(e^*(0)) = 0$. This equilibrium, which is the unique subgame perfect equilibrium, shares all the features of the asymmetric equilibria discussed in Proposition 2: rent dissipation is incomplete, and expected efforts are lower than without any investment.

But even if inequality (6) does not hold, there still is a subgame perfect equilibrium in which only the first mover invests. This is due to the fact that player 1 can choose an investment $e_{\lim}$ such that the best response of player 2 is not to invest at all.

To see that there is such an investment, note that

$$\max_e \left\{ V - \frac{c(e)}{c(e_1)} V - e \text{ s.t. } e \geq e_1 \right\}$$

decreases in e_1. Furthermore, the maximum is positive if $e_1 = 0$, and eventually gets negative as $e_1 \to \infty$. This implies that there is an unique $e_{\lim}$ such that

$$\max_e \left\{ V - \frac{c(e)}{c(e_1)} V - e \text{ s.t. } e \geq e_1 \right\} \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} 0 \text{ iff } e_1 \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} e_{\lim}.$$

By choosing $e_1 = e_{\lim}$, player 1 makes player 2 indifferent between investing zero and investing $\hat{e}(e_{\lim})$, where

$$\hat{e}(e_{\lim}) = \arg\max_{e} \left(V - \frac{c(e)}{c(e_{\lim})} V - e \text{ s.t. } e \geq e_{\lim} \right).$$

In both cases, player $2's$ expected utility is zero.

If player 1 invests $e_1 = e_{\lim}$ and player 2 invests zero, then the expected utility of player 1 is strictly positive. To see this, note that, since $V - \frac{c(e)}{c(0)} V - e$ is a concave function of e with a global maximum at $e = e^*(0) < e_{\lim}$, it is falling in e over the entire interval $[e_{\lim}, \hat{e}(e_{\lim})]$. Hence

$$E(u_1(e_{\lim}, 0)) = V - \frac{c(e_{\lim})}{c(0)} V - e_{\lim} \geq V - \frac{c(\hat{e}(e_{\lim}))}{c(0)} V - \hat{e}(e_{\lim}). \quad (8)$$

But

$$V - \frac{c(\hat{e}(e_{\lim}))}{c(0)} V - \hat{e}(e_{\lim}) > V - \frac{c(\hat{e}(e_{\lim}))}{c(e_{\lim})} V - \hat{e}(e_{\lim}) = 0, \quad (9)$$

where the inequality follows from $c(0) > c(e_{\lim})$ and the equality from the definition of $e_{\lim}$. Combining (8) and (9) we have $E(u_1(e_{\lim}, 0)) > 0$.

Proposition 4 *In the game with sequential timing of investment, there is a unique subgame perfect equilibrium. If inequality (6) holds, the first mover invests $e^*(0)$; otherwise he invests $e_{\lim}$. In any case, the second mover does not invest.*

Proof. The second mover chooses $e^*(e_1)$ in every subgame perfect equilibrium. If player 1 follows the strategy described above, clearly $e^*(e_1) = 0$.

If inequality (6) holds, $e^*(0)$ is obviously the best investment of decision player 1.

Now consider the case where (6) doesn't hold. If player 1 invests less than $e_{\lim}$, player 2 will invest more than player 1, and player $1's$ expected payoff will be $-e_1 \leq 0$. On the other hand, if $e_1 \geq e_{\lim}$, player 2 will not invest. Since $e_{\lim} > e^*(0)$ and $V - \frac{c(e)}{c(0)} V - e$ is a concave function of e with a global maximum at $e = e^* (0) < e_{\lim}$, player 1 has no incentive to invest more than $e_{\lim}$. ■

Thus, with sequential timing of investments the possibility to invest always lowers expected effort and rent dissipation.

If the investment increases the value of winning instead of lowering bid cost, then we have the socially optimal investment when inequality (6) holds; otherwise, we have overinvestment to deter the second mover from investing.

4.2.3 Endogenous timing of investment

Now I endogenize the order of moves. The investment stage consists of two periods, t=1 and t=2. In t=1, players can either invest or wait. In t=2, players observe all actions taken in t=1, and then those players who waited in t=1 can invest.

Proposition 5 *With endogenous timing of investment, there are always two asymmetric subgame perfect equilibria where players choose pure strategies in the investment stage which correspond to the equilibrium of the game with sequential play. In these equilibria, expected efforts and rent dissipation are lower than with investments exogenously fixed at zero. In addition, there exists a symmetric subgame perfect equilibrium where rent dissipation is complete.*

Proof. See Appendix A.3. ■

5 Extensions

5.1 Many players

In this section I show that the results do not depend on the assumption that there are only two players. I focus on the game with simultaneous investment; however, similar comments apply to the other cases as well. Suppose there are $n \geq 3$ players. Proposition 2 can be generalized easily. The condition for existence of subgame perfect equilibria where the players play pure strategies in the investment stage is still given by line (6), where $e^{*}(0)$ now denotes the optimal response of a player to zero investment by all his rivals. If condition (6) holds, there are n asymmetric subgame perfect equilibria. In each of these equilibria, exactly one player invests a positive amount. Expected effort and expected rent dissipation in these equilibria are reduced relative to the game without any investment.

As the following proposition shows, in the equilibria with mixing in the investment stage, we again have complete rent dissipation.

Proposition 6 *(a) There are $n-1$ different types of subgame perfect equilibria with mixing in the investment stage. These types differ in the number m of "active" players, where a player is called active if he invests a positive amount with a strictly positive probability, and $2 \leq m \leq n$.*
An active players does invest zero with probability

$$a_m := \left(-\frac{c(0)}{Vc'(0)}\right)^{\frac{1}{m-1}},$$

with the remaining probability, she randomizes according to the cumulative distribution function

$$F_m(e) = -\frac{a_m}{1-a_m} + \frac{1}{1-a_m}\left[\frac{1}{V}\left(e - \frac{c(e)}{c'(e)}\right)\right]^{\frac{1}{m-1}},$$

with support $[0, \bar{e}]$, where $\bar{e}$ is given in line (7) above.
The remaining $n-m$ players are inactive, that is, they invest zero with probability one.
(b) In each of these equilibria, rent dissipation is complete.

Proof. See Appendix A.4. ■

Proposition 6 shows that, for the discussed equilibria, the case $n > 2$ also exhibits complete rent dissipation.

5.2 Increasing returns to investment

The purpose of this section is to show that the main findings do not depend on the assumption that $c''(e) > 0$, again focussing on the game with simultaneous investments, and returning to the case $n = 2$. I will assume the following:

$$\exists e_{\max} : c'(e) < 0 \text{ and } c''(e) \leq 0 \ \forall e < e_{\max} \text{ and } c(e) = c(e_{\max}) > 0 \ \forall e \geq e_{\max}. \tag{10}$$

Assumption (10) says that there are weakly increasing returns to investment up to a certain maximum investment $e_{\max}$, but that the bid cost cannot be lowered under $c(e_{\max})$.

Given (10), $V - \frac{c(e_i)}{c(e_j)}V - e_i$ is convex in e_i at all $e_i \in [0, e_{\max}]$. Therefore each player will invest either zero or the maximal amount $e_{\max}$. Suppose that player i chooses $e_i = 0$. What is the optimal response of player j? If (5) holds, clearly $e^*(0) = e_{\max}$. But (5) is overly strong in this context, it is sufficient but not necessary for $e^*(0) = e_{\max}$. If (5) does not hold, but

$$V - \frac{c(e_{\max})}{c(0)}V - e_{\max} \geq 0, \tag{11}$$

to invest $e_{\max}$ is still an optimal response to $e_i = 0$. This leads to the following proposition.

Proposition 7 *Consider the game with observable simultaneous investments and assume that investment in cost reduction has weakly increasing returns up to some maximum investment* $e_{\max}$ *(i.e. (10) holds).*
(a) If (11) holds, there are two asymmetric subgame perfect equilibria where players use pure strategies in the investment stage. In each of these equilibria one player invests the maximal amount $e_{\max}$ *and the other player invests zero. In these equilibria, rent dissipation is incomplete and the expected effort is less than in the game without investment.*
(b) Furthermore, if (11) holds with strict inequality, there is a subgame perfect

equilibrium in which each player invests zero with probability

$$s = \frac{e_{\max}}{V - \frac{c(e_{\max})}{c(0)}V} \in (0,1)$$

and $e_{\max}$ *with the remaining probability* $1-s$*. In this equilibrium rent dissipation is complete.*
(c) If (11) does not hold there is a unique subgame perfect equilibrium with zero investment.

Proof. (a) and (c) follow from the discussion above. Concerning (b), first note that $s > 0$ since $e_{\max} > 0$, and $s < 1$ since by assumption (11) holds with strict inequality. Suppose player i behaves according to the described mixed strategy. Then the payoff of player j from investing $e_{\max}$ is

$$\frac{e_{\max}}{V - \frac{c(e_{\max})}{c(0)}V}\left(V - \frac{c\left(e_{\max}\right)}{c\left(0\right)}V - e_{\max}\right) + \left(1 - \frac{e_{\max}}{V - \frac{c(e_{\max})}{c(0)}V}\right)\left(-e_{\max}\right) = 0.$$

So he is indifferent between investing $e_{\max}$ and not investing at all. Clearly he has no incentive to invest an amount $e \in (0, e_{\max})$. ■

One interesting result is that expected bids in the mixed strategy equilibrium of the contest stage can be bigger or smaller than $V/c\left(0\right)$. This can be shown with a simple linear example.

Example 1 *Suppose* $c\left(e\right) = \max\left(k - e, k_{\min}\right)$.
Here $e_{\max} = k - k_{\min}$ *and* $c\left(e_{\max}\right) = k_{\min}$*. Inequality (11) holds iff* $V \geq k$*. In the mixed strategy equilibrium, a player doesn't invest with probability* $\frac{k}{V}$*. Note that this doesn't depend on* $k_{\min}$*. Expected bids equal*

$$E\left(x_1 + x_2\right) = \left(\frac{k}{V}\right)^2 \frac{V}{k} + 2\frac{k}{V}\left(1 - \frac{k}{V}\right)\frac{V}{k}\frac{k + k_{\min}}{2k} + \left(1 - \frac{k}{V}\right)^2 \frac{V}{k_{\min}}$$

Now if $k_{\min} \to 0$, $E\left(x_1 + x_2\right) \to \infty$*. If the minimal cost get very low, the expected bids for the case where both players happen to invest gets very high, and so expected bids are bigger than without investment. On the other hand, if* $V = 100$, $k = 90$ *and* $k_{\min} = 80$ *we get* $E\left(x_1 + x_2\right) = \frac{793}{720} < \frac{V}{k} = \frac{100}{90} \left(= \frac{800}{720}\right)$.

6 Conclusion

This paper studies investments that enhance the ability of contestants who engage in a perfectly discriminating contest. It distinguishes between observable and unobservable investments. If investments are not observable, the game collapses to a game in only one decision variable. As in the standard model of a completely discriminating contest, rent dissipation is complete. Surprisingly, expected bids can both be smaller or bigger than in a world where no investment is possible.

If investments are observable, they have an additional strategic effect. The paper considers three assumptions on the timing of the investment decisions: simultaneous moves, sequential moves with an exogenously predetermined order, and endogenous timing of moves. The paper finds that there is a strong tendency for the emergence of an endogenous asymmetry, even though players are identical ex ante. With simultaneous investments, equilibria in which players play a pure strategy in the investment stage are asymmetric, exhibit incomplete rent dissipation, and expected effort is reduced relative to the game without investment. There also are symmetric mixed strategy equilibria with complete rent dissipation: in these equilibria there is a positive probability that the players invest different amounts and hence also some tendency for an endogenous asymmetry. With sequential investment, the first mover always invests enough to deter the second mover from investing, and enjoys a first mover advantage. The game with endogenous timing of investments has multiple equilibria. There is an equilibrium where only one player invests, and rent dissipation is incomplete. There is also a symmetric equilibrium with mixing in the investment stage where rent dissipation is complete.

A Appendix

A.1 Proof of Proposition 1

Existence. Suppose that contestant i follows this strategy. Then, by bidding x, contestant j wins with probability $\int_0^x f(z)\,dz$ and thus his utility equals

$$E(u_j) = \int_0^x \frac{c(e^*(z))}{V} dz V - k(x).$$

By bidding $x_j = 0$ contestant j gets $u_j = 0$ since he loses with probability one and $k(0) = 0$. Also, by bidding $\bar{x}$ he gets $u_j = V - k(\bar{x}) = 0$. Now for all $x \in (0, \bar{x})$ expected utility is constant in x since

$$\frac{dE(u_j)}{dx} = c(e^*(x)) - k'(x) = 0$$

where the second equality follows by applying the envelope theorem on (3). Hence j is indifferent between all $x \in [0, \bar{x}]$. Bidding more than $\bar{x}$ is clearly suboptimal. Thus, the strategies are an equilibrium.

Uniqueness can be shown by a slight adaptation of the proof in Baye et al. (1996).

Expected utility of a contestant is equal to zero, thus rent dissipation is complete. Since the equilibrium is symmetric, in equilibrium each contestant wins with probability 1/2. Thus we must have $E[k(x)] = V/2$ or

$$E[c(e^*(x))x + e^*(x)] = \frac{V}{2}.$$

The expected bid of a contestant equals

$$E(x) = \int_0^{\bar{x}} x \frac{c(e^*(x))}{V} dx$$

Combining the last two equations, we find that

$$E(x) = \int_0^{\bar{x}} x \frac{c(e^*(x))}{V} dx = \frac{1}{2} - \frac{1}{V} E[e^*(x)].$$

Without investments, the expected bid of a contestant equals $\frac{V}{2c(0)}$. Thus, the expected sum of bids is higher with investment if

$$\frac{1}{2}-\frac{1}{V}E\left[e^{*}\left(x\right)\right]>\frac{V}{2c\left(0\right)}.$$

Expected bids can both be higher or lower than in a game where the investment is exogenously fixed to zero. Clearly, if $c\left(0\right)$ is infinite, then, without investment, expected bids are zero. Thus the bids will be higher in the game with investment. On the other hand, if $c\left(0\right)<V$, then investment lowers expected bids.

A.2 Proof of Proposition 3

First note that $\int_{0}^{\bar{e}}f\left(e\right)de=1$ and $f>0$, hence f is a probability density function. Also note that $r\geq 0$ since $c'\left(0\right)<0$, and $r<1$ by (5).

Suppose player j invests according to the mixed strategy described in the proposition. I will show that player i is indifferent between all $e_i\in\left[0,\bar{e}\right]$, and has no incentive to choose an $e_i>\bar{e}$. Clearly, if player i doesn't invest he gets a payoff of zero. If player i chooses an $e_i\in\left(0,\bar{e}\right]$ then his expected payoff from the contest stage depends on the investment of player j. If $e_j=0$, then i gets an expected payoff from the contest stage of $\left(1-\frac{c(e_i)}{c(0)}\right)V$. If player j invests $e_j<e_i$, player i gets an expected payoff from the contest stage of $\left(1-\frac{c(e_i)}{c(e_j)}\right)V$. Finally, if $e_j\geq e_i$ his expected payoff from the contest stage is zero. Therefore

$$\begin{aligned}
&E\left(u_i\left(e_i,e_j\right)\right)=\\
=\;&r\left(1-\frac{c\left(e_i\right)}{c\left(0\right)}\right)V+\left(1-r\right)\int_{0}^{e_i}\left(1-\frac{c\left(e_i\right)}{c\left(e_j\right)}\right)Vf\left(e_j\right)de_j-e_i=\\
=\;&r\left(1-\frac{c\left(e_i\right)}{c\left(0\right)}\right)V+\int_{0}^{e_i}\frac{c''\left(e_j\right)}{\left(c'\left(e_j\right)\right)^2}c\left(e_j\right)de_j-c\left(e_i\right)\int_{0}^{e_i}\frac{c''\left(e_j\right)}{\left(c'\left(e_j\right)\right)^2}de_j-e_i=\\
=\;&-\frac{c\left(0\right)}{c'\left(0\right)}\left(1-\frac{c\left(e_i\right)}{c\left(0\right)}\right)+\left[-\frac{c\left(e_j\right)}{c'\left(e_j\right)}+e_j\right]_{0}^{e_i}-c\left(e_i\right)\left[-\frac{1}{c'\left(e_j\right)}\right]_{0}^{e_i}-e_i=\\
=\;&0.
\end{aligned}$$

This shows that player i is indifferent between all $e_i \in [0, \bar{e}]$.

If player i chooses an investment $e_i > \bar{e}$, he gets

$$\begin{aligned} E\left(u_i\left(e_i, e_j\right)\right) &= -\frac{c\left(0\right)}{c'\left(0\right)V}\left(1-\frac{c\left(e_i\right)}{c\left(0\right)}\right)V \\ &\quad +\left(1+\frac{c\left(0\right)}{c'\left(0\right)V}\right)\int_0^{\bar{e}}\left(1-\frac{c\left(e_i\right)}{c\left(e_j\right)}\right)Vf\left(e_j\right)de_j - e_i \\ &= -\frac{c\left(\bar{e}\right)}{c'\left(\bar{e}\right)}+\bar{e}+c\left(e_i\right)\frac{1}{c'\left(\bar{e}\right)}-e_i \\ &= \frac{c\left(e_i\right)-c\left(\bar{e}\right)}{c'\left(\bar{e}\right)}+\bar{e}-e_i. \end{aligned}$$

The cost function $c(e)$ is strictly convex, hence $c(e_i) > c(\bar{e}) + c'(\bar{e})(e_i - \bar{e})$. Rearranging and using $c' < 0$ leads to

$$\frac{c\left(e_i\right)-c\left(\bar{e}\right)}{c'\left(\bar{e}\right)}+\bar{e}-e_i < 0 \quad \forall e_i > \bar{e}.$$

Hence $E(u_i(e_i, e_j)) < 0 \ \forall e_i > \bar{e}$. This shows that player i has no incentive to choose an $e_i > \bar{e}$.

Clearly the same considerations apply to the other player. Therefore the expected utility of both players is zero, and this implies that rent dissipation is complete.

A.3 Proof of Proposition 5

The following strategies constitute an asymmetric subgame perfect equilibrium.

- t=1: Player i invests $\max\{e^*(0), e_{\lim}\}$ and player j waits.
- t=2: If neither player invested in $t = 1$, then both play according to the symmetric mixed strategy equilibrium of the game with simultaneous investment. If one player waited and the other player invested $e > 0$ in $t = 1$, the first player reacts optimally by investing $e^*(e)$.

If player j behaves according to this strategy, player i cannot gain by deviating. If he waits in $t = 1$, then they reach a subgame which is actually the game with simultaneous investment. Since j then plays according to the mixed strategy equilibrium, $i's$ payoff is zero. If player i invests less then $\max\{e^*(0), e_{\text{lim}}\}$ in t=1, then in t=2 player j will invest more then e_i, and the payoff of i equals $-e_i \leq 0$. Further, if player i invests $\max\{e^*(0), e_{\text{lim}}\}$ in $t = 1$, player j cannot gain by deviating. So these strategies describe an equilibrium.

The following strategies constitute a symmetric subgame perfect equilibrium:

- t=1: Invest $\tilde{e} := \max(e^*(0), e_{\text{lim}})$ with probability

$$q = \frac{V - \frac{c(\tilde{e})}{c(0)}V - \tilde{e}}{V - \frac{c(\tilde{e})}{c(0)}V},$$

  and wait with probability $1 - q$.

- t=2: If neither player invested in $t = 1$, then both play according to the symmetric mixed strategy equilibrium of the game with simultaneous investment. If one player waited and the other player invested $e > 0$ in $t = 1$, the first player reacts optimally by investing $e^*(e)$.

Suppose j behaves according to these strategies. Then i can not gain from deviating in t=2 only. If he invests $e_i = \tilde{e}$ in t=1, he gets

$$q(-\tilde{e}) + (1-q)\left(V - \frac{c(\tilde{e})}{c(0)}V - \tilde{e}\right) = 0.$$

Investing $e_i = 0$ gives player i zero utility, too. Thus, i is indifferent between $e_i = 0$ and $e_i = \tilde{e}$. Investing more than $\tilde{e}$, or investing $e_i \in (0, \tilde{e})$, is worse than investing $\tilde{e}$. Thus, these strategies are an equilibrium. Moreover, expected utility is zero and hence rent dissipation complete.

A.4 Proof of Proposition 6

a) First I show that an active player (say, i) is indifferent between all $e \in [0, \bar{e}]$, given that all the other players behave according to the described strategies.

Clearly, the active player gets a payoff of zero if he doesn't invest. If he invests $e_i \in (0, \bar{e}]$, his expected payoff is

$$E\left(u_i^*\left(e_i\right)\right) = a_m^{m-1}\left(1 - \frac{c\left(e_i\right)}{c\left(0\right)}\right)V +$$
$$+\sum_{k=1}^{m-1}\left[\left(\begin{array}{c} m-1 \\ k \end{array}\right) a_m^{m-1-k}\left(1-a_m\right)^k \int_0^{e_i}\left(1 - \frac{c\left(e_i\right)}{c\left(e\right)}\right) V k F_m\left(e\right)^{k-1} F_m'\left(e\right) de\right] - e_i$$
$$= a_m^{m-1}\left(1 - \frac{c\left(e_i\right)}{c\left(0\right)}\right)V +$$
$$+\int_0^{e_i}\left(1 - \frac{c\left(e_i\right)}{c\left(e\right)}\right)\left[V\sum_{k=1}^{m-1}\left(\begin{array}{c} m-1 \\ k \end{array}\right) a_m^{m-1-k}\left(1-a_m\right)^k k F_m\left(e\right)^{k-1} F_m'\left(e\right)\right] de - e_i.$$

Now let us calculate the bracket $[\cdot]$ in the line above. By the definition of F_m, we have

$$\left[a_m + \left(1-a_m\right) F_m\left(e\right)\right]^{m-1} = \frac{1}{V}\left(e - \frac{c\left(e\right)}{c'\left(e\right)}\right).$$

Applying the binomic theorem, the left hand side equals

$$\left[a_m + \left(1-a_m\right) F\left(e\right)\right]^{m-1} = \sum_{k=0}^{m-1}\left(\begin{array}{c} m-1 \\ k \end{array}\right) a_m^{m-1-k}\left(\left(1-a_m\right) F_m\left(e\right)\right)^k.$$

Therefore

$$V\sum_{k=0}^{m-1}\left(\begin{array}{c} m-1 \\ k \end{array}\right) a_m^{m-1-k}\left(\left(1-a_m\right) F_m\left(e\right)\right)^k = e - \frac{c\left(e\right)}{c'\left(e\right)}.$$

Differentiating both sides with respect to e leads to

$$V\sum_{k=0}^{m-1}\left(\begin{array}{c} m-1 \\ k \end{array}\right) a_m^{m-1-k}\left(1-a_m\right)^k k F_m\left(e\right)^{k-1} F_m'\left(e\right) = \frac{c''\left(e\right) c\left(e\right)}{c'\left(e\right)^2}.$$

The left hand side is just the bracket $[\cdot]$ we wanted to calculate!

Plugging this back into player $i's$ utility, we get

$$E\left(u_i\left(e_i\right)\right)=a_m^{m-1}\left(1-\frac{c\left(e_i\right)}{c\left(0\right)}\right)V+\int_0^{e_i}\left(1-\frac{c\left(e_i\right)}{c\left(e\right)}\right)\frac{c''\left(e\right)c\left(e\right)}{c'\left(e\right)^2}de-e_i.$$

We already know from the proof of Proposition 3 that the right hand side of this equation is zero. Therefore we have shown that $E\left(u_i^*\left(e_i\right)\right)=0$ for all $e_i\in\left[0,\bar{e}\right]$. If the active player i invests $e_i>\bar{e}$ he gets a payoff

$$E\left(u_i\left(e_i\right)\right)=a_m^{m-1}\left(1-\frac{c\left(e_i\right)}{c\left(0\right)}\right)V+\int_0^{\bar{e}}\left(1-\frac{c\left(e_i\right)}{c\left(e\right)}\right)\frac{c''\left(e\right)c\left(e\right)}{c'\left(e\right)^2}de-e_i$$

which is negative by the proof of Proposition 3. So an active player has no incentive to deviate.

Further, no inactive player (say, j) has an incentive to invest a positive amount. This is shown by the following consideration. Suppose the first active player didn't invest. Then, conditional on the first active player not investing, the expected payoff of player j from investing $e_j\in\left(0,\bar{e}\right]$ is zero, since then player j is in the same position as one of the active players is ex ante. But, conditional on the first active player investing, player $i's$ utility has to be negative. Therefore his unconditional expected utility, which is a weighted sum of the two conditional expectations, is negative. And of course, there is never an incentive to invest more than $\bar{e}$.

b) We know from part a) that the expected utility of all players is zero. Therefore, rent dissipation is complete.

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