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Optimal Ownership Structures in the Presence of Investment Signals

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Abstract

The property-rights approach to the theory of the firm is extended by introducing distorted signals of the parties' investments. Investment incentives are then given in two ways, by allocating ownership rights and by tying pay to the signal realization. Optimal incentive strength, that is, the weight that a signal is optimally given in a wage contract, depends on two distortions, namely the distortion of the signal from the realized and from the disagreement benefit. Under the optimal ownership structure, the deviations of both investments from their first-best levels are relatively small implying that the relative importance of investment matters. Further, it is shown that most of the Grossman-Hart-Moore results are not robust to an introduction of investment signals.

Key words: Signal, Property rights, Integration, Distortion

JEL classification: D2, L2

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1 Introduction

Most projects of the National Aeronautics and Space Administration (NASA) have the following in common: They are very risky and require both, very high and mainly relationship-specific, investments. Therefore, the activities of NASA seem to nicely fit into the property-rights theory of the firm, which was originated by Grossman & Hart (1986), Hart & Moore (1990) and Hart (1995) (henceforth, GHM). Indeed, property-rights considerations seem to play an important role in protecting the investments of NASA's contractors and, accordingly, in providing investment incentives, as NASA spends about 90 percent of its annual budget on outsourced work.[1] However, NASA uses other instruments to induce investment incentives as well, namely performance-based contracts. According to part 37 of the Federal Acquisition Regulation (FAR), these performance-based contracts have to be used by NASA, whenever possible.

Notice that, besides the NASA example, there are many other situations, where asset ownership and performance-based contracts interact in providing investment incentives. Consider e.g. a salesperson selling products for a client. The performance of the salesperson usually depends on several efforts. It depends on how intensively he is searching for new potential customers, on how honestly he presents his client's products to these potential customers, on how hard he learns to get to know his client's products and so on. These efforts are hardly verifiable by a third party. Consequently, an important task is to align the salesperson's interests with those of his client. One possibility

[1] See report GAO-03-507 by the Government Accountability Office (GAO).

is to provide the salesperson with certain ownership rights (i.e. transacting with a self-employed person instead of transacting with an employee). A self-employed salesperson might e.g. have higher incentives to get to know his client's products, as, in case the parties break up, this knowledge might prove useful in selling related products of other clients. Hence, giving assets to a salesperson (for example an office) might improve the salesperson's effort decision. Yet, providing the salesperson with ownership rights is certainly not the only way to induce incentives. As there usually exists a contractible signal of the salesperson's performance - namely total sales achieved by this person - incentives might also be induced by means of pay tied to the signal realization. Similarly, the client oftentimes exerts some non-verifiable efforts (e.g. for marketing the products), too. Incentives might be given by asset ownership or, for instance, by tying the client's payment to his total monetary marketing expenses.[2]

As the examples indicate, an important question is, how incentives provided via ownership rights interact with incentives provided via payments based on signal realizations. Introducing contractible investment signals into a multi-tasking property-rights model, this paper tries to give an answer to this question.

I start with a benchmark model, where investment signals are not available. In such a situation, investments are always inefficiently low. Moreover, they are misallocated to single components, whenever the coefficient vec-

[2] The salesperson might e.g. be paid a lower fixed wage the more his client has spent for product marketing.

tors of the realized benefit, that is, the benefit when the parties agree to work together, and the disagreement benefit are not parallel. In this setting, providing a party with ownership rights always improves the party's investment decision, as each component of investment gets closer to the first-best. Having considered this benchmark model, investment signals are introduced. With investment signals, each party is incentivized in two ways, by allocating ownership rights (which I henceforth label "residual-rights" incentives) and by tying pay to the signal realization (which I label "contractual" incentives). Quite interesting is the optimal strength of contractual incentives. This strength will be shown to depend on two distortions, the signal distortions from the realized as well as from the disagreement benefit. This is because an optimal signal is usually distorted from the realized benefit, but undistorted from some linear combination of realized and disagreement benefit. Thus, the more the actual signal deviates from this linear combination, the lower is its weight in an optimal contract. Further, the optimal ownership structure minimizes the sum of the squared Euclidean distances of the induced investments from their respective first-best levels. That is, an ownership structure will be optimal, if it leads to investments, which both deviate relatively little from the respective first-best investments. The squaring of the Euclidean distances implies that an extreme deviation of one investment should be avoided, even if the other investment's deviation is very low. It follows that the party with the more important investment should be induced to choose a relatively higher investment. Finally, it is found that providing a party with ownership rights does no longer necessarily improve that party's investment decision. Possession of ownership rights might adversely affect contractual incentive provision and this might yield a worse investment. As a consequence, the results stated in Hart's (1995) famous Proposition 2 are not robust to an introduction of investment signals.

By introducing contractual incentives into the property-rights theory of the firm, this paper contributes to the literature on the interaction of asset ownership with other instruments in generating investment incentives. Two examples of this literature are Holmström & Milgrom (1994) and Holmström & Roberts (1998).[3] While Holmström & Milgrom show that asset ownership, contractual incentives and freedom from direct controls are often complementary, Holmström & Roberts analyze the determinants of different real-world "make or buy" decisions and conclude that a comprehensive theory of the firm must not focus on property-rights considerations alone.

Clearly, this paper is additionally related to the literature on multitasking, which was initiated by Holmström & Milgrom (1991) and further developed by e.g. Baker (1992, 2002), Feltham & Xie (1994) or Corts (2004). Assuming that one component of investment is directed at enhancing the value of an asset, Holmström & Milgrom determine the optimal ownership structure, too. They find that integration is more likely in quite uncertain settings. Under integration, it is optimal to set incentives lower than under non-integration to promote investment into the asset's value. In uncertain settings, non-integration yields high risk-premiums to be paid and is thus suboptimal. Besides the completely different modeling approach, this pa-

[3] Further examples include Holmström (1999), Baker et al. (2002) and Roider (2004).

per differs from Holmström & Milgrom in other ways. Most importantly, investment does not affect an asset's value, but is in human capital. As a consequence, providing a party with ownership rights does not only affect that party's incentive to invest into the asset, but may change the whole investment vector. Further, this paper deals with risk-neutral parties so that risk-considerations are neglected.[4]

Finally, the paper is also related to the work of Baker & Hubbard (2003, 2004). They analyze the impact of an exogenous change in contractibility of driver effort on ownership structures in the United States trucking industry. While their main focus is on an empirical analysis, they theoretically compare the cases, where one component of driver effort is contractible and where driver effort is completely non-contractible. It is shown that, in the latter case, the driver should be more likely to own the truck. The model of Baker & Hubbard can be understood as a special case of the current model, where the signal of a driver's investment corresponds to that component of driver effort being contractible.

The remainder of the paper is organized as follows: Section 2 contains the basic model with multidimensional investments. In Section 3, investment signals are introduced. Further, the optimal incentive schemes are derived and implications for the optimal ownership structure are given. A concluding discussion is offered in Section 4.

[4] As indicated before, the current model extends the property-rights approach to the theory of the firm. See, for a discussion emphasizing the commonalities and distinctions of this approach and the approach by Holmström & Milgrom (1991), Gibbons (2005).

2 The basic model

2.1 Economic Environment

To highlight the inefficiencies in investment behavior arising with multidimensional investments, I first present a model, where signaling of investment is impossible. The model is similar to the one in Hart (1995). There are two risk-neutral and wealth-unconstrained parties, a downstream (denoted as D) and an upstream party (denoted as U), and two assets, $as1$ and $as2$. U produces, in combination with $as2$, a single unit of an input. D, using $as1$, can transform this input into a final product that is sold on the output market.[5] We concentrate on the following three ownership-structures: Non-integration (D owns $as1$ and U owns $as2$), forward-integration (U owns both, $as1$ and $as2$), and backward-integration (D owns both, $as1$ and $as2$).[6] As the parties are wealth-unconstrained, they settle on the structure yielding the highest aggregate benefit.

Before starting the production process, both parties are given the opportunity to undertake certain non-contractible investments. Particularly, U is able to reduce the input's production costs (e.g. by costly acquiring skills that allow for a less tedious production), whereas D's investment might facilitate sale of the final product at a given price, or, similarly, allow D to

[5] Although a supplier-producer relationship is considered, the model can easily be reinterpreted to describe the producer-seller relationship from the introduction.

[6] The case of reversed non-integration is, different from property-rights models without signaling of investments, not necessarily suboptimal. It is therefore briefly discussed in Subsection 3.4. This subsection also deals with joint ownership of assets.

charge a higher price for it. Let $i = (i_1, ..., i_n) \geq 0$ ($e = (e_1, ..., e_m) \geq 0$) represent the level and $0.5 \sum_{k=1}^{n} (i_k)^2$ ($0.5 \sum_{j=1}^{m} (e_j)^2$) the costs of D's (U's) investment. Investments are assumed to be multidimensional. For example, acquiring skills allowing for a less tedious production could be twofold, via learning from courses or books and via learning by doing at the workplace. Investments are also assumed to be in human capital, i.e. they do not affect the assets' values.

It is further assumed that the parties' investments are (partly) relationship-specific. In other words, they will entail a higher benefit, if the parties agree to trade the input. Formally, suppose that, in case of trade between the two parties, D's revenue is given by $R(i) = a^T i$, with $a \in \Re_+^n$ and T denoting the transpose of a vector. His payoff then equals $R(i) - p - 0.5 \sum_{k=1}^{n} (i_k)^2$, where p is the agreed price for the input. In the event of disagreement D buys a non-specific input from the market at price $\bar{p}$. Due to the non-specificity of the input, D's revenue is lower and given by $r(i, A)$, where A represents the set of assets D owns. That is, we may either have $A = \{as1, as2\}$ (backward-integration), $A = \{as1\}$ (non-integration), or $A = \varnothing$ (forward-integration). Moreover, suppose that $r(i, as1, as2) = b^T i$, $r(i, as1) = c^T i$, $r(i, \varnothing) = d^T i$, with $b, c, d \in \Re_+^n$ and $a > b \geq c \geq d$.[7] D's payoff in case of buying the input

[7] These inequalities do not only ensure that investment is relationship-specific, but that this relationship-specificity also holds in a marginal sense. The marginal return from investment will be higher, if the parties agree to trade the input than if they do not. Further, this marginal return is also higher the more assets the investing party has access to. Although these assumptions might seem inappropriate in some practical settings, they should widely hold. Further, they enable a comparison of the results to be derived in

from the market is $r(i, A) - \bar{p} - 0.5 \sum_{k=1}^{n} (i_k)^2$.

Similarly, if the parties decide to trade, U's production costs will be given by $C(e) = -f^T e$, with $f \in \Re_+^m$, and his payoff by $p - C(e) - 0.5 \sum_{j=1}^{m} (e_j)^2$. If trade does not occur, U will go to the market and sell the input at price $\bar{p}$. However, before selling the input, U has to make some adjustments to turn the specific input into a general one. Hence, his production costs are higher in this case. They are given by $c(e, B)$, where B, the set of asset U owns, is either $\{as1, as2\}$ (forward-integration), $\{as2\}$ (non-integration), or $\varnothing$ (backward-integration). To be concrete, let $c(e, as1, as2) = -g^T e$, $c(e, as2) = -h^T e$ and $c(e, \varnothing) = -j^T e$, where $g, h, j \in \Re_+^m$ and $f > g \geq h \geq j$. U's payoff in case of disagreement is given by $\bar{p} - c(e, B) - 0.5 \sum_{j=1}^{m} (e_j)^2$.

To prevent the parties from writing an effective long term contract, assume that there is ex ante uncertainty about what type of input D requires. The parties therefore cannot specify the input price in advance since they do not know what type of input D needs. The uncertainty is resolved after investments are undertaken. Ex post, the parties are thus able to achieve an efficient bargaining outcome. This outcome is assumed to be the Nash-bargaining solution with equal bargaining power.[8]

The timing of the model is as follows. In the first stage, the parties decide on the ownership structure, that is, they choose an allocation of the assets.

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this paper with the results presented in Hart (1995) and so help to isolate the effects of introducing an investment signal.

[8]The assumed bargaining procedure is of great importance. As shown by de Meza & Lockwood (1998) or Chiu (1998), abstracting from Nash-bargaining may lead to very different results.

In the second stage, investments are undertaken, while, in the third stage, the parties decide on whether or not to trade, i.e. whether or not to supply the input.

2.2 Solution to the basic model

Before turning to the model solution, it is convenient to introduce the following definitions: (a) D's investment will be said to become relatively unproductive, if $R(i)$ is replaced by $\rho R(i) + (1-\rho)0.5\sum_{k=1}^{n}(i_k)^2$ and $r(i,A)$ by $\rho r(i,A) + (1-\rho)0.5\sum_{k=1}^{n}(i_k)^2$, where ρ is a small positive number. Similarly, U's investment will be said to become relatively unproductive, if $C(e)$ is replaced by $\rho C(e) - (1-\rho)0.5\sum_{j=1}^{m}(e_j)^2$ and $c(e,B)$ by $\rho c(e,B) - (1-\rho)0.5\sum_{j=1}^{m}(e_j)^2$. (b) Assets $as1$ and $as2$ will be independent, if $b = c$ and $g = h$. (c) Assets $as1$ and $as2$ will be strictly complementary, if either $c = d$ or $h = j$. (d) D's (U's) human capital will be essential, if $g = j$ $(b = d)$.

As a benchmark case, I start by deriving the first-best solution. In such a solution, the efficient trade possibility is always exploited. The net surplus to be maximized is therefore

$$R(i) - C(e) - 0.5\sum_{k=1}^{n}(i_k)^2 - 0.5\sum_{j=1}^{m}(e_j)^2 \tag{1}$$

As first-order conditions, we obtain[9]

$$\frac{\partial R(i)}{\partial i_k} - i_k = 0 \quad , \quad k = 1, ..., n \tag{2}$$
$$\left|\frac{\partial C(e)}{\partial e_j}\right| - e_j = 0 \quad , \quad j = 1, ..., m \tag{3}$$

As can be seen from (2) and (3), in a first-best solution, investment in each component is set equal to its marginal effect on revenue or costs, respectively.

Let us now turn to the model, which is solved by backward induction. I thus begin with stage three. As indicated before, there are always ex post gains from trade. These gains, in the amount of $R(i) - C(e) - (r(i, A) - c(e, B))$, are not achieved under the initial contract since, when the contract was signed, the parties did not know what type of input D requires. However, as mentioned before, the two parties realize the ex post gains through negotiation, where the gains are equally shared. That is, the input price p is determined such that the rent $R(i) - C(e) - (r(i, A) - c(e, B))$ is equally divided. Summarizing, the ex-post payoffs of D and U are given by

$$\pi_D = R(i) - p = r(i, A) - \bar{p} + 0.5(R(i) - C(e) - (r(i, A) - c(e, B))) \tag{4}$$
$$\pi_U = p - C(e) = \bar{p} - c(e, B) + 0.5(R(i) - C(e) - (r(i, A) - c(e, B))) \tag{5}$$

At stage two of the model, D and U choose their investments. Their maxi-

[9] Note that the Hessian matrix corresponding to the maximization problem has the entry -1 along the main diagonal and 0 elsewhere. Determining the leading principal minors of this matrix, it can easily be shown that the function in (1) is strictly concave. It follows that the expressions in (2) and (3) indeed maximize the function.

mization problems are given by (6) and (7), respectively

$$\underset{i \geq 0}{Max} \quad \pi_D - 0.5 \sum_{k=1}^{n} (i_k)^2 \tag{6}$$

$$\underset{e \geq 0}{Max} \quad \pi_U - 0.5 \sum_{j=1}^{m} (e_j)^2 \tag{7}$$

The corresponding first-order conditions are[10]

$$0.5 \frac{\partial R(i)}{\partial i_k} + 0.5 \frac{\partial r(i, A)}{\partial i_k} - i_k \quad = \quad 0 \quad , \quad k = 1, ..., n \tag{8}$$

$$0.5 \left| \frac{\partial C(e)}{\partial e_j} \right| + 0.5 \left| \frac{\partial c(e, B)}{\partial e_j} \right| - e_j \quad = \quad 0 \quad , \quad j = 1, ..., m \tag{9}$$

These first-order conditions look familiar from Hart (1995). The only difference is that there is now a first-order condition for each component of investment. Hence, the results presented in Hart (1995) should hold even with multidimensional investments. This is shown in Proposition 1:[11]

Proposition 1 *With multidimensional investments, the results presented in Hart (1995) remain valid. In particular, (a) if D's (U's) investment becomes relatively unproductive and* $\rho \to 0$*, forward-integration (backward-integration) will be optimal, (b) if the assets are independent, non-integration will be optimal, (c) if the assets are strictly complementary, one form of integration will be optimal, (d) if D's (U's) human capital is essential, backward-integration (forward-integration) will be optimal.*

[10] With a similar argumentation as for the first-best solution one can show that the functions in (6) and (7) are strictly concave so that the expressions in (8) and (9) indeed solve the maximization problems.

[11] Holmström (1999) provides an example, where Proposition 1 does not hold with multidimensional investment. The reason is that in his example overinvestment is possible, whereas in the current model it is not.

Proof. See Appendix 1. ■

Note that with multidimensional investments there are usually two inefficiencies. First, the parties invest inefficiently low (as in the one-dimensional case).[12] Second, whenever the coefficient vectors of the realized and the disagreement benefit are not parallel, there is misallocation of investment, i.e. the parties allocate the resources they are going to spend in an inefficient way. While judging the single ownership structures one has to consider both inefficiencies. In each of the cases (a) to (d) there is some change in ownership structure that leads to an increase in investment of one party, while leaving the other party's investment or the benefit from this investment unaffected. This change clearly mitigates the first inefficiency, on the other hand, it may intensify the second one. That is, we may have two countervailing effects on total benefit. However, as Proposition 1 shows, the first effect is always the dominating one, so the single ownership structures can be judged unambiguously. The reason is that investment in all components is inefficiently low, and the change brings each component closer to the first-best level.

3 Signaling of investments

3.1 The signals

It is now assumed that there exists some signal of each party's investment. Let the two signals be of the form $s(i) = k^T i$ and $t(e) = m^T e$, with $k \in \Re^n$

[12] Bearing the restrictions $a > b \geq c \geq d$ and $f > g \geq h \geq j$ in mind, this follows directly from a comparison of (2) with (8) and (3) with (9).

and $m \in \Re^m$. Note that it is without loss of generality that the benefit and the signal coefficient vectors consist of the same number of components. If the signal was responsive to only some components of investment, the other signal components would equal zero. Similarly, if some investment components affect the signal but not the benefit, the respective benefit coefficient vector components would be zero.[13] The signals are assumed to be contractible, hence they can be part of an enforceable contract and used to induce investment incentives. In Appendix 2, I demonstrate that linear contracts can replicate every outcome to be achieved with the most general contract. It is thus not restrictive at all to focus on linear incentive schemes contingent on the signals, where D (U) receives from U (D) $IS_D = \alpha_0 + \alpha s(i)$ ($IS_U = \beta_0 + \beta t(e)$), with $\alpha_0, \alpha, \beta_0, \beta \in \Re$.[14] Denote α and β as piece-rates. These piece-rates measure the strength of contractual incentives. The incentive schemes are determined after ownership rights are allocated, but before investments are chosen. The signal realizations are learned after investments are made, but before negotiations occur. Since both parties are risk-neutral and wealth unconstrained, it is reasonable to assume that the incentive schemes are determined such that the aggregate ex ante revenue of the two parties is maximized. Determining the lump-sum payments α_0 and

[13] The conditions $a > b$ and $f > g$ then must be replaced by $a_k > b_k$, for $a_k \neq 0$, and $f_j > g_j$, for $f_j \neq 0$.

[14] This double contracting seems a little unusual, as, in most economic transactions, only one party proposes an incentive contract for another one. However, note that the model also includes this case, when all components of the signal coefficient vector of one party equal zero.

β_0, D and U could allocate the revenue in a way that makes both better off.

3.2 Optimal investments and optimal incentive provision

As the introduction of pay tied to the signal realizations is the only difference from the model in Section 2, the two parties' ex ante payoffs change to (10) and (11) and the first-order conditions to (12) and (13), respectively[15]

$$
\begin{aligned}
& r(i,A) - \bar{p} + 0.5(R(i) - C(e) - (r(i,A) - c(e,B))) && (10)\\
& +\alpha_0 + \alpha s(i) - \beta_0 - \beta t(e) - 0.5\sum_{k=1}^{n}(i_k)^2 && \\
& \bar{p} - c(e,B) + 0.5(R(i) - C(e) - (r(i,A) - c(e,B))) && (11)\\
& +\beta_0 + \beta t(e) - \alpha_0 - \alpha s(i) - 0.5\sum_{j=1}^{m}(e_j)^2 &&
\end{aligned}
$$

$$
\begin{aligned}
0 &= 0.5\frac{\partial R(i)}{\partial i_k} + 0.5\frac{\partial r(i,A)}{\partial i_k} + \alpha\frac{\partial s(i)}{\partial i_k} - i_k \quad , \quad k = 1,...,n && (12)\\
0 &= 0.5\left|\frac{\partial C(e)}{\partial e_j}\right| + 0.5\left|\frac{\partial c(e,B)}{\partial e_j}\right| + \beta\frac{\partial t(e)}{\partial e_j} - e_j \quad , \quad j = 1,...,m && (13)
\end{aligned}
$$

Obviously, investments now depend on two kinds of incentives, incentives that are given by asset ownership and contractual incentives. In order to see how the second kind of incentives changes the results from Section 2, one needs to determine the optimal incentive schemes. While doing this, the two

[15]Notice that the Hessian matrices corresponding to the maximization problems are the same as in the Section 2. The solutions in (12) and (13) therefore maximize the functions in (10) and (11), respectively.

parties face the following maximization problem:[16]

$$\underset{i^*,e^*,\alpha,\beta}{Max} \quad R(i^*) - 0.5\sum_{k=1}^{n}(i_k^*)^2 - C(e^*) - 0.5\sum_{j=1}^{m}(e_j^*)^2 \tag{14}$$

$$\begin{aligned} s.t. \quad 0 &= 0.5\frac{\partial R(i^*)}{\partial i_k} + 0.5\frac{\partial r(i^*,A)}{\partial i_k} + \alpha\frac{\partial s(i^*)}{\partial i_k} - i_k^* \quad , \quad k = 1,...,n, \\ 0 &= 0.5\left|\frac{\partial C(e^*)}{\partial e_j}\right| + 0.5\left|\frac{\partial c(e^*,B)}{\partial e_j}\right| + \beta\frac{\partial t(e^*)}{\partial e_j} - e_j^* \quad , \quad j = 1,...,m \end{aligned}$$

Solving this problem, yields the subsequent proposition:

Proposition 2 *The optimal piece-rates are given by* $\alpha_y = \frac{\sqrt{a^Ta}\cos\gamma - \sqrt{y^Ty}\cos\phi_y}{2\sqrt{k^Tk}}$ *(if* $k \neq 0$*, and* $\alpha_y \in \Re$ *otherwise) and* $\beta_z = \frac{\sqrt{f^Tf}\cos\delta - \sqrt{z^Tz}\cos\theta_z}{2\sqrt{m^Tm}}$ *(if* $m \neq 0$*, and* $\beta_z \in \Re$ *otherwise), where* γ *is the angle between* a *and* k*,* $y = b, c, d$*,* ϕ_y *is the angle between* y *and* k*,* δ *is the angle between* f *and* m*,* $z = j, h, g$*, and* θ_z *is the angle between* z *and* m*.*

Proof. See Appendix 1. ■

Proposition 2 gives insight in the optimal incentive schemes of the two parties. First, note that the cosine of the angle between the benefit and the signal coefficient vector measures the distortion between these vectors. A higher value of the cosine corresponds to a signal that is less distorted from the benefit.[17] Unlike in previous work on multitasking, the strength

[16] Since they are only used to allocate revenue, we do not further consider the fixed payments. Their determination depends on the ex ante bargaining power of the two parties that we do not specify in this paper. Note, however, that, due to the fundamental transformation described by e.g. Williamson (1985), the ex ante bargaining power of a party may be different from its ex post bargaining power.

[17] See e.g. Baker (2002), Feltham & Xie (1994), or Schnedler (2003). Baker and Feltham & Xie develop different measures of effort misallocation (effort in these models corresponds

of incentives depends on two distortions, the distortion of the available signal from realized benefit and its distortion from disagreement benefit. The intuition behind this result is as follows: As shown in Section 2, ex ante investments are, in the absence of contractual incentive provision, inefficiently low and usually misallocated to the single components. While installing an incentive scheme, the parties try to mitigate both inefficiencies. Consider e.g. D's investment choice. Without contractual incentive provision, he chooses $i = 0.5a + 0.5y$, while first-best investment satisfies $i_{fb} = a$. Hence, optimally, contractual incentives would increase investment by $i_{fb} - i = 0.5a - 0.5y$. In order to achieve this, a signal with a coefficient vector parallel to $0.5a - 0.5y$ is needed. Stated differently, a desirable signal is of the form $s = k^{*T} i$, with $k^{*} = const\,(0.5a - 0.5y)$, and $const \neq 0$. Bearing this in mind, Proposition 2 is very intuitive. It simply states that the weight a signal is given in the wage contract should decrease, the more the signal deviates from the optimal signal $s = k^{*T} i$. Further, note that the piece-rate naturally depends on the lengths of the single coefficient vectors. If the signal coefficient vector is rather small compared to the benefit coefficient vectors, the signal is obviously given a higher weight (and vice versa).

The structure of the model allows us to give some nice geometric interpretation of the optimal piece-rate that proves useful, when turning to the

to investments in the current one). Schnedler proposes several properties such a misallocation measure should possess. While Feltham & Xie's measure fails to possess these properties, a slight variation of Baker's measure exhibits all these properties. Baker's measure is the same as used in this paper, namely the cosine of the angle between the benefit and the signal coefficient vector.

optimal ownership structure. From (12), we see that D's investment lies on the straight line $i = 0.5\nabla R(i) + 0.5\nabla r(i, A) + \alpha\nabla s(i)$, with ∇ denoting the gradient of the respective functions. By determining α, the parties select a particular point on the line. It can be shown that this is the point on the line, whose Euclidean distance to i_{fb} is minimal. In fact, minimizing $|\nabla R(i) - (0.5\nabla R(i) + 0.5\nabla r(i, A) + \alpha\nabla s(i))|$ over α leads to the same solution for the piece-rate as in Proposition 2. This result, however, depends on the specific forms of the benefit and cost functions. If e.g. marginal investment costs were not linear, the equivalence of the two optimization problems would disappear.

Let us now turn to the following corollary:

Corollary 3 *Suppose that there exist two different signals of a party's investment, but that the parties are only allowed to contract on one of them. They may then decide to contract on the more distorted signal (with respect to the realized benefit).*

We prove this corollary by giving an example. Suppose that U's investment is relatively unproductive and that $\rho \to 0$. That is, ownership rights should be allocated such that $R(i^*) - 0.5\sum_{k=1}^{n}(i_k^*)^2$ is maximized. Further, assume that $a = (1, 1)$, $b = (b_1, 0)$, with $b_1 \in (0, 1)$, $c = d \equiv 0$. Let $k_1 = (0, 1)$ and $k_2 = \left(\frac{1}{2}, 1\right)$ denote the coefficient vectors of the two available signals. After some calculations one can show that, in this example, it is optimal to give all ownership rights to D, that is, backward integration is optimal. Using the first signal, the parties would choose $\alpha_{b,1} = 0.5$

and the surplus was $\frac{7+2b_1-(b_1)^2}{8}$. Using the second signal, the corresponding piece-rate was $\alpha_{b,2} = \frac{3-b_1}{5}$ leading to surplus $\frac{39+4b_1-4(b_1)^2}{40}$. The parties will then contract on the first signal, if and only if $b_1 \geq \bar{b}_1 \approx 0.764$. Note that the second signal is the less distorted one with respect to $R(i)$ since $\cos\gamma_1 = 2^{-0.5} < \cos\gamma_2 = \frac{3}{5^{0.5}}2^{-0.5}$. In other words, for some parameter values the parties contract on the less distorted signal, while for others they contract on the more distorted one.

This result directly follows from Proposition 2. As an optimal signal has a coefficient vector that is parallel to $0.5a - 0.5y$, the parties do not only care for the signal distortion with respect to the realized benefit, but also for its distortion with respect to the disagreement benefit. Hence, without taking this second distortion into account, one cannot assess a signal's appropriateness.

Note that Corollary 3 also implies that the parties might not increase the quality of a certain signal with respect to the realized benefit, even if this were totally costless. This complements the findings in Schnedler (2005), who states, in a model neglecting property-rights considerations, that a signal being ceteris paribus less distorted with respect to benefit is always superior to the more distorted signal. To summarize, a signal that at first sight seems to be very appropriate (because of a low distortion from the realized benefit) might be less appropriate, if its distortion from the disagreement benefit is taken into account, too.

3.3 The optimal ownership structure

Before turning to the optimal ownership structure, denote by $ED_D(A)$ the Euclidean distance of D's investment $i = 0.5a + 0.5y + \alpha_y k$ from the first-best investment $i_{fb} = a$. Similarly, let $ED_U(B)$ denote the Euclidean distance of U's investment $e = 0.5f + 0.5z + \beta_z m$ from first-best investment $e_{fb} = f$. One can easily show that $ED_D(A) = \left(0.25a^T a - 0.5a^T y + 0.25y^T y - 0.25\frac{\left((a-y)^T k\right)^2}{k^T k}\right)^{0.5}$ and $ED_U(B) = \left(0.25f^T f - 0.5f^T z + 0.25z^T z - 0.25\frac{\left((f-z)^T m\right)^2}{m^T m}\right)^{0.5}$.

Suppose first that only one party's investment matters. Proposition 4 describes, in this case, the optimal ownership structure.

Proposition 4 *Given that only D's (U's) investment matters, the two parties select the ownership structure minimizing $ED_D(A)$ $(ED_U(B))$.*

Proof. See Appendix 1. ■

When judging the single ownership structures, the parties take into account both, the residual rights incentives and the contractual ones that each ownership structure entails. In particular, they choose to allocate the assets such that the induced investment lies closest to its first-best level. Assets are therefore allocated such that the single coefficient vectors best "fit" together. For example, if the signal shows little distortion from realized benefit, an ownership structure with a disagreement benefit vector, which is either short or shows little distortion from realized benefit, will be optimal.

Suppose now that the parties care for both investments. Proposition 5 describes the optimal ownership structure in this case:

Proposition 5 *If the parties care for both investments, they will select the ownership structure minimizing* $(ED_D(A))^2 + (ED_U(B))^2$.

Proof. See Appendix 1. ■

Note first that it will be a very easy task to determine the optimal ownership structure, if the ownership structure being optimal when only D's or U's investment is important, respectively, is the same. Technically, this means that there exists an allocation of assets that minimizes $ED_D(A)$ and $ED_U(B)$. Clearly, this allocation of assets minimizes $(ED_D(A))^2 + (ED_U(B))^2$, too. Of more interest is the case, where the ownership structure minimizing $ED_D(A)$ differs from the ownership structure minimizing $ED_U(B)$. Here, Proposition 5 implies that not the levels of the investments per se, but their deviations from the respective first-best investments are important. In particular, an ownership structure will be optimal, if it leads to investments, which both deviate relatively little from the respective first-best investments. Further, an extreme deviation of one investment should be avoided, even if the other investment's deviation is very low.[18] As a consequence, the relative importance of the two investments matters. Assume e.g. that D's investment is much more effective in increasing revenue than U's investment is in decreasing production costs. Then, the components in a

[18]This follows from the squaring of the Euclidean distances in Proposition 5. If the Euclidean distances were not squared, it would not matter, how the aggregate Euclidean distance is divided between the two investments. For example, an ownership structure yielding $ED_D(A) = 1$ and $ED_U(B) = 5$ were as good as another yielding $ED_D(A) + ED_U(B) = 3$. Applying the decision rule from Proposition 5, on the other hand, the second ownership structure dominates the first.

should be higher than the components in f, implying that the single components of i_{fb} are higher than the single components of e_{fb}. By allocating assets such that both investments differ relatively little from their first-best levels, it is thus important to induce D to invest higher into the single components than U.

Finally, we analyze whether or not the results of Proposition 1 are robust to an introduction of investment signals. As the following proposition shows, the answer is negative, i.e. the presence of investment signals changes the results.

Proposition 6 *With investment signals, either non-integration or some kind of integration will be optimal, even if (a) some party's investment is relatively unproductive and $\rho \to 0$, (b) the assets are independent, (c) the assets are strictly complementary, or (d) some party's human capital is essential.*

Proof. See Appendix 1. ■

In the presence of investment signals, the assumptions under (a) to (d) are not sufficient to rank the ownership structures unambiguously. The reason is that the effectiveness of contractual incentives may increase as residual rights incentives decrease. In other words, providing a party with ownership rights might worsen that party's investment choice. In this case, the results in Proposition 1 do no longer hold. To give an illustration, consider part (a) of Proposition 6. Assume that U's investment is relatively unproductive and that $\rho \to 0$. The parties then allocate ownership rights such that $R(i^*) - 0.5 \sum_{k=1}^{n} (i_k^*)^2$ is maximized. Assume further, that k is parallel to a.

As described before, an optimal ownership structure is one, where D's disagreement benefit coefficient vector is either short or shows little distortion from the realized benefit coefficient vector. Hence, backward-integration need not be optimal. Giving more assets to D might lead to a highly distorted, and also long disagreement benefit coefficient vector. Such a vector might prevent an efficient signal use so that backward-integration is dominated.

To summarize, the results in Proposition 1 should further hold, if allocating ownership rights does not extremely affect the signal use in incentive contracts. Otherwise, the results are likely to break down.

3.4 Reversed non-integration and joint ownership

In the property rights literature, reversed non-integration has not received much attention. Without investment signals, it is easily found that it is dominated by non-integration. Since D works primarily with $as1$ and U with $as2$, it is very likely that D is more productive with $as1$ than $as2$. Similarly, U is expected to be more productive with $as2$ than $as1$. Technically, this means that $\frac{\partial r(i,as1)}{\partial i_k} > \frac{\partial r(i,as2)}{\partial i_k}$, $\forall i$, $\forall k$, and $\left|\frac{\partial c(e,as2)}{\partial e_j}\right| > \left|\frac{\partial c(e,as1)}{\partial e_j}\right|$, $\forall e$, $\forall j$. Clearly, given these inequalities non-integration leads to higher investments than reversed non-integration. However, in the presence of investment signals, ownership structures are evaluated according to two criteria, the residual rights and the contractual incentives they entail. Therefore, even reversed non-integration may be an optimal ownership structure. If it allows the parties to better use the signals in incentive contracts compared to the other ownership structures, its weaknesses in providing high residual rights incentives might be outbalanced.

Similarly, different from the GHM results joint ownership of an asset[19] might be optimal, too. Under joint ownership, either side is able to veto the use of an asset. As a consequence, in the event of disagreement, the assets under joint ownership become unavailable for both parties. This entails lower residual rights incentives. On the other hand, the signals might be used more intensely in incentive contracts, so joint ownership might be preferred.

4 Concluding remarks

This paper extended the property-rights approach to the theory of the firm by introducing distorted signals of the parties' multidimensional investments. It was shown that this extension crucially affects the original results. The main findings are the following:

- The extension to multidimensional investments will usually lead to a second inefficiency, if investment signals are not available. Investments are then not only inefficiently low, but also misallocated to the different components. Yet, giving a party comprehensive ownership rights yields second-best investments of this party. Even if these ownership rights lead to a more distorted investment, the advantage of higher investment incentives that ownership rights entail will dominate. Therefore, the results presented in Hart (1995) are robust to a generalization to multidimensional investments.

- The strength of contractual incentives depends on both, the signal

[19] For alternative explanations of joint ownership of assets see e.g Cai (2003), Rosenkranz & Schmitz (2003) or Schmitz (forthcoming).

distortion from the realized benefit as well as from the disagreement benefit.

- As a direct consequence, a party might prefer tying the other party's remuneration to a signal of little quality (with respect to the realized benefit), even if the signal quality could be increased at zero costs.

- The optimal ownership structure depends on both, the residual rights incentives and the contractual incentives it entails. For this reason, the results presented in Hart (1995) are not robust to an introduction of investment signals. Different from Hart (1995), removing assets from a party might improve that party's investment decision. Thinking of the important role of investment signals in contract theory and their availability in many real-world settings, this result is of particular interest.

It is this wide availability of investment signals that makes the introduction of such signals into the property-rights theory of the firm so important. This paper provides a guide of how using investment signals in an incentive contract and of how optimal ownership structure should respond to the existence of investment signals. Of particular importance are the distortions of the single coefficient vectors. In order to make the theory applicable for predicting or explaining ownership structures being prevalent in practice, one has to think about how disagreement benefit vectors and, hence, the single distortions vary with ownership structure. Of course, a comprehensive answer to this question cannot be given. Yet, many real-world relationships should have the following property in common: Some components of investment should be fully worthless without having access to some asset or the other party's human capital. For example, suppose that U's investment may be divided into two groups, investment that generally decreases the production costs of a class of inputs and investment that is solely oriented at D's needs, i.e. investment that decreases the costs of producing the special input D requires. If both parties do not agree to trade and if U does not have access to $as1$, this second type of investment will be totally worthless. What does this observation mean in terms of the model? It implies that the distortion in a party's investment behavior due to residual rights incentives probably increases as the party has access to fewer assets. This has consequences for the use of signals in incentive contracts and for asset allocation. For example, it is less likely that a party with a totally undistorted signal should be given no ownership rights. The effectiveness of the signal in inducing investment incentives would in this case suffer from the distorted residual rights incentives. However, if the idea concerning the relative distortion of disagreement benefits is really true, has to be shown empirically. I leave this for future research.

Appendix 1

Proof of Proposition 1:

(a)

Consider the case, where U's investment is relatively unproductive and $\rho \to 0$. Replacing $\left|\frac{\partial C(e)}{\partial e_j}\right|$ by $\rho\left|\frac{\partial C(e)}{\partial e_j}\right| + (1-\rho)e_j$ and $\left|\frac{\partial c(e,B)}{\partial e_j}\right|$ by $\rho\left|\frac{\partial c(e,B)}{\partial e_j}\right| + (1-\rho)e_j$, it can be seen that U's first-order conditions remain the same and are independent of ρ. The aggregate benefit, on the other hand, changes to $R(i^*) - 0.5\sum_{k=1}^{n}(i_k^*)^2 - \rho C(e^*) - \rho 0.5\sum_{j=1}^{m}(e_j^*)^2$, which tends to $R(i^*) - 0.5\sum_{k=1}^{n}(i_k^*)^2$, as $\rho \to 0$. Thus, the parties allocate ownership rights such that $R(i^*) - 0.5 \sum_{k=1}^{n} (i_k^*)^2$ is maximized. The case, where D's investment is relatively unproductive is analogous. $R(i^*) - 0.5 \sum_{k=1}^{n} (i_k^*)^2$ can be rewritten as $a_1 i_1^* + ... + a_n i_n^* - 0.5 (i_1^*)^2 - ... - 0.5 (i_n^*)^2$ or $\sum_{k=1}^{n} \left(a_k i_k^* - 0.5 (i_k^*)^2 \right)$. Hence, if there is an ownership structure that yields higher investment in each component of i than the other ownership structures (notice that investment is always inefficiently low), this ownership structure will be optimal. Since $b \geq c \geq d$, backward integration is optimal.

(b)

Note that, with independent assets, giving D or U a second asset does not change their investment. As a consequence, giving D a second asset leaves i unchanged, but leads to an investment e that is dominated by the investment in case of non-integration since every investment component is at most as high as under non-integration. Hence, backward-integration is (weakly) dominated by non-integration. With a similar argument, forward-integration is (weakly) dominated by non-integration. Non-integration is therefore optimal.

(c)

Note that, with strictly complementary assets, giving D or U a first asset does not change their investment. Hence, changing from non-integration to backward integration does not change U's investment, but (weakly) increases each component of D's investment. Such a change is therefore welfare enhancing, and non-integration is (weakly) dominated.

(d)

If D's human capital is essential, from $g = j$ and $g \geq h \geq j$, it follows that $g = h = j$. Then, U chooses the same investment no matter what the allocation of assets looks like. Hence, ownership rights should be allocated such that $R(i^*) - 0.5\sum_{k=1}^{n}(i_k^*)^2$ is maximized. From part (a), we know that backward-integration is optimal in this case. Analogously, if U's human capital is essential, forward-integration will be optimal.

Proof of Proposition 2:

Note first that for $k \equiv 0$ (analogously for $m \equiv 0$) no contractual incentives can be provided for D (U), so that every piece-rate performs equally well.

This is no longer the case when $k \neq 0$ or $m \neq 0$ (or both). In this case, I explicitly derive the optimal piece-rates under backward-integration. The solutions under other ownership structures can be derived analogously. Inserting the optimal investments, maximization problem (14) can be written as

$$\begin{aligned} &\underset{\alpha,\beta}{Max}\quad 0.5a^T a + 0.5a^T b + \alpha a^T k - 0.5\sum_{k=1}^{n}(0.5a_k + 0.5b_k + \alpha k_k)^2 \\ &+0.5f^T f + 0.5f^T j + \beta f^T m - 0.5\sum_{j=1}^{m}(0.5f_j + 0.5j_j + \beta m_j)^2 \end{aligned}$$

The first-order conditions to this maximization problem are[20]

$$\begin{aligned} \frac{\partial}{\partial\alpha} &= a^T k - \sum_{k=1}^{n}(0.5a_k + 0.5b_k + \alpha k_k)\,k_k = 0 \\ \frac{\partial}{\partial\beta} &= f^T m - \sum_{j=1}^{m}(0.5f_j + 0.5j_j + \beta m_j)\,m_j = 0 \end{aligned}$$

[20] The corresponding Hessian matrix is $H = \begin{pmatrix} -k^T k & 0 \\ 0 & -m^T m \end{pmatrix}$. The function in maximization problem (14) is strictly concave, as $(-1)\left(-k^T k\right) > 0$ and $(-1)^2\left(-k^T k\right)\left(-m^T m\right) > 0$. The solution to be obtained is therefore indeed a maximum.

Simplifying these conditions yields

$$\begin{aligned}0.5a^Tk &= 0.5b^Tk + \alpha k^Tk \Longleftrightarrow \alpha = \frac{0.5a^Tk - 0.5b^Tk}{k^Tk} \\ &\Longleftrightarrow \alpha = \frac{0.5}{\sqrt{k^Tk}}\left(\sqrt{a^Ta}\frac{a^Tk}{\sqrt{a^Ta}\sqrt{k^Tk}} - \sqrt{b^Tb}\frac{b^Tk}{\sqrt{b^Tb}\sqrt{k^Tk}}\right)\end{aligned}$$

and

$$\begin{aligned}0.5f^Tm &= 0.5j^Tm + \beta m^Tm \Longleftrightarrow \beta = \frac{0.5f^Tm - 0.5j^Tm}{m^Tm} \\ &\Longleftrightarrow \beta = \frac{0.5}{\sqrt{m^Tm}}\left(\sqrt{f^Tf}\frac{f^Tm}{\sqrt{f^Tf}\sqrt{m^Tm}} - \sqrt{j^Tj}\frac{j^Tm}{\sqrt{j^Tj}\sqrt{m^Tm}}\right)\end{aligned}$$

This completes the proof of Proposition 2.

Proof of Proposition 4:

Suppose that only D's investment is of interest and compare the case of non-integration to the case of backward-integration. Other comparisons as well as the case, where only U's investment matters, are completely analogous. Total benefit in case of non-integration is given by

$$0.5a^Ta + 0.5a^Tc + a^Tk\frac{0.5a^Tk - 0.5c^Tk}{k^Tk} - 0.5\sum_{k=1}^{n}\left(0.5a_k + 0.5c_k + k_k\frac{0.5a^Tk - 0.5c^Tk}{k^Tk}\right)^2$$

whereas total benefit in case of backward-integration is

$$0.5a^Ta + 0.5a^Tb + a^Tk\frac{0.5a^Tk - 0.5b^Tk}{k^Tk} - 0.5\sum_{k=1}^{n}\left(0.5a_k + 0.5b_k + k_k\frac{0.5a^Tk - 0.5b^Tk}{k^Tk}\right)^2$$

As assumed the parties settle on the ownership structure that yields highest total benefit. Hence, non-integration will be preferred to backward-integration, if and only if the former benefit exceeds the latter. This can be rewritten as

$$
\begin{aligned}
& -\frac{(a^Tk)(c^Tk)}{4k^Tk} - \frac{c^Tc}{8} - \frac{\left(a^Tk - c^Tk\right)^2}{8k^Tk} + \frac{a^Tc}{4} - c^Tk\frac{a^Tk - c^Tk}{4k^Tk} \\
> \quad & -\frac{(a^Tk)(b^Tk)}{4k^Tk} - \frac{b^Tb}{8} - \frac{\left(a^Tk - b^Tk\right)^2}{8k^Tk} + \frac{a^Tb}{4} - b^Tk\frac{a^Tk - b^Tk}{4k^Tk} \\
\iff \quad & b^Tb - c^Tc - 2a^T(b-c) - \frac{1}{k^Tk}\left(2(a^Tk)\left((c^Tk) - (b^Tk)\right) - \left(c^Tk\right)^2 + \left(b^Tk\right)^2\right) > 0
\end{aligned}
$$

The last condition is equivalent to

$$
\begin{aligned}
& 0.25a^Ta + 0.25b^Tb - 0.5a^Tb - \frac{0.25}{k^Tk}\left((a-b)^T k\right)^2 \\
> \quad & 0.25a^Ta + 0.25c^Tc - 0.5a^Tc - \frac{0.25}{k^Tk}\left((a-c)^T k\right)^2
\end{aligned}
$$

or using the expressions for the Euclidean distances to

$$
ED_D(as1, as2) > ED_D(as1)
$$

This completes the proof of Proposition 4.

Proof of Proposition 5:

The proof of Proposition 5 is similar to the proof of Proposition 4. Again, we compare the case of non-integration with the case of backward-integration. Other comparisons are completely analogous. Total benefit in case of non-integration is

$$
\begin{aligned}
& 0.5a^Ta + 0.5a^Tc + a^Tk\frac{0.5a^Tk - 0.5c^Tk}{k^Tk} \\
& -0.5\sum_{k=1}^{n}\left(0.5a_k + 0.5c_k + k_k\frac{0.5a^Tk - 0.5c^Tk}{k^Tk}\right)^2 \\
& +0.5f^Tf + 0.5f^Th + f^Tm\frac{0.5f^Tm - 0.5h^Tm}{m^Tm} \\
& -0.5\sum_{j=1}^{m}\left(0.5f_j + 0.5h_j + m_j\frac{0.5f^Tm - 0.5h^Tm}{m^Tm}\right)^2
\end{aligned}
$$

while total benefit under backward-integration is

$$\begin{aligned}
&0.5a^T a + 0.5a^T b + a^T k \frac{0.5a^T k - 0.5b^T k}{k^T k} \\
&-0.5 \sum_{k=1}^{n} \left(0.5a_k + 0.5b_k + k_k \frac{0.5a^T k - 0.5b^T k}{k^T k} \right)^2 \\
&+0.5f^T f + 0.5f^T j + f^T m \frac{0.5f^T m - 0.5j^T m}{m^T m} \\
&-0.5 \sum_{j=1}^{m} \left(0.5f_j + 0.5j_j + m_j \frac{0.5f^T m - 0.5j^T m}{m^T m} \right)^2
\end{aligned}$$

In analogy to the proof of Proposition 4 one can show that the former benefit will exceed the latter, if the following condition holds:

$$\begin{aligned}
& 0.25a^T a + 0.25b^T b - 0.5a^T b - \frac{0.25}{k^T k} \left((a-b)^T k \right)^2 \\
& +0.25f^T f + 0.25j^T j - 0.5f^T j - \frac{0.25}{m^T m} \left((f-j)^T m \right)^2 \\
> \; & 0.25a^T a + 0.25c^T c - 0.5a^T c - \frac{0.25}{k^T k} \left((a-c)^T k \right)^2 \\
& +0.25f^T f + 0.25h^T h - 0.5f^T h - \frac{0.25}{m^T m} \left((f-h)^T m \right)^2
\end{aligned}$$

Using the expressions for the Euclidean distances, this is equivalent to

$$(ED_D(as1, as2))^2 + (ED_U(\emptyset))^2 > (ED_D(as1))^2 + (ED_U(as_2))^2$$

This completes the proof of Proposition 5.

Proof of Proposition 6:

I prove this proposition by giving examples, where each of the three considered ownership structures is optimal, respectively.

(a)

Suppose that U's investment is relatively unproductive and that $\rho \to 0$ so that ownership structure is chosen such that $R(i^*) - 0.5 \sum_{k=1}^{n} (i_k^*)^2$ is maximized. Further, assume that $a = (1,1)$, $b = \left(\frac{4}{5}, \frac{4}{5}\right)$, $c = \left(\frac{4}{5}, \frac{1}{2}\right)$ and $d = \left(0, \frac{1}{2}\right)$. Backward-integration will be optimal, if $k \equiv 0$, non-integration, if $k = \left(\frac{1}{5}, \frac{1}{2}\right)$ and forward-integration, if $k = \left(1, \frac{1}{2}\right)$. The case where D's investment is relatively unproductive is analogous.

(b)

Suppose that $a = (1,1)$, $b = c = \left(\frac{4}{5}, \frac{4}{5}\right)$ and $d = \left(0, \frac{1}{2}\right)$. Moreover, think that $f = (1,1)$, $g = h = \left(\frac{1}{2}, \frac{1}{2}\right)$, $j \equiv 0$ and $m = (1,1)$. Then, both, backward-integration and non-integration will be optimal, if $k = \left(\frac{1}{2}, \frac{1}{2}\right)$, while forward-integration will be optimal, if $k = \left(1, \frac{1}{2}\right)$.

(c)

Suppose that $a = (1,1)$, $b = \left(\frac{4}{5}, \frac{4}{5}\right)$ and $c = d = \left(0, \frac{1}{2}\right)$. Moreover, assume that $f = (1,1)$, $g = \left(\frac{1}{2}, \frac{1}{2}\right)$, $h = j \equiv 0$ and $m = (1,1)$. Then, backward-integration will be optimal, if $k = \left(\frac{1}{2}, \frac{1}{2}\right)$, whereas both, non-integration and forward-integration will be optimal, if $k = \left(1, \frac{1}{2}\right)$.

(d)

Suppose that D's human capital is essential. As shown before, ownership rights are then allocated such that $R(i^*) - 0.5 \sum_{k=1}^{n} (i_k^*)^2$ is maximized. The examples in part (a) of this proposition show that, in this case, each ownership structure may be optimal.

Appendix 2

In this Appendix, it is shown that linear contracts are able to replicate the outcome to be achieved with the most general contract. The proof is similar to a proof in Bhattacharyya & Lafontaine (1995), who also show that linear contracts are optimal in their model. For ease of exposition, we restrict attention to the case, where only U offers D a contract (an extension to the double contracting analyzed in Section 3 is straightforward). Let the contract be of the form $w(s(i), t(e))$, with w being differentiable in both arguments. D's investment is then given by $i = 0.5\nabla R(i) + 0.5\nabla r(i, A) + \frac{\partial w(s(i),t(e))}{\partial s(i)} \nabla s(i)$, as t is independent of i. The contract is set such that aggregate ex ante revenue is maximized. Under the optimal contract, D is induced to choose some investment i^*, which is given by $i^* = 0.5\nabla R(i^*) + 0.5\nabla r(i^*, A) + \frac{\partial w(s(i^*),t(e^*))}{\partial s(i)} \nabla s(i^*)$. Consider now the linear contract $w = \alpha_0 + \alpha_1 s(i)$ used in Section 3. By setting $\alpha_1 = \frac{\partial w(s(i^*),t(e^*))}{\partial s(i)}$, this contract leads to same investment and can thus replicate the outcome of the general contract. Similarly, the outcome to be achieved by a contract $v(s(i), t(e))$ that is not everywhere differentiable can be replicated by a linear contract, too. I show this for a very simple contract. It is very easy to apply the argument to more complex contracts. Suppose that the contract specifies some payment $\hat{v} > 0$ to be transferred from U to D, if $s(i)$ (weakly) exceeds some threshold value $\hat{s}$. Note that this contract will only make sense, if $s(0.5\nabla R(i) + 0.5\nabla r(i, A)) < \hat{s}$. Further, assume that $\hat{v}$ is so high that it is in D's interest to invest such that $s(i) \geq \hat{s}$. The Lagrangian to D's maximization problem is then $L = r(i, A) - \bar{p} + 0.5(R(i) - C(e) - (r(i, A) - c(e, B))) + \hat{v} - 0.5 \sum_{k=1}^{n} (i_k)^2 + \lambda (s(i) - \hat{s})$, where λ is a Lagrange multiplier. Notice that $s(0.5\nabla R(i) + 0.5\nabla r(i, A)) < \hat{s}$ implies that the constraint $s(i) \geq \hat{s}$ is never slack. Hence, $\lambda > 0$. Optimal investment is therefore given by $i = 0.5\nabla R(i) + 0.5\nabla r(i, A) + \lambda \nabla s(i)$. Under the optimal contract, we get a certain value for the parameter λ, say λ^*. It is easy to see that a linear contract with $\alpha = \lambda^*$ is again able to replicate the outcome of the non-differentiable contract. Q.E.D.

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