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Selection Tournaments, Sabotage, and Participation

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Abstract

This paper studies sabotage in tournaments with at least three contestants, where the contestants know each other well. Every contestant has an incentive to direct sabotage specifically against his most dangerous rival. In equilibrium, contestants who choose a higher productive effort are sabotaged more heavily. This might explain findings from psychology, where victims of mobbing are sometimes found to be overachieving. Further, sabotage equalizes promotion chances. The effect is most pronounced if the production function is linear in sabotage, and the cost function depends only on the sum of all sabotage activities: in an interior equilibrium, who will win is a matter of chance, even when contestants differ a great deal in their abilities. This, in turn, has adverse consequences for who might want to participate in a tournament. Since better contestants anticipate that they will be sabotaged more strongly, it may happen that the most able stay out and the tournament selects one of the less able with probability one. I also study the case where some contestants are easy victims, i. e. easier to sabotage than others.

JEL-Codes: M51, J41, J29

Keywords: tournament, contest, sabotage, selection

1 Introduction

Labor market tournaments have the double role of selecting the most able individuals and supplying incentives. Although many economists have voiced the opinion that the selection aspect is at least as important as the incentives aspect (e.g. Rosen 1986, Schlicht 1988, Glazer and Hassin 1988, Prendergast 1999), the focus of the bulk of research has clearly been on the latter. This paper explicitly addresses the selection aspect. The question is whether tournaments, and, more generally, relative comparison contests, tend to select the most able individuals.

In most tournament models, more able contestants have a greater chance of winning.[1] But the picture changes radically once we take into account that tournaments - like other relative comparison contests - give each contestant an incentive to sabotage his rivals (Lazear 1989, Konrad 2000, Chen 2003). Here "sabotage" is a catchall term for different kinds of activities that are intended to hinder the productive efforts of other contestants. These range from strategic withholding of information, less mutual help, to outright forms of mobbing and actual physical sabotage. There is one obvious problem in using a tournament for selection in the presence of sabotage. The result might be the promotion of the best saboteur - a contestant who might be not very good at working productively - and promoting the best saboteur is not necessarily in the interest of the firm. This is a particularly striking example of the more general point that ability is a multidimensional property, and that the abilities and personality traits needed to win a tournament are not always the same as those needed at a higher level in a hierarchy.

This paper focusses on a more subtle point. It starts with two observations. In many real world tournaments, there are more than two contestants who compete for a single prize; thus I will assume throughout that the number of contestants is at least three. And the contestants often know each other well, especially if they work closely and regularly together. In such a case each contestant knows who his most dangerous rival

[1] Lazear and Rosen (1981), Nalebuff and Stiglitz (1983), Green and Stokey (1983), Rosen (1986).

is. Intuitively, sabotaging a strong rival improves one's own chance of winning more than sabotaging a weaker rival does. Therefore, each contestant has an incentive to sabotage the most dangerous rival most strongly. The paper shows that this has an equalizing effect on the winning probabilities. The effect is most pronounced when the production functions are linear in sabotage, and the cost functions of the contestants depends only on the sum of the sabotage activities: in an interior equilibrium, each contestant has the same chance of winning. In other words, who will win the tournament is a matter of pure chance, even if some contestants are much more able than other contestants.

In fact, the selection properties of tournaments may be even much worse. Since the most able individuals are sabotaged most, they may well have a lower expected utility from participation in the tournament. Once we take into account the fact that participation in a tournament is endogenous, it turns out that only the least able individuals may want to participate. In that case, a tournament selects one of the least able with probability one.

These results are derived in a model in which all contestants are similar in their ability to cope with sabotage. However, due to different abilities, or different positions within the firm, some contestants may be easier to sabotage than others. Thus I also consider the case where some contestants are 'easy victims'. Easy victims are sabotaged more strongly and have lower chances of winning the tournament.

The incentives to sabotage were pointed out early in the tournament literature (Nalebuff and Stiglitz 1983, p. 40). The present paper is most closely related to Lazear (1989) and Chen (2003). Lazear (1989) considers the optimal tournament reward structure from the incentives aspect and shows that, in the presence of sabotage, the optimal prize structure is compressed. However, Lazear does not discuss the possibility of directing sabotage specifically against stronger rivals and the implications of this for the selection properties. Chen (2003) studies the implication of sabotage in selection tournaments. He points out the fact that some contestants may have a comparative advantage in sabotaging, and shows that stronger contestants do not necessarily have better chances of winning in equilibrium. In contrast to Chen (2003), I assume that the output produced by a contestant is additively separable in the sabotage activities of his rivals. This drives the equalizing effect of sabotage, which is not found in Chen (2003). Other papers that study sabotage include Drago and Turnbull (1991) (who study how bargaining between workers about effort and mutual help affects optimal incentive schemes), Chan (1996) (who studies external recruitment as a means of keeping sabotage incentives low), Kräkel (2000) (who considers the effect of relative deprivation in tournaments with sabotage), and Chen (2005) (who studies incentive effects of external recruitment).

My paper contributes to this literature in the following ways. First, I show that if production is additively separable, sabotage has an equalizing effect on the winning probabilities. In the classic benchmark where the production functions are linear and the cost function depends only on the sum of all sabotage activities, in an interior equilibrium, all contestants have equal chances of winning. Second, I show that only the weaker contestants my want to participate. A third contribution is the result that easy victims are sabotaged more heavily and thus have lower chances of winning in equilibrium. A fourth contribution of my paper is that it studies conditions under which interior equilibria exist. Due to the complexity of the problem, interior equilibria are assumed in most of the literature. In an example with a specific cost function, I derive necessary and sufficient conditions for existence of interior equilibria. It turns out to be crucial that the contestants do not differ extremely in their abilities. Hence, an analysis based on first order conditions for an interior equilibrium may be misleading. However, the effect that sabotage tends to make promotion chances more equal is robust even for equilibria with corner solutions.

There is considerable evidence for the importance of sabotage, both from field data (Drago and Garvey 1998)[2] and from experiments (Harbring and Irlenbusch 2004). Harbring et al. (2004) is an interesting experimental study of sabotage in an asymmetric contest. It gives support to the idea that more able contestants will be sabotaged more

[2]There are also papers that use data from sports: Becker and Huselid (1992) (auto racing) and Garicano and Palacios-Huerta (2000) (European soccer).

heavily. However, the model and experimental setup in Harbring et al. (2004) differ from the model used here.[3]

The present paper also sheds new light on findings from psychology, where in some studies, victims of bullying in the workplace are found to be "overachieving": more achievement oriented, punctual, accurate and conscientious than the control group (Zapf and Einarsen 2003, p. 178). While this is often explained with regard to group norms, the present paper offers another explanation. As I show below, people that choose higher productive effort are sabotaged more strongly. The reason is that they are more dangerous rivals in a contest for promotion. Another finding is that victims of bullying tend to be more vulnerable than the control groups, e.g. "low in social competencies, bad conflict managers, unassertive and weak personalities" (Zapf and Einarsen 2003, p. 174ff). I capture this in a stylized way by considering easy victims.

Sabotage-like activities have been studied in other contexts as well. Shubik's (1954) model of a "truel" (three person duel) is closely related to the present paper. Here the "truelist" with the lowest shooting ability may have the best chances of survival. The reason is that the contestants have an incentive to shoot at the truelist who is the best shot. Baumol (1992) considers sabotage in the process of innovation. Skaperdas and Grofman (1995) and Harrington and Hess (1996) model negative campaigning in election races. Konrad (2000) studies sabotage in rent seeking contests. Auriol et al. (2002) show that, when the principal cannot commit to long term contracts, career concerns in teams give the agents incentives to sabotage, even if they are not involved in a tournament scheme. The results of the present paper are also relevant to these other contests.

In addition, the paper also contributes to the small but growing literature on the selection properties of tournaments and other kinds of contests. One important paper in

[3] Harbring et al. (2004) consider a two stage game: on the first stage, sabotage is chosen, on the second stage productive effort; sabotage increases the victim's marginal cost of working productively; and sabotage of player i against an opponent j can take only two values. In contrast, in the present paper a one shot simultaneous move game is studied: sabotage and productive effort are chosen simultaneously; sabotage destroys part of the output produced by the victim; and sabotage of i against j can take any nonnegative real number.

this literature is Rosen (1986), who studied a sequential elimination tournament. Meyer (1991) works out how to design a repeated contest between the same contestants in order to get the most information about the contestants. Clark and Riis (2001) study a selection tournament in the case where performance is deterministic. They show that, by making the winner prize depend on which of two test standards are passed, the tournament can be designed to select the most able contestant as a winner. Hvide and Kristiansen (2003) consider risk taking in a selection contest. However, none of these papers considers sabotage.

The paper proceeds as follows. Section 2 sets out the model. Section 3 shows that sabotage tends to equalize promotion chances. Section 4 considers the decision whether to participate in a tournament. Section 5 studies easy victims. Section 6 gives necessary and sufficient conditions for existence of an interior equilibrium, albeit for a somewhat simplified example. Section 7 concludes. The appendix collects some of the longer proofs.

2 The model

There are $n \geq 3$ contestants. For simplicity, the contestants are assumed to be risk neutral. Contestant i chooses productive effort x_i and sabotage efforts $s_{i1}, ..., s_{i(i-1)}$, $s_{i(i+1)}, ..., s_{in}$, where s_{ij} denotes the sabotage of contestant i against contestant j. He has a personal cost of doing so which is given by

$$c_i \left(x_i, s_{i1}, ..., s_{i(i-1)}, s_{i(i+1)}, ..., s_{in} \right), \tag{1}$$

where $c_i : R^n \to R$ is increasing in all its arguments and convex. Contestants differ in their abilities, hence each contestant may have a different cost function. I will assume that the cost function is *symmetric in the sabotage activities*: exchanging s_{ij} and s_{ik}, while holding constant all other decision variables of i, does not change the costs of i. The cost functions are common knowledge among the contestants. This simplifying assumption captures the idea that work colleagues often know each other pretty well, while their superiors know considerably less about them.

The output produced by contestant i is denoted by q_i and assumed to be additively separable:

$$q_i = \phi(x_i) - \sum_{j \neq i} \psi(s_{ji}) + \varepsilon_i. \tag{2}$$

Here, ϕ and ψ are strictly increasing and weakly concave functions: ϕ is a production function, and similarly, ψ is a 'destruction function' which describes how easy or difficult it is to destroy output of a rival.[4] Equation (2) assumes that there are no cross effects between the sabotage activities. This is different from Chen (2003), where the marginal impact of sabotage against i depends on how much sabotage is inflicted on i by *all* his rivals.[5] Furthermore, (2) also presupposes that there are no cross effects between x_i and the sabotage directed against i. Finally, ε_i is an error term. The error terms $\varepsilon_1, \ldots, \varepsilon_n$ are identically and independently distributed with PDF f. Let F denote the CDF corresponding to f. I assume that F has full support and is strictly log-concave.[6]

The contestant with the highest output gets a winner prize w, which represents the monetary equivalent of a promotion. All the other contestants get a strictly lower loser prize which is normalized to zero. Let p_i denote contestant i's probability of winning. Then his payoff is

$$u_i = p_i w - c_i\left(x_i, s_{i1}, \ldots, s_{i(i-1)}, s_{i(i+1)}, \ldots, s_{in}\right).$$

[4] One could also assume that these functions differ across contestants, such that $q_i = \phi_i(x_i) - \sum_{j \neq i} \psi_j(s_{ji}) + \varepsilon_i$. This would not change Proposition 1 qualitatively.

[5] In the notation of the present paper, Chen (2003) assumes that $q_i = a_i x_i - \psi\left(\sum_j b_j s_{ji}\right) + \varepsilon_i$, where ψ is an increasing and strictly concave function, a_i is the productive ability of i, and b_j the ability of j to sabotage. An additional but minor difference between the present paper and Chen (2003) is that Chen (2003) assumes identical cost functions for all players and models different abilities with different production functions. Finally, Chen (2003) assumes a cost function which depends only on the sum of all the sabotage activities plus productive effort.

[6] The assumption of log-concavity is fulfilled by most commonly studied distribution functions, see Bagnoli and Bergstrom (1989).

Contestant i maximizes u_i subject to the non-negativity constraints $x_i \geq 0$ and $s_{ij} \geq 0$ for all $j \neq i$.

3 Sabotage equalizes promotion chances

Let

$$y_{il} := \phi(x_i) - \sum_{k \neq i} \psi(s_{ki}) - \left(\phi(x_l) - \sum_{k \neq l} \psi(s_{kl}) \right).$$

Note that $y_{il} = E(q_i) - E(q_l)$. Using this notation, we have

$$p_i = \int_{-\infty}^{\infty} \left[\Pi_{l \neq i} F(y_{il} + \varepsilon_i) \right] f(\varepsilon_i) \, d\varepsilon_i. \tag{3}$$

The results of the paper are built on the following lemma.

Lemma 1 *For all values of the decision variables,*

$$p_j > p_k \Leftrightarrow \frac{1}{\psi'(s_{ij})} \frac{\partial p_i}{\partial s_{ij}} > \frac{1}{\psi'(s_{ik})} \frac{\partial p_i}{\partial s_{ik}}.$$

Proof. Differentiating equation (3), we get

$$\begin{aligned}
\frac{\partial p_i}{\partial s_{ij}} &= \psi'(s_{ij}) \int_{-\infty}^{\infty} f(y_{ij} + \varepsilon_i) \left[\Pi_{l \neq i,j} F(y_{il} + \varepsilon_i) \right] f(\varepsilon_i) \, d\varepsilon_i, \\
\frac{\partial p_i}{\partial s_{ik}} &= \psi'(s_{ik}) \int_{-\infty}^{\infty} f(y_{ik} + \varepsilon_i) \left[\Pi_{l \neq i,k} F(y_{il} + \varepsilon_i) \right] f(\varepsilon_i) \, d\varepsilon_i.
\end{aligned}$$

Hence,

$$\begin{aligned}
\frac{1}{\psi'(s_{ij})} \frac{\partial p_i}{\partial s_{ij}} - \frac{1}{\psi'(s_{ik})} \frac{\partial p_i}{\partial s_{ik}} = &\int_{-\infty}^{\infty} \left[f(y_{ij} + \varepsilon_i) F(y_{ik} + \varepsilon_i) - f(y_{ik} + \varepsilon_i) F(y_{ij} + \varepsilon_i) \right] \\
&* \left[\Pi_{l \neq i,j,k} F(y_{il} + \varepsilon_i) \right] f(\varepsilon_i) \, d\varepsilon_i.
\end{aligned} \tag{4}$$

Suppose $p_j > p_k$. This is equivalent to $E(q_j) > E(q_k)$ and to $y_{ij} < y_{ik}$. Since $F(z)$ is strictly log-concave, $f(z)/F(z)$ decreases strictly in z. It follows that

$$\frac{f(y_{ij}+\varepsilon_i)}{F(y_{ij}+\varepsilon_i)} > \frac{f(y_{ik}+\varepsilon_i)}{F(y_{ik}+\varepsilon_i)},$$

or

$$f(y_{ij}+\varepsilon_i)\,F(y_{ik}+\varepsilon_i) > f(y_{ik}+\varepsilon_i)\,F(y_{ij}+\varepsilon_i) \tag{5}$$

for all ε_i. Therefore the integrand in equation (4) is strictly positive.

This proves that

$$p_j > p_k \Rightarrow \frac{1}{\psi'(s_{ij})}\frac{\partial p_i}{\partial s_{ij}} > \frac{1}{\psi'(s_{ik})}\frac{\partial p_i}{\partial s_{ik}}.$$

The converse statement can be proven similarly. ■

To understand Lemma 1, note that, in order to destroy one (marginal) unit of j 's output, i has to increase s_{ij} by $1/\psi'(s_{ij})$. Thus, $\frac{1}{\psi'(s_{ij})}\frac{\partial p_i}{\partial s_{ij}}$ measures how much i 's chances to win increase if i destroys one unit of j 's output. If $p_j > p_k$, Lemma 1 says that destroying one unit of the output of j increases the chances of i more than destroying one unit of the output of k does.

The basic insight behind the crucial line (5) in the proof is as follows. For a given ε_i, the probability that i has a higher output than j is $F(y_{ij}+\varepsilon_i)$. Destroying one unit of the output of j increases i 's chance to win against j by $f(y_{ij}+\varepsilon_i)$. However, winning against j is beneficial for i only if i simultaneously wins against all other contestants, including k. The probability to win against k is $F(y_{ik}+\varepsilon_i)$. Thus, for a given ε_i, the marginal benefit of destroying one unit of the output of j is proportional to $f(y_{ij}+\varepsilon_i)\,F(y_{ik}+\varepsilon_i)$. Similarly, the marginal benefit of destroying one unit of the output of k is proportional to $f(y_{ik}+\varepsilon_i)\,F(y_{ij}+\varepsilon_i)$. These two terms differ for two reasons. First, since $p_j > p_k$, $E(q_j) > E(q_k)$ and $y_{ij} < y_{ik}$; hence i is more likely to win against k than against j : $F(y_{ik}+\varepsilon_i)$ is greater than $F(y_{ij}+\varepsilon_i)$. Second, $f(y_{ij}+\varepsilon_i)$ may differ from $f(y_{ik}+\varepsilon_i)$. If the density is decreasing, $f(y_{ij}+\varepsilon_i)$ is greater than $f(y_{ik}+\varepsilon_i)$, and the second effect goes in the same direction as the first. If the density is increasing, the effects go in opposite directions. However, the assumption that F is log-concave ensures that the density is not increasing too rapidly, and thus the first effect dominates.

In any pure strategy equilibrium, the following first order condition for optimal sabotage must hold for all i and all $j \neq i$:

$$\frac{\partial p_i}{\partial s_{ij}} w - \frac{\partial c_i}{\partial s_{ij}} \leq 0, \; s_{ij} \geq 0, \; s_{ij} \left(\frac{\partial p_i}{\partial s_{ij}} w - \frac{\partial c_i}{\partial s_{ij}} \right) = 0.$$

Using Lemma 1, one can show from this first order condition that, in equilibrium, a contestant will sabotage a rival with a higher expected output more strongly.

Proposition 1 *In any equilibrium, if* $E(q_j) > E(q_k)$ *, then* $s_{ij} \geq s_{ik}$ *(with strict inequality unless* $s_{ij} = s_{ik} = 0$*) for all* $i \neq j, k$*.*

Proof. See appendix A.1. ∎

The intuition for this result is from Lemma 1: if j has a higher expected output than k, and hence a higher chance of winning, then for all the other contestants $i \neq j, k$ the incentives to sabotage j are higher.

It is interesting to compare Proposition 1 with the model of Chen (2003), where the contestant with the highest expected output is not necessarily sabotaged most (see Chen 2003, p. 132). The difference between the findings is due to the different assumptions on how sabotage diminishes the output of the victim: in the present paper, output is additively separable in sabotage (see equation (2)), which is not the case in Chen (2003).

Proposition 1 has interesting implications for the question of whether the more able contestants have greater chances of winning in equilibrium. Consider only the decisions of contestants j and k and assume that (disregarding the decisions of the other contestants) contestant j has a higher expected output and a hence better chance of winning. Corollary 1 below states a property of the reaction of the other contestants to such a situation: they will destroy more of the output of j than of k.

Corollary 1 *In any equilibrium, if*

$$\phi(x_j) - \psi(s_{kj}) \geq \phi(x_k) - \psi(s_{jk}), \tag{6}$$

then

$$\sum_{i \neq j,k} \psi(s_{ij}) \geq \sum_{i \neq j,k} \psi(s_{ik}), \tag{7}$$

but nevertheless $E(q_j) \geq E(q_k)$.

Proof. See appendix A.2. ■

Corollary 1 shows that the sabotage activities have an equalizing effect on the winning probabilities.[7] If, looking only at the decisions of j and k, player j has a higher expected output, then the other players will destroy more of the output of j than of k. However, Corollary 1 also shows that j still has a weakly higher expected output than k - thus, sabotage of the other players $i \neq j, k$ does not reverse the initial inequality.

Proposition 1 and Corollary 1 hold for all equilibria, even if they involve corner solutions. Additional results can be gained by focussing on interior equilibria, where all decision variables are strictly positive. Given the complexity of the problem, it is very difficult to derive general conditions for the existence of interior equilibria. Therefore, in what follows I can do no better than to simply assume the existence of an interior equilibrium.[8] However, Section 6 below I also provide, for a simplified example, necessary and sufficient conditions for an interior equlbrium.

Assuming an interior equilibrium, one can show that if (6) holds with strict inequality, then (7) holds with strict inequality, too. However, in case of corner solutions it may happen that neither j nor k are sabotaged at all by the remaining contestants.

[7] This does not mean that promotion chances are necessarily more equal in a model with sabotage than in a model where sabotage is exogenously fixed at zero. Consider the case where the productive abilities of the players are equal, but their sabotage activities differ immensely.

[8] Most of the literature does this, e.g. Chen (2003).

Clearly, the effect described in Corollary 1 hinges on the presence of at least three players. It is most pronounced in the following special case. Suppose that output is linear in sabotage:

$$q_i = \phi(x_i) - \sum_{j \neq i} s_{ji} + \varepsilon_i. \tag{8}$$

Further, suppose that the cost function depends only on the sum of the sabotage activities - not on the distribution of this sum over one's rivals:

$$c_i\left(x_i, \sum_{j \neq i} s_{ji}\right) \tag{9}$$

where $c_i : R^2 \rightarrow R$ is an increasing and convex function. For future reference, I will call the cost function in (9) *total-sabotage-dependend*.[9]

Proposition 2 *Suppose that output is linear in sabotage as in (8) and that the cost function is total-sabotage-dependend as in (9). In an interior equilibrium, every contestant* $i = 1, ..., n$ *wins with the same probability* $p_i = 1/n$.

Proof. In an interior equilibrium i sabotages all his rivals. If $p_j > p_k$, it follows from Lemma 1 and $\psi'(s_{ij}) = 1$ that $\frac{\partial p_i}{\partial s_{ij}} > \frac{\partial p_i}{\partial s_{ik}}$. Now i can decrease s_{ik} by a small amount and, at the same time, increase s_{ij} by the same amount. By (9), his cost is unchanged, but his probability of winning is higher than before, so the initial situation cannot have been an equilibrium. Therefore, we must have $p_j = p_k$ for all $j, k \neq i$ in an equilibrium where i sabotages all his rivals. If all contestants sabotage all their rivals, it follows that $p_1 = ... = p_n = 1/n$. ■

Proposition 2 says that, in an interior equilibrium, who will win the tournament is a matter of pure chance.[10] Those contestants who produce more are sabotaged more

[9] See Auriol et al. (2002) for a discussion about different specifications of the cost function in a related setting.

[10] One can make Proposition 2 a bit stronger: we do not have to restrict attention to interior equilibria where literally *all* contestants sabotage *all* their rivals. I show in appendix A.3 that, if at least one of

strongly. As mentioned in the introduction, this is in line with some recent results from psychology on mobbing. The intuition behind Proposition 2 is simple: if (say) contestant 1 had a higher probability of winning than contestant 2, then it would be better for contestant 3 to increase s_{31} by a small amount and, at the same time, decrease s_{32} by the same amount. By Lemma 1, this would increase his chance of getting the promotion. And if the cost function is total-sabotage-dependend, it does not change his costs.

4 Participation

So far it was assumed that any player can have a completely different cost function. While this shows the generality of the results, the drawback is that there is no straightforward way to compare the abilities of different players. In order to compare the incentives to participate in a tournament, it is helpful to have some more specific assumptions on the cost functions. Thus, consider the following special case of a total-sabotage-dependend cost function:

$$c_i\left(x_i, s_{i1}, ..., s_{i(i-1)}, s_{i(i+1)}, ..., s_{in}\right) = \alpha_i C\left(x_i\right) + \beta_i S\left(\sum_{j \neq i} s_{ij}\right) \tag{10}$$

where C and S are strictly increasing and strictly convex functions. Here, each contestant is characterized by two parameters. A high value of α_i means that i has high costs of working productive and hence is of low productive ability. Similarly, a high value of β_i means that i has high costs of sabotaging and hence has a low ability to sabotage.

Suppose that output is linear in sabotage (8) and the cost functions are given by (10).

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the following conditions holds in an equilibrium, then $p_i = 1/n$ for all $i = 1, ..., n$ in this equilibrium:

1. There are at least two contestants who sabotage all their rivals.
2. Each contestant is sabotaged by at least two rivals.
3. The contestants can be renumbered so that $s_{i(i+1)} > 0$ for $i = 1, .., n-1$ and $s_{n1} > 0$.

Then, in an interior equilibrium the first order conditions reduce to

$$\begin{aligned} wg\phi'(x_i) &= \alpha_i C'(x_i) \\ \frac{wg}{(n-1)} &= \beta_i S'\left(\sum_{j\neq i} s_{ij}\right), \end{aligned} \tag{11}$$

where

$$g := \frac{\partial p_i}{\partial x_i}\Big|_{y_{ij}=0\forall j\neq i} = (n-1)\int_{-\infty}^{\infty} F(\varepsilon)^{n-2} f(\varepsilon)^2 \, d\varepsilon.$$

Due to the additive separability of the cost functions and the production function, x_i does not depend on β_i, and $\sum_{j\neq i} s_{ij}$ does not depend on α_i. As the following proposition shows, i will be sabotaged more than j if and only if i has a higher productive ability.

Proposition 3 *Assume that output is linear in sabotage as in (8) and the cost functions are total-sabotage-dependend as in (10). In an interior equilibrium, a player who has a higher productive ability is sabotaged more.*

Proof. By Proposition 2, $p_i = p_j = 1/n$ in an interior equilibrium, and hence $E(q_i) = E(q_j)$ or

$$\sum_{k\neq i} s_{ki} - \sum_{k\neq j} s_{kj} = \phi(x_i) - \phi(x_j). \tag{12}$$

From (11),

$$\frac{dx_i}{d\alpha_i} = \frac{C'(x_i)}{wg\phi''(x_i) - \alpha_i C''(x_i)} < 0. \tag{13}$$

Suppose i has a higher productive ability, i. e. a lower cost parameter $\alpha_i < \alpha_j$. From (13), we have $x_i > x_j$; together with (12) this implies that $\sum_{k\neq i} s_{ki} > \sum_{k\neq j} s_{kj}$. ■

Proposition 3 is closely related to Theorem 1 in Chen (2003). In fact, if all players have the same ability in sabotage, the results are identical (see Corollary 1 in Chen 2003). However, Proposition 3 says that a player will be sabotaged more heavily if he has a higher *absolute* ability in production - independent of the abilities to sabotage - whereas in Chen (2003), a player is subject to more total attack if he has a higher *relative* ability in production.

In an interior equilibrium, the probability of winning is independent of the individual cost parameters. Thus, equilibrium utilities differ only because the contestants incur different costs. A higher cost parameter α_i has two effects. First, the direct effect that, for any given effort, the associated costs are higher. Second, the indirect effect that a player with a higher α_i chooses a lower effort, which ceteribus paribus leads to lower costs. Which of the effects dominates depends on the cost function and on the production function. If the cost function $C(\cdot)$ is very convex, or the production function $\phi(\cdot)$ very concave, then a change in α_i has only a small effect on the effort chosen, and thus the first effect dominates. Proposition 4 makes this precise. Define

$$\rho(x) := \frac{C'(x)}{\phi'(x)}.$$

Proposition 4 *In an interior equilibrium, utility is decreasing in productive ability if*

$$\frac{C(x_i)}{C'(x_i)} < \frac{\rho(x_i)}{\rho'(x_i)}; \tag{14}$$

utility is decreasing in sabotage ability if the cost-of-sabotage-function $S(\cdot)$ is strictly log-concave.

Proof. See appendix A.4. ■

To illustrate, consider the case where $\phi(x) = x$. Then any log-concave cost function (for example the class of functions $C(x) = x^\eta$, where η is a parameter) implies that a higher productive ability leads to lower equilibrium utility. On the other hand, if $C(x) = \exp(x)$, the two effects exactly offset each other and equilibrium utility is independent of α_i. Finally, if $C(x) = \exp(\exp(x))$, equilibrium utility is increasing in α_i.

Propositions 3 and 4 indicate that the firm may have problems retaining the more productive contestants, since they are victims of more sabotage and may have a lower equilibrium utility from participating in the tournament. Of course, the principal can always induce participation by all types of agents by offering a fixed payment for participation. In that case, the contestants of lower ability would have a rent, while the participation constraint of the higher ability contestants are binding.

5 Easy victims

Some contestants may be easier to sabotage than others. This can be due to personal differences between the contestants. People differ in their ability to cope with a hostile environment. Or it may be due to different positions or experience within the firm. For example, workers who are relatively new depend more strongly on the help of other workers, if only to get information about the job and the firm. They are therefore more vulnerable to sabotage.

To capture this is in the model, I will for simplicity return to a specification where output is linear in the sabotage activities, with the additional twist that some contestants are easier to sabotage than others:

$$q_i = \phi\left(x_i\right) - b_i \sum_{j \neq i} s_{ji} + \varepsilon_i \tag{15}$$

where a high value of b_i means that i is an easy victim.

Proposition 5 *Suppose production functions are given by (15) and the cost functions are total-sabotage-dependent as in (9). In an interior equilibrium, contestant j has a higher chance of winning than contestant k if and only if $b_j < b_k$.*

Proof. See appendix A.5. ■

Proposition 5 says that contestants who are easy to sabotage will have lower chances of winning in an interior equilibrium. The tournament will select *only* on the basis of the ability to cope with sabotage. If $b_1 < b_2 < ... < b_n$, then $p_1 > ... > p_n$. This may or may not be in the interest of the firm. In particular, there is no reason to assume that low vulnerability to sabotage on the one hand, and ability to work productively on the other, always go together.

By Proposition 5, if two contestants behave equally in an equilibrium, the one who is an easier victim will be sabotaged more heavily. As mentioned in the introduction, this fits with some results from the psychological literature on mobbing or bullying. Basically, within the model, there are two reasons why a contestant might become a victim: being an overachiever and therefore a dangerous rival, and being an easy victim.

6 Existence of interior equilibria: an example

Existence of interior equilibria chances is not automatically ensured. There can be two types of corner solutions. First, there might be no sabotage at all in equilibrium. This is especially likely when the marginal cost of the first unit of sabotage is high, and if the number of contestants is high (see Konrad (2000)). The reason is that sabotage involves a positive externality to all the contestants except the one who is sabotaged. This externality is more important when there are many contestants, and sabotage is therefore less attractive.

However, even if there is some sabotage in equilibrium, there can still be corner solutions of a second type. For example, if there is one contestant ("she") who is much better than all her rivals, she will have a higher chance of winning in the equilibrium even though only she is sabotaged by all the other contestants. In such a situation, it doesn't pay for the other contestants to sabotage anyone except her, so they direct all sabotage against her. Intuitively, one would expect corner solutions of this type if the contestants are very different. Given the complexity of the problem, it is very difficult to derive general conditions for existence of interior equilibria.[11] However, some important

[11] Thus, Chen (2003) assumes existence of interior equilibria.

lessons can be learned by considering the following example.

Example 1 *Output is linear in sabotage and productive effort*

$$q_i = x_i - \sum_{j \neq i} s_{ji} + \varepsilon_i.$$

The cost functions are given by

$$c_i \left(x_i, \sum_{k \neq i} s_{ik} \right) = \frac{\gamma_i}{2} \left(x_i^2 + \left(\sum_{k \neq i} s_{ik} \right)^2 \right).$$

There are two types of contestants: l low cost contestants with $\gamma_i = 1$, and $h = n - l$ high cost contestants with $\gamma_i = \gamma > 1$.

In this example, the contestants differ only in one parameter. This parameter γ is a natural way to measure how different the contestants are. Higher values of γ imply greater differences between contestants. Further, there will always be some sabotage in equilibrium, since a contestant who does not sabotage at all would have zero marginal cost of sabotaging.

Proposition 6 *Consider example 1.*

a) A necessary condition for the existence of an interior equilibrium is

$$\gamma \leq \begin{cases} 1 + \frac{n}{l(n-2)}, & \text{if } l \geq 2, \\ 1 + \frac{n(n-2)}{n^2-2n+2}, & \text{if } l = 1. \end{cases} \tag{16}$$

b) If, in addition, the inequality

$$\max_z \left(\frac{\partial^2}{\partial z^2} \int_{-\infty}^{\infty} F(z+\varepsilon)^{n-1} f(\varepsilon)\, d\varepsilon \right) < \frac{(n-1)^2}{n^2 - 2n + 2} \frac{1}{w} \tag{17}$$

holds, then existence of interior equilibria is ensured.

Proof. See appendix A.6. ■

Inequality (17) serves to rule out problems related to possible non - concavities of the objective function.[12] To give an example, if the error terms follow a Gumbel distribution, it can be shown that inequality (17) holds if the variance of the error terms is high enough.

While Proposition 6 confirms the intuition that there will not be an interior equilibrium if the contestants are very different, it also shows that contestants can differ substantially and nevertheless have the same chance of winning in equilibrium. For example, if there are two low cost contestants and one high cost contestant, then interior equilibria exist if $\gamma \leq 5/2$. The high cost contestant can have a cost function which is more than twice those of the low cost contestants, and still have the same chance of winning in the equilibrium!

Proposition 6 implies that, if the number of contestants is large, corner solutions are more likely. This is as should be expected. With many contestants sabotage is less attractive, and it therefore plays a less important role. So the range of the parameter γ for which sabotage completely equalizes promotion chances gets smaller.

Proposition 6 also shows that the case of a single low cost contestant ($l = 1$) is different from the other cases ($l \geq 2$). The reason for this is as follows. If $l \geq 2$, and the contestants are very different, then there will be a corner solution where *no one* will sabotage a high cost contestant. On the other hand, if $l = 1$, the single low cost contestant will always sabotage high cost contestants, because he has no other rivals. Here, in a corner solutions all the high cost contestants sabotage only the single low cost contestant.

The equilibrium is not unique. In fact, there is a continuum of interior equilibria, where only the total amount of sabotage that contestant $i = 1, ..., n$ chooses ($\sum_{j \neq i} s_{ij}$), and the

[12] This problem is common in tournament models. See, among others, Lazear and Rosen (1981), p. 845 fn. 2; Nalebuff and Stiglitz (1983), p. 29; Lazear (1989), p. 565 fn. 3; Kräkel (2000), p. 398 fn. 17; McLaughlin (1988), p. 236 and p. 241.

total amount of sabotage that contestant i suffers ($\sum_{j \neq i} s_{ji}$) is determined. This can be illustrated as follows. Suppose that every contestant $i = 1, ..., n-1$ sabotages contestant $i+1$ one unit more, and contestant n sabotages 1 more. In addition, $i = 2, ..., n$ sabotages $i-1$ one unit less and contestant 1 sabotages contestant n less. Then the total amount of sabotage against any contestant is unchanged, and so are all the marginal benefits of working and sabotaging. Further, the total amount of sabotage chosen by a contestant is the same as before, and so are the marginal costs. Therefore, if the previous situation was an equilibrium, then the new situation is an equilibrium, too. Basically, the game is a coordination game where there are many ways to coordinate.

7 Conclusion

This paper studied sabotage in selection tournaments with heterogeneous contestants. Sabotage can lead to equalization of promotion chances, even if the contestants differ a lot in their abilities. Furthermore, it may happen that only the least productive individuals participate. Therefore, using a tournament for selection can result in selecting (with probability one) someone who is among the least productive.

One might think that a sequential elimination tournament as studied in Rosen (1986) solves the problem that sabotage equalizes promotion chances. In such a tournament, contestants are paired in each round. One winner emerges from each pair and moves on to the next round. So in any given round, each contestant has only one rival, and it might be thought that the equalizing effect of sabotage is not at work in such a sequential elimination tournament. But contestants do not only care about moving on to the next round, they are also interested in meeting weak rivals in the coming rounds. This gives them an incentive to interfere with the other paired contests in any given round. Consider, for example, the incentives of the semi-finalists. By helping the weaker contestant in the semi-final contest in which one is not directly involved, and by sabotaging the stronger one, one increases the probability of meeting a weaker rival in the final round. Therefore, there is some equalizing effect of sabotage at work in a sequential elimination tournament, too.

Lazear (1989, p. 557) has argued that contestants can be separated with the right design of promotion tracks and sabotage can thus be made more difficult and hence less important. Lazear's point is that separating contestants is good for the firm because sabotage decreases the valuable output of the contestants. The results of this paper show that, in addition, separating contestants also helps to make better promotion decisions. People who do not work with each other closely and regularly are less likely to know each other well, and so they cannot direct sabotage against their strongest rivals. Therefore, the effect that sabotage equalizes promotion chances does not apply.

The results are relevant for other types of contest as well. For example, in rent seeking contests, yardstick competition between regulated firms, or political election contests, sabotage can equalize the probabilities of winning the contest. In rent-seeking contests, the heterogeneity between contestants often takes the form of different valuations of winning the contest. Applied to this setting, the results of the paper imply that there is no guarantee that the contestants with the highest valuations will win most often.

A related problem of tournaments in the presence of sabotage is that sabotage reduces the incentives for productivity-enhancing investments in human capital. Since the contestants know that the better they are, the more they will be sabotaged, they have little incentive to invest in their human capital. This adds to the potential severity of the problems described.

Further research should investigate the case of multiple prizes. In addition, it would be interesting to extend the analysis to dynamic tournaments and to tournaments with multiple rounds.

A Appendix

A.1 Proof of Proposition 1

The proof uses the following Lemmas 2 and 3 which state properties of functions that satisfy a symmetry property like the cost functions.

Lemma 2 *Let* $h : R^2 \rightarrow R$ *be a convex function of two variables and suppose that* $h(x, y) = h(y, x)$ *for all* x *and* y*. Then* $y \geq x$ *implies* $h_2(x, y) \geq h_1(x, y)$, *where* h_i *denotes the partial derivative with respect to the* $i - th$ *argument.*

Proof. A convex function is underestimated by a linear approximation, hence

$$h(y, x) \geq h(x, y) + h_1(x, y)(y - x) + h_2(x, y)(x - y).$$

Since $h(x, y) = h(y, x)$, we have

$$0 \geq (h_1(x, y) - h_2(x, y))(y - x)$$

If $y > x$, it follows that $h_2(x, y) \geq h_1(x, y)$.

Now consider the case $y = x$. Since $h(x, y) = h(y, x)$ for all x and y, we have $h_1(x, x) = h_2(x, x)$. Thus $y = x$ implies $h_1(x, y) = h_2(x, y)$. ■

Using Lemma 2, the following fact about the cost functions is established.

Lemma 3 *If*

$$\frac{1}{\psi'(s_{ij})} \frac{\partial c_i}{\partial s_{ij}} > \frac{1}{\psi'(s_{ik})} \frac{\partial c_i}{\partial s_{ik}},$$

then $s_{ij} > s_{ik}$.

Proof. Suppose that $s_{ik} \geq s_{ij}$. Since the cost function is convex and symmetric in the sabotage activities, Lemma 2 implies that $\frac{\partial c_i}{\partial s_{ik}} \geq \frac{\partial c_i}{\partial s_{ij}}$. Since ψ is concave, we also have

$\psi'(s_{ik}) \leq \psi'(s_{ij})$. This shows that

$$s_{ik} \geq s_{ij} \Longrightarrow \frac{1}{\psi'(s_{ik})}\frac{\partial c_i}{\partial s_{ik}} \geq \frac{1}{\psi'(s_{ij})}\frac{\partial c_i}{\partial s_{ij}}.$$

Equivalently,

$$\frac{1}{\psi'(s_{ij})}\frac{\partial c_i}{\partial s_{ij}} > \frac{1}{\psi'(s_{ik})}\frac{\partial c_i}{\partial s_{ik}} \Longrightarrow s_{ij} > s_{ik}.$$

■

Lemma 3 says that, if i sabotages k more than i sabotages j, then i's cost of destroying an additional unit of k 's output are higher than i's cost of destroying an additional unit of j 's output.

Proof of Proposition 1. $E(q_j) > E(q_k)$ implies $p_j > p_k$. Thus, by Lemma 1,

$$\frac{1}{\psi'(s_{ij})}\frac{\partial p_i}{\partial s_{ij}} > \frac{1}{\psi'(s_{ik})}\frac{\partial p_i}{\partial s_{ik}} \quad \forall i \neq j, k. \tag{18}$$

Next I argue that inequality (18) implies $s_{ij} \geq s_{ik}$. This follows directly from the non-negativity constraints if $s_{ik} = 0$. Thus, suppose $s_{ik} > 0$. Then the following first order condition has to hold:

$$\begin{aligned} \frac{\partial p_i}{\partial s_{ik}} w &= \frac{\partial c_i}{\partial s_{ik}}, & (19) \\ \frac{\partial p_i}{\partial s_{ij}} w &\leq \frac{\partial c_i}{\partial s_{ij}}. & (20) \end{aligned}$$

Using (20), (18), and (19) (in that order),

$$\frac{1}{\psi'(s_{ij})}\frac{\partial c_i}{\partial s_{ij}} \geq \frac{1}{\psi'(s_{ij})}\frac{\partial p_i}{\partial s_{ij}} w > \frac{1}{\psi'(s_{ik})}\frac{\partial p_i}{\partial s_{ik}} w = \frac{1}{\psi'(s_{ik})}\frac{\partial c_i}{\partial s_{ik}}.$$

By Lemma 3, this implies that $s_{ij} > s_{ik}$. ■

A.2 Proof of Corollary 1

Line (7) is derived by contradiction. Suppose that (6) holds but $\sum_{i\neq j,k}\psi(s_{ij})<\sum_{i\neq j,k}\psi(s_{ik})$. Then $E(q_j)>E(q_k)$. By Proposition 1, this implies $s_{ij}\geq s_{ik}$ for all $i\neq j,k$. Using the fact that ψ is increasing, and summing over i, gives $\sum_{i\neq j,k}\psi(s_{ij})\geq\sum_{i\neq j,k}\psi(s_{ik})$, contradiction.

It remains to show that $E(q_j)\geq E(q_k)$. To see this, suppose to the contrary that $E(q_j)<E(q_k)$. By Proposition 1, it follows that $s_{ij}\leq s_{ik}$ for all $i\neq j,k$, hence $\sum_{i\neq j,k}\psi(s_{ij})\leq\sum_{i\neq j,k}\psi(s_{ik})$. Together with inequality (7), this implies $\sum_{i\neq j,k}\psi(s_{ij})=\sum_{i\neq j,k}\psi(s_{ik})$. By (6), we have $E(q_j)\geq E(q_k)$, contradiction.

A.3 Proof of stronger version of Proposition 2

1) If there are two contestants i,j who sabotage all their rivals, then it follows that for all $k\neq i,j$: $p_k=p_i$ (since j sabotages both k and i) and $p_k=p_j$ (since i sabotages both k and j). Therefore we have $p_k=p_i=p_j=1/n$.

2) Suppose in an equilibrium every contestant is sabotaged by two rivals. That is, we have $\forall i\exists j_i,k_i: i\neq j_i\neq k_i\neq i, s_{(j_i)i}>0$ and $s_{(k_i)i}>0$. Then $p_i\geq p_l$ for all $l\neq j_i$ since i is sabotaged by j_i. Also, since i is sabotaged by k_i, we have $p_i\geq p_l$ for all $l\neq k_i$. Putting things together, $p_i\geq p_l$ for all $l\neq i$. Since this holds for all i, we have $p_i=1/n$ for all $i=1,...,n$.

3) If, in an equilibrium, we have $s_{(k-1)k}>0$, then it follows that $p_k\geq p_{k+1}$, since $k-1$ sabotages k. Therefore, if the contestants can be renumbered so that $s_{i(i+1)}>0$ for $i=1,..,n-1$ and $s_{n1}>0$, we have $p_2\geq p_3\geq ...\geq p_n\geq p_1\geq p_2$ or $p_i=1/n$ for all $i=1,...,n$.

A.4 Proof of Proposition 4

Clearly, $\rho'(x)>0$ and hence an inverse function ρ^{-1} exists and is increasing. From the first order condition (11), $x_i=\rho^{-1}(wg/\alpha_i)$. Thus, in equilibrium i 's cost of working productively is $\alpha_i C\left(\rho^{-1}\left(wg/\alpha_i\right)\right)$. Differentiating,

$$\frac{\partial}{\partial \alpha_i}\left(\alpha_i C\left(\rho^{-1}\left(\frac{wg}{\alpha_i}\right)\right)\right) =$$
$$= C\left(\rho^{-1}\left(\frac{wg}{\alpha_i}\right)\right) - \alpha_i C'\left(\rho^{-1}\left(\frac{wg}{\alpha_i}\right)\right)\frac{1}{\rho'\left(\rho^{-1}\left(\frac{wg}{\alpha_i}\right)\right)}\frac{wg}{\alpha_i^2} =$$
$$= C\left(x_i\right) - C'\left(x_i\right)\frac{\rho\left(x_i\right)}{\rho'\left(x_i\right)}.$$

If (14) holds, equilibrium costs are decreasing in α_i. Thus, higher ability (lower α_i) leads to higher costs and thus lower utility.

Similarly, i 's cost of sabotage is $\beta_i S\left(S'^{-1}\left(wg/\left(\beta_i\left(n-1\right)\right)\right)\right)$. Differentiating,

$$\frac{\partial}{\partial \beta_i}\left(\beta_i S\left(S'^{-1}\left(\frac{wg}{\beta_i\left(n-1\right)}\right)\right)\right) =$$
$$= S\left(\sum_{j\neq i} s_{ij}\right) - \beta_i S'\left(\sum_{j\neq i} s_{ij}\right)\frac{1}{S''\left(\sum_{j\neq i} s_{ij}\right)}\frac{wg}{\beta_i^2\left(n-1\right)} =$$
$$= S\left(\sum_{j\neq i} s_{ij}\right) - \frac{\left(S'\left(\sum_{j\neq i} s_{ij}\right)\right)^2}{S''\left(\sum_{j\neq i} s_{ij}\right)}.$$

This is strictly negative if $S\left(\cdot\right)$ is strictly log-concave:

$$\frac{d^2}{dt^2}\left(\ln S\left(t\right)\right) \equiv \frac{S\left(t\right)S''\left(t\right) - \left(S'\left(t\right)\right)^2}{\left(S\left(t\right)\right)^2} < 0.$$

Thus, a higher ability to sabotage (lower β_i) leads to higher costs and hence lower utility.

A.5 Proof of Proposition 5

In an interior equilibrium it must be true that $\frac{\partial p_i}{\partial s_{ij}} w = \frac{\partial c_i}{\partial s_{ij}}$ and $\frac{\partial p_i}{\partial s_{ik}} w = \frac{\partial c_i}{\partial s_{ik}}$. By (9), $\frac{\partial c_i}{\partial s_{ij}} = \frac{\partial c_i}{\partial s_{ik}}$, hence $\frac{\partial p_i}{\partial s_{ij}} = \frac{\partial p_i}{\partial s_{ik}}$ or

$$b_j \int_{-\infty}^{\infty} f\left(y_{ij}+\varepsilon\right)\left[\Pi_{l\neq i,j} F\left(y_{il}+\varepsilon\right)\right] f\left(\varepsilon\right) d\varepsilon = b_k \int_{-\infty}^{\infty} f\left(y_{ik}+\varepsilon\right)\left[\Pi_{l\neq i,k} F\left(y_{il}+\varepsilon\right)\right] f\left(\varepsilon\right) d\varepsilon.$$

If $b_j < b_k$ this implies

$$\int_{-\infty}^{\infty} f(y_{ij}+\varepsilon)\left[\Pi_{l\neq i,j} F(y_{il}+\varepsilon)\right] f(\varepsilon)\,d\varepsilon > \int_{-\infty}^{\infty} f(y_{ik}+\varepsilon)\left[\Pi_{l\neq i,k} F(y_{il}+\varepsilon)\right] f(\varepsilon)\,d\varepsilon$$

which, by the same reasoning as in the proof of Lemma 1, implies $p_j > p_{k.}$ Conversely, if $b_j \geq b_k$, we get $p_j \leq p_{k.}$.

A.6 Proof of Proposition 6

A.6.1 Part a)

The proof is by contradiction. Suppose there is an interior equilibrium. By Proposition 2, $p_i = 1/n$ for all $i = 1, ..., n$. Hence $y_{ij} = 0$ for all i and $j \neq i$. Denote the set of all low cost contestants by L, and the set of all high cost contestants by H. The following first order conditions have to hold in the supposed equilibrium:

$$\begin{aligned} x_i &= wg \text{ for all } i \in L, &(21)\\ x_i &= \frac{wg}{\gamma} \text{ for all } i \in H, &(22)\\ \sum_{k\neq i} s_{ik} &= \frac{wg}{n-1} \text{ for all } i \in L, &(23)\\ \sum_{k\neq i} s_{ik} &= \frac{wg}{(n-1)\gamma} \text{ for all } i \in H. &(24)\end{aligned}$$

Further, from $p_i = p_j = 1/n$ we get

$$x_i - \sum_{k\neq i} s_{ki} = x_j - \sum_{k\neq j} s_{kj} \text{ for all } i \text{ and } j. \tag{25}$$

Equations (21) and (25) imply that $\sum_{k\neq i} s_{ki} = \sum_{k\neq j} s_{kj}$ for all $i, j \in L$. That is, all low cost types endure the same amount of sabotage. Call this amount S_l :

$$\sum_{k\neq i} s_{ki} = \sum_{k\neq j} s_{kj} =: S_l \text{ for all } i, j \in L. \tag{26}$$

In the same way it follows from equations (22) and (25) that all high cost types endure the same amount of sabotage:

$$\sum_{k\neq i} s_{ki} = \sum_{k\neq j} s_{kj} =: S_h \text{ for } i,j \in H. \tag{27}$$

Now let us calculate S_l and S_h. Summing over equations (23) and (24) we find that the total amount of sabotage equals

$$\sum_{i=1}^{n} \sum_{j\neq i} s_{ji} = l\frac{wg}{n-1} + h\frac{wg}{\gamma(n-1)} = lS_l + hS_h, \tag{28}$$

where the second equality follows from equations (26) and (27). From equations (21) to (27) we get

$$wg - S_l = \frac{wg}{\gamma} - S_h. \tag{29}$$

Combining equations (28) and (29) and using $l + h = n$, we finally get

$$S_h = wg\frac{n - l(n-2)(\gamma-1)}{\gamma n(n-1)}, \tag{30}$$

$$S_l = wg\frac{n + (\gamma-1)(l + h(n-1))}{\gamma n(n-1)}. \tag{31}$$

If $\gamma > 1 + \frac{n}{l(n-2)}$, equation (30) implies $S_h < 0$, a contradiction. This completes the proof for the case $l > 1$.

If there is only one low cost contestant ($l = 1$), this contestant directs all his sabotage against high cost contestant. We can calculate the total amount of sabotage that high cost contestants inflict on high cost contestants as the difference between the total amount of sabotage suffered by high cost contestants, hS_h, and the amount of sabotage chosen by the low cost contestant, $wg/(n-1)$:

$$\sum_{i\in H}\sum_{j\in H,j\neq i} s_{ij} = hS_h - \frac{wg}{n-1} = wg\frac{n(n-2)-(\gamma-1)(n^2-2n+2)}{\gamma n(n-1)} \tag{32}$$

This is non-negative if and only if $\gamma \leq 1 + \frac{n(n-2)}{n^2-2n+2}$. This completes the proof.

A.6.2 Part b)

This section develops the sufficient condition for existence of interior equilibria in example 1. Existence is proved by direct construction of such equilibria. I focus on symmetric equilibria, in which

$$s_{ik} = \left\{ \begin{array}{c} s_{ll} \\ s_{lh} \\ s_{hl} \\ s_{hh} \end{array} \right\}, \text{ if } i \in \left\{ \begin{array}{c} L \\ L \\ H \\ H \end{array} \right\} \text{ and } j \in \left\{ \begin{array}{c} L \\ H \\ L \\ H \end{array} \right\}.$$

Let us first derive candidates for symmetric interior equilibria, and check afterwards that they are really equilibria. In an interior equilibrium, the effort choices x_i have to be given by equations (21) and (22). Further, a contestant $i \in H$ sabotages l low cost contestants and $h-1$ high cost contestants. The total amount of sabotage that a contestant $i \in H$ chooses in a symmetric equilibrium is therefore

$$\sum_j s_{ij} = ls_{hl} + (h-1)\, s_{hh} = \frac{wg}{\gamma(n-1)} \text{ for all } i \in H. \tag{33}$$

The second equality follows from equation (24). Similarly

$$\sum_j s_{ij} = (l-1)\, s_{ll} + hs_{lh} = \frac{wg}{(n-1)} \text{ for all } i \in L. \tag{34}$$

The total amount of sabotage suffered by contestant i is

$$\sum_j s_{ji} = ls_{lh} + (h-1)\, s_{hh} = S_h \text{ if } i \in H, \tag{35}$$

$$\sum_j s_{ji} = (l-1)\, s_{ll} + hs_{hl} = S_l \text{ if } i \in L, \tag{36}$$

where S_h and S_l are given by equations (30) and (31). Equations (33) to (36) are four equations in the four unknowns s_{ll}, s_{lh}, s_{hl} and s_{ll}. However, if $l \geq 2$ and $h \geq 2$ (the remaining cases will be considered later), equations (33) to (36) are linearly dependent. Using s_{lh} as a free variable, equations (33) to (36) can be solved to get

$$s_{ll} = \frac{wg}{(n-1)(l-1)} - \frac{n-l}{(l-1)} s_{lh}, \tag{37}$$

$$s_{hh} = wg \frac{n - (\gamma - 1)\, l\, (n-2)}{\gamma n\, (n-1)(n-l-1)} - \frac{ls_{lh}}{(n-l-1)}, \tag{38}$$

$$s_{hl} = \frac{(n-2)(\gamma-1)\, wg}{(n-1)\, \gamma n} + s_{lh}. \tag{39}$$

If the condition (16) given in Proposition 6 a) is satisfied, and

$$s_{lh} \in \left[0, \min\left\{\frac{wg}{(n-1)(n-l)}, wg\frac{n-(\gamma-1)\, l\, (n-2)}{\gamma n\, (n-1)\, l}\right\}\right], \tag{40}$$

then all the variables given in equations (37) to (40) are non-negative. Moreover, note that condition (16) implies that $(\gamma - 1)\, l\, (n-2) \leq n$, and thus the interval in line (40) is not empty.

In what follows, I show that, if the conditions given in Proposition 6 are satisfied, then there exists a continuum of interior equilibria given by (21), (22), and (37) to (40).

Consider the maximization problem of contestant i, given that the other contestants behave according to one of these candidate equilibria. Contestant i chooses

$$\left(x_i, s_{i1}, ..., s_{i(i-1)}, s_{i(i+1)}, ..., s_{in}\right)$$

to maximize u_i, subject to the non-negativity constraints $x_i \geq 0$ and $s_{ij} \geq 0$ for all $j \neq i$. As a first step, I will ignore for the moment the constraints and solve the unconstrained problem. We will check afterwards that the constraints are satisfied.

The unconstrained problem certainly has a solution. This can be seen as follows. It is never optimal to choose very high values of the decision variables. Therefore we can consider the problem

$$\text{maximize } u_i \quad \text{s.t. } -k \leq x_i \leq k \text{ and } -k \leq s_{ij} \leq k \text{ for all } j \neq i \tag{41}$$

for some sufficiently high $k \in R$. By the Weierstrass theorem, a solution to problem (41) exists. If k is high enough, the solution to problem (41) also solves the unconstrained problem.

The following lemma allows the n-dimensional optimization problem to be reduced to a one-dimensional one:

Lemma 4 *Let $l \geq 2$ and $h \geq 2$. Suppose all contestants except i behave symmetrically according to equations (21), (22), and (37) to (40). In the optimum of the unconstrained optimization problem of contestant i, the following conditions have to hold.*

a) Contestant i sabotages all his low cost rivals equally:

$$s_{ij} = s_{ik} =: s_{il} \text{ for all } j, k \in L, j, k \neq i,$$

and i also sabotages all his high cost rivals equally:

$$s_{ij} = s_{ik} =: s_{ih} \text{ for all } j, k \in H, j, k \neq i.$$

b) Contestant i sabotages his high cost and his low cost rivals so that they have the same chance of winning:

$wg-(l-2)\,s_{ll}-hs_{hl}-s_{il}=\frac{wg}{\gamma}-(l-1)\,s_{lh}-(h-1)\,s_{hh}-s_{ih}$, if $i\in L$, and

$wg-(l-1)\,s_{ll}-(h-1)\,s_{hl}-s_{il}=\frac{wg}{\gamma}-ls_{lh}-(h-2)\,s_{hh}-s_{ih}$, if $i\in H$.

c) All rivals of i have the same chance of winning: $y_{ij}=y_{ik}$ for all j,k.

d)

$$x_i=\begin{cases}(n-1)\left((l-1)\,s_{il}+hs_{ih}\right), & \text{if } i\in L,\\ (n-1)\left(ls_{il}+(h-1)\,s_{ih}\right), & \text{if } i\in H.\end{cases}$$

Proof. a) Suppose there are $j,k\in L$ $(j,k\neq i)$ such that $s_{ij}>s_{ik}$. Since j and k are treated in the same way by all other contestants, and choose the same $x_j=x_k=wg$, this implies $y_{ij}>y_{ik}$. Now contestant i could decrease s_{ij} a little and increase s_{ik} by the same amount. By Lemma 1, this increases p_i, while the costs of contestant i are unchanged. Therefore, it cannot be optimal to choose $s_{ij}>s_{ik}$.

The case $j,k\in H$ and part b) are proved in the same way as a). Part c) is obvious from a) and b).

d) It follows from equation (3) that, in Example 1, $\sum_{j\neq i}\frac{\partial p_i}{\partial s_{ij}}=\frac{\partial p_i}{\partial x_i}$. Using c) we have $\frac{\partial p_i}{\partial s_{ij}}=\frac{\partial p_i}{\partial s_{ik}}$ for all $j,k\neq i$ and hence

$$(n-1)\frac{\partial p_i}{\partial s_{ij}}=\frac{\partial p_i}{\partial x_i}.$$

In the optimum of the unconstrained problem, the first order conditions $w\frac{\partial p_i}{\partial x_i}=\gamma_i x_i$ and $w\frac{\partial p_i}{\partial s_{ij}}=\gamma_i\sum_{j\neq i}s_{ij}$ have to hold with equality. Putting things together, $x_i=(n-1)\sum_{j\neq i}s_{ij}$. Finally, using a) completes the proof. ■

Lemma 4 establishes that, in the optimum of the unconstrained problem, certain relations between $s_{i1},...,s_{in}$ and x_i must hold. It allows us to express the unconstrained problem as a one-dimensional problem, where the contestant i maximizes only over x_i. Denote the objective function in this reduced problem by $\hat{u}_i(x_i)$.

Take the case $i\in L$. Straightforward but tedious omitted calculations show that

$$\hat{u}_i(x_i)=\int_{-\infty}^{\infty}F\left(\kappa\left(x_i-wg\right)+\varepsilon\right)^{n-1}f(\varepsilon)\,d\varepsilon w-\frac{1}{2}\kappa x_i^2, \tag{42}$$

where $\kappa := 1 + \frac{1}{(n-1)^2}$.

Lemma 5 *If inequality (17) holds, $\hat{u}_i(x_i)$ given in (42) is strictly concave.*

Proof. The objective function $\hat{u}_i(x_i)$ is strictly concave for all x_i if

$$\max_{x_i}\left(\frac{\partial^2}{\partial x_i^2}\int_{-\infty}^{\infty} F(\kappa(x_i - wg) + \varepsilon)^{n-1} f(\varepsilon)\, d\varepsilon w\right) < \kappa. \tag{43}$$

Note that

$$\begin{aligned} &\frac{\partial^2}{\partial x_i^2}\int_{-\infty}^{\infty} F(\kappa(x_i - wg) + \varepsilon)^{n-1} f(\varepsilon)\, d\varepsilon = \\ = \; &\kappa^2\left(\frac{\partial^2}{\partial z^2}\int_{-\infty}^{\infty} F(z + \varepsilon)^{n-1} f(\varepsilon)\, d\varepsilon\right)\Bigg|_{z=\kappa(x_i - wg)}. \end{aligned}$$

Therefore, inequality (43) is equivalent to

$$\max_{z}\left(\frac{\partial^2}{\partial z^2}\int_{-\infty}^{\infty} F(z + \varepsilon)^{n-1} f(\varepsilon)\, d\varepsilon\right) < \frac{w}{\kappa} = \frac{(n-1)^2}{n^2 - 2n + 2}w$$

which is inequality (17). ■

By Lemma 5, the solution to $\max_{x_i} \hat{u}_i(x_i)$ can be found simply as the solution of the first order condition $\frac{d\hat{u}_i(x_i)}{dx_i} = 0$, which is, of course, unique and as given by equation (21): $x_i = wg$.

By using Lemma 4, we can verify that $s_{ij} = s_{ll}$ for all $j \in L$ (where s_{ll} is given in (37)), and $s_{ij} = s_{lh}$ for all $j \in H$, solve the unconstrained maximization problem of contestant i. Again, these calculations are straightforward but tedious and hence omitted.

Finally, we have to check whether all decision variables satisfy the non-negativity constraints. This is guaranteed by the condition $\gamma \leq 1 + \frac{n}{l(n-2)}$. Therefore, we have shown that no $i \in L$ has an incentive to deviate from any of the symmetric candidate equilibria.

In the same way, it can be shown that no $i \in H$ has an incentive to deviate. For an

$i \in H$, the objective function of the reduced problem turns out to be

$$\hat{u}_i(x_i) = \int_{-\infty}^{\infty} F\left(\kappa\left(x_i - \frac{wg}{\gamma}\right) + \varepsilon\right)^{n-1} f(\varepsilon)\, d\varepsilon w - \frac{1}{2}\gamma\kappa x_i^2.$$

The condition ensuring concavity is

$$\max_z \left(\frac{\partial^2}{\partial z^2}\int_{-\infty}^{\infty} F(z+\varepsilon)^{n-1} f(\varepsilon)\, d\varepsilon\right) < \frac{\gamma}{\kappa w}. \tag{44}$$

The only difference from inequality (17) is the γ on the right hand side. Since $\gamma > 1$, if (17) holds, so does (44). This completes the proof of Proposition 6 b) for the case that $l \geq 2$ and $h \geq 2$.

The two remaining cases where there is only one low cost contestant, or only one high cost contestant, can be dealt with similarly. In these cases there is a unique symmetric equilibrium. If $l = 1$,

$$\begin{aligned} s_{lh} &= \frac{wg}{(n-1)^2}, \\ s_{hl} &= wg\frac{(2+n(n-2))(\gamma-1)+n}{\gamma n(n-1)^2}, \\ s_{hh} &= wg\frac{n(n-2)-(\gamma-1)(n^2-2n+2)}{\gamma n(n-1)}\frac{1}{h(h-1)}. \end{aligned}$$

Of course, s_{hh} is non-negative if, and only if, the inequality given in Proposition 6a for the case $l = 1$ holds. Also note that, as it must be the case, $h(h-1)s_{hh} = \sum_{i\in H}\sum_{j\in H, j\neq i} s_{ij}$ where the right hand side is given in equation (32) above.

Finally, if $h = 1$,

$$\begin{aligned} s_{hl} &= \frac{wg}{\gamma (n-1)^2}, \\ s_{ll} &= wg \frac{(2\gamma - 1)(n-1)^2 - 1}{\gamma n (n-1)^2 (n-2)}, \\ s_{lh} &= wg \frac{n - (\gamma - 1)(n-1)(n-2)}{\gamma n (n-1)(n-1)}. \end{aligned}$$

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